IE 5545 Changing the Network Structure

Adding links to a game of strategic substitutes can change the equilibrium structure in unexpected ways. Consider the "Best-shot" public goods game in which a player will receive a benefit of 1 if she or any of her neighbors take action 1 (but she prefers that one of her neighbors take action 1). In this game the utility to player i for taking action 0 or 1 is

$$u_i(1, S_{N_i}) = 1 - c$$
$$u_i(0, S_{N_i}) = \begin{cases} 1 & \text{if } a_j = 1 \text{ for some } j \in N_i \\ 0 & \text{if } a_j = 0 \text{ for all } j \in N_i \end{cases}$$

where N_i are the neighbors of player i, S_{N_i} are the strategies of the neighbors of player i, a_j is the action taken by player j, and $0 < c < 1$ is the cost for taking action 1. The following situation is a pure strategy Nash equilibrium for this game. Now, add a link that connects the two centrally located nodes and then change the players' behavior (i.e. choice of action) in a sequential manner until a new equilibrium is achieved.