

# Compressive Privacy Generative Adversarial Networks

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Place: EE-II Room 504



# Outline

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- Introduction
- Related works
  - Attack schemes in Machine Learning Model
  - Privacy preserving mechanism
    - Differential privacy
    - Homomorphic encryption
    - Compressive privacy
    - Gan-inspired model: GAP and RAN
- Methodology
  - Architecture
  - Objective Function and Algorithm.
  - Theoretical analysis
- Empirical results
- Conclusion and Future Works (include one page summary)

# Introduction

- Machine Learning as a service (MLaaS) raises the serious privacy issue.

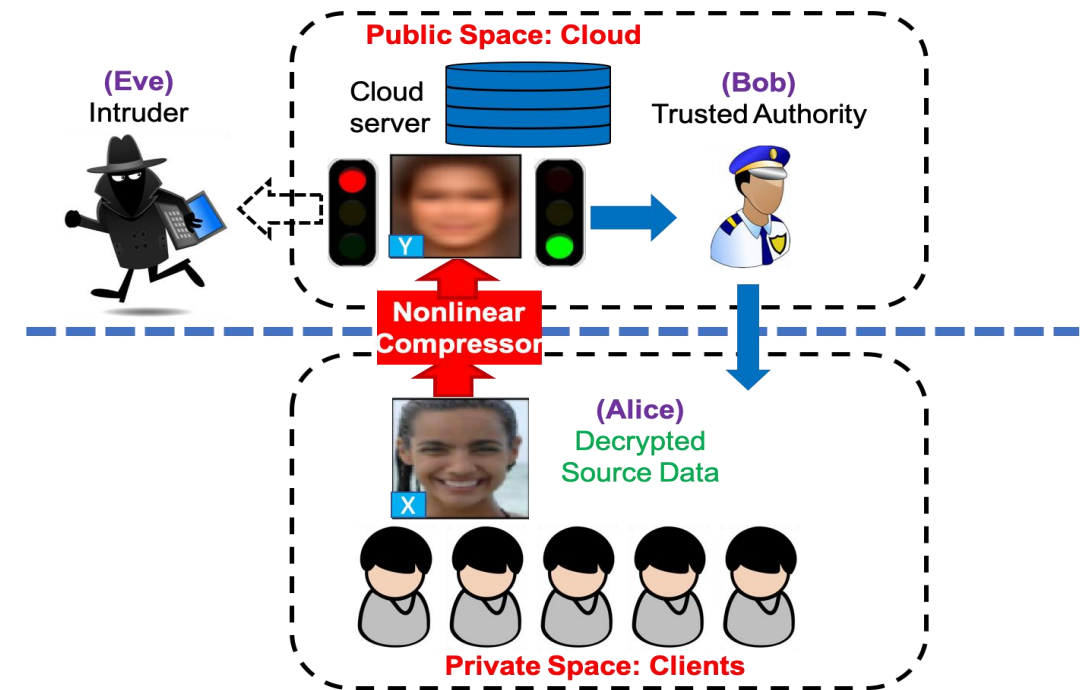
**In real world application:**



**Output:**  
Lipton's  
product

**Yellow box in this picture leaks  
user's sensitive information.**

**Privatization mechanism must be applied in  
collaborative learning system**



Kung, S. Y. (2018). A Compressive Privacy approach to Generalized Information Bottleneck and Privacy Funnel problems. *Journal of the Franklin Institute*, 355(4), 1846-1872.

# How Important ?

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FORTUNE

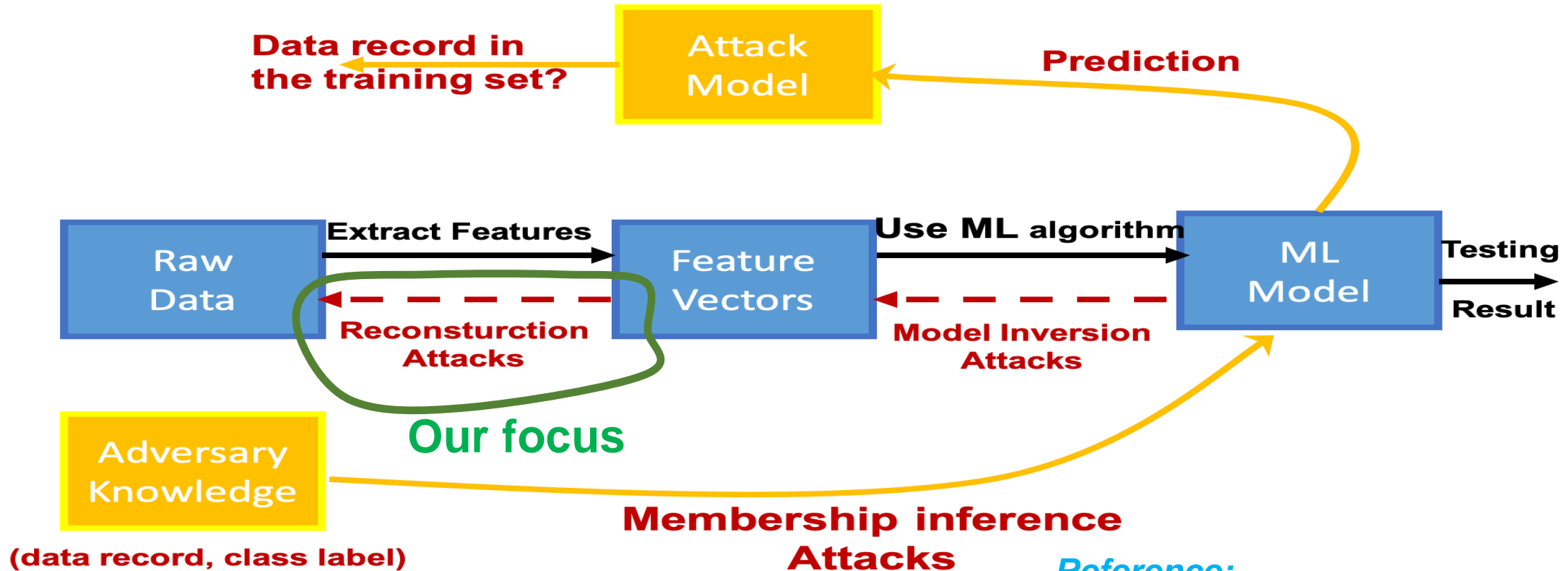
TECH • THE FUTURE OF WORK

## AI Has a Big Privacy Problem and Europe's New Data Protection Law Is About to Expose It

-> [Europe's new General Data Protection Regulation \(GDPR\)](#)

Privacy is a **fundamental** human **right!**

# Attack schemes

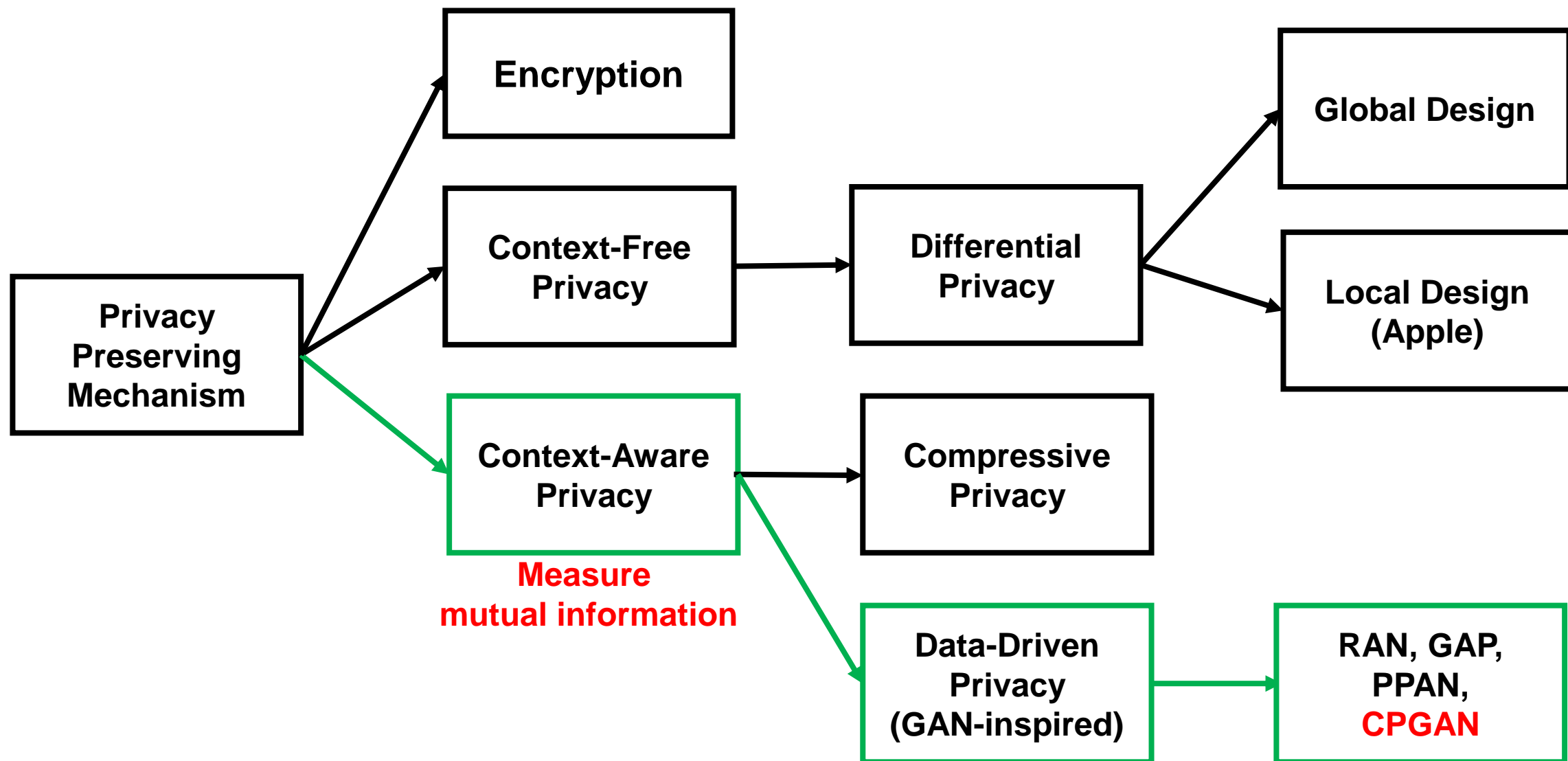


## Reference:

R. Shokri, M. Stronati, C. Song, and V. Shmatikov, "Membership inference attacks against machine learning models," in *2017 IEEE Symposium on Security and Privacy (SP)*, pp. 3–18, May 2017.

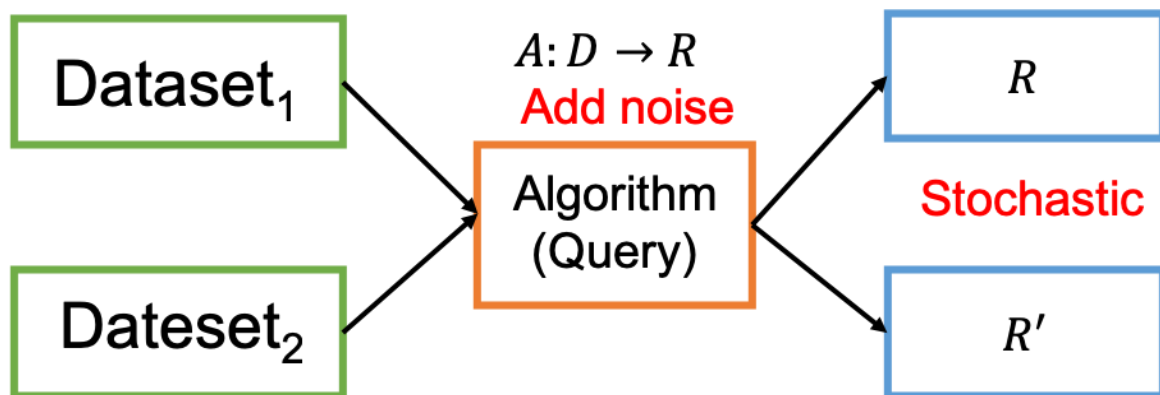
M. Fredrikson, S. Jha, and T. Ristenpart, "Model inversion attacks that exploit confidence information and basic countermeasures," in *Computer and Communications Security*, pp. 1322–1333, ACM, 2015.

# Privacy Preserving mechanisms



# Differential Privacy (DP)

**Global: if the aggregator is trustable.**



**Privacy guarantee formulation:**

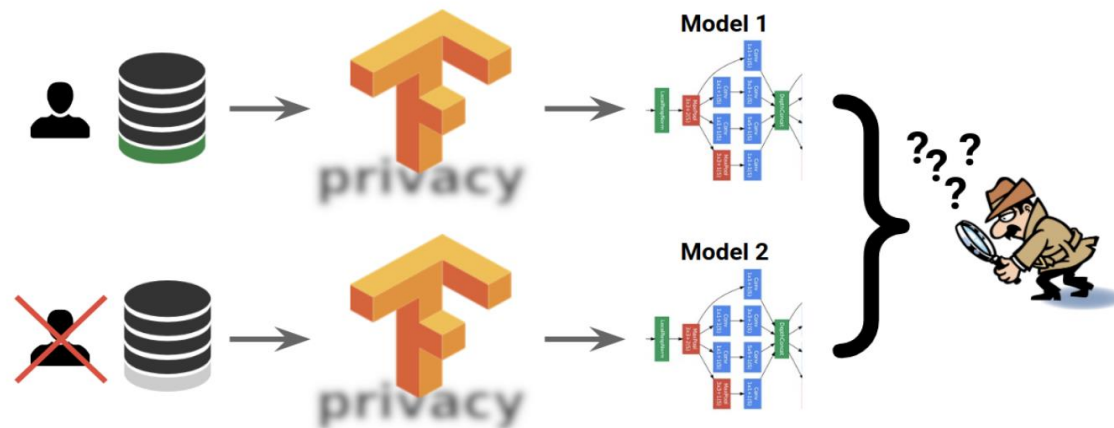
$$P(A(D_1) = o) \leq e^\epsilon P(A(D_2) = o)$$

- $|D_1 - D_2| = 1$
- Popular used mechanism: Laplician( $\frac{\Delta f}{\epsilon}$ )

**Composability (LDP):**

- Each  $A_i$  satisfies  $\epsilon$ -differential privacy, then for the  $n$  DP-mechanisms, it must become  $n\epsilon$ -differential privacy.

**DP in Deep Learning model**



- May drop the **utility** of the model trained by DP optimization.

**Reference:**

- <https://medium.com/tensorflow/introducing-tensorflow-privacy-learning-with-differential-privacy-for-training-data-b143c5e801b6>





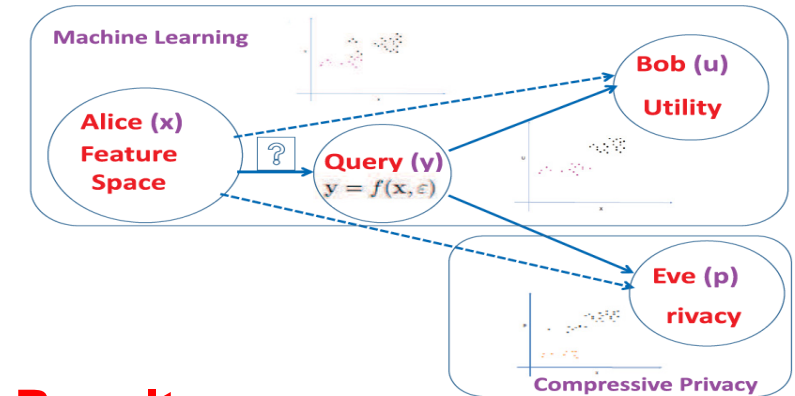
# Compressive Privacy (DCA · KDCA)

**Target:** Explore the low dimension representations ( $y$ ) that retain high utility but low privacy information.

## Reference:

S. Kung, T. Chanyaswad, J. Chang, and P.Y.Wu, "Collaborative pca/dca learning methods for compressive privacy," *ACM Transactions on Embedded Computing Systems (TECS)*, vol. 16, p. 76, 7 2017.

M. Al, T. Chanyaswad, and S. Y. Kung, "Multi-kernel, deep neural network and hybrid models for privacy preserving machine learning," in *2018 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, April 2018, pp. 2891–2895.



## Results:

## DCA formulation:

$$\max_{F: F^T [\bar{S} + \rho I] F = I} \text{Tr}(F^T S_{B_U} F)$$

## KDCA formulation:

- $\max_{F: F^T [\bar{K}^2 + \rho \bar{K}] F = I} \text{Tr}(F^T K_{B_U} F)$
- Basic idea of Kernel Method:** Map the original data to the RKHS space before applying DCA projection. And the inner product in RKHS is defined as  $k(x, y) = \phi(x)^T \phi(y)$

## Shift invariant Kernel Function:

- RBF kernel**

$$k(x, y) = e^{-\frac{\|x-y\|^2}{2\sigma^2}}$$

- Laplacian kernel:**

$$k(x, y) = e^{-\frac{\|x-y\|}{2\sigma}}$$

## DCA

## KDCA

HAR: Experiment II		
	Utility (%)	Privacy (%)
Random guess	16.67	5.00
Compressive single linear kernel	51.02	5.19
Compressive single RBF kernel	86.20	6.48
Compressive single Laplacian kernel	90.83	5.00
Compressive single sigmoid kernel	82.59	7.04
Compressive uniform multi-kernel	90.65	6.57
Compressive alignment-based multi-kernel	91.30	6.57
Compressive SNR-based multi-kernel $\rho_{snr} = 0$	89.35	6.85
Compressive SNR-based multi-kernel $\rho_{snr} = 0.1$	91.39	5.00

TABLE II  
HAR: UTILITY AND PRIVACY CLASSIFICATION ACCURACIES

# Generative Adversarial Privacy (GAP)

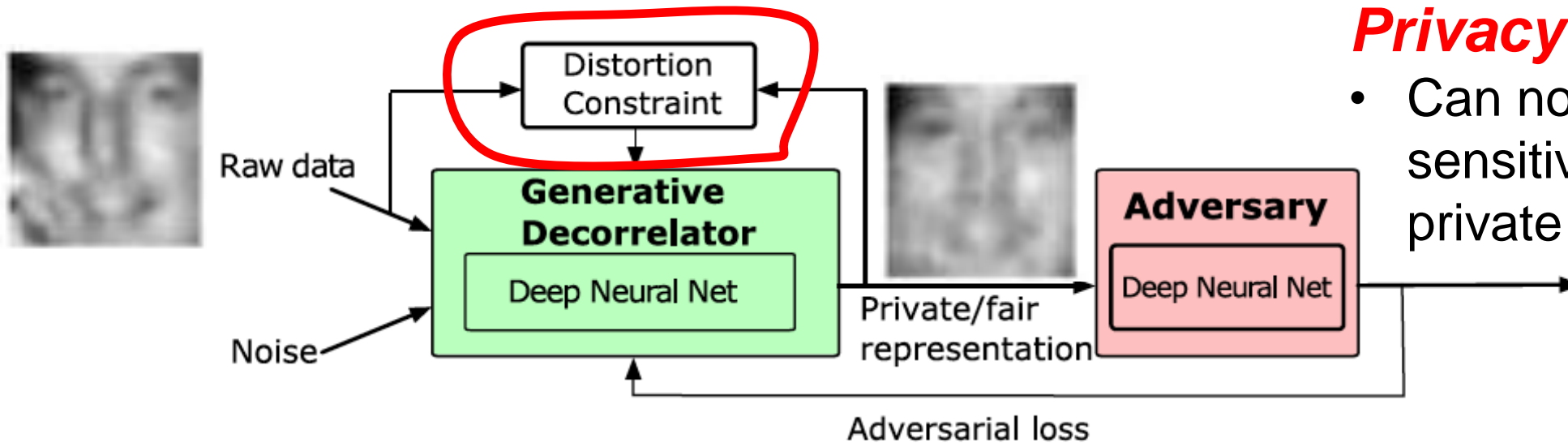
## GAN-inspired data-driven based model.

### Reference:

C. Huang, P. Kairouz, X. Chen, L. Sankar, and R. Rajagopal, "Generative adversarial privacy," *arxiv preprint arXiv:1807.05306*, 2018. [Online]. Available: <http://arxiv.org/abs/1807.05306>

### Utility

- Distortion constraint



### Privacy:

- Can not infer the specified sensitive attribute of this private images.

Figure 1: Generative adversarial model for privacy and fairness

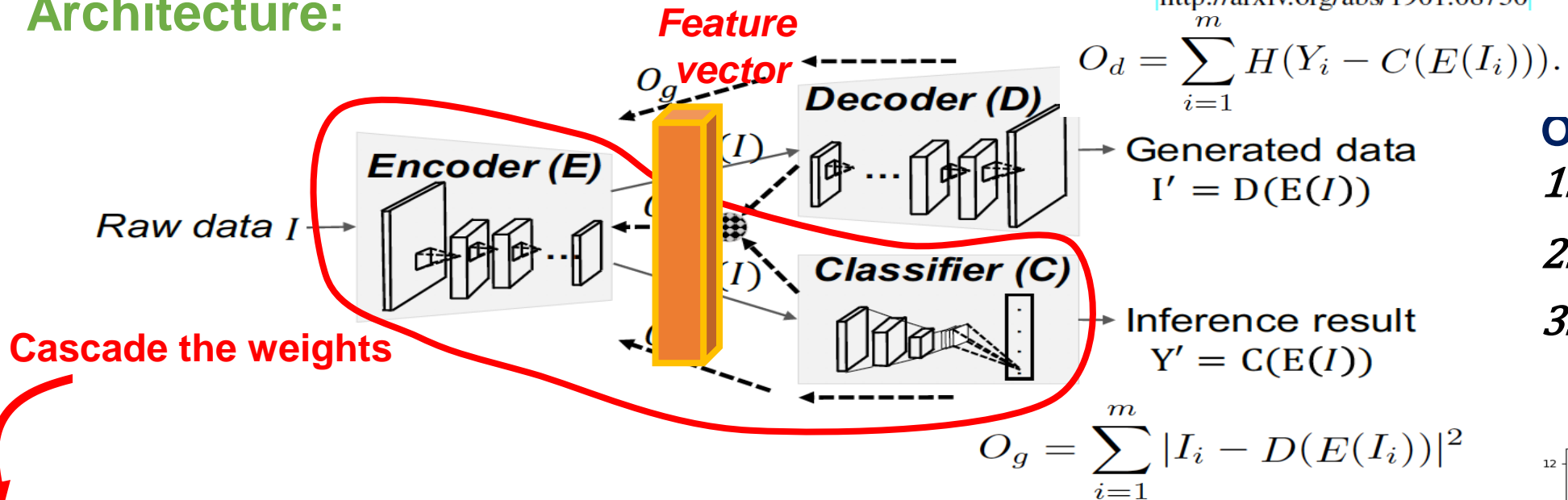
# Reconstructive adversarial network (RAN)

## GAN-inspired data-driven based model.

### Reference:

S. Liu, A. Shrivastava, J. Du, and L. Zhong, "Better accuracy with quantified privacy: representations learned via reconstructive adversarial network," *arXiv preprint arXiv:1901.08730*, 2019. [Online]. Available: <http://arxiv.org/abs/1901.08730>

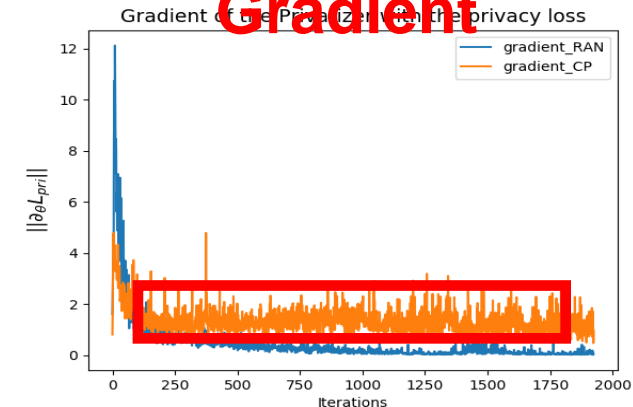
### Architecture:



### Optimization:

1.  $\min_{E,C} O_d$
2.  $\min_D O_g$
3.  $\min_{E,C} \lambda O_g - (1 - \lambda) O_d$

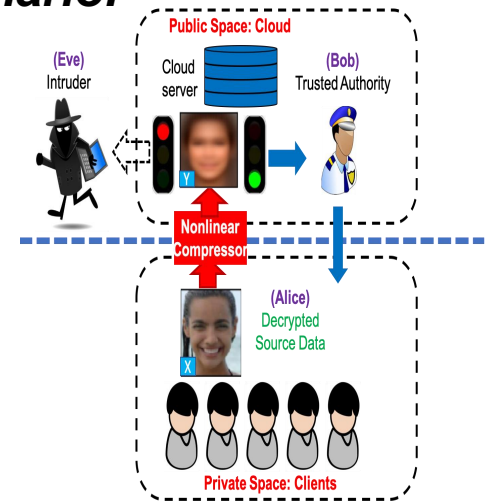
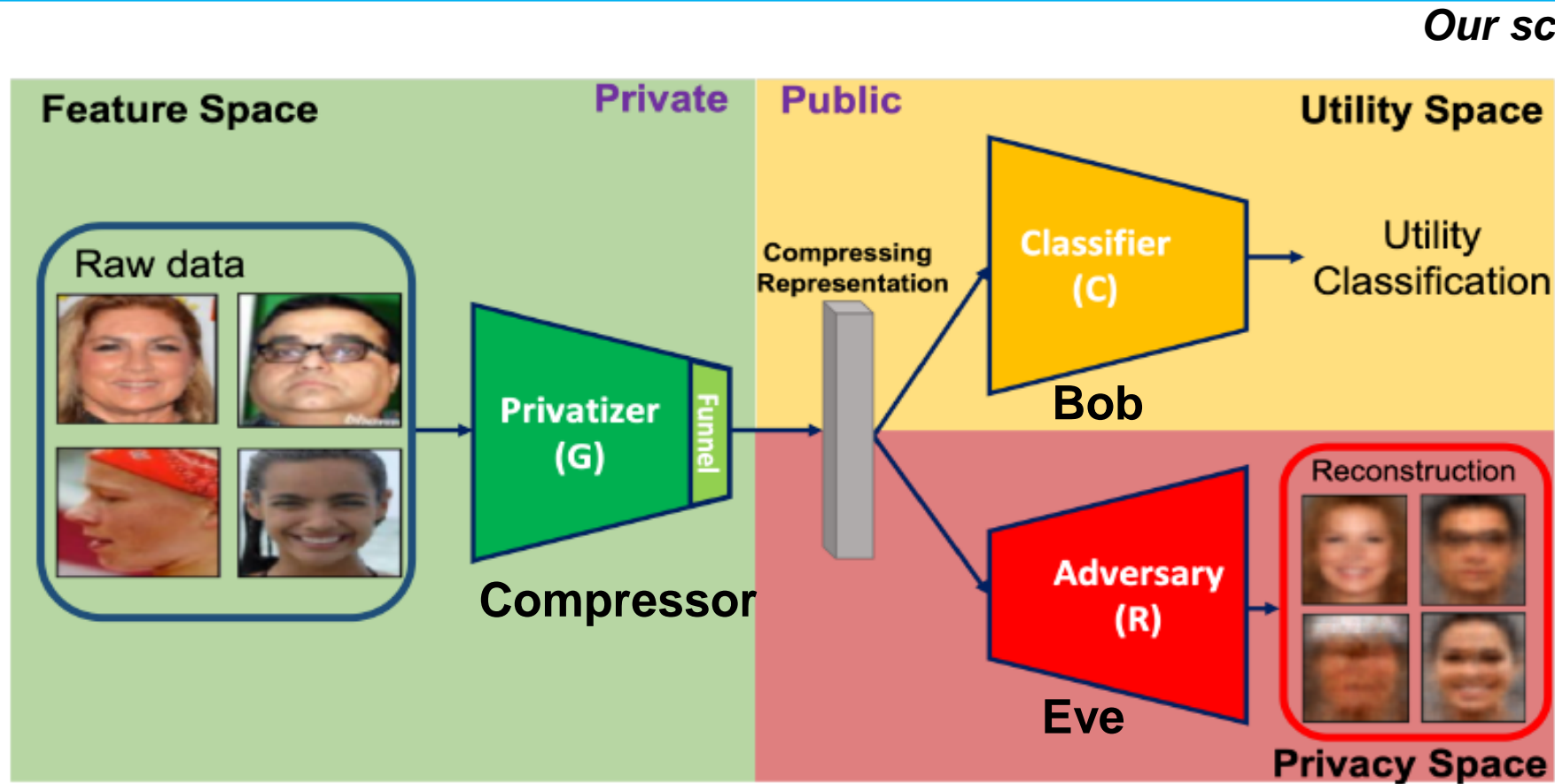
### Gradient



### Problems in RAN:

- Gradient flowing to the encoder (E) is too weak.
- Without considering the dimension of the encoded vectors.
- Only apply neural network to the decoder (adversary).

# CPGAN architecture



Kung, S. Y. (2018). A Compressive Privacy approach to Generalized Information Bottleneck and Privacy Funnel problems. *Journal of the Franklin Institute*, 355(4), 1846-1872.

CPGAN's scenario thus can be formulated as GAN's min-max function  $\min_G \max_D [\log P(D(x)) + \log P(1 - D(G(z)))]$ , where  $z \sim \text{Gaussian}$  and  $x \sim P_x$

# Formulate Proposed CPGAN

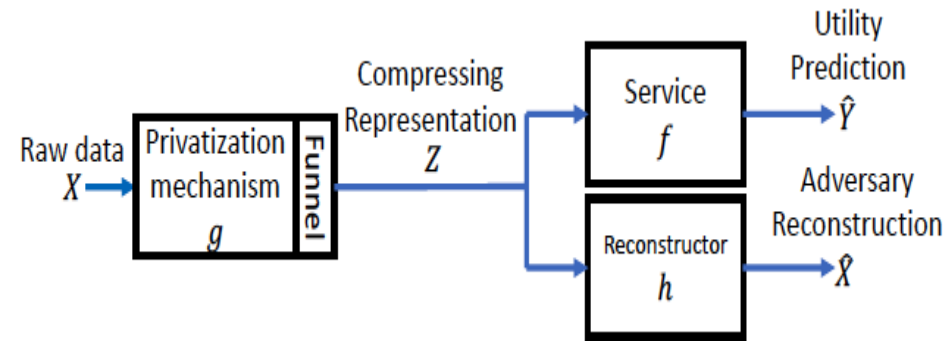
- We follow GAN's objective function to formulate CPGAN's architecture.
- Let  $(X, Y) \sim P_{x,y}$ ,  $Z|X \sim P_{g_\theta}(\cdot | X)$ ,  $\hat{Y}|Z \sim P_{f_\tau}(\cdot | Z)$

$$L_{\text{util}}(P_f(\cdot | Z), Y) = \mathbb{E}[-\log P_f(Y|Z)]$$

**It's equal to cross entropy**

$$L_{\text{adv}} = \mathbb{E}_{\hat{X} \sim P_h(\cdot | Z)} [\|X - \hat{X}\|_2^2]$$

**Mean square error**



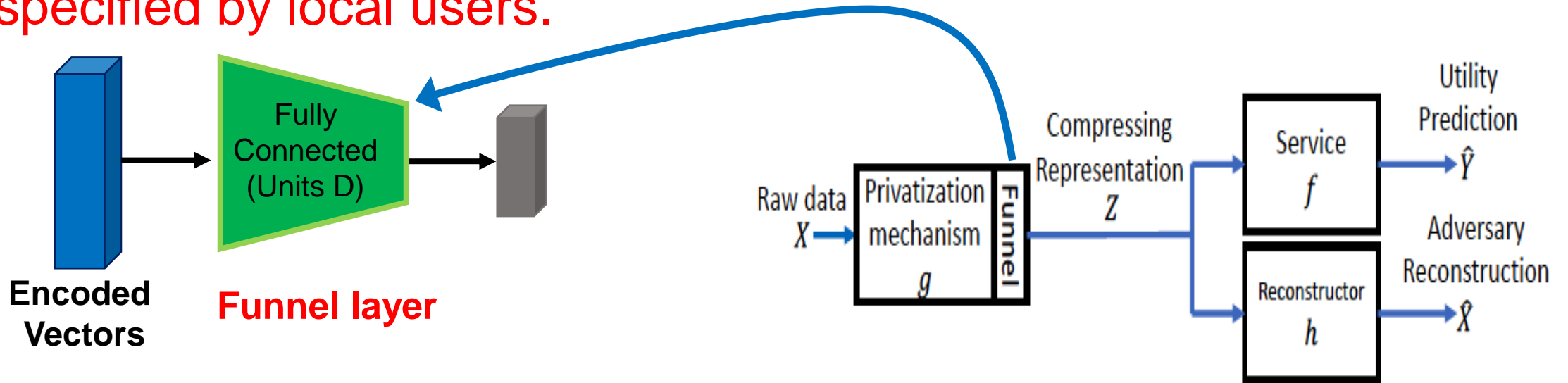
$$\max_g (\min_h L_{\text{adv}}(g, h) - \lambda \min_f L_{\text{util}}(g, f))$$

**Explore the best service ( $f$ ) and reconstructor( $h$ )**

**The privatizer( $g$ ) targets at attaining better accuracy ( $f$ ) while fooling the reconstructor( $h$ )**

# Design of the Privitizer

- Funnel layer:
  - Compress the data into the dimension specified by local users.

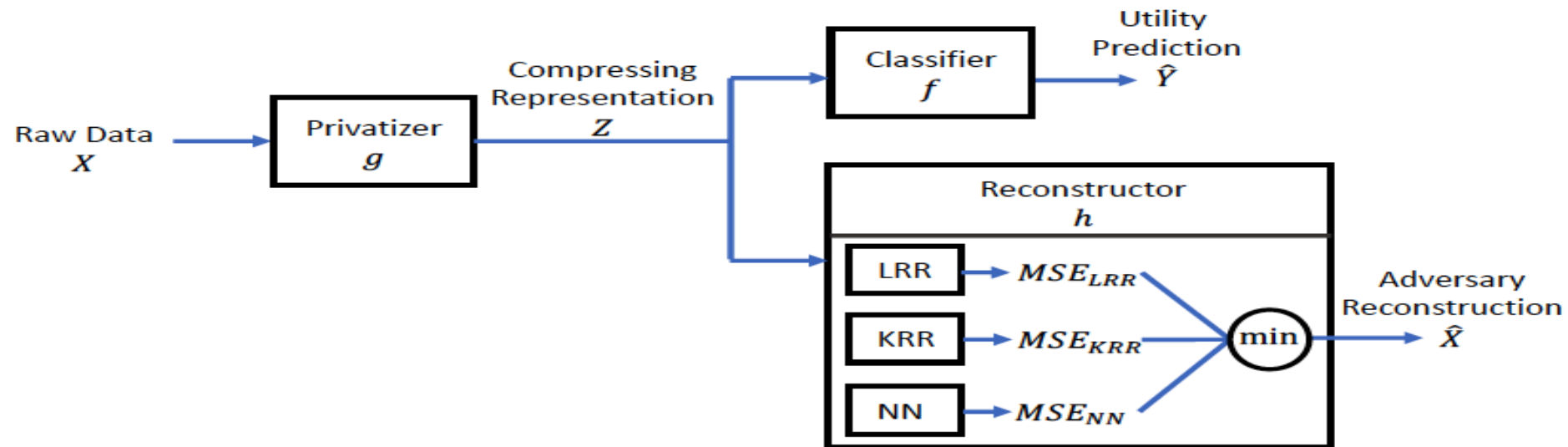


- Light-weight design:
  - It's thus applicable for the limiting computation resource, such as mobile device.

# Multiple adversaries scheme

- Why?
  - It is well known that the optimization of the nonlinear neural network is intractable, furthermore, it is questionable whether NN achieves the global optimum or saddle point.

## Architecture:



**LRR: Linear Ridge Regression, KRR: Kernel Ridge Regression**

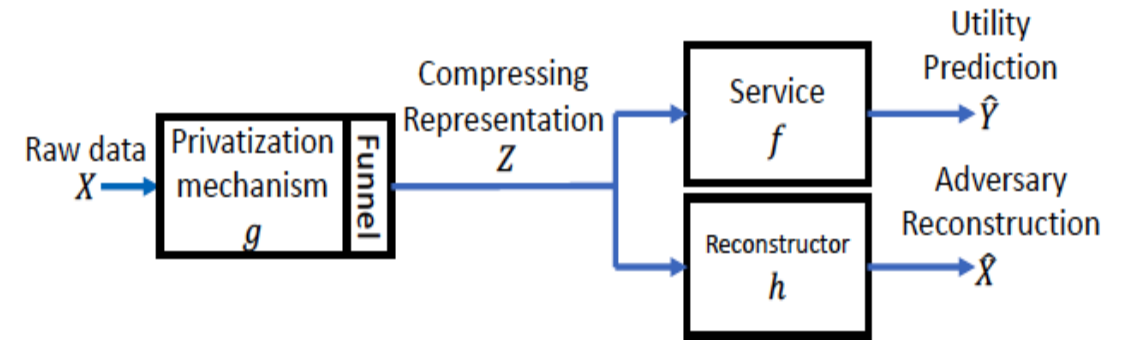


# The close-form solution of LRR and KRR

Assuming  $Z$  and  $X$  is center-adjusted, where  $\rho$  is the regularization term.

- For Linear Ridge Regression:

$$\underline{W_{LRR} = (ZZ^T + \rho I)^{-1}ZX}$$



- For Kernel Ridge Regression:

$$\begin{aligned} W_{LRR} &= (\phi(Z)\phi(Z)^T + \rho I)^{-1}\phi(Z)X \\ &= \phi(Z)(\phi(Z)^T\phi(Z) + \rho I)^{-1}X \end{aligned}$$



Learning subspace property:  
 $W = \phi(Z)A$

$$\underline{A = W_{KRR} = (K + \rho I)^{-1}X}$$

$\phi: R^m \rightarrow R^n$ , where  $m < n$   
Intrinsic space



Empirical space

Reference:

S. Y. Kung, *Kernel Methods and Machine Learning*. Cambridge University Press, 2014.



# Random Fourier Feature (RFF)

- Caused by the high computation cost on kernel matrix  $O(N^2)$ .
- RFF is inspired from Bochner's theorem:
  - The expectation of the inner product of two mapping points is the **unbiased approximation** of the **shift-invariant** kernel. (i.e.  $k(x, y) = k(x - y)$ )

$$k(\mathbf{x} - \mathbf{y}) = \int_{\mathcal{R}^d} p(\omega) e^{j\omega'(\mathbf{x}-\mathbf{y})} d\omega = E_{\omega} [\zeta_{\omega}(\mathbf{x}) \zeta_{\omega}(\mathbf{y})^*],$$

- Some parameters used in LRR and KRR adversary:

Table I. Parameters of KRR on different dataset

	Synthetic dataset	MNIST	HAR	GENKI-4K	SVHN	CIFAR-10	CelebA
Ridge	1	0.001	1	1	0.001	0.001	0.001
Mapping dimension	10000	500	5000	2048	5000	5000	2000
Gamma	0.001	0.001	0.001	0.001	0.001	0.001	0.001

## Reference:

A. Rahimi and B. Recht, "Random features for large-scale kernel machines," in *Proceedings of the 20th International Conference on Neural Information Processing Systems, NIPS'07*, (USA), pp. 1177–1184, Curran Associates Inc., 2007.

# Neural Network (NN)

## Our implementation detail of the adversary(NN), privatizer and classifier.

Table IV. Implementation detail of proposed CPGAN on SVHN and CIFAR-10 dataset

	SVHN				CIFAR-10			
	Layers	Units	Optimizer	Learning rate	Layers	Units	Optimizer	Learning rate
Privatizer	13-layer Residual Network [59], [63]		Adam	0.001	13-layer Residual Network		Adam	0.001
Reconstructor	Conv-T, stride=1 Conv-T, stride=1 Conv-T, stride=1 Conv-T, stride=1	128 64 32 3	Adam	0.001	Conv-T, stride=1 Conv-T, stride=1 Conv-T, stride=1 Conv-T, stride=1	128 64 32 3	Adam	0.001
Classifier	16-8 Wide Residual Networks [57]		Adam	0.01 <sup>2</sup>	26-2x32d Shake-shake Regularization [54]		Momentum [64]	0.01 <sup>2</sup>
Epochs	160				1800			

<sup>1</sup> The notation "Conv-t" means the deconvolution layers (upsampling).

<sup>2</sup> We apply cosine learning rate decay [54].

Table V. Parameters and computation cost on SVHN and CIFAR-10 dataset.

	SVHN			CIFAR-10		
	Parameters	Addition	Multiplication	Parameters	Addition	Multiplication
Privatizer	1647	1575963	1575954	1647	1575963	1575954
Classifier	2923162	438281548	439272780	10961834	1547703315	1547703309

Table VI. Implementation detail of proposed CPGAN on CelebA dataset

Table VI. Implementation detail of proposed CPGAN on CelebA dataset

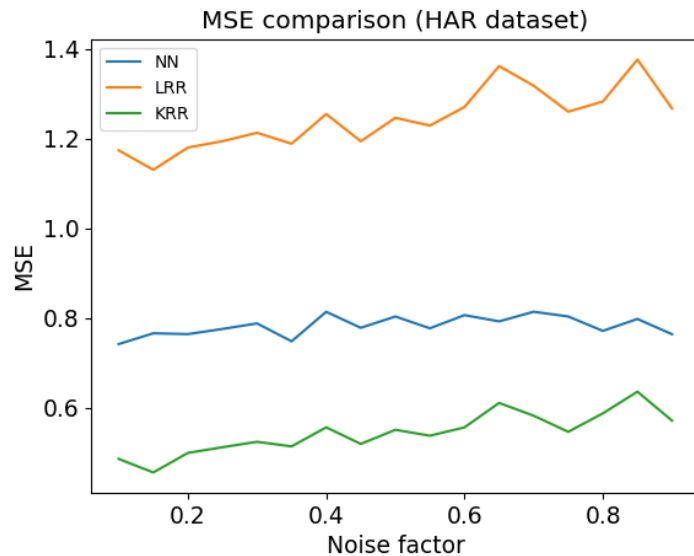
	Single task CelebA					Multiple task CelebA				
	Layers	Units	Optimizer	Learning rate	Parameter	Layers	Units	Optimizer	Learning rate	Parameter
Privatizer	Conv, stride=2 Conv, stride=2 Conv, stride=2 Conv, stride=2 Fully Connected	64 128 256 512 compressive-d <sup>1</sup>	Adam [65]	0.001	414466	From "conv_1" to "concat" in ATNET_GT [60] Fully Connected with compressive-d units		Adam	0.001	673600
Reconstructor	Fully Connected Reshape Conv-T, stride=2 Conv-T, stride=2 Conv-T, stride=2 Conv-T, stride=2	192 128 64 32 3	Adam	0.001		Fully Connected Batch Norm [66] Reshape Conv-T, stride=2 Conv-T, stride=2 Conv-T, stride=2 Conv-T, stride=2 Conv-T, stride=2	5*5*128 128 128 64 32 3	Adam	0.001	
Classifier	Fully Connected Batch Norm Fully Connected Fully Connected	256 256 1	Adam	0.001	68098	Fully Connected Batch Norm Fully Connected Fully Connected (40 branches)	64 64 1	Adam	0.001	30160
Epochs	30					30				

<sup>1</sup> The notation "compressive-d" means that the dimension of the compressing representations.

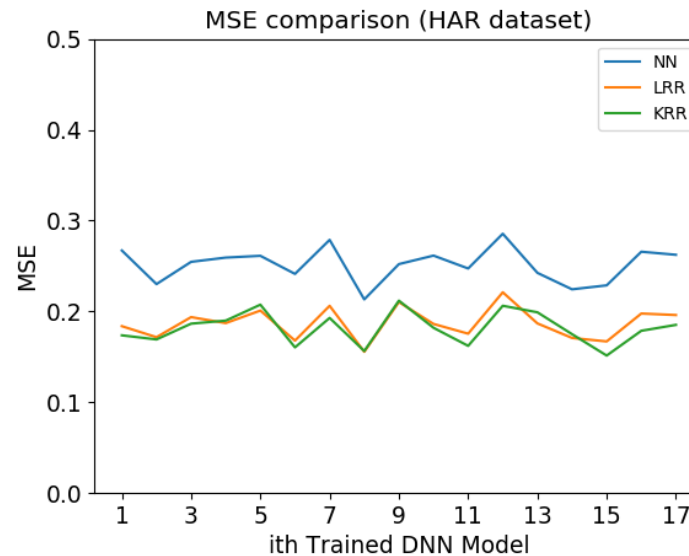
<sup>2</sup> The notation "Conv-t" means the deconvolution layers (upsampling).

# Comparison Between the Adversaries

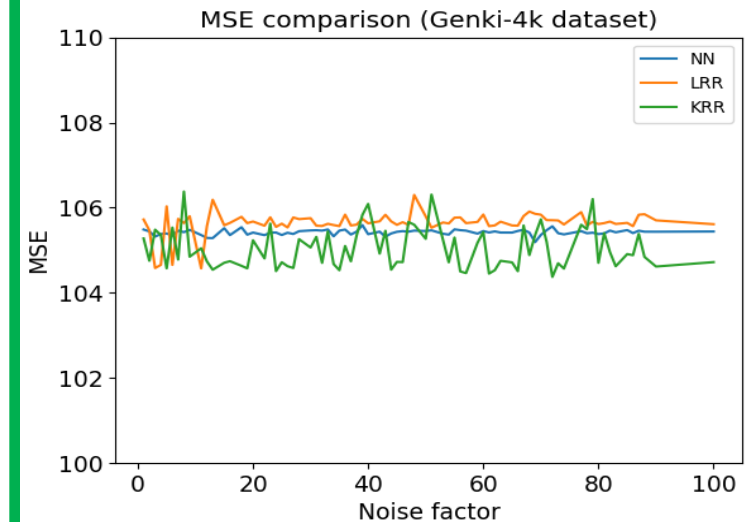
## HAR dataset:



**DNN (Resize) and DNN architecture**



## Genki-4K dataset:



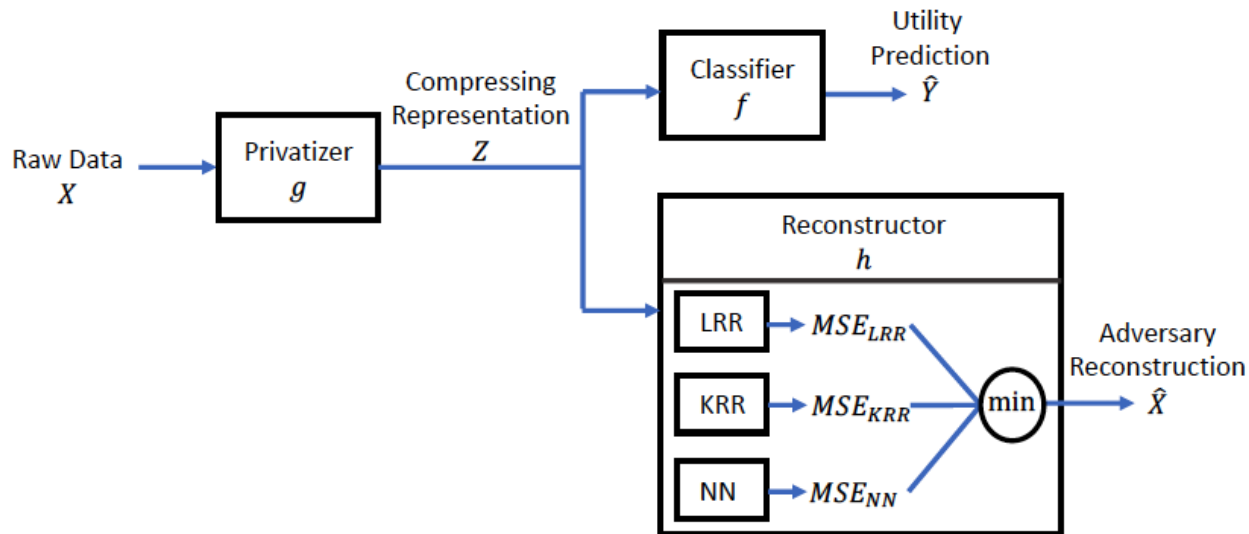
**CPGAN architecture**

These figures indicate the neural network can not **guarantee to achieve the best reconstruction error** in the evaluation phase.

# Algorithm

**We follow GAN's training strategy !!!**

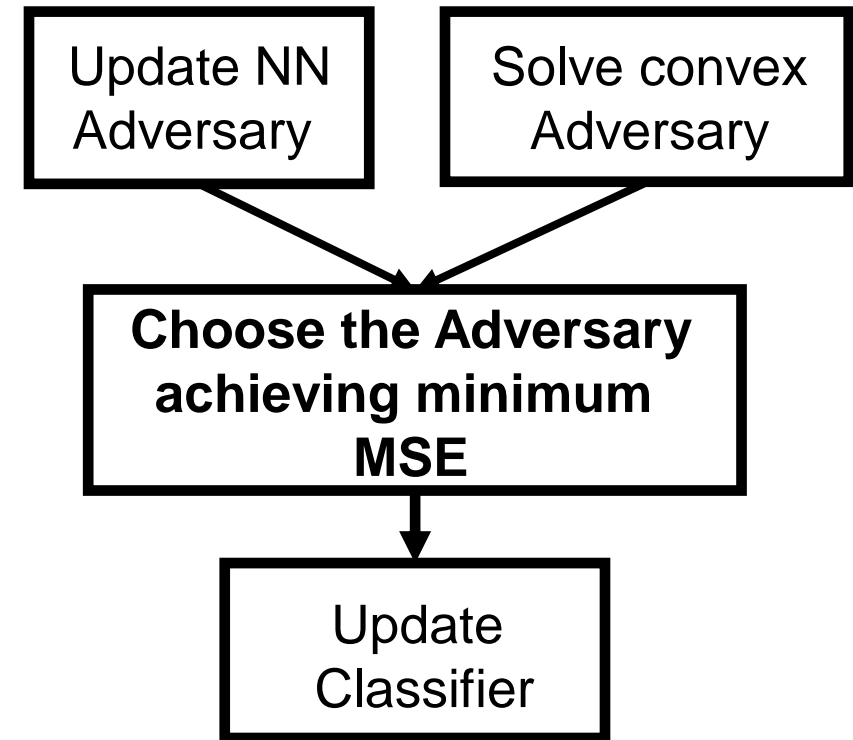
## Architecture



## Objective function:

$$\max_g (\min_h L_{adv}(g, h) - \lambda \min_f L_{util}(g, f))$$

## First Stage:



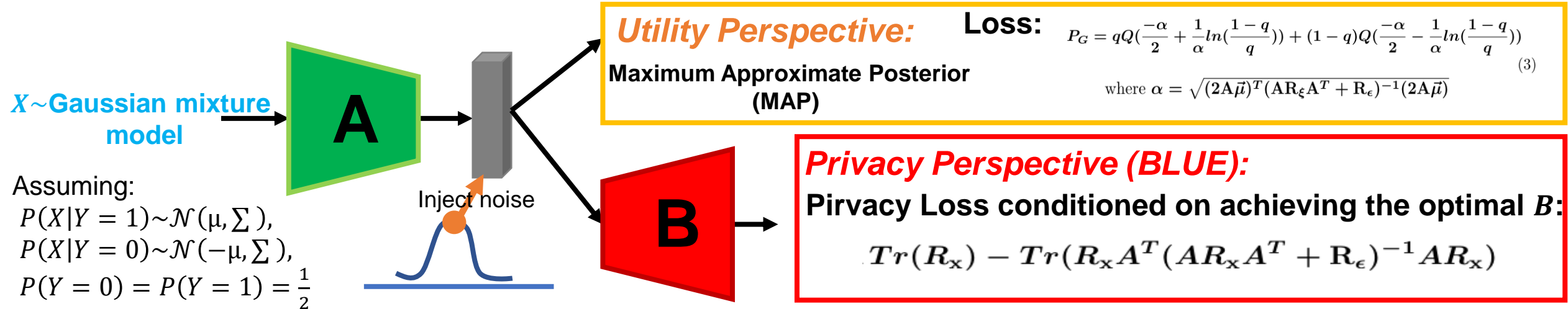
## Second Stage



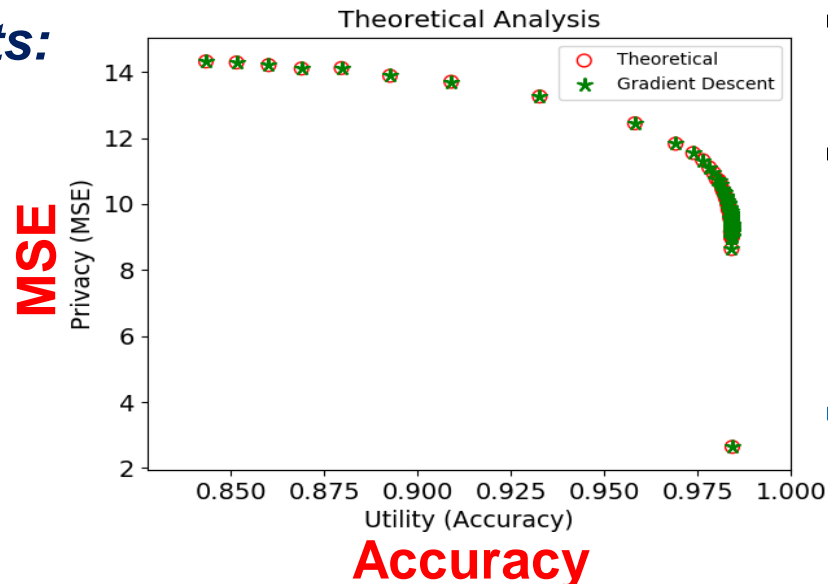
# Theoretical Analysis For CPGAN

Reference:

C. Huang, P. Kairouz, L. S. Xiao Chen, and R. Rajagopal, "Generative adversarial privacy," in *arXiv preprint arXiv:1807.05306*, 2018.



## Results:



- It's intractable to optimize with the linear combination of privacy and utility loss (i.e.  $\lambda L_{uti} - L_{pri}$ )
- Alternative way:
  - Use gradient descent to determine the solution (A).
  - Substitute CPGAN's privatizer with A.
  - Train the Classifier and Reconstructor, respectively.
- Conclusion:
  - CPGAN achieves the trade-off approximate to the theoretical solution.

# Distinction between RAN and CPGAN

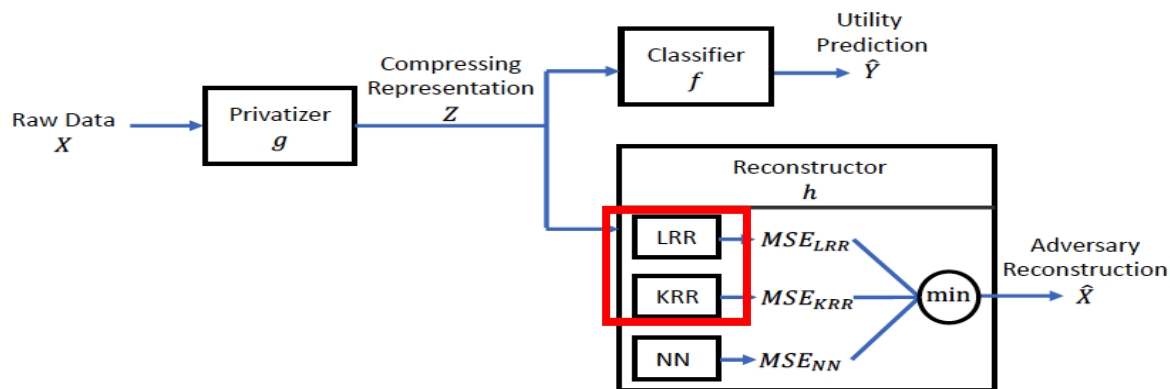
- Two tuning factors for the trade-off between privacy and utility.

- $$\max_g (\min_h L_{adv}(g, h) - \lambda \min_f L_{util}(g, f))$$

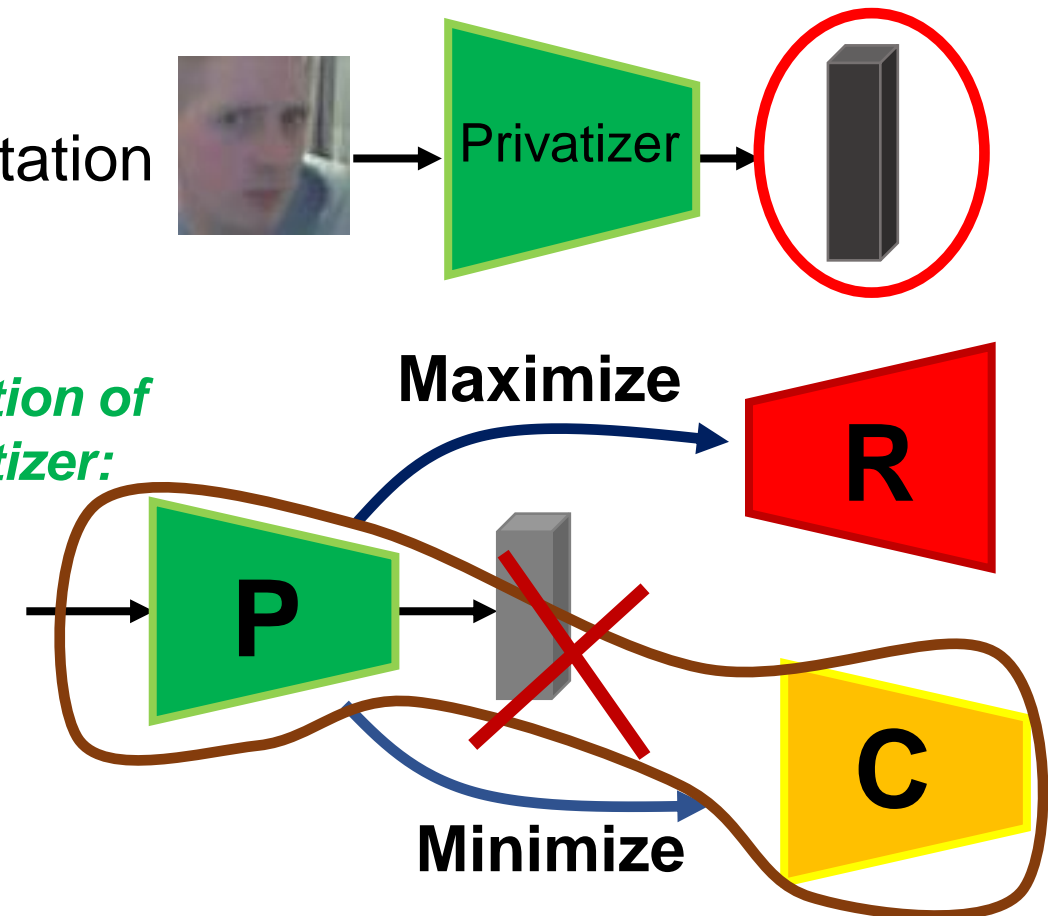
- Dimension** of the compressing representation

- Architecture and training strategy


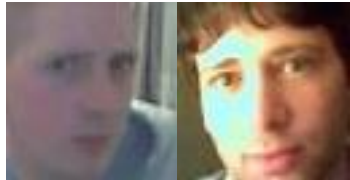
*Multiple adversaries Strategy in training/evaluation:*



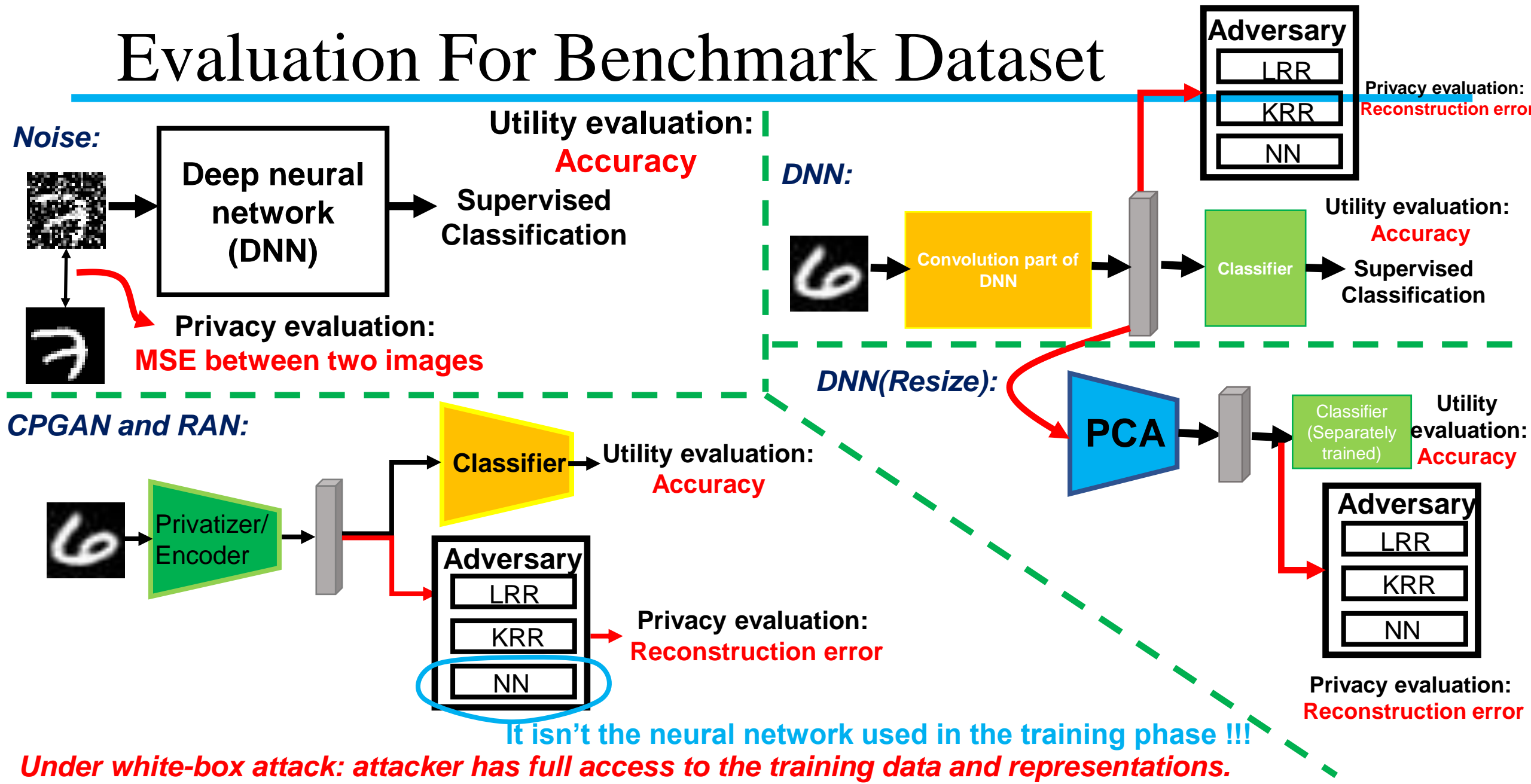
*Optimization of the Privatizer:*



# CPGAN for Benchmark Dataset

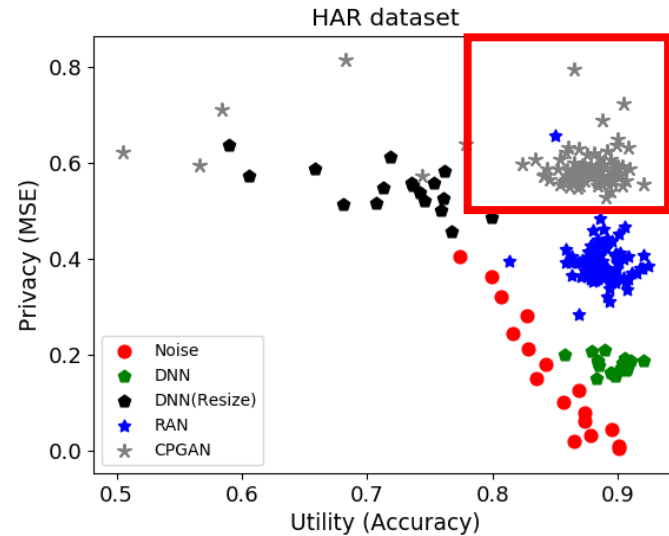
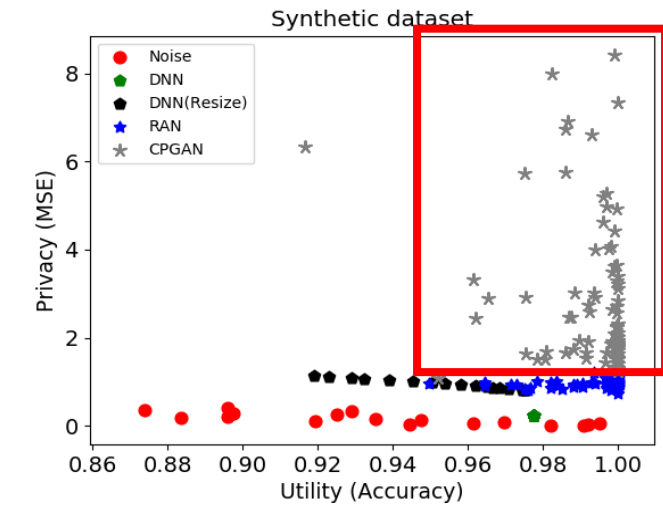
- Synthetic dataset:
  - Sampled from Gaussian mixture data model with binary class.
  - Training/testing samples: 20K/2K
- MNIST:
  - Training/testing samples: 55000/10000
  - Examples 
- UCI Human activity recognition (HAR) dataset
  - Given the time-series sensor record from ten identities.
  - Six activities: walking, sitting, standing etc.
- Genki-4K dataset:
  - Face images with 4000 sample. Detect the expression of this image.
  - Example: 

# Evaluation For Benchmark Dataset





# Quantitative Analysis



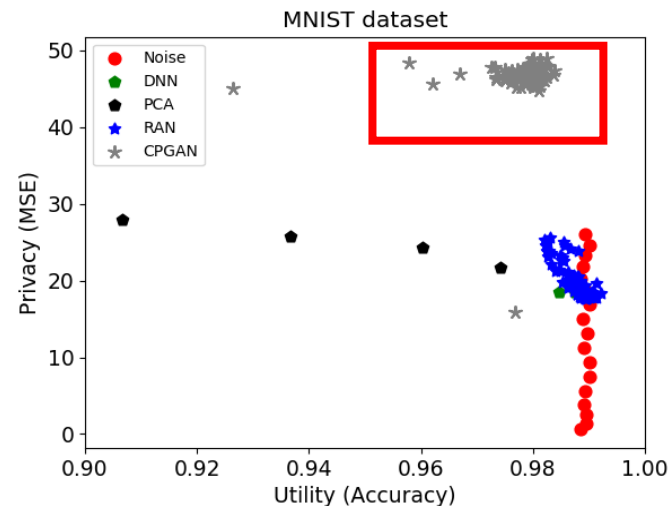
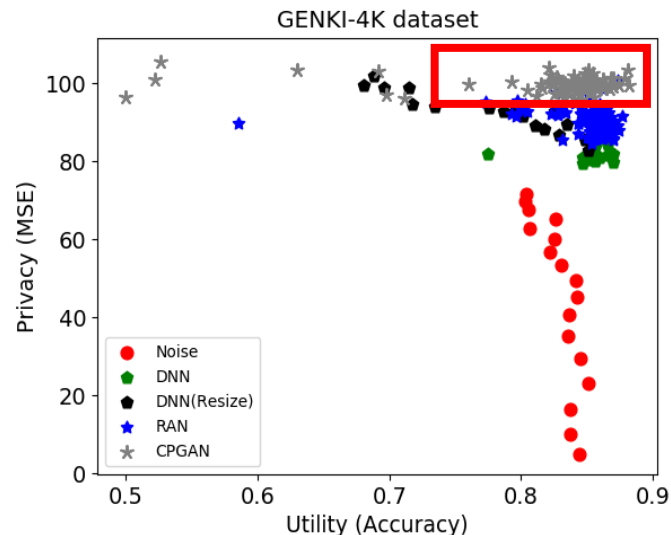
**X-axis -> Utility Accuracy**  
**Y-axis -> Privacy MSE**

*Privacy Perspective:*

- Outperform than other methods






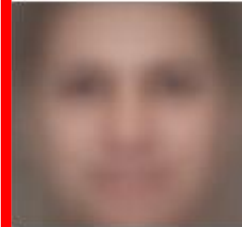





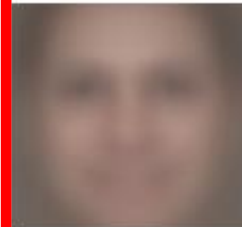
*Utility Perspective:*

- Drop by 1% on MNIST dataset.
- Get comparable accuracy on Synthetic, HAR and Genkl-4K dataset.



# Qualitative Analysis

Table II. Reconstructed images from five privacy preserving mechanisms on GENKI-4K dataset.

	Original	Noise	DNN	DNN (Resize)	RAN	CPGAN
Average Accuracy (%)		69.83	85.19	77.70	84.89	84.93
Image1						
Image2						

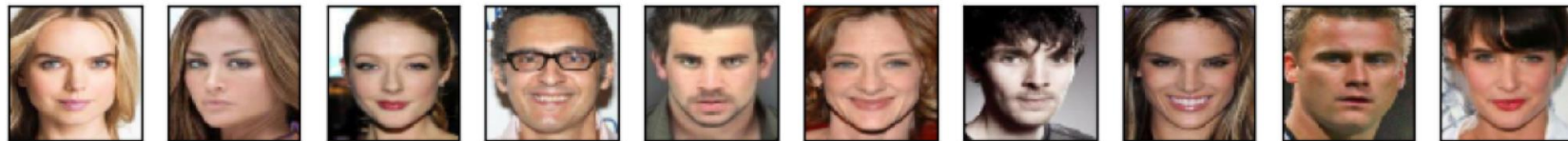
**Reconstructed images from our CPGAN is the most unrecognizable  
and the average accuracy only drops by 0.26%**

# CPGAN for Real Dataset

- CelebA

- 202599 images ( $218 \times 178 \times 3$ ), 10122 identities, each image has forty attributes.
- Image is cropped to  $175 \times 175 \times 3 / 112 \times 112 \times 3$  for multi/single attribute classification

- Example:



- CIFAR-10

- There are 50000/10000 images for training/test, each image size is  $32 \times 32 \times 3$ .
- Ten classes (such as cat, airplane, .. etc.)

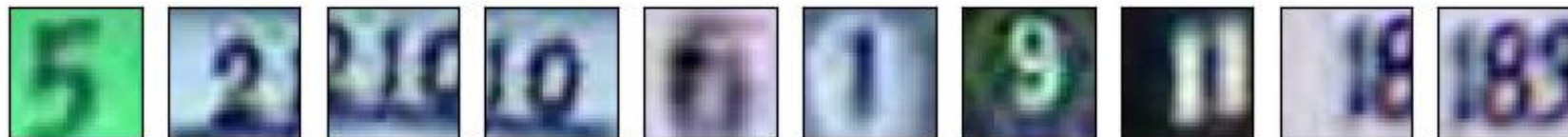
- Example:



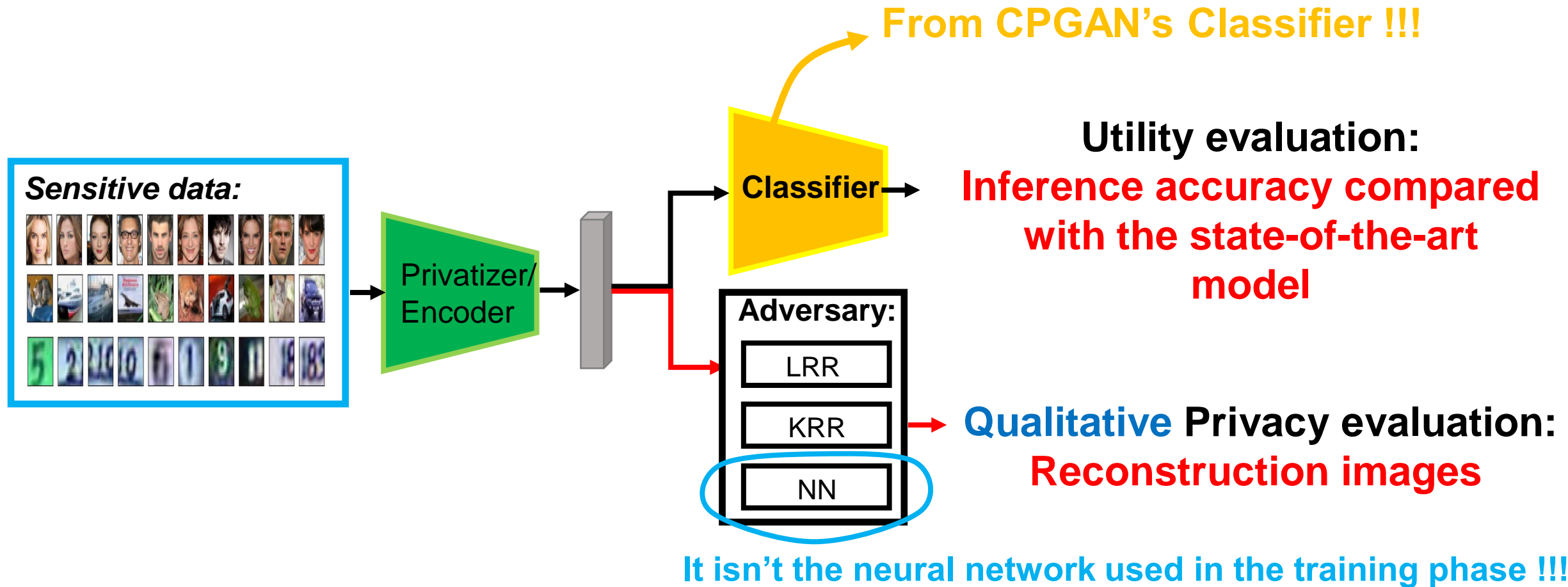
- SVHN

- 604388/26032 images for training/testing, each image size is  $32 \times 32 \times 3$ .
- Ten classes (from 0 to 9)

- Example:



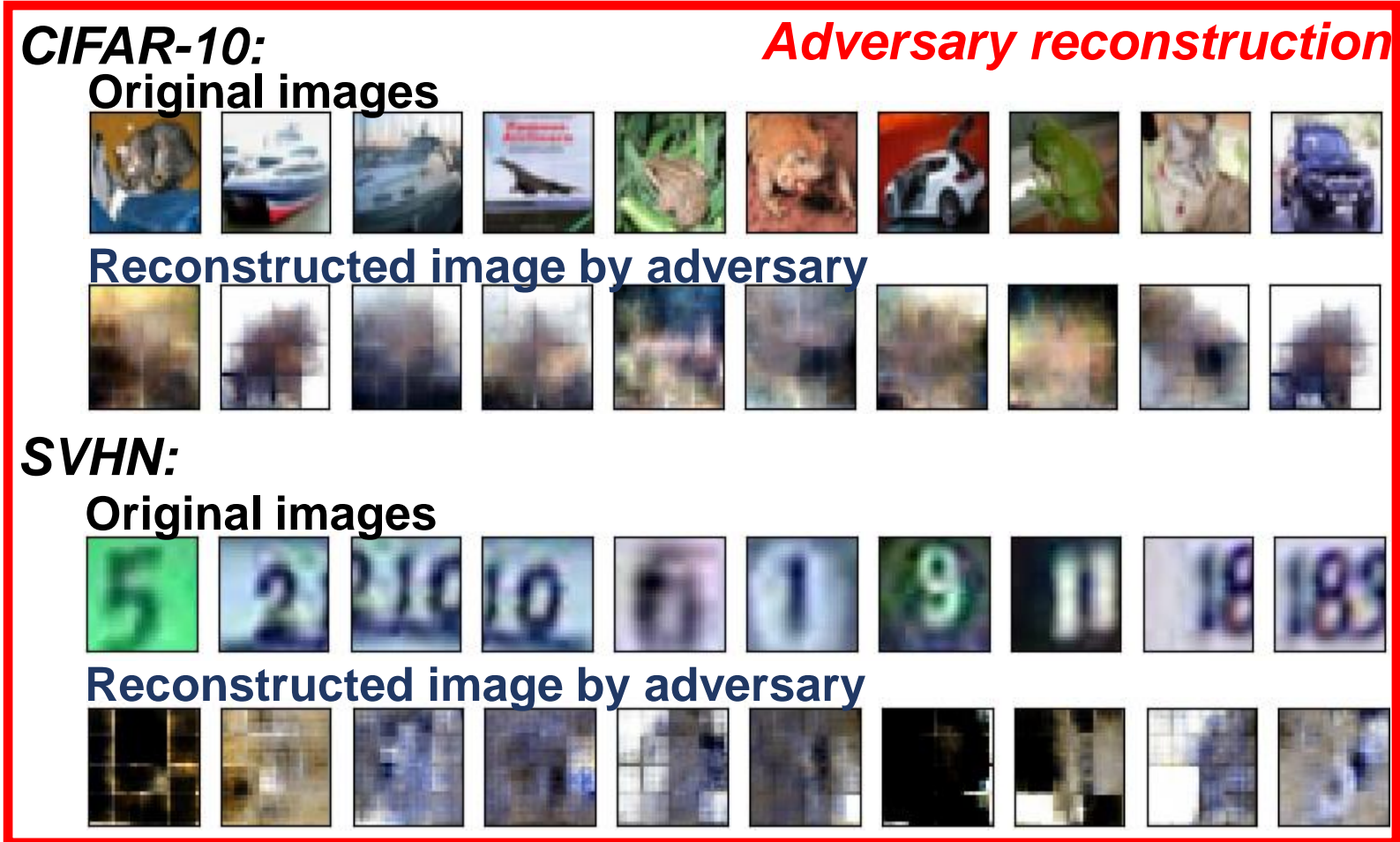
# Evaluation For Real Dataset



**Under white-box attack: attacker has full access to the training data and compressing representations.**



# Results on CIFAR-10 and SVHN



*Utility accuracy:*

	CIFAR-10	SVHN
<b>Classify by compressed data</b>		
CPGAN	93.87%	97.68%
ResNet-20 [2]	92.28%	97.70%
Xavier [3]	96.45%	98.6%
Zagoruyko [4]	95.83%	98.3%

*Classify by original image*

**CPGAN defends the *reconstruction attack under white-box attack* while achieving satisfactory *utility performance***

# Results on CelebA

## Utility accuracy:

Table V. Average accuracy of Single attribute CPGAN

	LNet+ANets [58]	Zhong [66]	CPGAN
Accuracy	87.30%	89.97%	89.92%

Classified by original image

Classified by  
Compressed data

Table VI. Average accuracy of multiple attribute CPGAN

	Han [64]	ATNET_GT [63]	CPGAN
Accuracy	92.52%	90.18%	90.30%

Classified by original image

Classified by  
Compressed data

Accuracy of 40 attributes:

	5 o Clock Shadow	Arched Eyebrows	Attractive	Bags Under Eyes	Bald	Bangs	Big Lips	Big Nose	Black Hair	Blond Hair	Blurry	Brown Hair	Bushy Eyebrows	Chubby	Double Chin	Eyeglasses	Goatee	Gray Hair	Heavy Makeup	High Cheekbones
LNet+ANets [39]	91	79	81	79	98	95	68	78	88	95	84	80	90	91	92	99	95	97	90	87
Zhong [40]	93	83	81	82	98	96	70	83	86	95	96	84	92	95	96	100	97	98	90	86
Hu [42]	95	86	83	85	99	99	96	85	91	96	96	88	92	96	97	99	99	98	92	88
ATNET_GT [41]	92	81	81	84	99	96	71	83	89	95	96	87	92	94	96	99	97	98	90	86
Single CPGAN	92	82	80	83	98	95	71	83	89	95	95	85	90	95	96	99	96	98	90	85
Multi CPGAN	93	82	82	84	98	95	71	83	88	96	96	88	92	95	96	99	97	98	91	86

	Male	Mouth S. Open	Mustache	Narrow Eyes	No Beard	Oval Face	Pale Skin	Pointy Nose	Receding Hairline	Rosy Cheeks	Sideburns	Smiling	Straight Hair	Wavy Hair	Wearing Earrings	Wearing Hat	Wearing Lipstick	Wearing Necklace	Wearing Necktie	Young
LNet+ANets [39]	98	92	95	81	95	66	91	72	89	90	96	92	73	80	82	99	93	71	93	87
Zhong [40]	98	93	97	87	95	71	97	76	92	94	97	92	80	77	87	99	92	86	94	88
Hu [42]	98	94	97	90	96	78	97	78	94	96	98	94	85	87	91	99	93	89	97	90
ATNET_GT [41]	97	93	97	86	94	76	97	75	93	95	97	92	80	82	89	99	93	86	96	88
Single CPGAN	100	93	97	89	91	72	96	75	94	95	96	92	79	78	88	99	92	83	94	87
Multi CPGAN	96	93	96	87	96	74	97	76	93	95	97	91	82	81	88	99	93	86	96	87

**CELEBA: Single attribute classification**

Original images



Reconstructed images by adversary



**CELEBA: Multiple attribute classification**

Original images



Reconstructed images by adversary





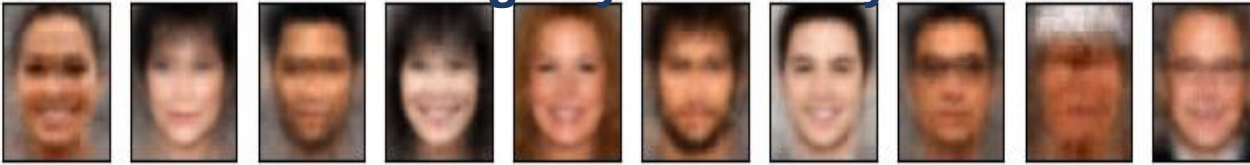
# Privacy leakage

**Reference:** D. Gao, P. Yuan, N. Sun, X. Wu, and Y. Cai, "Face attribute prediction with convolutional neural networks," in *2017 IEEE International Conference on Robotics and Biomimetics (ROBIO)*, pp. 1294–1299, Dec 2017.

**CELEBA: Multiple attribute classification**  
Original images

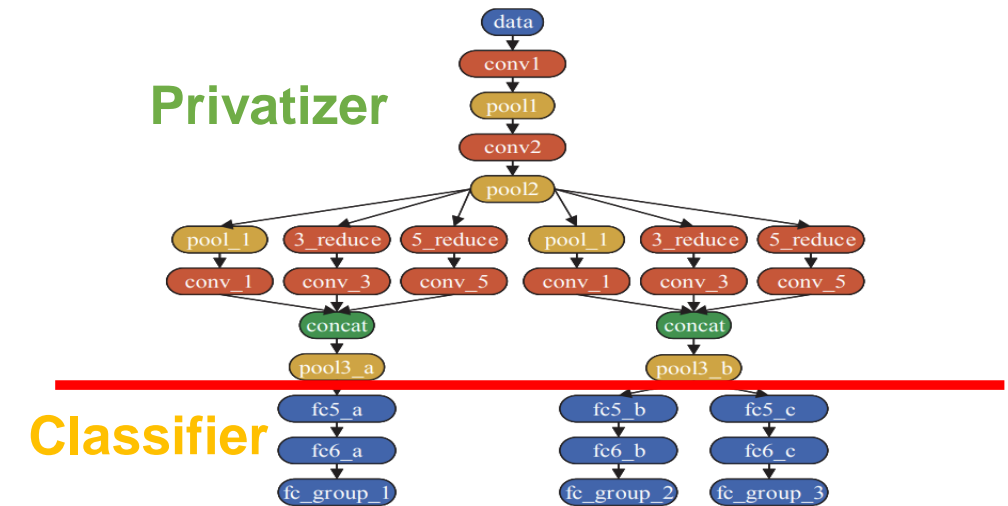


Reconstructed image by adversary



- Privacy issue:
  - Adversaries are capable of attaining the information corresponding to 40 attributes.
- How to solve?
  - Tune the dimension of the compressing representations.









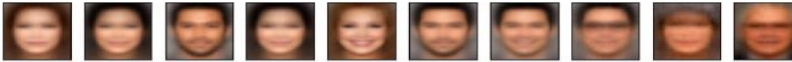

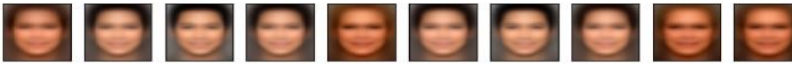
**Model architecture:**



Group	Attribute
group 1	black hair, blond hair, blurry, eyeglasses, gray hair, pale skin, straight hair, wearing hat
group 2	attractive, bangs, brown hair, heavy makeup, high cheekbones, mouth slightly open, no beard, oval face, pointy nose, rosy cheeks, smiling, wavy hair, wearing lipstick, young
group 3	5 o'clock shadow, arched eyebrows, bags under eyes, bald, big lips, big nose, bushy eyebrows, chubby, double chin, goatee, male, mustache, narrow eyes, receding hairline, sideburns, wearing earrings, wearing necklace, wearing necktie

# Enhance CPGAN

Table VII. Privacy and utility trade-off among different compressive dimensions

Compressive Dimension	Accuracy	Reconstructed Images
Raw images		
G=Identity <sup>a</sup>	90.81%	
1728*2 <sup>b</sup>	90.21%	
128*2 <sup>b</sup>	90.21%	
64*2 <sup>b</sup>	90.19%	
32*2 <sup>b</sup>	89.92%	
16*2 <sup>b</sup>	87.63%	
8*2 <sup>b</sup>	87.21%	
4*2 <sup>b</sup>	87.06%	
2*2 <sup>b</sup>	85.92%	
1*2 <sup>b</sup>	80.5%	
Majority Classifier <sup>c</sup>	80.52%	

<sup>a</sup> The notation "G=identity" is that the model without privacy preserving mechanism.

<sup>b</sup> The reason that the dimension is multiplied by 2 is that the model of multiple attribute classification generates two compressing sent to the cloud.

<sup>c</sup> Majority classifier always outputs the class that is in the majority in the training set.

**Accuracy**  
90.21%



80.5%

**Reconstruction:**  
Images remaining  
sensitive information

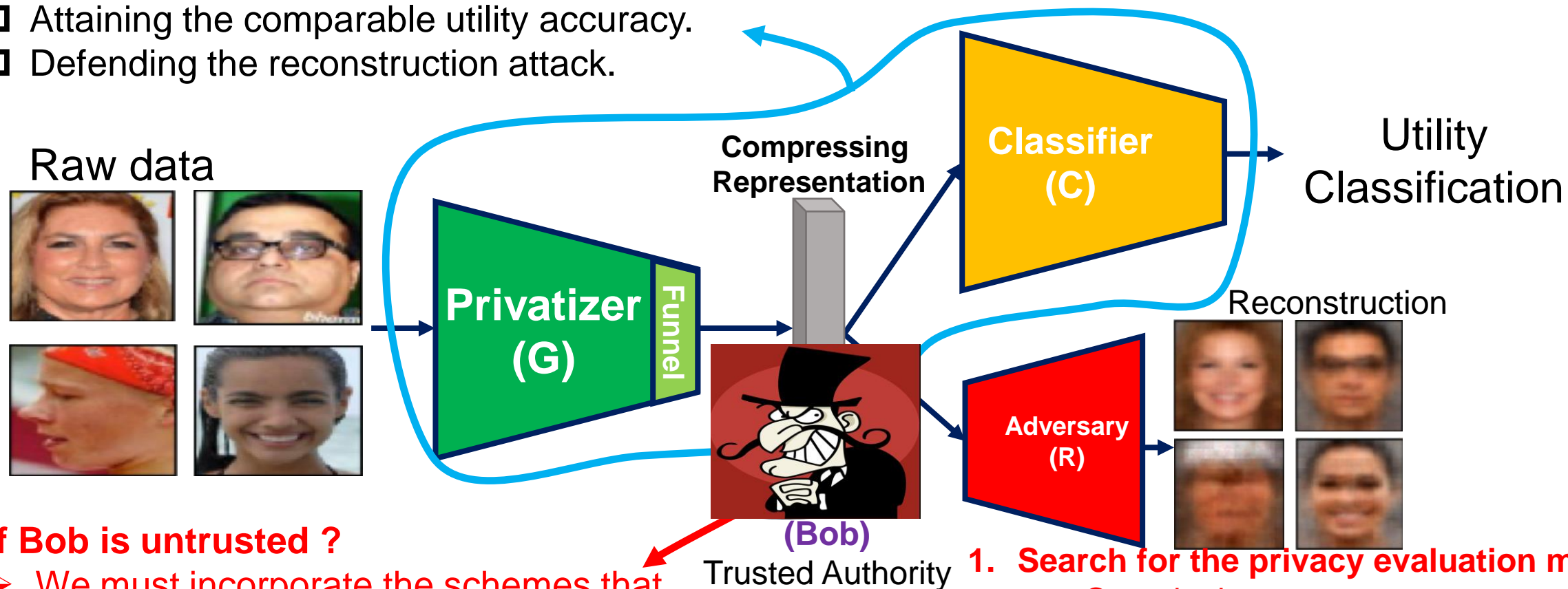


Unrecognizable  
Images



# Conclusions and Future Works

- We develop the local compression network that prevents sensitive data from getting exposed to public.
- We confirm that the compressing representation is capable of
  - ❑ Attaining the comparable utility accuracy.
  - ❑ Defending the reconstruction attack.



## 2. If Bob is untrusted ?

- We must incorporate the schemes that can protect the training data to CPGAN.

## 1. Search for the privacy evaluation metric

- Quantitative
- Human's vision perception.

# Reference

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- [1] Sicong Liu et al., “Better accuracy with quantified privacy: representations learned via reconstructive adversarial network,” arXiv, 2017.
- [2] Kaiming He et al., “Identity Mappings in Deep Residual Networks,” ECCV, 2016.
- [3] Xavier Gastaldi, “Shake-Shake regularization,” arXiv, 2017.
- [4] Sergey Zagoruyko et al., “Wide Residual Networks,” arXiv, 2017.
- [5] Doudou Gao et al., “Face attribute Prediction with Convolutional Neural Networks,” IEEE conference, 2018.
- [6] Hu Han et al., “Heterogeneous Face Attribute Estimation: A Deep Multi-Task Learning Approach,” IEEE, 2018.
- [7] Chong Huang et al., “Generative Adversarial Privacy,” ICML workshop, 2018

# Compressive Privacy Generative Adversarial Networks

34

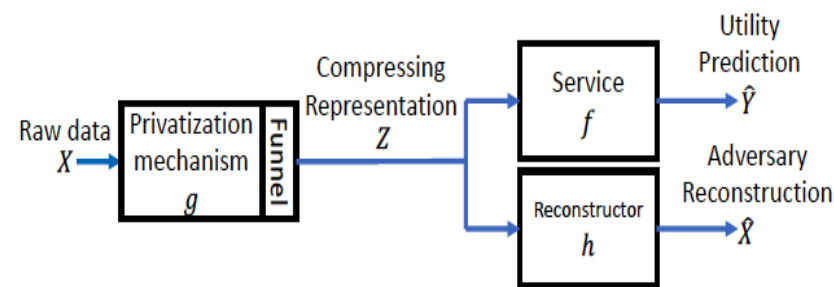
*Bo-Wei Tseng, Pei-Yuan Wu*

**Why?** -> Solve the privacy issue (reconstruction attack) occurring in the MLaaS model.

**What?** -> Develop the local privacy preserving mechanism (privatizer) to prevent the sensitive data from getting exposed to the cloud.

**How?** -> Incorporate the multiple adversaries strategy to adversarial learning scheme.

**CPGAN architecture:**



**CPGAN objective function:**

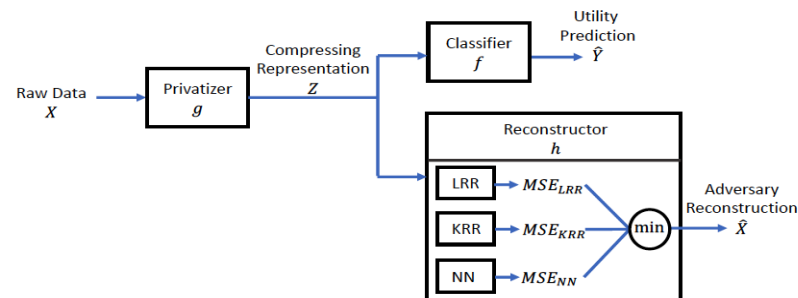
$$\max_g (\min_h L_{adv}(g, h) - \lambda \min_f L_{util}(g, f))$$

$$L_{adv} = \mathbb{E}_{\hat{X} \sim P_h(\cdot | Z)} [\|X - \hat{X}\|_2^2]$$

$$L_{util}(P_f(\cdot | Z), Y) = \mathbb{E}[-\log P_f(Y | Z)]$$

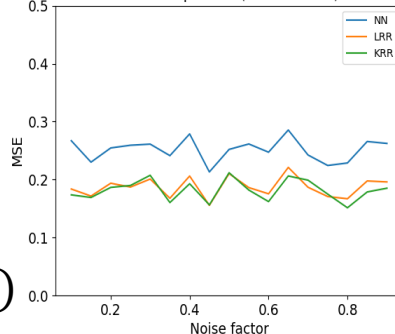
$$(X, Y) \sim P_{X, Y}, Z | X \sim P_{g_\theta}(\cdot | X), \hat{Y} | Z \sim P_{f_\tau}(\cdot | Z)$$

**Multiple adversaries strategy for training/evaluation:**

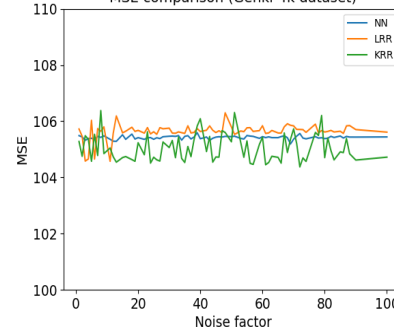


**Verification:**

MSE comparison (HAR dataset)



MSE comparison (Genki-4k dataset)



**Results on GENKI-4K dataset:**

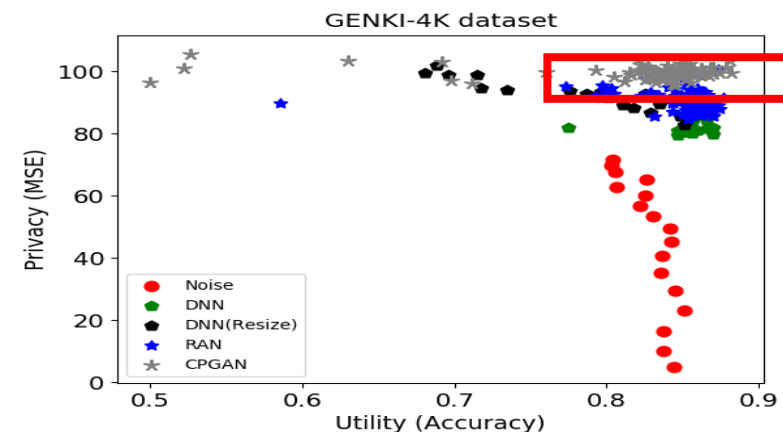
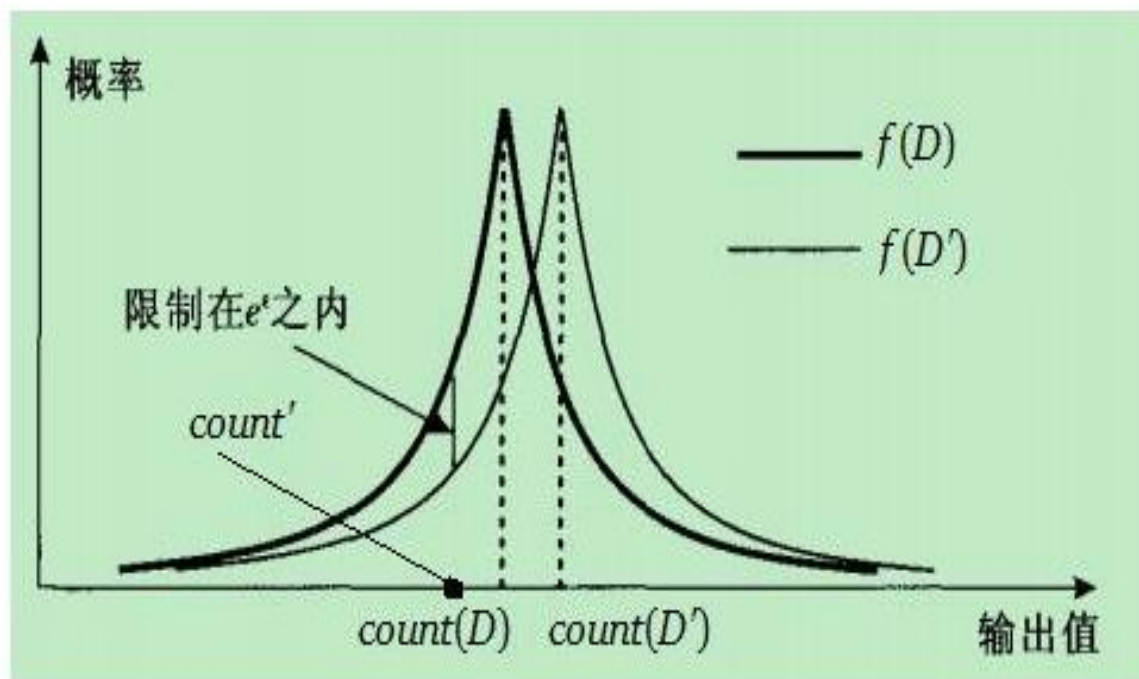


Table II. Reconstructed images from five privacy preserving mechanisms on GENKI-4K dataset

	Original	Noise	DNN	DNN (Resize)	RAN	CPGAN
Average Accuracy (%)		69.83	85.19	77.70	84.89	84.93
Image1						
Image2						

**Thank You !!!**

# Differential Privacy



## Differential privacy in Deep Learning:

The general steps for adding differential privacy to any learning algorithm are as follows:

1. Initialize learning parameters randomly.
2. Take a random sample.
3. Compute gradient on that random sample.
4. Clip the gradient.
5. Add noise.
6. Descent.
7. Compute the overall privacy cost using a privacy accountant.

$$\Pr[\mathcal{A}(D_1) \in S] \leq e^\epsilon \times \Pr[\mathcal{A}(D_2) \in S],$$

# Gradient reconstruction

- Take one neural network for example (i.e. regression problem), then the objective function is:
- Its derivations:

$$\begin{aligned}
 J(W, b, x, y) &\stackrel{\text{def}}{=} (h_{W,b}(x) - y)^2 \\
 \eta_k &\stackrel{\text{def}}{=} \frac{\delta J(W, b, x, y)}{\delta W_k} = 2(h_{W,b}(x) - y) \frac{\delta h_{W,b}(x)}{\delta W_k} = 2(h_{W,b}(x) - y) \frac{\delta f(\sum_{i=1}^d W_i x_i + b)}{\delta W_k} \\
 &= 2(h_{W,b}(x) - y) f'(\sum_{i=1}^d W_i x_i + b) \cdot x_k \\
 \eta &\stackrel{\text{def}}{=} \frac{\delta J(W, b, x, y)}{\delta b} = 2(h_{W,b}(x) - y) \frac{\delta h_{W,b}(x)}{\delta b} = 2(h_{W,b}(x) - y) \frac{\delta f(\sum_{i=1}^d W_i x_i + b)}{\delta b} \\
 &= 2(h_{W,b}(x) - y) f'(\sum_{i=1}^d W_i x_i + b) \cdot 1.
 \end{aligned}$$

**Thus,**  $\eta_k / \eta = x_k.$

# DCA Formulation

From derivation maximum utility mutual information

$$\begin{aligned}
 I(u; y) &= H(u) - H(u|y) \\
 &= \frac{1}{2} \log_2 |\Sigma_u| + \frac{\mu}{2} \log_2 2\pi e - \frac{1}{2} \log_2 |\Sigma_{\hat{u}}| + \frac{\mu}{2} \log_2 2\pi e \\
 &= \frac{1}{2} \log_2 |\Sigma_u| - |\Sigma_{\hat{u}}| \\
 &= \frac{-1}{2} \log_2 |\Sigma_{\hat{u}}| - |\Sigma_u| \\
 &= \frac{-1}{2} \log_2 (|\Sigma_u + (\Sigma_{\hat{u}} - \Sigma_u)| - |\Sigma_u|) \\
 \text{Derivative:} \quad &\cong \frac{-1}{2} \text{Tr}(\Sigma_u^{-1}(\Sigma_{\hat{u}} - \Sigma_u)) \\
 &= \frac{1}{2} \text{Tr}(\Sigma_u^{-1}(U^T \Sigma_x U - U^T \Sigma_{\hat{x}} U)) \\
 &= \frac{1}{2} \text{Tr}(\Sigma_u^{-1}(U^T (\Sigma_x - \Sigma_{\hat{x}}) U)) \\
 &= \frac{1}{2} \text{Tr}(\Sigma_u^{-1}(U^T (\Sigma_x U^T (F^T (\Sigma_x + \Sigma_\epsilon) F)^{-1} F^T) \Sigma_x U)) \\
 &= \frac{1}{2} \text{Tr}((F^T (\Sigma_x + \Sigma_\epsilon) F)^{-1} F^T \Sigma_x U \Sigma_u^{-1} U^T \Sigma_x F)) \\
 &= \frac{1}{2} \text{Tr}((F^T (\Sigma_x + \Sigma_\epsilon) F)^{-1} F^T \Omega F))
 \end{aligned} \tag{8}$$

# RFF theory

**Theorem 1** (Bochner [13]). *A continuous kernel  $k(\mathbf{x}, \mathbf{y}) = k(\mathbf{x} - \mathbf{y})$  on  $\mathcal{R}^d$  is positive definite if and only if  $k(\delta)$  is the Fourier transform of a non-negative measure.*

If a shift-invariant kernel  $k(\delta)$  is properly scaled, Bochner's theorem guarantees that its Fourier transform  $p(\omega)$  is a proper probability distribution. Defining  $\zeta_\omega(\mathbf{x}) = e^{j\omega' \mathbf{x}}$ , we have

$$k(\mathbf{x} - \mathbf{y}) = \int_{\mathcal{R}^d} p(\omega) e^{j\omega'(\mathbf{x} - \mathbf{y})} d\omega = E_\omega[\zeta_\omega(\mathbf{x}) \zeta_\omega(\mathbf{y})^*], \quad (2)$$

so  $\zeta_\omega(\mathbf{x}) \zeta_\omega(\mathbf{y})^*$  is an unbiased estimate of  $k(\mathbf{x}, \mathbf{y})$  when  $\omega$  is drawn from  $p$ .



# MAP (In detail.)

## 1 Assumption

Let  $\mathbf{x} = \boldsymbol{\tau} + \boldsymbol{\xi}$ , where  $\boldsymbol{\tau} \in \{-\boldsymbol{\mu}, \boldsymbol{\mu}\}$ ,  $\mathbf{z} = \mathbf{A}\mathbf{x} + \boldsymbol{\epsilon}$ ,  $\hat{\mathbf{x}} = \mathbf{B}\mathbf{z}$ , where  $\boldsymbol{\tau}, \boldsymbol{\xi}, \boldsymbol{\epsilon}$  are independent r.v.s with zero mean, and  $\boldsymbol{\xi}, \boldsymbol{\epsilon}$  is sampled from Gaussian distribution. Thus,  $\mathbf{R}_{\boldsymbol{\tau}} = \mathbb{E}[\boldsymbol{\tau}\boldsymbol{\tau}^T]$ ,  $\mathbf{R}_{\boldsymbol{\xi}} = \mathbb{E}[\boldsymbol{\xi}\boldsymbol{\xi}^T]$ . Note that if  $s=0$  then  $\boldsymbol{\tau} = \boldsymbol{\mu}$ ,  $s=1$  is on the contrary, where  $s$  denotes the utility label (binary):

- Since the diagonal covariance matrix can make the analysis simpler, and shift the mean vector to the form (such as  $[\alpha, 0, 0, 0, 0, \dots]$ ). Note that we assume the mean vector is  $(\boldsymbol{\mu}, -\boldsymbol{\mu})$  in the following discussion.

$$\begin{aligned}
 \frac{Q(X|s=0)}{Q(X|s=1)} &\underset{s_0}{\overset{s_1}{\geq}} \frac{q}{1-q} \\
 \frac{e^{-(\vec{x}-\vec{\mu})^T \Sigma_D^{-1} (\vec{x}-\vec{\mu})}}{e^{-(\vec{x}+\vec{\mu})^T \Sigma_D^{-1} (\vec{x}+\vec{\mu})}} &= \frac{q}{1-q} \\
 2\vec{x}^T \Sigma_D^{-1} \vec{\mu} + 2\vec{\mu}^T \Sigma_D^{-1} \vec{x} &= 2\ln\left(\frac{q}{1-q}\right) \\
 \text{Let } c &= \Sigma_D^{-1} \vec{\mu} \\
 \therefore c^T \vec{x} &= \frac{\ln(\frac{q}{1-q})}{2}
 \end{aligned} \tag{1}$$

therefore, the  $\vec{x} = [x_1, x_2, x_3, \dots]$  only has the deterministic solution in  $x_1$ , the other has infinite solution. The following is the 2-dimension example:

$$\int_{-\infty}^{\infty} e^{-\frac{(x_1+\alpha)^2}{2}} dx_1 \int_{-\infty}^{\infty} e^{-\frac{(x_2)^2}{2}} dx_2 \tag{2}$$

# Privacy Loss

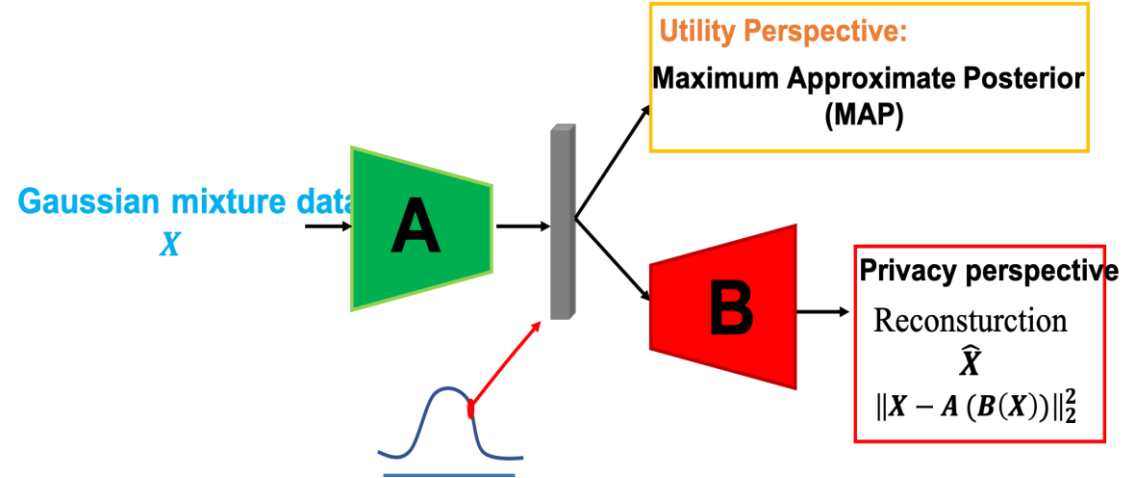
$$\begin{aligned}
 L_{rec} &= \mathbb{E}_{X \sim P_{data}} \|X - \hat{X}\|_2^2 \\
 &= \mathbb{E}_{X \sim P_{data}} \|X - B(AX + \epsilon)\|_2^2 \\
 &= \mathbb{E}_{X \sim P_{data}} \|(I - BA)X - B\epsilon\|_2^2 \\
 &= \text{Tr}(\mathbb{E}_x [(I - BA)X - B\epsilon][(I - BA)X - B\epsilon]^T)
 \end{aligned}$$

- Zero gradient with respect to B:

$$\begin{aligned}
 -2R_x A^T + 2BR_\epsilon + 2B(AR_x A^T) &= 0 \\
 \therefore B &= R_x A^T (AR_x A^T + R_\epsilon)^{-1}
 \end{aligned}$$

Substitute the solution into B, then find A follows the loss below:

$$\max_A \text{Tr}(R_x) - \text{Tr}(R_x A^T (AR_x A^T + R_\epsilon)^{-1} AR_x)$$



# Theory MSE

- Since the Q function is increasing with the alpha. Out Optimization becomes:

## 1.3 Combination

- The combination of the alpha and MSE loss above is:

$$\max_A \text{Tr}(R_x) - \text{Tr}(R_x A^T (A R_x A^T + R_\epsilon)^{-1} A R_x) - \lambda (2A\mu)^T (A R_\xi A^T + R_\epsilon)^{-1} (2A\mu) \quad (7)$$

- And zero gradient with respect to A (assuming  $R_\epsilon = 0$ )

$$\begin{aligned} 0 = & -3((A R_x A^T)^{-1} A (R_x)^2 + (R_x)^2 A^T (A R_x A^T)^{-1} + \\ & -2(A R_x A^T)^{-1} A R_x^2 A^T (A R_x A^T)^{-1} A R_x + \\ & 4\lambda((A \Sigma_\xi A^T)^{-1} A \vec{\mu} \vec{\mu}^T + 2\vec{\mu} \vec{\mu}^T (A \Sigma_\xi A^T)^{-1} A A^T (A \Sigma_\xi A^T)^{-1} A \Sigma_\xi) \end{aligned}$$

**This is really intractable !!!**

# LRR and KRR formulation

Assuming that training data matrix  $\mathbf{X}$  and output value matrix  $\mathbf{Y}$  are both zero mean. Thus,  $\mathbf{S} = \mathbf{X}\mathbf{X}^T$  and the bias term is no longer useful.

$$\begin{aligned} E_{LSE} &= \left\| \mathbf{X}^T \mathbf{W} - \mathbf{Y} \right\|_2^2 \\ &= \text{Tr}((\mathbf{X}^T \mathbf{W} - \mathbf{Y})(\mathbf{X}^T \mathbf{W} - \mathbf{Y})^T) \end{aligned} \tag{11}$$

zero gradient with respect to  $\mathbf{W}$ , we get:

$$\begin{aligned} \mathbf{0} &= 2\mathbf{X}\mathbf{X}^T \mathbf{W} - 2\mathbf{X}\mathbf{Y} + 2\rho \mathbf{W} \\ \mathbf{W} &= (\mathbf{S} + \rho \mathbf{I})^{-1} \mathbf{X}\mathbf{Y} \end{aligned} \tag{12}$$

# LFW Accuracy Comparison

	Raw images	Reconstruction images
Recognition Accuracy	67.2% (train) 7% (validation) 1% (testing)	0% (train) 0% (validation) 0% (testing)
LFW accuracy	91.83%	72%