A Research Program for State-Dependent DTS Scaling in Corporate Credit Spreads

Part II: Shock Heterogeneity, Industry Exposure, and Multi-Factor Decomposition

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Abstract

Note: This paper is work in progress. All Empirical Results in this Paper are expectations and await empirical confirmation or refutation

This document extends the research program developed in Part I by addressing a fundamental limitation in the standard Duration-Times-Spread (DTS) framework: the implicit assumption that all firms experience identical firm value shocks when macroeconomic news arrives. We develop a two-layer theoretical framework that cleanly separates (i) the *Merton elasticity* $\lambda(s, T)$—governing how spread level and maturity determine the translation of firm value changes into spread changes—from (ii) *shock exposure* $\beta_k^{(m)}$—governing how macroeconomic shock type m translates into firm value changes for industry k.

The key insight is that when an employment report surprises to the upside, a Consumer Cyclicals firm experiences a larger increase in firm value ($\Delta V/V > 0$) than an Electric Utilities firm, *before* any consideration of how Merton elasticity translates this firm value change into spread changes. Because positive firm value shocks *tighten* spreads, the Consumer Cyclicals bond tightens more—and this differential response is distinct from any maturity or spread-level effects captured by Merton elasticity. The standard DTS framework conflates these two sources of heterogeneity, leading to biased estimates of structural parameters and incomplete risk attribution.

We propose: (1) explicit modeling of industry-specific macro betas using the Bloomberg BCLASS3 classification standard for corporate bonds; (2) a properly orthogonalized shock decomposition separating real economy, inflation, monetary policy, and risk appetite shocks; (3) a covariance-adjusted composite beta that accounts for shock correlations; (4) enhanced empirical tests that isolate Merton elasticity from shock exposure; (5) a principal component analysis framework that validates the multiplicative two-layer structure. The research program delivers both academic contribution—the first clean separation of credit spread mechanics from economic exposure—and practical tools for risk attribution, hedging, and relative value analysis.

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1 Introduction and Motivation

1.1 The limitation identified in Part I

Part I of this research program developed a comprehensive framework for enhancing Duration-Times-Spread (DTS) through Merton-based elasticity adjustments. The core empirical specification was:

$$\frac{\Delta s_{i,t}}{s_{i,t-1}} = \lambda_i(s,T) \cdot f_{DTS,t} + \varepsilon_{i,t} \tag{1}$$

where $\lambda_i(s,T)$ captures how spread level s and maturity T affect bond i's sensitivity to a common DTS factor $f_{DTS,t}$, constructed as the market-value-weighted average proportional spread change across the index.

The empirical findings from Part I were instructive but not entirely clean:

- **Within-issuer tests** showed strong support for Merton elasticity predictions: when comparing two bonds from the same issuer with different maturities, their relative sensitivities aligned well with $\lambda(s,T)$ ratios.
- **Cross-industry comparisons** revealed significant "sector effects": Financial sector bonds appeared to have 30-40% higher sensitivity than Industrial bonds, even after controlling for spread and maturity.
- **Bucket-level regressions** produced noisier estimates than within-issuer tests, with residual variation that Merton elasticity alone could not explain.

At the time, we interpreted the sector effects as potential evidence that the Merton model's elasticity predictions might vary by industry—perhaps due to differences in asset volatility dynamics or capital structure. This interpretation, while plausible, missed a more fundamental explanation.

1.2 The deeper issue: Heterogeneous firm value shocks

The DTS factor $f_{DTS,t}$ is not a single homogeneous shock. When we observe the aggregate credit spread index moving on a given day, this movement reflects a mixture of:

- Real economy surprises (employment, GDP, consumption)
- Inflation surprises (CPI, wage growth)
- Monetary policy shocks (Fed funds surprises, forward guidance)
- Risk appetite shifts (VIX changes, flight-to-quality flows)
- Sector-specific news (oil prices, financial stress indicators)

Each of these shock types affects firm values *differentially* across industries. When nonfarm payrolls surprise to the upside:

- Consumer Cyclicals firms see increased expected revenues and higher firm value
- Electric Utilities firms, with inelastic demand, see minimal firm value change
- Banking firms benefit from improved loan quality and potentially steeper yield curves

This heterogeneity in firm value response exists *before* any consideration of how the Merton model translates firm value changes into spread changes. The standard DTS framework implicitly assumes all firms experience the same $\Delta V/V$ when the aggregate factor moves—an assumption that is clearly violated.

1.3 The two-layer heterogeneity framework

The true data-generating process involves two distinct layers of heterogeneity. For bond i issued by firm j in industry k:

Layer 1: Shock Exposure (Firm Value Response)

$$\frac{\Delta V_{j,t}}{V_{j,t-1}} = \sum_{m=1}^{M} \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} + \nu_{j,t} \tag{2}$$

where $\beta_k^{(m)}$ is industry k's sensitivity to shock type m, $\eta_t^{(m)}$ is the shock realization, and $\nu_{j,t}$ is firm-specific idiosyncratic news.

Layer 2: Merton Elasticity (Spread Response to Firm Value)

$$\frac{\Delta s_{ij,t}}{s_{ij,t-1}} = -\lambda_{ij}(s,T) \cdot \frac{\Delta V_{j,t}}{V_{j,t-1}} \tag{3}$$

where $\lambda_{ij}(s,T) > 0$ is the Merton elasticity from Part I, and the negative sign reflects that higher firm value implies lower default probability and tighter spreads.

Sign convention (used throughout this paper): We define $\beta_k^{(m)} > 0$ to mean that a positive shock $\eta_t^{(m)} > 0$ *increases* firm value. For example, $\beta_{ConsumerCyclicals}^{NFP} > 0$ because a positive employment surprise is good news for consumer discretionary firms. We define $\lambda_{ij} > 0$ as the absolute elasticity, with the negative sign in equation (3) making explicit that firm value increases cause spread decreases.

Combining the two layers:

$$\boxed{\frac{\Delta s_{ij,t}}{s_{ij,t-1}} = -\lambda_{ij}(s,T) \cdot \left[\sum_{m=1}^{M} \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} + \nu_{j,t}\right]} \tag{4}$$

This is the core equation of Part II. It shows that spread changes depend on:

1. The **Merton elasticity** $\lambda_{ij}(s,T)$, which varies with spread level and maturity (Part I contribution)
2. The **industry shock betas** $\beta_k^{(m)}$, which vary across industries and shock types (Part II contribution)
3. The **shock realizations** $\eta_t^{(m)}$, which vary over time
4. The **idiosyncratic component** $\nu_{j,t}$, which is firm-specific

1.4 Reinterpretation of Part I findings

The two-layer framework provides a unified explanation for Part I's empirical patterns:

Why within-issuer tests worked well: When comparing two bonds from the same issuer j, both bonds have identical shock exposures $\{\beta_{k(j)}^{(m)}\}$ and experience the same idiosyncratic shock $\nu_{j,t}$. Taking the ratio:

$$\frac{\Delta s_{ij,t}/s_{ij,t-1}}{\Delta s_{i'j,t}/s_{i'j,t-1}} = \frac{\lambda_{ij}(s,T)}{\lambda_{i'j}(s,T)} \tag{5}$$

The shock exposure terms cancel completely, isolating pure Merton elasticity differences.

Why sector effects appeared: The finding that Financials have higher sensitivity than Industrials is likely driven by $\beta^{composite}_{Banking} > \beta^{composite}_{Industrial}$—i.e., Banking firms have higher exposure to the mix of shocks that drive aggregate spread movements. This is *not* evidence that the Merton model's elasticity predictions differ by sector; it is evidence of differential shock exposure that was being misattributed to elasticity differences.

Why bucket-level tests were noisy: Cross-issuer comparisons within a bucket mix together: (i) true Merton elasticity variation across spread/maturity combinations, (ii) within-industry variation in shock exposure, and (iii) idiosyncratic firm effects. Without controlling for industry, the estimated "elasticity" coefficients are biased.

1.5 Contribution and roadmap

This paper makes the following contributions:

1. **Theoretical:** We formally separate the two layers of heterogeneity and derive testable predictions for each layer independently.

2. **Methodological:** We develop a properly orthogonalized shock taxonomy and a covariance-adjusted composite beta that accounts for shock correlations.

3. **Empirical:** We propose tests that (a) estimate industry shock betas from equity data, (b) validate that these betas explain the sector effects from Part I, (c) confirm that Merton elasticity predictions hold within industries once shock exposure is controlled.

4. **Practical:** We provide production formulas for enhanced DTS that incorporate both elasticity and exposure adjustments, enabling more accurate risk attribution and hedging.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework in detail, including proper treatment of the idiosyncratic component and conditions under which the multiplicative structure holds. Section 3 presents the shock taxonomy, discusses identification and orthogonalization, and addresses endogeneity concerns. Section 4 develops the industry beta estimation methodology using the Bloomberg BCLASS3 classification. Section 5 derives the covariance-adjusted composite beta. Section 6 presents the principal component analysis framework for model validation. Sections 7-9 detail the empirical research program. Section 10 provides production implementation specifications. Section 11 concludes.

2 Theoretical Framework: Two-Layer Heterogeneity

2.1 Layer 1: Merton elasticity revisited

From the Merton (1974) structural model, the credit spread s on a zero-coupon bond with face value F and maturity T is:

$$s(V,T) = -\frac{1}{T}\ln\left[\Phi(d_2) + \frac{V}{Fe^{-rT}}\Phi(-d_1)\right] \tag{6}$$

where d_1 and d_2 are the standard Black-Scholes terms depending on firm value V, face value F, risk-free rate r, maturity T, and asset volatility σ_V.

The elasticity of the spread with respect to firm value is:

$$\varepsilon_s \equiv \frac{\partial s}{\partial V} \cdot \frac{V}{s} \tag{7}$$

In Part I, we showed that this elasticity can be approximated as:

$$\varepsilon_s \approx -\frac{\phi(d_2)}{T \cdot s \cdot \Phi(-d_2)} \tag{8}$$

which is negative (spread falls when firm value rises) and varies systematically with spread level and maturity.

Definition 1 (Merton Elasticity)**.** *We define the **Merton elasticity** $\lambda(s,T) > 0$ as the absolute value of the spread-to-firm-value elasticity:*

$$\lambda(s,T) \equiv |\varepsilon_s| = \frac{\phi(d_2)}{T \cdot s \cdot \Phi(-d_2)} \tag{9}$$

With this convention, a firm value increase of 1% causes a spread decrease of approximately $\lambda(s,T)$%.

The key findings from Part I regarding $\lambda(s,T)$:

- λ is **decreasing in maturity**: Short-maturity bonds are more sensitive to firm value changes than long-maturity bonds (for given spread level).
- λ is **decreasing in spread**: Low-spread (high-quality) bonds are more sensitive than high-spread bonds (for given maturity).
- The reference point $\lambda = 1$ corresponds approximately to 5-year maturity at 100 bps spread.

2.2 Layer 2: Shock exposure heterogeneity

The firm value of issuer j in industry k evolves according to:

$$\frac{\mathrm{d}V_{j,t}}{V_{j,t}} = \mu_j \,\mathrm{d}t + \sum_{m=1}^{M} \beta_{k(j)}^{(m)} \,\mathrm{d}\eta_t^{(m)} + \tilde{\beta}_j^{(m)} \,\mathrm{d}\eta_t^{(m)} + \sigma_j \,\mathrm{d}W_{j,t} \tag{10}$$

For empirical purposes, we work with discrete returns:

$$\frac{\Delta V_{j,t}}{V_{j,t-1}} = \underbrace{\sum_{m=1}^{M} \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)}}_{\text{Industry-systematic}} + \underbrace{\sum_{m=1}^{M} \tilde{\beta}_j^{(m)} \cdot \eta_t^{(m)}}_{\text{Firm deviation from industry}} + \underbrace{\nu_{j,t}}_{\text{Idiosyncratic}} \tag{11}$$

Definition 2 (Industry Shock Beta)**.** *The **industry shock beta** $\beta_k^{(m)}$ measures the sensitivity of firm value in industry k to a one-unit shock of type m:*

$$\beta_k^{(m)} \equiv \frac{\partial}{\partial \eta^{(m)}} \mathbb{E}\left[\frac{\Delta V_j}{V_j} \,\middle|\, j \in \mathit{industry}\ k\right] \tag{12}$$

We normalize $\beta_k^{(m)} > 0$ when positive shocks are "good news" for firm value in industry k.

2.3 The combined two-layer model

Combining equations (3) and (2):

$$\begin{aligned}
\frac{\Delta s_{ij,t}}{s_{ij,t-1}} &= -\lambda_{ij} \cdot \frac{\Delta V_{j,t}}{V_{j,t-1}} \\
&= -\lambda_{ij} \cdot \left[\sum_{m=1}^{M} \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} + \sum_{m=1}^{M} \tilde{\beta}_j^{(m)} \cdot \eta_t^{(m)} + \nu_{j,t} \right] \\
&= \underbrace{-\sum_{m=1}^{M} \lambda_{ij} \cdot \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)}}_{\text{Industry-systematic spread change}} \underbrace{-\sum_{m=1}^{M} \lambda_{ij} \cdot \tilde{\beta}_j^{(m)} \cdot \eta_t^{(m)}}_{\text{Firm-specific systematic}} - \underbrace{\lambda_{ij} \cdot \nu_{j,t}}_{\text{Idiosyncratic}} \qquad (13)
\end{aligned}$$

Remark 1 (Idiosyncratic Term Scaling)**.** *Note that the idiosyncratic term $\nu_{j,t}$ is also scaled by λ_{ij}. This is economically correct: if a firm experiences idiosyncratic good news (e.g., wins a major contract), the resulting firm value increase translates into a spread decrease, and the magnitude of this spread response depends on where the bond sits on the Merton surface. A short-maturity, low-spread bond will respond more to the same idiosyncratic news than a long-maturity, high-spread bond from the same issuer.*

2.4 Relationship to the standard DTS model

The aggregate DTS factor $f_{DTS,t}$ is constructed as:

$$f_{DTS,t} = \sum_{i \in \text{Index}} w_i \cdot \frac{\Delta s_{i,t}}{s_{i,t-1}} \qquad (14)$$

where w_i are market-value weights. Substituting our two-layer model:

$$\begin{aligned}
f_{DTS,t} &= \sum_i w_i \cdot \left(-\lambda_i \cdot \sum_m \beta_{k(i)}^{(m)} \cdot \eta_t^{(m)} - \lambda_i \cdot \nu_{i,t} \right) \\
&= -\sum_m \underbrace{\left(\sum_i w_i \cdot \lambda_i \cdot \beta_{k(i)}^{(m)} \right)}_{\equiv \gamma^{(m)}} \cdot \eta_t^{(m)} + \text{idiosyncratic terms} \\
&\approx -\sum_m \gamma^{(m)} \cdot \eta_t^{(m)} \qquad (15)
\end{aligned}$$

The weight $\gamma^{(m)}$ on shock type m in the aggregate DTS factor depends on:

- The market-value weights w_i of bonds in the index
- The Merton elasticities λ_i of those bonds
- The industry composition of the index
- The industry shock betas $\beta_k^{(m)}$

Proposition 1 (Conflation in Standard DTS Regression)**.** *When we estimate the standard DTS regression:*

$$\frac{\Delta s_{i,t}}{s_{i,t-1}} = \alpha + \hat{\lambda}_i \cdot f_{DTS,t} + \varepsilon_{i,t} \tag{16}$$

the estimated coefficient $\hat{\lambda}_i$ is:

$$\operatorname{plim} \hat{\lambda}_i = \frac{\lambda_i \cdot \sum_m \beta_{k(i)}^{(m)} \cdot \gamma^{(m)} \cdot \operatorname{Var}(\eta^{(m)})}{\sum_m (\gamma^{(m)})^2 \cdot \operatorname{Var}(\eta^{(m)})} \tag{17}$$

This is not *equal to λ_i unless all industries have identical shock exposures ($\beta_k^{(m)} = \beta^{(m)}$ for all k).*

Proof sketch: The regression coefficient is the covariance of $\Delta s_i/s_i$ with $f_{DTS,t}$ divided by the variance of $f_{DTS,t}$. Both the covariance and variance depend on the shock betas, and they do not cancel unless betas are homogeneous. □

2.5 Conditions for clean separation

Proposition 2 (Within-Issuer Identification)**.** *For two bonds i and i' from the same issuer j, the ratio of proportional spread changes identifies the ratio of Merton elasticities:*

$$\frac{\Delta s_{ij,t}/s_{ij,t-1}}{\Delta s_{i'j,t}/s_{i'j,t-1}} = \frac{\lambda_{ij}}{\lambda_{i'j}} \tag{18}$$

regardless of the shock composition on date t or the industry of issuer j.

Proof: Both bonds share the same issuer j, hence the same industry $k(j)$, the same industry betas $\{\beta_{k(j)}^{(m)}\}$, and the same idiosyncratic shock $\nu_{j,t}$. The ratio of spread changes is:

$$\frac{-\lambda_{ij} \cdot [\sum_m \beta_k^{(m)} \eta_t^{(m)} + \nu_{j,t}]}{-\lambda_{i'j} \cdot [\sum_m \beta_k^{(m)} \eta_t^{(m)} + \nu_{j,t}]} = \frac{\lambda_{ij}}{\lambda_{i'j}} \tag{19}$$

The bracketed terms cancel. □

This proposition explains why Part I's within-issuer tests provided clean validation of Merton elasticity predictions even though the cross-industry tests were contaminated by shock exposure heterogeneity.

2.6 When does the multiplicative structure hold?

The two-layer model assumes a multiplicative structure: $\Delta s/s = \lambda \times$ (firm value shock). Under what conditions might this fail?

Non-multiplicative interaction: If the transmission of firm value shocks to spread changes depended on the *source* of the shock (not just its magnitude), we would have:

$$\frac{\Delta s_{ij,t}}{s_{ij,t-1}} = -\sum_m \lambda_{ij}^{(m)} \cdot \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} \tag{20}$$

with shock-type-specific elasticities $\lambda_{ij}^{(m)}$.

The Merton model does not predict such shock-type dependence: a 1% firm value change should produce the same spread response regardless of whether it came from employment news or oil prices. However, this is an empirical question, and we propose tests in Section 9.

Testable restriction: The multiplicative structure implies that we can factor:

$$\frac{\mathrm{Cov}(\Delta s_{ij}/s_{ij}, \eta^{(m)})}{\mathrm{Cov}(\Delta s_{i'j'}/s_{i'j'}, \eta^{(m)})} = \frac{\lambda_{ij} \cdot \beta_{k(j)}^{(m)}}{\lambda_{i'j'} \cdot \beta_{k(j')}^{(m)}} \tag{21}$$

for all shock types m. If we find that this ratio varies systematically with m, the multiplicative structure is rejected.

3 Shock Taxonomy, Identification, and Measurement

3.1 Classification of macroeconomic shock types

We classify systematic shocks into five categories based on their economic nature and expected differential impact across industries:

Category 1: Real Economy Shocks

- η_t^{NFP}: Nonfarm payrolls surprise (actual minus consensus)
- η_t^{GDP}: GDP growth surprise
- η_t^{ISM}: ISM manufacturing index surprise
- η_t^{CONS}: Personal consumption surprise

These shocks primarily affect cyclically-sensitive industries. Expected pattern: $\beta^{NFP}_{ConsumerCyclicals} > \beta^{NFP}_{Electric}$.

Category 2: Inflation Shocks

- η_t^{CPI}: Core CPI surprise
- η_t^{PPI}: Producer price index surprise
- η_t^{WAGE}: Average hourly earnings surprise

Inflation surprises have ambiguous effects on firm value: higher inflation may increase nominal revenues but also costs. Expected pattern: Mixed, with potential positive effects for Energy (pricing power) and negative for industries with fixed-price contracts.

Category 3: Monetary Policy Shocks

- η_t^{FF}: Fed funds rate surprise (actual minus futures-implied)
- η_t^{FG}: Forward guidance shock (change in expected path)
- η_t^{QE}: Quantitative easing announcement effects

Monetary tightening generally reduces firm values, but the magnitude varies: Banking may benefit from steeper yield curves while interest-sensitive industries (REITs, Electric Utilities) suffer more.

Category 4: Risk Appetite Shocks

- η_t^{VIX}: Change in VIX index
- η_t^{CRP}: Credit risk premium shock (e.g., excess bond premium from Gilchrist-Zakrajšek)

Risk-off shocks are generally negative for all firm values but affect high-beta industries more.

Category 5: Sector-Specific Shocks

- η_t^{OIL}: Oil price change
- η_t^{FIN}: Financial stress indicator (e.g., TED spread)
- η_t^{TECH}: Technology sector news

These have concentrated effects: oil price increases benefit Energy but hurt Transportation.

3.2 Shock construction methodology

For announcement-based shocks (NFP, CPI, FOMC), we use the standard event-study approach:

$$\eta_t^{(m)} = \text{Actual}_t^{(m)} - \text{Consensus}_t^{(m)} \tag{22}$$

where consensus is from Bloomberg economic surveys. This ensures shocks are *unexpected* and thus informative about how markets revise valuations.

For market-based shocks (VIX, oil), we use daily changes:

$$\eta_t^{VIX} = \Delta \ln(VIX_t), \quad \eta_t^{OIL} = \Delta \ln(WTI_t) \tag{23}$$

Normalization: We standardize all shock series to have zero mean and unit variance over the estimation sample, facilitating comparison of beta magnitudes across shock types.

3.3 Orthogonalization and identification

A critical issue is that the raw shocks $\{\eta_t^{(m)}\}$ are correlated. For example, strong employment reports often coincide with Fed tightening expectations. If we estimate industry betas using correlated shocks, we cannot cleanly attribute firm value responses to individual shock types.

Orthogonalization procedure:

1. Order shocks by exogeneity: We place real economy shocks first (most exogenous), followed by inflation, monetary policy, and finally risk appetite shocks (most endogenous/reactive).

2. Apply Cholesky decomposition: Let $\boldsymbol{\eta}_t = (\eta_t^{(1)}, \ldots, \eta_t^{(M)})'$ be the vector of raw shocks with covariance matrix Σ_η. Compute the Cholesky factor L such that $\Sigma_\eta = LL'$. The orthogonalized shocks are:
$$\tilde{\boldsymbol{\eta}}_t = L^{-1}\boldsymbol{\eta}_t \tag{24}$$
These have identity covariance matrix.

3. Re-estimate industry betas using orthogonalized shocks:
$$r_{k,t}^{equity} = \alpha_k + \sum_m \tilde{\beta}_k^{(m)} \cdot \tilde{\eta}_t^{(m)} + \epsilon_{k,t} \tag{25}$$

Alternative: Principal component shocks. Rather than orthogonalizing observed shocks, we can extract principal components from the shock covariance matrix and work with PC-based factors. This is more agnostic about ordering but less interpretable.

3.4 Endogeneity concerns

Several of our shock variables are not exogenous to credit markets:

VIX: Changes in VIX are partially driven by credit market stress. A banking crisis widens credit spreads *and* raises VIX, creating spurious correlation. We address this by:

- Using high-frequency (intraday) windows around macro announcements
- Instrumenting VIX with exogenous volatility predictors (e.g., geopolitical events)
- Conducting robustness tests excluding VIX from the shock set

Fed policy: The Fed responds to economic conditions, including credit spreads. However, the *surprise* component (actual minus futures-implied) should be orthogonal to public information, mitigating endogeneity.

Oil prices: Oil prices respond to growth expectations, creating correlation with real economy shocks. We partially address this through the Cholesky ordering (real shocks before oil).

Remark 2. *Perfect identification of causal effects is not required for our purposes. We need industry betas that* predict *how spreads respond to the mix of shocks that drive aggregate DTS movements. Even if our shock decomposition is imperfect, improved prediction relative to the naive DTS model is valuable.*

3.5 Summary statistics and shock properties

Table 1: Shock Series Properties (Illustrative)

Shock	**Frequency**	**Std. Dev. (%)**	**Autocorr.**	**Corr. with NFP**
NFP surprise	Monthly	0.15	0.02	1.00
Core CPI surprise	Monthly	0.08	0.05	0.12
Fed funds surprise	8x/year	0.04	-0.10	0.25
ΔVIX	Daily	4.2	-0.05	-0.18
ΔOil	Daily	2.1	0.01	0.22

Note: Values are illustrative and will be populated with actual data in the empirical implementation.

4 Industry Beta Estimation

4.1 Industry classification: Bloomberg BCLASS3

For corporate bond analysis, we use the Bloomberg BCLASS3 industry classification rather than GICS. This classification is standard in fixed income markets and aligns with how corporate bond indices are constructed and reported. Table 2 shows the BCLASS3 industries and their approximate weights in the Bloomberg US Investment Grade Corporate Bond Index.

The BCLASS3 classification has 18 industries, providing sufficient granularity to capture meaningful differences in shock exposure while maintaining adequate sample sizes for estimation within each industry. For some analyses, we may aggregate to BCLASS2 (broader categories) or disaggregate to BCLASS4 (finer categories) as robustness checks.

Table 2: Bloomberg BCLASS3 Industry Classification

Industry	**Index Weight (%)**	**Expected** $\beta^{composite}$
Banking	22.4	High
Consumer Non-Cyclicals	15.0	Very Low
Technology	9.7	High
Electric	8.9	Very Low
Energy	7.3	High (commodity-sensitive)
Communications	7.3	Mixed (subsector-dependent)
Consumer Cyclicals	7.1	High
Capital Goods	5.5	Medium-High
Insurance	4.9	Low-Medium (subsector-dependent)
REITs	2.8	Medium (rate-sensitive, high dispersion)
Basic Industry	2.5	High
Transportation	2.0	High (airlines) / Medium (other)
Brokerage, Asset Managers, Exchanges	1.6	Mixed (see note)
Financial Companies	1.5	High
Natural Gas	0.8	Medium (fee-based, less commodity-sensitive)
Industrial Other	0.5	Medium
Utility Other	0.2	Very Low
Financial Other	0.1	Medium-High

Note: "Brokerage, Asset Managers, Exchanges" encompasses three distinct business models: Exchanges (very low beta, utility-like), Asset Managers (low-medium beta), and Broker-Dealers (high beta, bank-like). Communications similarly varies: Towers/Fiber (very low), Legacy Telecom (medium), Media (high). Insurance varies by type: Life (low), P&C (medium with jump risk), Reinsurance (high).

4.2 Why estimate betas from equity returns?

We estimate industry shock betas $\beta_k^{(m)}$ using *equity* returns rather than credit spread changes. This choice is motivated by several considerations:

Theoretical alignment: The shock beta $\beta_k^{(m)}$ is defined as the sensitivity of *firm value* to shock type m. Equity prices are a direct (though noisy) proxy for firm value changes:

$$\frac{\Delta E_j}{E_j} \approx \Omega_j \cdot \frac{\Delta V_j}{V_j} \tag{26}$$

where Ω_j is the equity-to-firm-value elasticity (equity "omega"). For firms not in distress, Ω_j is relatively stable, so equity returns are proportional to firm value returns.

Avoiding simultaneity: If we estimated $\beta_k^{(m)}$ from credit spread changes, we would be estimating $\lambda_k \cdot \beta_k^{(m)}$ rather than $\beta_k^{(m)}$ alone. This conflates the two layers we are trying to separate.

Higher signal-to-noise: Equity markets are more liquid and informationally efficient than corporate bond markets. Equity prices respond within minutes to macro announcements, while credit spreads may take hours or days to fully adjust.

Larger cross-section: Many corporate bond issuers have publicly traded equity, allowing us to estimate industry betas from a broader sample than would be available using bonds alone.

Remark 3 (Limitations of Equity-Based Estimation)**.** *Using equity returns introduces potential biases:*

1. ***Growth options:*** *Equity values include growth options that may not affect credit risk. A technology firm's equity may respond strongly to growth news that has minimal impact on its ability to service existing debt.*

2. ***Leverage differences:*** *The equity omega Ω_j varies with leverage. High-leverage firms have higher equity betas for a given firm value beta.*

3. ***Sample selection:*** *Firms with traded equity may differ systematically from private issuers in the bond market.*

We address these through robustness checks in Section 9, including direct validation that equity-estimated betas predict credit spread responses.

4.3 Estimation methodology

For each BCLASS3 industry k, we estimate shock betas using industry-level equity returns. Let $r_{k,t}$ denote the return on the market-cap-weighted equity portfolio for industry k (e.g., using sector ETFs or constructed from CRSP/Compustat).

Baseline specification:

$$r_{k,t} = \alpha_k + \sum_{m=1}^{M} \beta_k^{(m)} \cdot \eta_t^{(m)} + \epsilon_{k,t} \tag{27}$$

This regression is estimated using OLS with heteroskedasticity-robust standard errors. For announcement-based shocks (NFP, CPI, FOMC), we use daily returns on announcement days only. For continuous shocks (VIX, oil), we use the full daily sample.

Using orthogonalized shocks: To obtain cleanly interpretable betas, we estimate using the orthogonalized shock series $\{\tilde{\eta}_t^{(m)}\}$ from Section 3.3:

$$r_{k,t} = \alpha_k + \sum_{m=1}^{M} \tilde{\beta}_k^{(m)} \cdot \tilde{\eta}_t^{(m)} + \epsilon_{k,t} \tag{28}$$

The orthogonalized betas $\tilde{\beta}_k^{(m)}$ have a cleaner interpretation: $\tilde{\beta}_k^{(NFP)}$ measures the response to the component of NFP surprise that is orthogonal to all other shocks.

High-frequency identification: For the cleanest identification of announcement effects, we can use intraday windows:

$$r_{k,[t-5\text{min},t+30\text{min}]} = \alpha_k + \beta_k^{(m)} \cdot \eta_t^{(m)} + \epsilon_{k,t} \tag{29}$$

where the return window spans from 5 minutes before to 30 minutes after the announcement. This minimizes contamination from other news but requires high-frequency equity data.

4.4 Expected beta patterns by industry

Based on economic intuition, we expect the following patterns:

Note: Values are illustrative expectations based on economic intuition. Actual estimates will be obtained from data.

Interpretation of signs:

- $\beta^{NFP} > 0$: Positive employment surprise increases firm value (all industries, but cyclicals more)
- $\beta^{CPI} < 0$ (generally): Inflation surprise is negative for most firms (cost increases), except Energy/Basic Industry with pricing power

Table 3: Expected Industry Beta Patterns by Shock Type

BCLASS3 Industry	β^{NFP}	β^{CPI}	β^{FF}	β^{VIX}	β^{OIL}
Banking	$+1.2$	-0.3	-1.5	-1.8	$+0.2$
Consumer Non-Cyclicals	$+0.3$	-0.1	-0.3	-0.4	-0.1
Technology	$+1.2$	-0.4	-1.2	-1.8	-0.2
Electric	$+0.1$	-0.2	-0.4	-0.2	0.0
Energy	$+0.6$	$+0.3$	-0.6	-1.0	$+1.8$
Communications (blended)	$+0.5$	-0.2	-0.6	-0.8	-0.1
Consumer Cyclicals	$+1.4$	-0.4	-0.8	-1.5	-0.3
Capital Goods	$+1.1$	-0.2	-0.7	-1.2	$+0.1$
Insurance (blended)	$+0.4$	-0.2	-0.6	-0.8	$+0.1$
REITs	$+0.5$	-0.4	-1.2	-1.0	0.0
Basic Industry	$+1.2$	$+0.1$	-0.6	-1.4	$+0.4$
Transportation	$+1.0$	-0.3	-0.7	-1.4	-1.0
Brokerage/Asset Mgrs (blended)	$+0.7$	-0.2	-0.9	-1.2	$+0.1$
Financial Companies	$+0.8$	-0.3	-1.1	-1.5	$+0.1$
Natural Gas	$+0.3$	$+0.1$	-0.4	-0.6	$+0.8$

Note: Values are illustrative expectations based on fundamental analysis. "Blended" entries mask substantial within-sector heterogeneity—see text for subsector breakouts. Key changes from naive expectations: Electric and Consumer Non-Cyclicals have very low betas across all shocks (defensive); Technology has high VIX beta reflecting equity-volatility linkage; Transportation has large negative oil beta (fuel costs); Natural Gas/Pipelines has lower commodity sensitivity than E&P due to fee-based contracts.

- $\beta^{FF} < 0$: Fed tightening surprise reduces firm values (higher discount rates, tighter financial conditions)
- $\beta^{VIX} < 0$: Increase in VIX (risk-off) reduces firm values
- β^{OIL}: Sign varies—positive for Energy, negative for Transportation

4.5 Granularity considerations: Bias-variance tradeoff

Using finer industry classifications (BCLASS4) provides more precise shock exposure estimates but increases estimation noise due to smaller samples. Using coarser classifications (BCLASS2) provides more stable estimates but may miss important within-category heterogeneity.

Proposition 3 (Optimal Granularity)**.** *Let $\hat{\beta}_k$ be the estimated shock beta for industry k with n_k firms. The mean squared error of $\hat{\beta}_k$ as a predictor of true firm-level betas is:*

$$MSE(\hat{\beta}_k) = \underbrace{\frac{\sigma_\epsilon^2}{n_k \cdot T}}_{\textit{Estimation variance}} + \underbrace{\mathrm{Var}(\beta_j | j \in k)}_{\textit{Within-industry heterogeneity}} \tag{30}$$

Finer classifications reduce the second term but increase the first.

Our baseline uses BCLASS3, which balances these considerations. Robustness checks will examine BCLASS2 and BCLASS4.

5 Covariance-Adjusted Composite Beta

5.1 The need for covariance adjustment

For production use, we need a single "composite beta" $\beta_k^{composite}$ that summarizes industry k's overall sensitivity to the mix of shocks that drive aggregate spread movements. A naive approach would be:

$$\beta_k^{naive} = \sqrt{\sum_m \left(\beta_k^{(m)}\right)^2 \cdot \omega_m} \tag{31}$$

where ω_m is the variance share of shock type m.

This formula is **incorrect** when shocks are correlated. It implicitly assumes $\text{Cov}(\eta^{(m)}, \eta^{(n)}) = 0$ for $m \neq n$, which is empirically false. For example, strong employment reports often coincide with Fed tightening expectations, creating positive correlation between η^{NFP} and η^{FF}.

5.2 Derivation of the correct composite beta

Let $\boldsymbol{\beta}_k = (\beta_k^{(1)}, \ldots, \beta_k^{(M)})'$ be the vector of industry k's shock betas, and let Σ_η be the $M \times M$ covariance matrix of shocks.

The variance of industry k's systematic firm value change is:

$$\text{Var}\left(\sum_m \beta_k^{(m)} \cdot \eta_t^{(m)}\right) = \boldsymbol{\beta}_k' \Sigma_\eta \boldsymbol{\beta}_k \tag{32}$$

Definition 3 (Covariance-Adjusted Composite Beta)**.** *The* ***composite beta*** *for industry k is:*

$$\boxed{\beta_k^{composite} = \sqrt{\boldsymbol{\beta}_k' \Sigma_\eta \boldsymbol{\beta}_k}} \tag{33}$$

This equals the standard deviation of industry k's systematic firm value shock.

Expanding the quadratic form:

$$\beta_k^{composite} = \sqrt{\sum_m \left(\beta_k^{(m)}\right)^2 \cdot \text{Var}(\eta^{(m)}) + 2 \sum_{m<n} \beta_k^{(m)} \beta_k^{(n)} \cdot \text{Cov}(\eta^{(m)}, \eta^{(n)})} \tag{34}$$

The covariance terms can be substantial. If $\beta_k^{NFP} > 0$ and $\beta_k^{FF} < 0$, and $\text{Cov}(\eta^{NFP}, \eta^{FF}) > 0$ (Fed tightens when employment is strong), then the cross-term $\beta_k^{NFP} \cdot \beta_k^{FF} \cdot \text{Cov}(\eta^{NFP}, \eta^{FF}) < 0$, which *reduces* the composite beta. This reflects the natural hedge: strong employment (good for firm value) tends to coincide with Fed tightening (bad for firm value).

5.3 Relationship to aggregate DTS factor

The composite beta has a natural interpretation in terms of the aggregate DTS factor. Recall from equation (15) that:

$$f_{DTS,t} \approx -\sum_m \gamma^{(m)} \cdot \eta_t^{(m)} \tag{35}$$

where $\gamma^{(m)} = \sum_i w_i \cdot \lambda_i \cdot \beta_{k(i)}^{(m)}$ is the index-weighted sensitivity to shock m.

The covariance between bond ij's spread change and the DTS factor is:

$$\begin{aligned}\operatorname{Cov}\left(\frac{\Delta s_{ij}}{s_{ij}}, f_{DTS}\right) &= \operatorname{Cov}\left(-\lambda_{ij}\sum_m \beta_k^{(m)}\eta^{(m)}, -\sum_n \gamma^{(n)}\eta^{(n)}\right) \\ &= \lambda_{ij}\cdot \boldsymbol{\beta}_k' \Sigma_\eta \boldsymbol{\gamma} \end{aligned} \tag{36}$$

where $\boldsymbol{\gamma} = (\gamma^{(1)}, \ldots, \gamma^{(M)})'$.

Proposition 4 (DTS Beta Decomposition)**.** *The bond's DTS beta (regression coefficient on $f_{DTS,t}$) is:*

$$\beta_{ij}^{DTS} = \frac{\operatorname{Cov}(\Delta s_{ij}/s_{ij}, f_{DTS})}{\operatorname{Var}(f_{DTS})} = \lambda_{ij}\cdot \frac{\boldsymbol{\beta}_k'\Sigma_\eta\boldsymbol{\gamma}}{\boldsymbol{\gamma}'\Sigma_\eta\boldsymbol{\gamma}} \tag{37}$$

This is proportional to λ_{ij} times a "projection" of the industry beta vector onto the index beta vector.

For the special case where all industries have proportional beta vectors ($\boldsymbol{\beta}_k = c_k \cdot \boldsymbol{\gamma}$ for some scalar c_k), the DTS beta simplifies to:

$$\beta_{ij}^{DTS} = \lambda_{ij}\cdot c_k = \lambda_{ij}\cdot \frac{\beta_k^{composite}}{\gamma^{composite}} \tag{38}$$

where $\gamma^{composite} = \sqrt{\boldsymbol{\gamma}'\Sigma_\eta\boldsymbol{\gamma}}$ is the composite beta of the index.

5.4 Estimation of the shock covariance matrix

The shock covariance matrix Σ_η is estimated from the historical shock series:

$$\hat{\Sigma}_\eta = \frac{1}{T-1}\sum_{t=1}^{T}(\boldsymbol{\eta}_t - \bar{\boldsymbol{\eta}})(\boldsymbol{\eta}_t - \bar{\boldsymbol{\eta}})' \tag{39}$$

For mixed-frequency shocks (monthly NFP, daily VIX), we aggregate to the lowest common frequency or use appropriate scaling.

Shrinkage estimation: With many shock types (M large) and limited observations, the sample covariance matrix may be poorly conditioned. We apply Ledoit-Wolf shrinkage toward a structured target:

$$\hat{\Sigma}_\eta^{shrunk} = \delta\cdot\hat{\Sigma}_\eta + (1-\delta)\cdot \operatorname{diag}(\hat{\Sigma}_\eta) \tag{40}$$

where $\delta \in [0, 1]$ is chosen to minimize expected loss.

5.5 Illustrative composite beta calculations

Table 4 presents illustrative composite betas based on the expected patterns from Table 3.

Note: Values are illustrative. The "Ratio" column shows $\beta^{composite}/\beta^{naive}$, which is less than 1 due to negative covariance terms (natural hedges between shock types).

Key observations:

- Brokerage/Asset Managers and Banking have the highest composite betas, reflecting high sensitivity to both real economy and risk appetite shocks.
- Consumer Non-Cyclicals and Electric have the lowest composite betas, reflecting defensive characteristics.

Table 4: Illustrative Composite Betas by Industry

BCLASS3 Industry	β^{naive}	$\beta^{composite}$	**Ratio**	**Rank**
Banking	2.48	2.31	0.93	2
Consumer Non-Cyclicals	0.55	0.51	0.93	17
Technology	2.35	2.18	0.93	3
Electric	0.50	0.47	0.94	18
Energy	2.21	2.05	0.93	4
Communications (blended)	1.15	1.06	0.92	12
Consumer Cyclicals	2.18	2.02	0.93	5
Capital Goods	1.72	1.59	0.92	7
Insurance (blended)	1.10	1.02	0.93	13
REITs	1.62	1.52	0.94	9
Basic Industry	1.91	1.78	0.93	6
Transportation	1.95	1.75	0.90	6
Brokerage/Asset Mgrs (blended)	1.55	1.44	0.93	10
Financial Companies	2.02	1.87	0.93	5
Natural Gas	1.05	0.97	0.92	14

Note: Values are illustrative. Key observations: (1) Electric and Consumer Non-Cyclicals have the lowest composite betas, consistent with their defensive characteristics and low macro sensitivity. (2) Technology now ranks among the highest, reflecting strong equity-volatility linkage. (3) Transportation's large adjustment (10%) reflects offsetting growth (positive) and oil (strongly negative) exposures. (4) "Blended" sectors mask substantial heterogeneity—e.g., within Brokerage/Asset Mgrs, Exchanges would rank near Electric while Broker-Dealers would rank near Banking.

- The covariance adjustment reduces composite betas by 6-10% on average, reflecting natural hedges.
- Transportation has the largest adjustment (10%) due to offsetting effects of growth (positive) and oil prices (negative).

6 Principal Component Analysis Framework

6.1 Motivation: Model-free factor identification

Principal Component Analysis (PCA) provides a model-free approach to identifying the factor structure of credit spread movements. This serves multiple purposes:

1. **Factor counting:** How many systematic factors are needed to explain spread co-movement? If one factor explains 90% of variance, the single-factor DTS model (with adjustments) is sufficient. If multiple factors each explain substantial variance, a multi-factor model may be needed.
2. **Theory validation:** The two-layer model predicts specific patterns in factor loadings. Testing whether empirical loadings match predictions validates the model.
3. **Residual structure:** Examining residuals after removing predicted factors reveals whether important systematic variation remains unexplained.

6.2 Construction of the spread change panel

We construct a panel of proportional spread changes at the bucket level:

Bucket definition: Each bucket b is defined by the intersection of:

- Industry (BCLASS3): 18 categories
- Rating (letter rating): 7 categories (AAA, AA, A, BBB, BB, B, CCC)
- Maturity: 4 categories (1-3y, 3-5y, 5-7y, 7-10y)

This yields up to $18 \times 7 \times 4 = 504$ potential buckets. In practice, many buckets are empty or have insufficient bonds; we require at least 10 bonds per bucket, reducing the effective number to approximately 150-200 buckets for IG and 100-150 for HY.

Bucket-level spread changes: For each bucket b on date t:

$$Y_{b,t} = \frac{\Delta \overline{OAS}_{b,t}}{\overline{OAS}_{b,t-1}} \tag{41}$$

where $\overline{OAS}_{b,t}$ is the market-value-weighted average OAS of bonds in bucket b.

Data matrix: The analysis uses the $B \times T$ matrix $\mathbf{Y}$ where B is the number of buckets and T is the number of time periods.

6.3 PCA implementation

Standardization: We demean each bucket's time series but do *not* standardize to unit variance. This preserves the information that some buckets have higher spread volatility than others (which is part of what we want to explain).

Eigendecomposition: Compute the $B \times B$ covariance matrix:

$$\hat{\Sigma}_Y = \frac{1}{T-1}\mathbf{Y}\mathbf{Y}' \tag{42}$$

and its eigendecomposition:

$$\hat{\Sigma}_Y = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}' \tag{43}$$

where $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_B]$ are eigenvectors (factor loadings) and $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_B)$ are eigenvalues in descending order.

Principal components: The k-th principal component is:

$$PC_{k,t} = \mathbf{v}_k'\mathbf{Y}_t = \sum_b v_{b,k} \cdot Y_{b,t} \tag{44}$$

Variance explained: The fraction of variance explained by the first K components is:

$$R_K^2 = \frac{\sum_{k=1}^{K} \lambda_k}{\sum_{k=1}^{B} \lambda_k} \tag{45}$$

6.4 Theoretical predictions for factor structure

The two-layer model generates specific predictions about the factor structure:

Proposition 5 (PC1 Loading Prediction)**.** *Under the two-layer model, if there is a single dominant systematic shock (or if all systematic shocks are perfectly correlated), the first principal component loadings satisfy:*

$$v_{b,1} \propto \lambda_b^{Merton} \cdot \beta_{k(b)}^{composite} \tag{46}$$

where λ_b^{Merton} is the average Merton elasticity for bucket b (based on its spread and maturity) and $\beta_{k(b)}^{composite}$ is the composite beta for bucket b's industry.

Proof: Under a single-factor structure, $Y_{b,t} = -\lambda_b \cdot \beta_{k(b)} \cdot \eta_t + \epsilon_{b,t}$. The covariance between buckets b and b' is:

$$\text{Cov}(Y_b, Y_{b'}) = \lambda_b \lambda_{b'} \beta_{k(b)} \beta_{k(b')} \text{Var}(\eta) \tag{47}$$

This is a rank-1 matrix with eigenvector proportional to $\lambda_b \cdot \beta_{k(b)}$. □

Proposition 6 (Multi-Factor Structure)**.** *If there are M imperfectly correlated shock types, the covariance matrix has rank at most M, and the factor loadings satisfy:*

$$v_{b,k} = \sum_m \lambda_b^{Merton} \cdot \beta_{k(b)}^{(m)} \cdot \phi_{m,k} \tag{48}$$

where $\phi_{m,k}$ are the loadings of shock type m on principal component k.

6.5 Testing theoretical predictions

Test 1: Do PC1 loadings match theory?

Regress empirical PC1 loadings on predicted loadings:

$$\hat{v}_{b,1} = \alpha + \gamma \cdot \left(\lambda_b^{Merton} \cdot \beta_{k(b)}^{composite}\right) + u_b \tag{49}$$

Predictions:

- If two-layer model is correct: $\gamma \approx 1$ (or at least $\gamma > 0$ with high R^2)
- If only Merton matters (no industry heterogeneity): Regression of $\hat{v}_{b,1}$ on λ_b^{Merton} alone should have high R^2
- If only industry matters (no Merton): Regression on $\beta_{k(b)}^{composite}$ alone should have high R^2

Model comparison:

$$\text{Model 1 (Merton only):} \quad \hat{v}_{b,1} = \alpha + \gamma_\lambda \cdot \lambda_b^{Merton} + u_b \tag{50}$$

$$\text{Model 2 (Industry only):} \quad \hat{v}_{b,1} = \alpha + \gamma_\beta \cdot \beta_{k(b)}^{composite} + u_b \tag{51}$$

$$\text{Model 3 (Additive):} \quad \hat{v}_{b,1} = \alpha + \gamma_\lambda \cdot \lambda_b^{Merton} + \gamma_\beta \cdot \beta_{k(b)}^{composite} + u_b \tag{52}$$

$$\text{Model 4 (Multiplicative):} \quad \hat{v}_{b,1} = \alpha + \gamma \cdot \lambda_b^{Merton} \cdot \beta_{k(b)}^{composite} + u_b \tag{53}$$

The two-layer theory predicts Model 4 should dominate: higher R^2 than Models 1-3, and the multiplicative specification should fit better than the additive specification (Model 3).

Test 2: How many factors are needed?

- If PC1 explains > 85% of variance: Single-factor DTS with two-layer adjustment is sufficient
- If PC1 explains 70-85%: Consider whether PC2 has interpretable structure
- If PC1 explains < 70%: Multi-factor model likely needed

Test 3: Is PC2 interpretable?
If a second factor is important, we examine whether PC2 loadings correspond to:

- Cyclical vs. defensive industries (growth factor)
- Rate-sensitive vs. rate-insensitive industries (duration factor)
- High-spread vs. low-spread buckets (credit quality factor)

6.6 Expected results

Based on prior literature and economic intuition, we expect:

Table 5: Expected PCA Results

Metric	Expected Value	Interpretation
PC1 variance share	65-80%	Strong single-factor structure
PC2 variance share	8-15%	Secondary factor (cyclicality?)
PC3+ variance share	<5% each	Noise/idiosyncratic
R^2 of Model 4	>70%	Two-layer model validated
R^2 improvement (Model 4 vs 1)	15-25 pp	Industry betas add value
R^2 improvement (Model 4 vs 3)	5-10 pp	Multiplicative beats additive

7 Reinterpretation of Part I Findings

7.1 Unified explanation of empirical patterns

Part I documented several empirical patterns that were interpreted primarily through the lens of Merton elasticity. The two-layer framework provides a unified reinterpretation:

7.2 Quantifying the sector effects

Part I found that Financial sector bonds had approximately 33% higher DTS sensitivity than Industrial sector bonds, controlling for spread and maturity. Under the two-layer model:

$$\frac{\text{Financial sensitivity}}{\text{Industrial sensitivity}} = \frac{\lambda_{Fin} \cdot \beta_{Fin}^{composite}}{\lambda_{Ind} \cdot \beta_{Ind}^{composite}} \tag{54}$$

If we controlled for spread and maturity (so $\lambda_{Fin} \approx \lambda_{Ind}$), the ratio should equal:

$$\frac{\beta_{Fin}^{composite}}{\beta_{Ind}^{composite}} \approx 1.33 \tag{55}$$

Table 6: Reinterpretation of Part I Findings

Part I Finding	**Original Interpretation**	**Two-Layer Reinterpretation**
Within-issuer tests confirm Merton	Merton elasticity varies with s and T as predicted	**Confirmed and strengthened**: Within-issuer design controls for shock exposure, providing clean identification of Merton elasticity
Financials 30-40% more sensitive	Possible industry-specific elasticity differences?	**Shock exposure**: $\beta_{Banking}^{composite} \approx 1.3 \times \beta_{avg}$, not an elasticity difference
Bucket-level tests noisier than within-issuer	Measurement error, smaller samples	**Omitted variable**: Cross-industry variation in $\beta_k^{composite}$ not controlled
Residual sector effects after Merton adjustment	Unexplained—possible model misspecification	**Explained**: These *are* the industry shock betas

Using our illustrative composite betas from Table 4:

$$\frac{\beta_{Banking}^{composite}}{\beta_{avg}^{composite}} = \frac{2.31}{1.50} \approx 1.54 \tag{56}$$

This is in the right ballpark, though somewhat higher than the Part I estimate. The discrepancy could reflect:

- Imperfect spread/maturity controls in Part I
- Our illustrative betas being too extreme
- Part I's "Industrial" category being a mix of BCLASS3 industries

7.3 Within-industry validation of Merton

A key prediction of the two-layer model is that Merton elasticity should work cleanly *within* industries, because all bonds in the same industry share the same $\beta_k^{(m)}$.

Test specification: For each BCLASS3 industry k, estimate:

$$\frac{\Delta s_{ij,t}}{s_{ij,t-1}} = \alpha_k + \gamma_k \cdot \lambda_{ij}^{Merton} \cdot f_{DTS,t}^{(k)} + \epsilon_{ij,t}, \quad \text{for } j \in \text{industry } k \tag{57}$$

where $f_{DTS,t}^{(k)}$ is the industry-specific DTS factor (average proportional spread change within industry k).

Prediction: $\gamma_k \approx 1$ for all industries k.

If we find that γ_k varies substantially across industries (e.g., $\gamma_{Banking} = 0.8$ while $\gamma_{Electric} = 1.2$), this would suggest either:

- The Merton model's elasticity predictions are industry-specific (problematic for the theory)
- Within-industry heterogeneity in shock exposure is important (need to go to BCLASS4 or firm-level)

7.4 Explaining the noise in bucket-level tests

Part I found that bucket-level DTS regressions (grouping bonds by spread × maturity) produced noisier coefficient estimates than within-issuer tests. The two-layer model explains this:

Within each spread × maturity bucket, there is variation in industry composition. A "5y, 100bps" bucket might contain:

- 30% Banking bonds ($\beta^{composite} \approx 2.3$)
- 25% Technology bonds ($\beta^{composite} \approx 1.7$)
- 20% Consumer Non-Cyclicals bonds ($\beta^{composite} \approx 0.8$)
- 25% other industries

The bucket's average sensitivity is:

$$\bar{\beta}^{composite}_{bucket} = \sum_k w_k \cdot \beta^{composite}_k \tag{58}$$

where w_k is the weight of industry k in the bucket.

This creates *within-bucket heterogeneity* that appears as residual variance in the regression. The noise is not measurement error—it is economically meaningful variation that the Merton-only model cannot capture.

7.5 Implications for Part I's empirical strategy

The two-layer framework suggests refinements to Part I's empirical strategy:

1. **Industry controls are essential:** Any cross-sectional test of Merton elasticity should include industry fixed effects or explicit industry beta controls.
2. **Within-issuer remains the gold standard:** For clean identification of Merton elasticity, within-issuer variation is ideal because it perfectly controls for shock exposure.
3. **Sector "effects" are features, not bugs:** The sector effects documented in Part I are not evidence against Merton—they are evidence of shock exposure heterogeneity that complements Merton.
4. **Time-varying sector composition matters:** If the industry composition of spread/maturity buckets changes over time (e.g., more Tech issuance in recent years), this will affect estimated DTS relationships even if underlying elasticities are stable.

8 Extended Empirical Research Program

This section details the empirical research program for validating and implementing the two-layer DTS framework. The program is organized into stages that build sequentially, with each stage producing deliverables that feed into subsequent analyses.

8.1 Stage 0E: Shock exposure estimation

Stage 0E establishes the empirical foundation by estimating industry shock betas and validating that they explain the sector effects documented in Part I.

8.1.1 Stage 0E.1: Construct shock surprise series

Objective: Build a clean dataset of macroeconomic shock surprises.

Data sources:

- Bloomberg economic calendar and consensus forecasts (NFP, CPI, GDP, ISM)
- Federal Reserve Bank of New York shadow rate estimates
- Chicago Fed National Financial Conditions Index
- CBOE VIX index
- WTI crude oil futures (front month)

Construction:

1. For announcement-based shocks: $\eta_t^{(m)} = \text{Actual}_t - \text{Consensus}_t$
2. Standardize each series to zero mean, unit variance
3. Apply Cholesky orthogonalization with ordering: Real $\rightarrow$ Inflation $\rightarrow$ Monetary $\rightarrow$ Risk appetite $\rightarrow$ Sector-specific
4. Compute shock covariance matrix $\hat{\Sigma}_\eta$ (with Ledoit-Wolf shrinkage)

Deliverables:

- Time series of raw and orthogonalized shocks (2000–2024)
- Correlation matrix and covariance matrix of shocks
- Summary statistics table (means, std devs, autocorrelations, cross-correlations)

Computation time: 2–4 hours

8.1.2 Stage 0E.2: Estimate industry betas from equity returns

Objective: Estimate $\beta_k^{(m)}$ for each BCLASS3 industry k and shock type m.

Data:

- Industry equity returns: Constructed from CRSP/Compustat using Bloomberg BCLASS3 mapping, or use sector ETFs as proxies (e.g., XLF for Financials, XLU for Utilities)

- Daily frequency, matched to shock dates

Estimation:

$$r_{k,t} = \alpha_k + \sum_{m=1}^{M} \beta_k^{(m)} \cdot \eta_t^{(m)} + \epsilon_{k,t} \tag{59}$$

Estimate separately using:

1. Raw shocks (for comparison)
2. Orthogonalized shocks (baseline)
3. High-frequency windows around announcements (robustness)

Deliverables:

- Table of industry shock betas (18 industries × 5–8 shock types)
- Standard errors and t-statistics
- Heatmap visualization of beta patterns
- Composite betas $\beta_k^{composite}$ using equation (33)

Computation time: 4–6 hours

8.1.3 Stage 0E.3: Validate that industry betas explain Part I sector effects

Objective: Test whether composite betas account for the 30–40% Financial vs. Industrial sensitivity difference found in Part I.

Approach:

1. Replicate Part I's cross-sector regression with explicit industry controls
2. Compare three specifications:

$$\text{Model 1 (Part I):} \quad y_{ij,t} = \alpha + \gamma_1 \cdot \lambda_{ij}^{Merton} \cdot f_{DTS,t} + \sum_k \delta_k \cdot \mathbf{1}_{k(j)=k} + \epsilon_{ij,t} \tag{60}$$

$$\text{Model 2 (Two-layer):} \quad y_{ij,t} = \alpha + \gamma_2 \cdot \lambda_{ij}^{Merton} \cdot \beta_{k(j)}^{composite} \cdot f_{DTS,t} + \epsilon_{ij,t} \tag{61}$$

$$\text{Model 3 (Nested):} \quad y_{ij,t} = \alpha + \gamma_3 \cdot \lambda_{ij}^{Merton} \cdot \beta_{k(j)}^{composite} \cdot f_{DTS,t} + \sum_k \delta_k \cdot \mathbf{1}_{k(j)=k} + \epsilon_{ij,t} \tag{62}$$

Predictions:

- Model 2 should fit nearly as well as Model 1 (composite betas capture sector effects)
- In Model 3, the sector dummies δ_k should be jointly insignificant
- $\gamma_2 \approx 1$ in Model 2

Deliverables:

- Regression output for all three models
- F-test for joint significance of sector dummies in Model 3
- Scatter plot: Part I sector effects vs. composite betas

Computation time: 4–6 hours

8.1.4 Stage 0E.4: Within-industry Merton validation

Objective: Confirm that Merton elasticity works cleanly within each industry.

Specification: For each BCLASS3 industry k:

$$y_{ij,t} = \alpha_k + \gamma_k \cdot \lambda_{ij}^{Merton} \cdot f_{DTS,t}^{(k)} + \epsilon_{ij,t}, \quad j \in \text{industry } k \tag{63}$$

where $f_{DTS,t}^{(k)}$ is the within-industry DTS factor.

Prediction: $\gamma_k \approx 1$ for all industries.

Deliverables:

- Table of $\hat{\gamma}_k$ estimates with standard errors for each industry
- Test of $H_0 : \gamma_k = 1$ for each industry
- Test of $H_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_{18}$ (homogeneity)

Computation time: 4–6 hours

8.1.5 Stage 0E.5: Principal component analysis

Objective: Validate the two-layer model using model-free factor analysis.

Implementation:

1. Construct bucket-level spread change panel (Industry $\times$ Rating $\times$ Maturity)
2. Compute covariance matrix and eigendecomposition
3. Extract PC1 and PC2 loadings
4. Test equation (49): regress PC1 loadings on $\lambda^{Merton} \times \beta^{composite}$

Deliverables:

- Scree plot showing variance explained by each PC
- Table of variance shares for PC1–PC5
- Regression results for Models 1–4 from Section 6.5
- Scatter plot: Actual PC1 loadings vs. predicted ($\lambda \times \beta$)
- Interpretation of PC2 (if substantial variance share)

Computation time: 3–5 hours

8.2 Stage AE: Variance decomposition

Objective: Quantify how much of DTS beta variation is explained by Merton elasticity vs. industry exposure.

Approach: Decompose the cross-sectional variance of estimated DTS betas:

$$\text{Var}(\hat{\beta}_b^{DTS}) = \underbrace{\text{Var}(\lambda_b)}_{\text{Merton}} \cdot \bar{\beta}^2 + \underbrace{\text{Var}(\beta_{k(b)})}_{\text{Industry}} \cdot \bar{\lambda}^2 + 2\text{Cov}(\lambda_b, \beta_{k(b)}) \cdot \bar{\lambda}\bar{\beta} + \text{Residual} \tag{64}$$

Implementation:

1. Estimate bucket-level DTS betas from time-series regressions
2. Regress on Merton elasticity, composite beta, and their interaction:

$$\hat{\beta}_b^{DTS} = \alpha + \gamma_\lambda \cdot \lambda_b + \gamma_\beta \cdot \beta_{k(b)} + \gamma_{int} \cdot \lambda_b \cdot \beta_{k(b)} + u_b \tag{65}$$

3. Compute partial R^2 contributions

Deliverables:

- Variance decomposition table (percentage attributed to each component)
- Regression output with partial R^2 values
- Visualization of DTS beta surface over (λ, β) space

Computation time: 2–4 hours

8.3 Stage BE: Core two-layer theory tests

Objective: Formally test the two-layer model's quantitative predictions.

8.3.1 Stage BE.1: Aggregate test

Core specification:

$$y_{ij,t} = \alpha + \gamma \cdot \lambda_{ij}^{Merton} \cdot \beta_{k(j)}^{composite} \cdot f_{DTS,t} + \epsilon_{ij,t} \tag{66}$$

Prediction: $\gamma = 1$.

Estimation: Panel regression with bond and time fixed effects, standard errors clustered by issuer and date.

8.3.2 Stage BE.2: Shock-specific tests

For each shock type m:

$$y_{ij,t} = \alpha + \gamma^{(m)} \cdot \lambda_{ij}^{Merton} \cdot \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} + \epsilon_{ij,t} \tag{67}$$

Prediction: $\gamma^{(m)} = 1$ for all shock types.

Key question: Does the two-layer model work equally well for all shock types, or are some shocks transmitted differently?

8.3.3 Stage BE.3: Multiplicative vs. additive structure

Compare:

$$\text{Multiplicative:} \quad y_{ij,t} = \alpha + \gamma \cdot (\lambda_{ij} \times \beta_{k(j)}) \cdot f_t + \epsilon \tag{68}$$

$$\text{Additive:} \quad y_{ij,t} = \alpha + \gamma_\lambda \cdot \lambda_{ij} \cdot f_t + \gamma_\beta \cdot \beta_{k(j)} \cdot f_t + \epsilon \tag{69}$$

Prediction: Multiplicative should dominate (higher R^2, better out-of-sample fit).

Deliverables:

- Table of $\hat{\gamma}$ and $\hat{\gamma}^{(m)}$ estimates with confidence intervals

- Tests of $H_0 : \gamma = 1$ and $H_0 : \gamma^{(m)} = 1$
- Model comparison statistics (AIC, BIC, out-of-sample R^2)
- Tests of multiplicative restriction

Computation time: 6–10 hours

8.4 Stage CE: Stability analysis

Objective: Test whether the two-layer parameters are stable over time.

8.4.1 Stage CE.1: Rolling estimation of industry betas

Estimate industry betas using 5-year rolling windows:

$$\hat{\beta}_{k,\tau}^{(m)} = \text{estimate from } [t-60, t] \text{ months} \tag{70}$$

Questions:

- Do industry betas change significantly over time?
- Are changes driven by changing industry composition or true beta shifts?
- Do betas increase during crises (time-varying risk exposure)?

8.4.2 Stage CE.2: Structural break tests

Test for breaks in the two-layer relationship around:

- 2008 Financial Crisis
- 2011 European Debt Crisis
- 2015 Oil Crash
- 2020 COVID Crisis
- 2022 Rate Hiking Cycle

8.4.3 Stage CE.3: Time-varying shock composition

Part I found some time-variation in the Merton coefficient. Test whether this is explained by changing shock composition:

$$\gamma_t = \gamma_0 + \phi \cdot \text{ShockMix}_t + \xi_t \tag{71}$$

where ShockMix_t measures the relative importance of different shock types in period t.

Deliverables:

- Time series of rolling industry beta estimates
- Structural break test results
- Analysis of whether Part I's time-variation is explained by shock composition

Computation time: 4–8 hours

8.5 Stage DE: Enhanced robustness

8.5.1 Stage DE.1: Shock-specific tail behavior

Test whether the two-layer model works in the tails using quantile regression:

$$Q_\tau(y_{ij,t}) = \alpha_\tau + \gamma_\tau \cdot \lambda_{ij} \cdot \beta_{k(j)} \cdot f_t \tag{72}$$

for $\tau \in \{0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95\}$.

Questions:

- Is γ_τ constant across quantiles (symmetric response)?
- Do tails show higher or lower elasticity than the median?
- Do different shock types show different tail behavior?

8.5.2 Stage DE.2: Industry-specific Merton transmission

Test whether Merton elasticity varies by industry:

$$y_{ij,t} = \alpha + \sum_k \gamma_k \cdot \lambda_{ij} \cdot \beta_k \cdot f_t \cdot \mathbf{1}_{k(j)=k} + \epsilon_{ij,t} \tag{73}$$

Prediction: $\gamma_k = 1$ for all k (Merton is industry-invariant).

Alternative: If $\gamma_k \neq \gamma_{k'}$, this suggests industry-specific transmission mechanisms beyond the multiplicative model.

8.5.3 Stage DE.3: Firm-level heterogeneity

Test whether firm-specific betas add explanatory power beyond industry:

$$y_{ij,t} = \alpha + \gamma_1 \cdot \lambda_{ij} \cdot \beta_{k(j)} \cdot f_t + \gamma_2 \cdot \lambda_{ij} \cdot \tilde{\beta}_j \cdot f_t + \epsilon_{ij,t} \tag{74}$$

where $\tilde{\beta}_j = \hat{\beta}_j^{equity} - \beta_{k(j)}$ is the firm's deviation from industry average.

Deliverables:

- Quantile regression coefficients across the distribution
- Tests of industry-specific Merton coefficients
- Incremental R^2 from firm-level betas
- Recommendations for when firm-level adjustment is warranted

Computation time: 6–10 hours

9 Production Implementation

9.1 Final production formula

The enhanced DTS measure incorporating both Merton elasticity and industry shock exposure is:

$$\boxed{DTS^{**}_{ij} = \lambda^{Merton}_{ij}(s,T) \times \beta^{composite}_{k(j)} \times DTS_{ij}} \tag{75}$$

where:

- $DTS_{ij} = OAS_{ij} \times OASD_{ij}$ is the standard Duration-Times-Spread
- $\lambda_{ij}^{Merton}(s, T)$ is the Merton elasticity adjustment from Part I
- $\beta_{k(j)}^{composite}$ is the industry composite beta from Part II

9.2 Merton elasticity lookup table

Table 7 provides the Merton elasticity adjustment $\lambda^{Merton}(s, T)$ as a function of spread level and maturity. Values are normalized so that $\lambda = 1.00$ at the reference point (5-year maturity, 100 bps spread).

Table 7: Merton Elasticity Lookup Table $\lambda^{Merton}(s, T)$

Spread (bps)	**1y**	**2y**	**3y**	**5y**	**7y**	**10y**	**15y**
25	4.12	2.21	1.62	1.15	0.91	0.71	0.54
50	3.62	1.98	1.47	1.07	0.86	0.68	0.53
75	3.32	1.84	1.38	1.02	0.82	0.66	0.52
100	3.08	1.73	1.31	1.00	0.80	0.64	0.51
150	2.72	1.56	1.20	0.94	0.77	0.62	0.50
200	2.45	1.44	1.13	0.90	0.75	0.61	0.50
300	2.08	1.27	1.02	0.84	0.71	0.59	0.49
400	1.82	1.15	0.94	0.79	0.68	0.57	0.48
500	1.62	1.06	0.88	0.75	0.65	0.55	0.47

Note: For spreads and maturities between grid points, use bilinear interpolation.

9.3 Industry composite beta lookup table

Table 8 provides the composite beta $\beta_k^{composite}$ for each BCLASS3 industry. Values are normalized so that the market-value-weighted average equals 1.00.

Note: Values are illustrative and will be updated based on empirical estimation. The "Category" column provides qualitative guidance for unmapped industries.

9.4 Implementation pseudo-code

```
function compute_enhanced_DTS(bond):
    """
    Compute the two-layer enhanced DTS for a single bond.

    Parameters:
        bond: object with attributes OAS, OASD, maturity, industry

    Returns:
        DTS_enhanced: float
    """
    # Standard DTS
    DTS_standard = bond.OAS * bond.OASD
```

Table 8: Industry Composite Beta Lookup Table

BCLASS3 Industry	**Index Wt (%)**	$\beta^{composite}$	**Category**
Banking	22.4	1.38	High
Consumer Non-Cyclicals	15.0	0.55	Very Low
Technology	9.7	1.42	High
Electric	8.9	0.50	Very Low
Energy	7.3	1.35	High
Communications	7.3	0.95	Mixed
Consumer Cyclicals	7.1	1.40	High
Capital Goods	5.5	1.15	Medium-High
Insurance	4.9	0.90	Low-Medium
REITs	2.8	1.05	Medium
Basic Industry	2.5	1.25	High
Transportation	2.0	1.30	High
Brokerage/Asset Mgrs	1.6	1.10	Mixed
Financial Companies	1.5	1.32	High
Natural Gas	0.8	0.85	Medium
Industrial Other	0.5	1.00	Medium
Utility Other	0.2	0.48	Very Low
Financial Other	0.1	1.15	Medium-High
Index Average	100.0	**1.00**	—

Note: Values are illustrative and will be updated based on empirical estimation. Key features consistent with fundamental analysis: (1) Electric and Consumer Non-Cyclicals are defensive with very low betas; (2) Technology joins Banking and Consumer Cyclicals in the high-beta group due to equity-volatility sensitivity; (3) Natural Gas/Pipelines has lower beta than Energy due to fee-based business model; (4) Insurance is lower than Financials due to less funding risk; (5) The "Category" column provides guidance for unmapped industries. For heterogeneous sectors (Communications, Insurance, Brokerage/Asset Mgrs), consider subsector-level betas where data permits.

```
    # Layer 1: Merton elasticity (from Part I lookup table)
    lambda_merton = interpolate_merton_lambda(
        spread=bond.OAS,
        maturity=bond.maturity,
        table=MERTON_LOOKUP_TABLE
    )

    # Layer 2: Industry composite beta (from Part II lookup table)
    beta_composite = INDUSTRY_BETA_TABLE[bond.industry]

    # Combined adjustment
    DTS_enhanced = lambda_merton * beta_composite * DTS_standard

    return DTS_enhanced


function compute_portfolio_DTS(portfolio):
```

```
    """
    Compute aggregate enhanced DTS for a portfolio.
    """
    total_DTS = 0
    total_weight = 0

    for bond in portfolio:
        weight = bond.market_value / portfolio.total_market_value
        DTS_enhanced = compute_enhanced_DTS(bond)
        total_DTS += weight * DTS_enhanced
        total_weight += weight

    return total_DTS


function interpolate_merton_lambda(spread, maturity, table):
    """
    Bilinear interpolation of Merton lambda from lookup table.
    """
    # Find bracketing spread levels
    spread_low = max(s for s in table.spreads if s <= spread)
    spread_high = min(s for s in table.spreads if s >= spread)

    # Find bracketing maturities
    mat_low = max(m for m in table.maturities if m <= maturity)
    mat_high = min(m for m in table.maturities if m >= maturity)

    # Bilinear interpolation
    if spread_low == spread_high and mat_low == mat_high:
        return table[spread_low, mat_low]

    # ... (standard bilinear interpolation formula)
    return interpolated_lambda
```

9.5 Recalibration protocol

The lookup tables should be recalibrated periodically to account for:

- Changes in industry shock sensitivities over time
- Structural shifts in the macroeconomic environment
- Changes in index composition

Recommended recalibration frequency:

- **Merton elasticity table:** Annual review; update only if significant changes in average volatility or leverage
- **Industry beta table:** Quarterly update using trailing 5-year estimation window

- **Shock covariance matrix:** Quarterly update; consider regime-switching specification for crisis periods

  Recalibration triggers:

- Major index reconstitution affecting industry weights by >2%
- Sustained change in cross-industry correlation structure
- Model residuals showing systematic patterns by industry
- Significant structural break in shock covariance matrix

10 Practical Applications

10.1 Enhanced risk attribution

The two-layer model enables precise attribution of spread changes to specific economic drivers.

Single-bond attribution:

$$\Delta s_{ij,t} = s_{ij,t-1} \times \lambda_{ij}^{Merton} \times \Bigg[\underbrace{\beta_{k(j)}^{NFP} \cdot \eta_t^{NFP}}_{\text{Employment}} + \underbrace{\beta_{k(j)}^{CPI} \cdot \eta_t^{CPI}}_{\text{Inflation}} + \underbrace{\beta_{k(j)}^{FF} \cdot \eta_t^{FF}}_{\text{Fed policy}} + \underbrace{\beta_{k(j)}^{VIX} \cdot \eta_t^{VIX}}_{\text{Risk appetite}} + \underbrace{\beta_{k(j)}^{OIL} \cdot \eta_t^{OIL}}_{\text{Oil}} + \underbrace{\nu_{j,t}}_{\text{Idiosyncratic}} \Bigg] \tag{76}$$

Example: A 5-year BBB-rated Consumer Cyclicals bond with 150 bps spread moves by +25 bps in a month. The attribution might be:

Table 9: Example Attribution for Consumer Cyclicals Bond

Driver	$\boldsymbol{\beta_k^{(m)}}$	$\boldsymbol{\eta_t^{(m)}}$	**Contribution (bps)**
Employment (NFP miss)	+1.35	-0.8σ	+8.2
Fed policy (hawkish)	−0.80	$+0.5\sigma$	+3.0
Risk appetite (VIX up)	−1.50	$+1.2\sigma$	+13.5
Oil (down)	−0.30	-0.3σ	−0.7
Idiosyncratic	—	—	+1.0
Total			**+25.0**

Note: Contributions computed as $s_{t-1} \times \lambda^{Merton} \times \beta_k^{(m)} \times \eta_t^{(m)}$.

Portfolio-level attribution:

$$\Delta s_{port,t} = \sum_{ij \in port} w_{ij} \times s_{ij,t-1} \times \lambda_{ij} \times \left[\sum_m \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} + \nu_{j,t} \right] \tag{77}$$

This can be reorganized by shock type:

$$\Delta s_{port,t} = \sum_m \underbrace{\left(\sum_{ij} w_{ij} \times s_{ij,t-1} \times \lambda_{ij} \times \beta_{k(j)}^{(m)} \right)}_{\text{Portfolio sensitivity to shock } m} \times \eta_t^{(m)} + \text{Idiosyncratic} \tag{78}$$

10.2 Cross-industry hedging

The two-layer model provides correct hedge ratios for cross-industry trades.

Problem: Hedge a long position in Bond A (industry k_A) using a short position in Bond B (industry k_B).

Naive DTS hedge ratio:

$$h^{naive} = \frac{DTS_A}{DTS_B} \tag{79}$$

Two-layer hedge ratio:

$$h^{full} = \frac{\lambda_A^{Merton} \cdot \beta_{k_A}^{composite} \cdot DTS_A}{\lambda_B^{Merton} \cdot \beta_{k_B}^{composite} \cdot DTS_B} = \frac{DTS_A^{**}}{DTS_B^{**}} \tag{80}$$

Example: Long \$10mm of a 3-year Banking bond (OAS = 80 bps, OASD = 2.8 years). Hedge with a 7-year Electric Utility bond (OAS = 90 bps, OASD = 6.2 years).

Table 10: Cross-Industry Hedge Ratio Example

	Banking (Long)	**Electric (Short)**
OAS (bps)	80	90
OASD (years)	2.8	6.2
DTS (bp-years)	224	558
λ^{Merton}	1.38	0.75
$\beta^{composite}$	1.42	0.62
DTS^{**}	439	259

$$h^{naive} = \frac{224}{558} = 0.40 \quad \Rightarrow \quad \text{Short \$4.0mm} \tag{81}$$

$$h^{full} = \frac{439}{259} = 1.69 \quad \Rightarrow \quad \text{Short \$16.9mm} \tag{82}$$

The two-layer hedge ratio is **4.2 times larger** than the naive ratio. This reflects:

- The Banking bond's short maturity increases its Merton elasticity ($\lambda = 1.38$ vs. 0.75)
- The Banking bond's high industry beta increases its shock exposure ($\beta = 1.42$ vs. 0.62)

Using the naive hedge ratio would leave substantial unhedged exposure to systematic shocks.

10.3 Shock-specific hedging

For targeted hedging of specific risk factors, use shock-specific betas:

Example: Hedge employment risk only.

$$h^{NFP} = \frac{\lambda_A \cdot \beta_{k_A}^{NFP} \cdot DTS_A}{\lambda_B \cdot \beta_{k_B}^{NFP} \cdot DTS_B} \tag{83}$$

This allows construction of portfolios that are hedged against specific macro risks while retaining exposure to others.

10.4 Relative value analysis

The two-layer model provides a framework for identifying mispriced bonds.

Fair value spread change: Given observed shocks, the model-implied spread change for bond ij is:

$$\left(\frac{\Delta s_{ij}}{s_{ij}}\right)^{model} = -\lambda_{ij} \cdot \sum_{m} \beta_{k(j)}^{(m)} \cdot \eta_t^{(m)} \tag{84}$$

Residual (alpha signal):

$$\alpha_{ij,t} = \left(\frac{\Delta s_{ij}}{s_{ij}}\right)^{actual} - \left(\frac{\Delta s_{ij}}{s_{ij}}\right)^{model} \tag{85}$$

Bonds with large positive residuals (spreads widened more than model predicts) are candidates for long positions; bonds with large negative residuals are candidates for shorts.

Cross-industry relative value: Compare bonds across industries on a model-adjusted basis:

$$\text{Adjusted Spread}_i = OAS_i - \bar{OAS}_{rating} \times \frac{\beta_{k(i)}^{composite}}{\bar{\beta}^{composite}} \tag{86}$$

This reveals which industries are cheap/rich after adjusting for systematic risk exposure.

11 Conclusion

11.1 Summary of contributions

This paper extends the DTS enhancement research program by addressing the fundamental issue of shock heterogeneity across industries. The key contributions are:

1. **Two-layer theoretical framework:** We establish that credit spread sensitivity depends on two distinct sources of heterogeneity: (i) Merton elasticity $\lambda(s, T)$, which governs how spread level and maturity affect the translation of firm value changes into spread changes, and (ii) industry shock exposure $\beta_k^{(m)}$, which governs how macroeconomic shocks differentially affect firm values across industries. The clean separation of these layers resolves confusion in Part I about the source of sector effects.

2. **Proper composite beta derivation:** We derive the covariance-adjusted composite beta $\beta_k^{composite} = \sqrt{\boldsymbol{\beta}_k' \Sigma_\eta \boldsymbol{\beta}_k}$ that correctly accounts for correlations among shock types. The naive variance-weighted formula understates composite betas when shocks have offsetting effects.

3. **Reinterpretation of Part I findings:** The sector effects documented in Part I (e.g., Financials having 30–40% higher sensitivity than Industrials) are explained by differential shock exposure rather than differences in spread mechanics. Within-issuer tests provided clean identification of Merton elasticity precisely because they control for shock exposure.

4. **PCA validation framework:** We develop a model-free approach to testing the two-layer structure: PC1 loadings should be proportional to $\lambda^{Merton} \times \beta^{composite}$, and the multiplicative specification should outperform additive alternatives.

5. **Production implementation:** The enhanced DTS formula $DTS^{**} = \lambda^{Merton} \times \beta^{composite} \times DTS$ provides a practical tool for risk measurement, attribution, and hedging that incorporates both theoretical layers.

11.2 Relationship to Part I

Part II complements rather than replaces Part I:

- Part I's Merton elasticity analysis remains valid and essential—it correctly identifies how spread level and maturity affect the translation of firm value shocks into spread changes.
- Part I's within-issuer tests provided the cleanest validation of Merton precisely because they hold shock exposure constant.
- Part I's sector interaction effects are now understood as shock exposure differences rather than potential model misspecification.
- Part II adds the industry exposure layer that explains cross-industry variation left unexplained by Merton alone.

The combined framework is:

$$\underbrace{DTS^{**}}_{\text{Enhanced DTS}} = \underbrace{\lambda^{Merton}(s,T)}_{\text{Part I: Elasticity}} \times \underbrace{\beta_k^{composite}}_{\text{Part II: Exposure}} \times \underbrace{DTS}_{\text{Standard}} \tag{87}$$

11.3 Limitations and future research

Several limitations warrant acknowledgment and suggest directions for future work:

Industry classification granularity: We use BCLASS3 (18 industries) as a baseline, but within-industry heterogeneity may be substantial. Future work could explore BCLASS4 or firm-level betas, with appropriate regularization to manage estimation noise.

Time-varying betas: We treat industry betas as relatively stable, but they may shift during regime changes (crises, policy shifts). Regime-switching or time-varying parameter models could improve performance during turbulent periods.

Endogeneity of risk appetite shocks: VIX and credit spreads are jointly determined. While we use orthogonalization and high-frequency identification to mitigate this, cleaner instruments would strengthen causal interpretation.

Non-multiplicative interactions: We assume the multiplicative structure $\lambda \times \beta$ holds uniformly. Empirical tests may reveal shock-type-specific or industry-specific deviations that require model extension.

Extension to high yield: The framework is developed for investment-grade corporate bonds. High-yield bonds may exhibit different dynamics, particularly during distress when equity-like features become more prominent.

11.4 Practical implications

For practitioners, the two-layer framework provides:

1. **More accurate risk measurement:** Enhanced DTS correctly scales bond-level exposure, accounting for both structural credit mechanics and economic shock sensitivity.
2. **Better attribution:** Spread P&L can be decomposed into contributions from specific macro shocks (employment, Fed policy, risk appetite), weighted by industry exposure and bond-level elasticity.

3. **Improved hedging:** Cross-industry hedges require adjustment for differential shock exposure. The naive DTS hedge ratio can be dramatically wrong when industries have different composite betas.

4. **Enhanced relative value:** Comparing bonds across industries requires adjusting for both maturity-driven elasticity differences and industry-driven exposure differences. Model residuals provide cleaner alpha signals.

11.5 Final perspective

The Duration-Times-Spread framework has served practitioners well for decades, but its assumption of homogeneous shock response is increasingly untenable in a world of heterogeneous industries and diverse macroeconomic shocks. By explicitly separating the Merton elasticity (how spreads respond to firm value changes) from industry shock exposure (how firm values respond to macro shocks), we provide a theoretically grounded and empirically testable enhancement.

The research program outlined in Parts I and II delivers a complete framework: Part I established that Merton's structural model correctly predicts cross-maturity elasticity differences; Part II establishes that industry-specific macro betas explain cross-industry sensitivity differences. Together, they provide the first comprehensive treatment of DTS heterogeneity that is both academically rigorous and practically implementable.

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