Communication

Microwave Lens Using Periodic Dielectric Sheets for Antenna-Gain Enhancement

Q. L. Li, S. W. Cheung, Di Wu, and T. I. Yuk

***Abstract*— A microwave lens designed using only 13 periodic dielectric sheets is proposed for gain enhancement of linearly polarized antenna. A 3-D-patch antenna with dual-fed and operating in the frequency band from 8–9 GHz is used for studies. The microwave lens is placed above the patch antenna to form a microwave-lens antenna. The final design is fabricated and measured using the Satimo Starlab system. Results show that the microwave lens has little effect on impedance bandwidth and efficiency of the patch antenna, but can increase the boresight gain by 11.9 dB from 5.1 to 17 dBi in simulation and by 11.6 dB from 5.2 to 16.8 dBi in measurement at the center frequency of 8.5 GHz. Compared with commercially available standard-gain horn antennas having the same center frequency of 8.5 GHz, the proposed microwave-lens antenna has a much smaller volume but higher gain.**

***Index Terms*— Dual-feeding, gain enhancement, microwave lens, patch antenna, periodic dielectric sheets.**

I. INTRODUCTION

Microwave-lens antennas with the ability to achieve high gain and directivity have attracted much attention [1]. In a microwave-lens antenna, the microwave lens transforms the quasi-spherical wave emitted by a source antenna into near-plane wave, producing a high gain. Different types of microwave lens such as homogeneous dielectric lens, artificial-dielectric-delay lens and accelerate lens have been proposed for antenna-gain enhancement [1]–[4].

Materials with gradual variations of refractive index can be used to make gradient-index (GRIN) lenses which may have flat surfaces or may not have spherical shapes. Different methods have been proposed to design microwave lenses using GRIN [5]–[15]. In [10], a flat microwave lens using GRIN was studied for gain enhancement, which had a circular shape with a diameter of only one λ_0, where λ_0 is the free space wavelength. It achieved a gain of only 5.5 dBi. In [11], a 3-D-lens antenna composed of multilayer microstrip square-ring arrays to achieve radial GRIN was studied. A compact 3-D-flat Luneburg-lens antenna with GRIN was designed in [12] using a suite of all-dielectric materials. In [13], the required surface-refractive index of a metasurface-Luneburg lens was realized using quasi-conformal mapping. In [14], the relative-permittivity profile with GRIN of a 3-D-Luneburg-lens antenna was realized using different sizes of plastic blocks centered at the junctions of a plastic-rod space frame. A flat microwave lens consisted of ten layers of GRIN dielectric in the radial direction and five layers in the longitudinal direction was designed in [15]. All the aforementioned microwave lenses require complex manufacturing processes to make.

In this communication, a compact microwave lens designed using 13 layers of purely dielectric sheets in a parallel and periodic way is proposed for high-gain enhancement of linearly polarized antennas. With the authors' best knowledge, this idea of using purely dielectric sheets for antenna-gain enhancement has never been used or studied before. Compared with other microwave lenses studied in [1]–[15], the proposed microwave lens has a very simple structure and a compact volume of only $1.7\lambda_0 \times 1.6\lambda_0 \times 1.7\lambda_0$ and is easy for fabrication. To demonstrate the gain-enhancement capability of the proposed microwave lens, a linearly polarized 3-D-patch antenna consisting of a radiating patch with dual-fed [16] is designed and used as the source antenna. The antenna generates a polarized signal with high purity. The preliminary study of the design using simulation was initially reported in [17] which only illustrated the basic idea. Here the microwave lens is analyzed using the GRIN function and studied using the electromagnetic simulation tool CST in detail. The prototype is fabricated for measurement using the antenna measurement equipment, the Satimo Starlab system [18]. Simulated results show that the microwave lens can significantly increase the boresight gain of the patch antenna by 11.9 dB from 5.1 to 17 dBi at 8.5 GHz, and significantly reduce the beamwidth of the radiation patterns in both the E- and H-planes. Compared with two commercially available standard horn antennas in [19] and [20], also 3-D antennas with high gain having the same center frequency of 8.5 GHz, the microwave lens-antenna has a much higher gain enhancement but with a much smaller volume.

II. MICROWAVE LENS USING PARALLEL DIELECTRIC SHEETS

The proposed microwave lens is shown in Fig. 1 which consists of a number of parallel dielectric sheets in a parallel and periodic way, with a total volume of width $\times$ depth $\times$ thickness $= W \times D \times T = 60 \times 56.9 \times 60$ mm^3. The dielectric sheets have a relative permittivity of $\varepsilon_r = 10.2$ and thickness of hs_1, and are separated by an air gap of d_1. For high-gain enhancement, the microwave lens is placed at distance F above an isotropic source, as shown in Fig. 1(a), which emits linearly polarized waves along the x-axis. To achieve the lens property for the linearly polarized waves, the microwave lens should have the refractive index $n(r)$ followed the GRIN function along the y-axis from the middle ($r = 0$) of the lens to the edge ($r = D/2$) [10], [11], i.e., the required effective-refractive index at a distance r from the middle should be [10]

$$n(r) = n(0) - \frac{\sqrt{F^2 + r^2} - F}{T}. \tag{1}$$

In Fig. 1(a), the distance r can be expressed as $r = F\tan\alpha$, so (1) can also be rewritten as a function of the incident angle α as

$$n(\alpha) = n(0) - \frac{\sqrt{F^2 + (F\tan\alpha)^2} - F}{T}. \tag{2}$$

Note that the required refractive index $n(\alpha)$ given by (2) depends only on F and T and does not take into account the thickness hs_1 and relative permittivity ε_r of the dielectric sheets, the air gap d_1 separating the substrates or the operating frequency. Since the characteristics of GRIN lenses are determined by the variations of refractive index, and not the absolute values, without loss of

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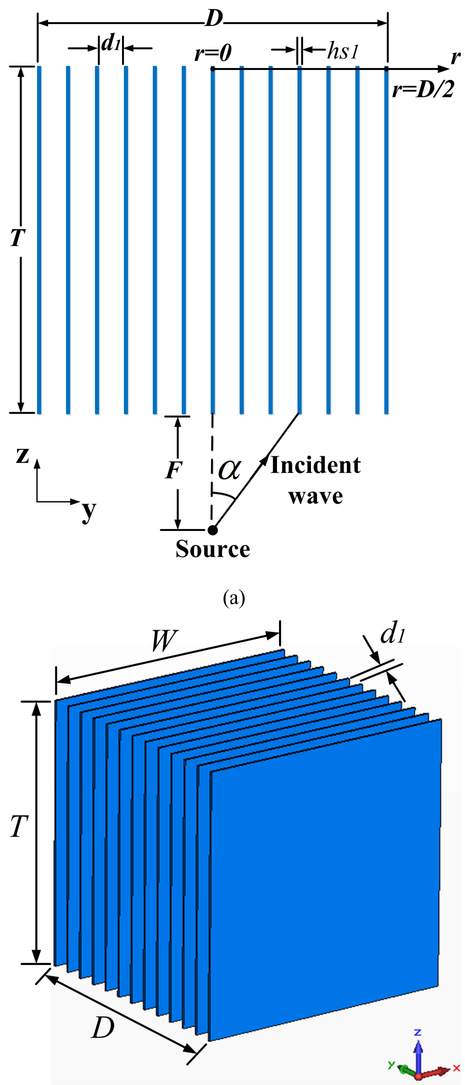

(a)

(b)

Fig. 1. Proposed microwave lens using parallel dielectric sheets. (a) Side view. (b) 3-D view.

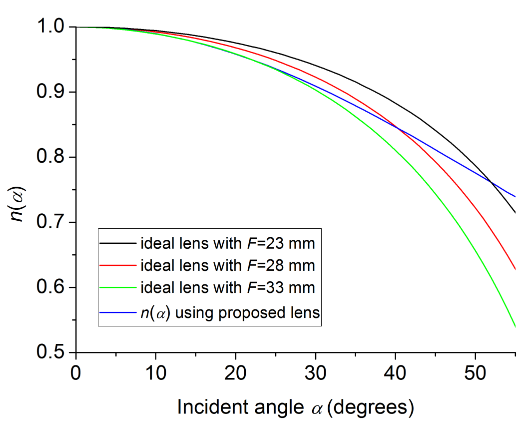

Fig. 2. Normalized refractive index $n(\alpha)$ of ideal lens with different values of F and proposed microwave lens.

generality and for convenience of comparison, the effective-refractive index given by (2) is normalized to 1 at $\alpha = 0°$ as

$$n(\alpha) = 1 - \frac{\sqrt{F^2 + (F\tan\alpha)^2} - F}{T}. \tag{3}$$

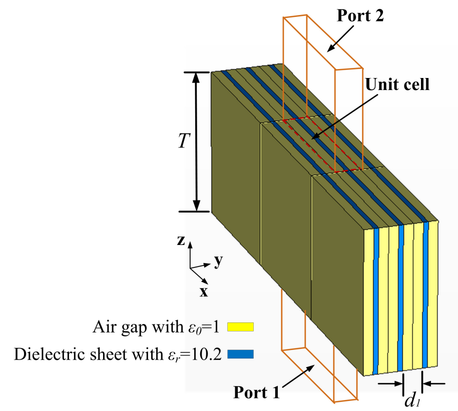

Fig. 3. Model of microwave lens for analysis using periodic boundary.

With the use of (3), the normalized $n(\alpha)$ of the ideal lens from $\alpha = 0$ to $55°$ with $T = 60$ mm and $F = 23$, 28, and 33 mm are computed and shown in Fig. 2.

The proposed microwave lens can be analyzed using the model shown in Fig. 3 through the use of the frequency-selective surface template in CST. Assuming that the microwave lens has an infinite bottom and top surface areas (aperture), a small volume of the lens, as indicated in Fig. 3, can be considered as a unit cell having one dielectric sheet sandwiched by two layers of air gap. Applying a linearly polarized plane wave with polarization along the x-direction and an angle of incidence α to the unit cell via port 1, the S-parameters between ports 1 and 2 of the unit cell can be computed using periodic boundary in CST [10] and used to compute the effective-refractive index as follows. From [21]–[24], we have

$$X = 1/\left[2S_{21}\left(1/S_{11}^2 + S_{21}^2\right)\right] \tag{4}$$

$$e^{jn(\alpha)k_0t} = X \pm j\sqrt{1 - X^2} \tag{5}$$

$$k_0 = 2\pi f/c \tag{6}$$

where k_0, f, c, T, and $n(\alpha)$ are the wavenumber, operating frequency, speed of light, thickness of the microwave lens, and effective-refractive index at an incident angle α. Substituting (6) and (4) into (5) yields the effective-refractive index

$$n(\alpha) = \frac{1}{k_0T}\left[\mathrm{Re}\left[\ln(X \pm j\sqrt{1 - X^2})\right] - j\mathrm{Im}\left[\ln\left(X \pm j\sqrt{1 - X^2}\right)\right]\right] \tag{7}$$

where Re(.) and Im(.) are the real and imaginary parts, respectively, of the function (.). Thus, we can use the computed S-parameters to obtain X in (4), and then compute the effective-refractive index $n(\alpha)$ of the unit cell using (5)–(7). Note that there are two possible values for each $n(\alpha)$, and from [23] we select the value with the condition that the imaginary part Im(n) >0. With the use of $T = 60$ mm, $d_1 = 4.2$ mm, and our dielectric sheets (with $hs_1 = 0.5$ mm and $\varepsilon_r = 10.2$) in the model of Fig. 3, the S_{11} and S_{21} and hence the corresponding $n(\alpha)$ of the unit cell for $\alpha = 0$ to $55°$ at 8.5 GHz are computed using (4)–(7). Results are shown in Fig. 2 for comparison. It can be seen that, the proposed lens designed using the unit-cell model has a good approximation to the ideal lens with $F = 33$ mm for about $\alpha < 30°$, with $F = 28$ mm for about $\alpha < 45°$. As $F = 23$ mm does not have a good approximation and $F = 33$ mm would lead to a larger size and a smaller aperture for gain enhancement, we select $F = 28$ mm in our design and so the lens will have a good approximation for $\alpha < 45°$. With this selection, the parameter D is 2×28 mm $= 56$ mm. Using $hs_1 = 0.5$ mm and $h_1 = 4.2$ mm, the number of dielectric sheets used is computed to be 13. Note that if

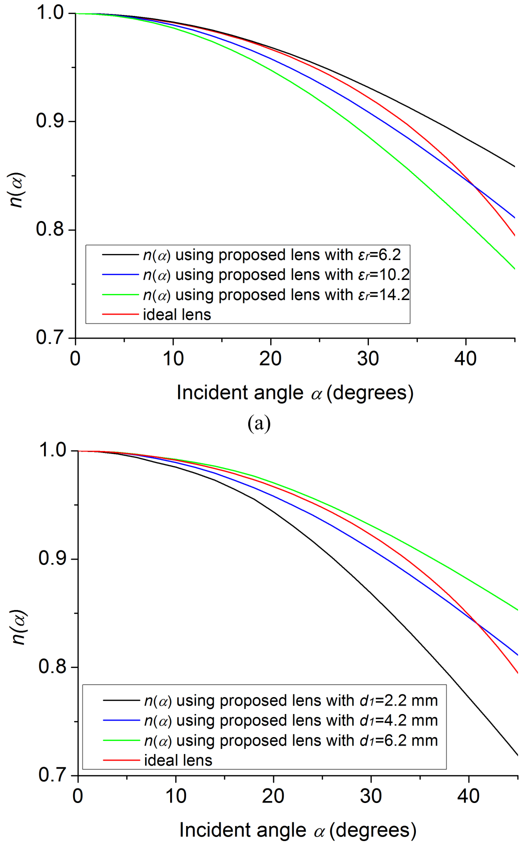

Fig. 4. Normalized refractive index $n(\alpha)$ of ideal lens and the proposed microwave lens with different values. (a) ε_r. (b) d_1.

another set of T, d_1, F, and ε_r is used, the analysis is exactly the same, but another set of curves similar to those in Fig. 2 will be obtained, so the number of dielectric sheets used will be different.

Study has been used to investigate the effect of relative permittivity ε_r on $n(\alpha)$. The normalized $n(\alpha)$ of the proposed microwave lens using (4)–(7) with $\varepsilon_r = 6.2$, 10.2, and 14.2 are shown in Fig. 4(a). It can be seen that the curve with $\varepsilon_r = 10.2$ has the closest approximation for $\alpha < 45°$, which is expected because the antenna is designed using $\varepsilon_r = 10.2$. Further, Fig. 4(a) shows that a higher ε_r leads to a lower normalized $n(\alpha)$ and hence a larger change of effective-refractive index $\Delta n(\alpha)$ for $\alpha = 0°$ to 45°. However, from (2) or (3), this change can also be expressed as

$$\Delta n(\alpha) = n(\alpha)_{\alpha=0°} - n(\alpha)_{\alpha=45°} = \frac{\sqrt{F^2 + (F \tan 45°)^2} - F}{T} \quad (8)$$

which indicates that a large change $\Delta n(\alpha)$ requires a smaller T for a given F. This means if we want to design a thinner lens, we need to use a large ε_r for the dielectric sheets. The effect of the air gap d_1 on $n(\alpha)$ of the proposed lens is also studied. With $\varepsilon_r = 10.2$, the normalized $n(\alpha)$ with $d_1 = 2.2$, 4.2, and 6.2 mm are shown in Fig. 4(b). It can be seen that increasing d_1 increases the normalized $n(\alpha)$ and hence leads to smaller $\Delta n(\alpha)$. The curve with $d_1 = 4.2$ mm is the closest approximation for $\alpha < 45°$, as expected because the lens is designed using this value.

In our design, we propose to use dielectric sheets in a parallel to approximate the property of an ideal lens for high-gain enhancement. Of course, the better the approximation, the higher is the gain enhancement. The results in Fig. 4(a) and (b) show that both the permittivity ε_r used in the dielectric sheets and the air gap d_1 between the dielectric sheets affect $n(\alpha)$. This means that we can use different values of ε_r and d_1 in the dielectric sheets to obtain a better approximation to the ideal curve. However, these two methods will complicate the design and increase the cost. For easy fabrication, we use a constant ε_r and d_1, but then the refractive-index profile $n(\alpha)$ in Fig. 2 does not exactly agree with the curve of the ideal lens.

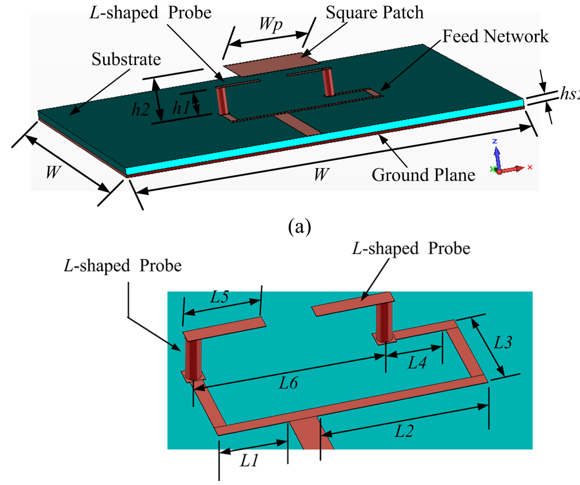

Fig. 5. Geometry of 3-D-view. (a) Source antenna. (b) Enlarged feeding network.

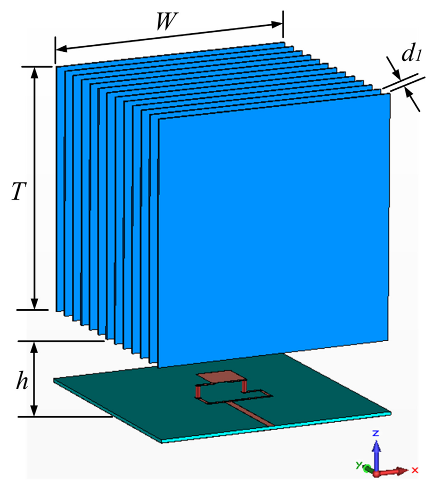

Fig. 6. Patch antenna with microwave lens.

III. Antenna Design

To study the gain enhancement of the proposed microwave lens, a patch antenna shown in Fig. 5(a) is used. The patch antenna has a 3-D-structure and consists of a square radiating patch, two L-shaped probes, a square common ground with a size of $W \times W = 60 \times 60$ mm^2 and a feeding network. The radiating patch is made of copper, having an area of 9×9 mm^2 and a thickness of 0.035 mm, and is placed at a height of $h_2 = 4.5$ mm above a substrate with a thickness of $h_s = 0.8$ mm, a relative permittivity of 3.5 and a loss tangent of 0.004. The feeding network has a 3-D-structure, as shown in Fig. 5(b), and is designed on the top side of the substrate, composing of two identical L-shape probes connected to two microstrip lines. The L-shape probes are used to coupled feed the square radiating patch. The antenna is designed for differential feeding with an operating band of 8–9 GHz. The difference in length of the two microstrip lines is $L_2 + L_4 - L_1 = 10.5$ mm. At the center frequency of 8.5 GHz, this length difference is equivalent to about

TABLE I
DIMENSIONS OF LENS ANTENNA (mm)

W	Wp	h_1	h_2	h	hs_1	hs_2	L_1
60	9	3	4.5	17	0.5	0.8	4.5
L_2	L_3	L_4	L_5	L_6	T	D	d_1
11	5	4	5	12.2	60	56.9	4.2

TABLE II
COMPARISON OF PROPOSED LENS WITH OTHER GRIN LENS

Ref.	Gain Enhancement	Gain	Aperture Size
[8]	3 dB	20.2 dBi	$\pi\times2\lambda_0\times2\lambda_0$
[9]	2.3 dB	6.9 dBi	$1.2\lambda_0\times1.2\lambda_0$
[10]	5.5 dB	10.7 dBi	$\pi\times\lambda_0/2\times\lambda_0/2$
[11]	4.7 dB	20.9 dBi	$\pi\times1.6\lambda_0\times1.6\lambda_0$
[12]	6 dB	17 dBi	$\pi\times1.6\lambda_0\times1.6\lambda_0$
Proposed	11.9 dB	17 dBi	$1.7\lambda_0\times1.6\lambda_0$

$\lambda_g/2$ (where λ_g is the guide wavelength), resulting in a 180°-phase difference in the signals to the two L-shaped probes. With differential feeding, the patch antenna produces highly linearly polarized waves along the x-direction.

For antenna-gain enhancement, the microwave lens of Fig. 1 is placed at a distance h above the ground plane of the patch antenna, as shown in Fig. 6, to form a microwave-lens antenna. In our previous analysis, we used an isotropic source at a distance F from the lens, as shown in Fig. 1(a), and selected $F = 28$ mm. Here the patch antenna is not an isotropic source, so in our design, we assume it is an isotropic source radiating waves from its phase center. Thus we use h to align the phase center to the position of the isotropic source in Fig. 1. It has been found using CST that the patch antenna has a phase center at 11 mm from the back, so h is computed as $h = F - 11 = 17$ mm. Detailed dimensions of the microwave-lens antenna are listed in Table I and used to fabricate the prototype, as shown in Fig. 7, for measurement. In the prototype, small pieces of foams are used to fix the separation gaps for the dielectric sheets.

IV. SIMULATION AND MEASUREMENT RESULTS

Both patch antenna and microwave-lens antenna of Figs. 5 and 6, respectively, have been studied using simulation, and the prototyped antennas have been measured for verification of simulation results. The simulated and measured S_{11} of the two antennas are shown in Fig. 8, which indicates good agreements. It can be seen that both antennas have similar simulated and measured impedance bandwidths (IMBWs) for $S_{11} < -10$ dB, thus using the microwave lens for gain enhancement does not affect much the IMBW. The simulated IMBWs for the microwave-lens antenna and patch antenna, for $S_{11} < -10$ dB, are 7.98–9.06 and 7.94–9.14 GHz, respectively, while the corresponding measured IMBWs are 7.82–9.15 and 7.88–9.02 GHz.

The antenna measurement equipment, Satimo StarLab system [18], is used to measure the efficiencies of the antennas. The results in simulated and measured efficiencies and boresight gains are shown in Figs. 9 and 10. It can be seen in Fig. 9 that, in the frequency band of 8–9 GHz, the simulated efficiencies of both patch antenna and microwave-lens antenna are more than 85%, while the measured efficiencies are more than 80%. Thus using the microwave lens for gain enhancement does not affect much the antenna efficiency. Fig. 10 shows that, at the center frequency of 8.5 GHz, the microwave-lens antenna and patch antenna have the simulated boresight gains of 17 and 5.1 dBi, respectively, thus the microwave lens substantially increases the boresight gain by 11.9 dB. For measurement, the microwave lens increases the boresight gain from 5.2 to 16.6 dBi by 11.4 dB. Both the simulated and measured results show that the 1-dB gain bandwidths can cover the whole operating bandwidth of 8–9 GHz.

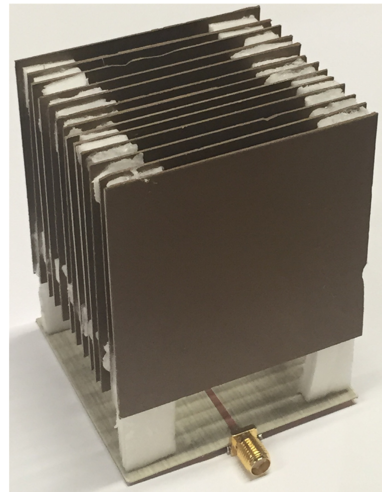

Fig. 7. Photograph of prototyped lens antenna.

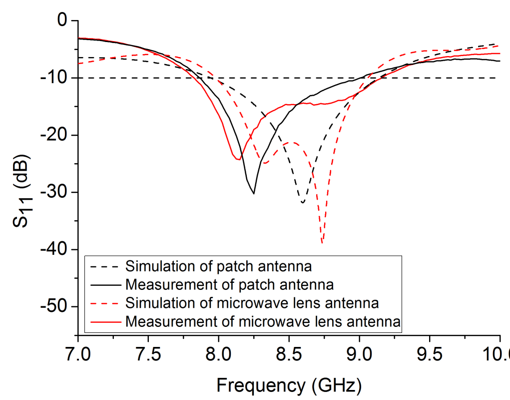

Fig. 8. Simulated and measured S_{11} of patch antenna and microwave-lens antenna.

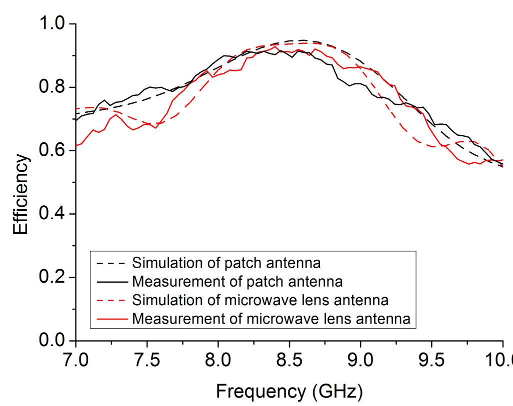

Fig. 9. Simulated and measured efficiencies of patch antenna and microwave-lens antenna.

The proposed microwave-lens antenna has a simulated boresight gain of 17 dBi which is higher than the directivity of an aperture with the same size given by [1]

$$D_0 = \frac{4\pi}{\lambda_0^2}A_{\text{physical}} = \frac{4\pi}{\lambda_0^2}(1.7^*1.6)\lambda_0^2 = 34.16 \rightarrow 15.30 \text{ dB}. \quad (9)$$

This because (9) is used to evaluate the gain of an aperture antenna having radiation mostly from the aperture. The proposed

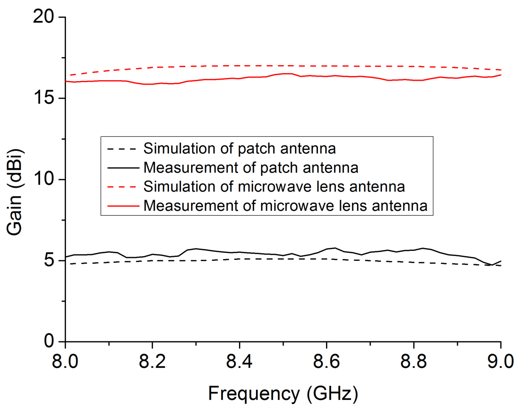

Fig. 10. Simulated and measured boresight gains of patch antenna and microwave-lens antenna.

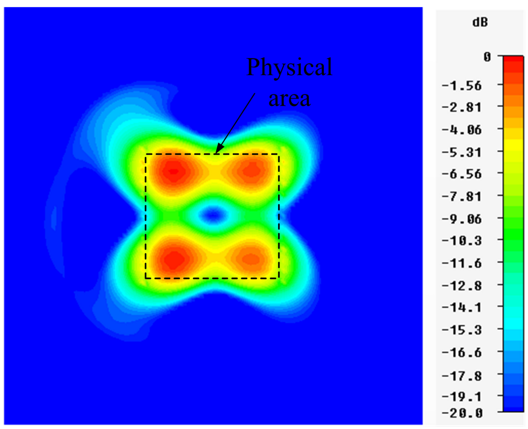

Fig. 11. E-field distribution on lens aperture.

microwave-lens antenna is not a pure aperture-type antenna and has radiation from the lens edges as well as the aperture [25], which results in a higher directivity. E-field distribution has been used to further investigate this. Fig. 11 shows the simulated E-field distribution on the aperture of the microwave-lens antenna. It can be seen that some E-fields are emitted from outside the physical aperture of the lens. This increases the directivity and results in a gain higher than that given by (9). However, to achieve the additional gain, as can be seen in Fig. 11, some clearances at the lens edges are required. More simulation results have shown that a clearance of 25 mm (0.7 λ_0) at the four edges is needed for the boresight gain of 17 dBi. If no clearance is used, the boresight gain is only about 13 dBi. Thus although the physical size of the antenna is $1.7\lambda_0 \times 1.6\lambda_0 \times 1.7\lambda_0$, the effective size of the antenna is $3.1\lambda_0 \times, 3\lambda_0 \times 1.7\lambda_0$.

The radiation patterns of both antennas at the center frequency of 8.5 GHz in the E- and H-planes are shown in Fig. 12. It can be seen in Fig. 12(a) that the simulated and measured half-power beam widths (HPBWs) of the patch antenna in the E-plane are 97.5° and 94°, respectively. While the microwave-lens antenna has a much higher directive beam, with the simulated and measured HPBWs of 20.6° and 22°, respectively. Fig. 12(b) shows that the simulated and measured HPBWs of the patch antenna in the H-plane are 85.7° and 80°, respectively. However, the simulated and measured HPBWs of the microwave-lens antenna in the H-plane are 21.9° and 22°, respectively, which are much narrower. These results show that the microwave lens can significantly reduce the HPBW in both the E- and H-planes, hence providing significant gain enhancement. Note that cross polarizations in the E- and H-planes are less than −30 dB

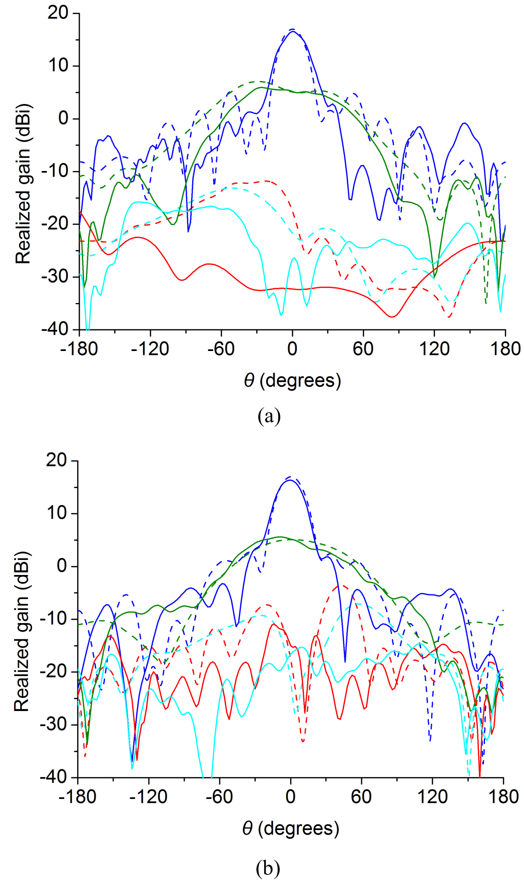

Fig. 12. Simulated radiation patterns. (a) E-plane (*xz* plane) and (b) H-plane (*yz* plane) (simulated co-pol with lens, measured co-pol with lens, simulated co-pol without lens, measured co-pol without lens, simulated cross-pol with lens, measured cross-pol with lens, simulated cross-pol without lens, and measured cross-pol without lens).

within the main lobe in both cases, which is due to the use of differential-feeding [16].

As the proposed microwave lens antenna has three dimensions and high gain, similar to horn antennas, we compare it with two commercial standard-gain horn antennas in [19] and [20]. The standard-gain horn antenna in [19] has a gain of 15 dBi, an operating band of 7.05–10 GHz, and a total volume of $57 \times 77 \times 198$ mm^3 = 869 022 mm^3. The standard-gain horn antenna in [20] has a gain of 15 dBi, an operating band of 7.05–10 GHz, and a total volume of $55 \times 74 \times 166$ mm^3 = 675 620 mm^3. Both antennas have a center frequency of about 8.5 GHz. However, the proposed microwave-lens antenna has a higher gain of 17 dBi at the center frequency of 8.5 GHz, but a smaller volume (including the patch antenna) of only $60 \times 57 \times 77$ mm^3 =263 340 mm^3, much smaller than the two standard-gain horn antennas.

Finally, we make a comparison of our work with other GRIN lenses in terms of gain enhancement and results in Table II. It can be seen that our lens have a gain enhancement of 11.9 dB, much higher than those of other GRIN lenses found in literature [8]–[12].

V. Conclusion

A microwave lens, with a compact volume of only $1.7\lambda_0 \times 1.6\lambda_0 \times 1.7\lambda_0$ and a simple structure, designed using 13 parallel dielectric sheets, has been proposed for antenna-gain enhancement. The microwave lens has been analyzed in detail using the GRIN function. A patch antenna designed to operate in the frequency band from 8 to 9 GHz has been designed for studying the gain enhancement of the microwave lens. Simulated and measured results have shown that the microwave lens can significantly increase the boresight gain of the patch antenna by 11.9 and 11.4 dB, respectively, at the center frequency of 8.5 GHz, through reducing the HPBWs in both the E- and H-planes.

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