Math 201, Assignment 3
Due at the beginning of tutorial on June 24, 2015
Illegible or disorganized solutions will receive no credit! Please, for the sake of our marker, be neat!

1) Solve the following homogeneous equations

    a)
    $$y'' + 3y' + 2y = 0$$

    b)
    $$y''' = o$$

    c)
    $$y^{(4)} + 8y'' + 16y = 0$$

2) Solve the following IVP

$$y'' + 2y' + 4y = 0 \quad y(0) = 1, \quad y'(0) = -1 + 2\sqrt{3}$$

3) When a spring is stretched or compressed, its restoring force is directly proportional to its change in length. If x represents the displacement of the weight from its equilibrium position, then by Hooke's Law

$$F = -kx \quad k > 0$$

where the minus sign indicates that the restoring force F is always opposite in direction to the displacement. Combining this with Newton's Second Law gives,

$$m\frac{d^2x}{dt^2} = -kx$$

or

$$m\frac{d^2x}{dt^2} + kx = 0$$

   a) If the spring is initially displaced to $x(0) = 5$ and it's initial velocity is zero (i.e. $x'(0) = 0$), find $x(t)$ for $t > 0$.

b) Sketch your solution in a).

c) This approximation is decent, but doesn't really reflect what happens in real life; in reality, the amplitude of the oscillations decreases until eventually the mass is at rest. Thus, we need to include another term which acts as air resistance. At slow speeds, the force of air resistance on an object is well approximated by $F_a = -c\frac{dx}{dt}$, where $c > 0$ and $\frac{dx}{dt}$ is the speed of the object. Thus our differential equation becomes

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

Assume $c^2 < 4mk$ and find $x(t)$ in terms of c, m and k assuming the same initial conditions as in a).

d) Sketch your solution in c).

4) Find the general solution of

$$y''' + 4y' = x^2 + sinx$$