IE 5545 A Network Game of Substitutes

In a game of substitutes a player has a *decreasing* incentive to choose an action as more neighbors choose the action. Suppose that there are only two actions for each player, 0 or 1. In the "Best-shot" public goods game, a player will receive a benefit of 1 if she or any of her neighbors take action 1. For example, if you or any of your neighbors buy a drill you will benefit (assuming your neighbors will lend you the drill) but there is a cost for purchasing the drill so you prefer that one of your neighbors buys the drill. In this game the utility to player i is

$$u_i(1, S_{N_i}) = 1 - c$$

$$u_i(0, S_{N_i}) = \begin{cases} 1 & \text{if } a_j = 1 \text{ for some } j \in N_i \\ 0 & \text{if } a_j = 0 \text{ for all } j \in N_i \end{cases}$$

where N_i are the neighbors of player i, S_{N_i} are the strategies of the neighbors of player i, a_j is the action taken by player j, and $0 < c < 1$ is the cost for taking action 1. Find a pure strategy Nash equilibrium for the Best-shot public goods game shown below that is different from the equilibrium given in class for this game.