May 20, 2015
Math 201, Tutorial 2

1) Modelling pollution in a lake.
Starting at $t = 0$ (where times are measured in months) a factory starts to discharge pollution into a lake at the constant rate of 300 tonnes/month. There is a stream which runs out of the lake and it carries pollution away at a variable rate, which at any time $t > 0$ is one-tenth of the total amount of pollution present in the lake, per month. So for example, if there are 2000 tonnes in the lake at time t, the stream is carrying pollution away at the instantaneous rate of .1 x 2000 tonnes/month = 200 tonnes/month.

   a) Set up the ODE and the initial condition for this problem. Let $A(t)$ be the amount of pollution present in the lake at time $t > 0$. In other words, the independent variable is time, t, and the dependent variable is amount of pollution, A.

   b) This ODE is autonomous, so find the equilibrium solutions.

   c) Sketch the direction field, and from that sketch what the solution curve looks like for the initial condition you came up with in a) and also one for $A(0) = 4000$. Sketch the one-dimensional phase portrait and classify the equilibrium.

   d) This ODE is in fact separable and we can solve it explicitly. Solve the IVP that you came up with in a) and also solve the same ODE with IC $A(0) = 4000$.

2) Solve the linear equation $2y' - 4y = 16e^x$ and verify that your answer is a solution by direct substitution. In this case, you can take the interval for your solution to be $(-\infty, \infty)$.