May 28, 2015
Math 201, Practice Midterm Questions

1) a) State sufficient conditions for the existence and uniqueness of a solution on some interval, I, including the point x_o of the IVP

$$\frac{dy}{dx} = f(x, y) \quad \text{with} \quad \mathrm{y(x_o) = y_o}$$

where x_o, $y_o \in \mathbb{R}$.

b) For what values of x_o, $y_o \in \mathbb{R}$ is the IVP

$$\frac{dy}{dx} = y^{2/3}\sqrt{x-2} \quad \text{with} \quad \mathrm{y(x_o) = y_o}$$

guaranteed to have a unique solution on some interval containing the point x_o?

2) Find an implicit family of solutions to

$$\frac{dy}{dx} = \frac{x^3}{\mathrm{cosy}}$$

3) Find the general solution to the IVP (i.e. find the 1 parameter family of solutions to the IVP)

$$\frac{dy}{dx} + 2xy = x^3$$

4) Verify that the following differential equation is exact (you will want to quote a theorem from the text, which you should know ,in order to verify this) and then find a family of implicit solutions.

$$(3x^2y + 8xy^2)dx + (x^3 + 6y^2 + 8x^2y)dy$$

5) Solve the homogeneous differential equation

$$(y^2 + xy)dx - x^2dy = 0$$

using the substitution $y = ux$.

6)

$$y^2\mathrm{dx} + (4\mathrm{xy} + 1)\mathrm{dy} = 0$$

is a nonseparable differential equation which can be made exact by multiplying through by an appropriate integrating factor. Find the integrating factor, verify that the new equation is in fact exact and then find a family of implicit solutions.

7) Identify the type of D.E. (Linear, Bernoulli, Exact, Homogeneous, Separable)

a)

$$\frac{dy}{dx} + 3x^4y = xy^3$$

b)

$$(x^3 + xy^2)dx + (y^3)dy = 0$$

c)

$$\frac{dy}{dx} + (x^3\mathrm{sin(x)})\mathrm{y} = \mathrm{e}^{4\mathrm{x}}$$

d)

$$(e^{2y} - y)\mathrm{cos(x)}\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{y}}\mathrm{sin(x)}$$

e)

$$12x^3y^5dx + (15x^4y^4 + 6y)dy = 0$$

8) Consider the autonomous differential equation:

$$\frac{dy}{dx} = (y^3 + y^2 - 12y)e^y$$

Find and classify the equilibrium solutions and draw the 1 dimensional phase portrait.

9) Suppose the rate at which a student complete his or her homework is proportional to the amount of homework he or she has remaining. If $h(t)$ represents the amount of homework remaining, write a differential equation which describes the system. If the student begins with H_o amount of homework, and after 50 hours has $\frac{4H_o}{5}$ remaining, find an explicit solution for $h(t)$ for $t > 0$. How long will it take for the student to finish $\frac{9}{10}$ of the total homework?