BLACKBOARD PROOFS

CSE202 – WEEK 7

1. Expected number of comparisons for a successful search in hashing with separate chaining

With the notations of slide 11, since the search is successful, there is an integer $i \in \{1, \ldots, n\}$ such that the searched element was the ith one that was inserted in the table. Each of these values can be attained with probability $1/n$. Decomposing the expected number of comparisons C_n according to that index i gives

$$\begin{aligned}
\mathbb{E}(C_n) &= 1 + \sum_{i=1}^{n} \frac{1}{n} \mathbb{E}(C_n \mid \text{searched element has index } i) \\
&= 1 + \frac{1}{n} \sum_{i=1}^{n} \frac{i-1}{m} \\
&= 1 + \frac{\alpha}{2} - \frac{1}{2m}.
\end{aligned}$$

The first 1 comes from the comparison where the key is found. On top of this comparison, the number of comparisons for an element that was inserted with index i is equal to the number of comparisons that were used when it was inserted. This was proved before to be α, for $\alpha = (i-1)/m$ the filling ratio of the table when the ith element was inserted.

2. Expected number of probes for an unsuccessful search in hashing with linear probing

This is a detailed explanation of Slide 14.

The first line is the definition of the expectation. The first 1 corresponds to the final test finding an empty slot.

The second line is obtained by summing over the possible value $i \in \{0, \ldots, m-1\}$ taken by the hashed value of the searched element, obtained with probability $1/m$ by hypothesis on the hash function. Next, the number of probes needed by the algorithm is upper bounded by the size of the cluster to which i belongs.

The third line is obtained by giving a cost k^2 for each starting point of a cluster and 0 to the other ones, which does not change the sum.

The next line is obtained by summing m times $1/m$ and using the bound of the next slide.