Linear Algebra Group Project - Bethany Wu & Alex Stapely

Problem 1: Problem Formulation

Parts (a) & (b)

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Linear Algebra (Group) Assignment

$-u''(x)+V(x)u'(x)=f(x)$, $x \in [0,1]$ $\quad u(0)=u(1)=0$

$V(x)=\gamma \quad f(x)=1$

1. Problem Foundation

a) Write this boundary value problem as a variational problem: $A(u,\phi)=F(\phi)$ for some test function ϕ.

Multiply by the test function ϕ and integrate over x.

$$\underbrace{-\int_0^1 dx\, \phi u''}_{-\phi u'']_0^1 \,+\, \int_0^1 dx\, \phi' u'} + \int_0^1 V(x)\phi u'\, dx = \int_0^1 dx \cdot \phi \cdot f$$

$-\phi u'']_0^1 + \int_0^1 dx\, \phi' u' \equiv A_1[u,\phi]$ bilinear form

$$\int_0^1 dx\, \phi' u' + \int_0^1 dx\, \gamma \phi u' = \int_0^1 dx\, f \phi$$

$$A_1[\phi,u] + A_2[\phi,u] = F[\phi]$$

b) Take ϕ_i to be the "hat" function discussed in class and approximate u as a linear combination of these basis vectors: $u(x)=\Sigma_i u_i \phi_i(x)$. Show that the variational problem becomes a linear algebra problem of the form $Ax=b$.

$$A_1[\phi_j, \Sigma_i u_i \phi_i(x)] + A_2[\phi_j, \Sigma_i u_i \phi_i(x)] = F[\phi_j] \quad \forall j$$

This gives m equations and m unknowns. The m equations are all linear. We can use that to rewrite...

$$\Sigma_{i=1}^{m} \underbrace{u_i}_{\text{"x"}} \underbrace{[A_1(\phi_j,\phi_i) + A_2(\phi_j,\phi_i)]}_{A_{ji}} = \underbrace{F[\phi_j]}_{b_j} \rightarrow Ax=b$$

Part (c)

```
[A1,b1] = BVPtoVar(5,1)
```

```
A1 = 5×5
    0.3333    0.3333         0         0         0
   -0.6667    0.3333    0.3333         0         0
         0   -0.6667    0.3333    0.3333         0
         0         0   -0.6667    0.3333    0.3333
         0         0         0   -0.6667    0.3333
b1 = 5×1
     6
     6
     6
     6
     6
```

```
function [A,b] = BVPtoVar(n,gamma)
    dx = n+1;
    % Forming A1
    side_A1 = ones(n-1,1)*(-1/(dx));
    diag_A1 = ones(n,1)*(2/dx);
    A1 = diag(side_A1,-1)+diag(diag_A1)+diag(side_A1,1);
    % Forming A2
    side_A2 = ones(n-1,1)*(gamma/2);
    A2 = diag(-side_A2,-1)+diag(side_A2,1);

    A = A1+A2;
    b = ones(n,1)*dx;
end
```