1 logit: Logistic Regression for Dichotomous Dependent Variables

Logistic regression specifies a dichotomous dependent variable as a function of a set of explanatory variables.

1.0.1 Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "logit", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out, x1 = NULL)</pre>
```

1.0.2 Additional Inputs

In addition to the standard inputs, zelig() takes the following additional options for logistic regression:

• robust: defaults to FALSE. If TRUE is selected, zelig() computes robust standard errors via the sandwich package (see [8]). The default type of robust standard error is heteroskedastic and autocorrelation consistent (HAC), and assumes that observations are ordered by time index.

In addition, robust may be a list with the following options:

- method: Choose from
 - * "vcovHAC": (default if robust = TRUE) HAC standard errors.
 - * "kernHAC": HAC standard errors using the weights given in [1].
 - * "weave": HAC standard errors using the weights given in [5].
- order.by: defaults to NULL (the observations are chronologically ordered as in the original data). Optionally, you may specify a vector of weights (either as order.by = z, where z exists outside the data frame; or as order.by = ~z, where z is a variable in the data frame)
 The observations are chronologically ordered by the size of z.
- ...: additional options passed to the functions specified in method.
 See the sandwich library and [8] for more options.

1.0.3 Examples

1. Basic Example

Attaching the sample turnout dataset:

> data(turnout)

Estimating parameter values for the logistic regression:

```
> z.out1 <- zelig(vote ~ age + race, model = "logit", data = turnout)
```

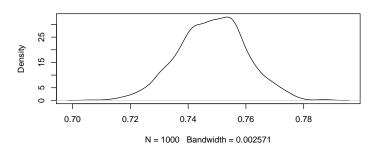
Setting values for the explanatory variables:

> x.out1 <- setx(z.out1, age = 36, race = "white")</pre>

Simulating quantities of interest from the posterior distribution.

- > s.out1 <- sim(z.out1, x = x.out1)
- > summary(s.out1)
- > plot(s.out1)

Expected Values: E(Y|X)



Predicted Values: Y|X

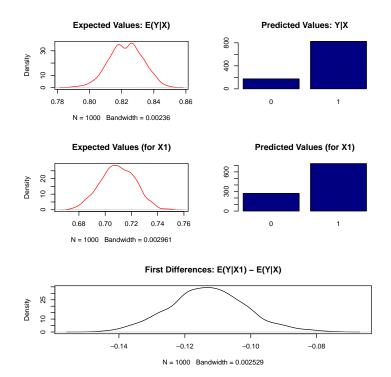


2. Simulating First Differences

Estimating the risk difference (and risk ratio) between low education (25th percentile) and high education (75th percentile) while all the other variables held at their default values.

```
> z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)
> x.high <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.75))
> x.low <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.25))
> s.out2 <- sim(z.out2, x = x.high, x1 = x.low)
> summary(s.out2)
```

> plot(s.out2)

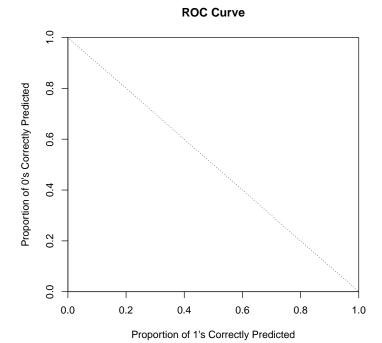


3. Presenting Results: An ROC Plot

One can use an ROC plot to evaluate the fit of alternative model specifications. (Use demo(roc) to view this example, or see King and Zeng (2002).)

```
> z.out1 <- zelig(vote ~ race + educate + age, model = "logit",
+    data = turnout)
> z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)</pre>
```

> rocplot(z.out1\$y, z.out2\$y, fitted(z.out1), fitted(z.out2))



1.0.4 Model

Let Y_i be the binary dependent variable for observation i which takes the value of either 0 or 1.

• The stochastic component is given by

$$Y_i \sim \operatorname{Bernoulli}(y_i \mid \pi_i)$$

= $\pi_i^{y_i} (1 - \pi_i)^{1 - y_i}$

where $\pi_i = \Pr(Y_i = 1)$.

• The *systematic component* is given by:

$$\pi_i \ = \ \frac{1}{1 + \exp(-x_i \beta)}.$$

where x_i is the vector of k explanatory variables for observation i and β is the vector of coefficients.

1.0.5 Quantities of Interest

• The expected values (qi\$ev) for the logit model are simulations of the predicted probability of a success:

$$E(Y) = \pi_i = \frac{1}{1 + \exp(-x_i \beta)},$$

given draws of β from its sampling distribution.

- The predicted values (qi\$pr) are draws from the Binomial distribution with mean equal to the simulated expected value π_i .
- The first difference (qi\$fd) for the logit model is defined as

$$FD = Pr(Y = 1 \mid x_1) - Pr(Y = 1 \mid x).$$

• The risk ratio (qi\$rr) is defined as

$$RR = Pr(Y = 1 \mid x_1) / Pr(Y = 1 \mid x).$$

• In conditional prediction models, the average expected treatment effect (att.ev) for the treatment group is

$$\frac{1}{\sum_{i=1}^{n} t_i} \sum_{i:t_i=1}^{n} \left\{ Y_i(t_i=1) - E[Y_i(t_i=0)] \right\},\,$$

where t_i is a binary explanatory variable defining the treatment $(t_i = 1)$ and control $(t_i = 0)$ groups. Variation in the simulations are due to uncertainty in simulating $E[Y_i(t_i = 0)]$, the counterfactual expected value of Y_i for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

• In conditional prediction models, the average predicted treatment effect (att.pr) for the treatment group is

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where t_i is a binary explanatory variable defining the treatment $(t_i = 1)$ and control $(t_i = 0)$ groups. Variation in the simulations are due to uncertainty in simulating $Y_i(\widehat{t_i} = 0)$, the counterfactual predicted value of Y_i for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

1.0.6 Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run z.out <- zelig(y ~ x, model = "logit", data), then you may examine the available information in z.out by using names(z.out), see the coefficients by using z.out\$coefficients, and a default summary of information through summary(z.out). Other elements available through the \$ operator are listed below.

- From the zelig() output object z.out, you may extract:
 - coefficients: parameter estimates for the explanatory variables.
 - residuals: the working residuals in the final iteration of the IWLS fit.
 - fitted.values: the vector of fitted values for the systemic component, π_i .
 - linear.predictors: the vector of $x_i\beta$
 - aic: Akaike's Information Criterion (minus twice the maximized loglikelihood plus twice the number of coefficients).
 - df.residual: the residual degrees of freedom.
 - df.null: the residual degrees of freedom for the null model.
 - data: the name of the input data frame.
- From summary(z.out), you may extract:
 - coefficients: the parameter estimates with their associated standard errors, p-values, and t-statistics.
 - cov.scaled: a $k \times k$ matrix of scaled covariances.
 - cov.unscaled: a $k \times k$ matrix of unscaled covariances.
- From the sim() output object s.out, you may extract quantities of interest arranged as matrices indexed by simulation × x-observation (for more than one x-observation). Available quantities are:
 - qi\$ev: the simulated expected probabilities for the specified values
 - qi\$pr: the simulated predicted values for the specified values of x.
 - qi\$fd: the simulated first difference in the expected probabilities for the values specified in x and x1.
 - qi\$rr: the simulated risk ratio for the expected probabilities simulated from x and x1.
 - qi\$att.ev: the simulated average expected treatment effect for the treated from conditional prediction models.
 - qi\$att.pr: the simulated average predicted treatment effect for the treated from conditional prediction models.

How to Cite the Logit Model

Kosuke Imai, Olivia Lau, and Gary King. logit: Logistic Regression for Dichotomous Dependent, 2011

How to Cite the Zelig Software Package

To cite Zelig as a whole, please reference these two sources:

- Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," http://GKing.harvard.edu/zelig.
- Imai, Kosuke, Gary King, and Olivia Lau. (2008). "Toward A Common Framework for Statistical Analysis and Development." Journal of Computational and Graphical Statistics, Vol. 17, No. 4 (December), pp. 892-913.

See also

The logit model is part of the stats package by (author?) [7]. Advanced users may wish to refer to help(glm) and help(family), as well as [6]. Robust standard errors are implemented via the sandwich package by (author?) [8]. Sample data are from [3].

References

- [1] Donald W.K. Andrews. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59(3):817–858, May 1991.
- [2] Kosuke Imai, Olivia Lau, and Gary King. logit: Logistic Regression for Dichotomous Dependent, 2011.
- [3] Gary King, Michael Tomz, and Jason Wittenberg. Making the most of statistical analyses: Improving interpretation and presentation. *American Journal of Political Science*, 44(2):341–355, April 2000. http://gking.harvard.edu/files/abs/making-abs.shtml.
- [4] Gary King and Langche Zeng. Improving forecasts of state failure. World Politics, 53(4):623–658, July 2002. http://gking.harvard.edu/files/abs/civil-abs.shtml.
- [5] Thomas Lumley and Patrick Heagerty. Weighted empirical adaptive variance estimators for correlated data regression. *jrssb*, 61(2):459–477, 1999.
- [6] Peter McCullagh and James A. Nelder. *Generalized Linear Models*. Number 37 in Monograph on Statistics and Applied Probability. Chapman & Hall, 2nd edition, 1989.
- [7] William N. Venables and Brian D. Ripley. *Modern Applied Statistics with S.* Springer-Verlag, 4th edition, 2002.
- [8] Achim Zeileis. Econometric computing with hc and hac covariance matrix estimators. *Journal of Statistical Software*, 11(10):1–17, 2004.