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Math 201, Tutorial 5

A damped spring mass system which is acted on by an external force satisfies the linear differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

where $F(t)$ is the applied force, $k > 0$ is the spring constant, $c > 0$ is the drag coefficient and m is the mass of the weight. Suppose our applied force satisfied $F(t) = F_o cos(\omega t)$ where F_o is the constant amplitude and ω is the angular frequency. Our D.E. becomes

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_o cos(\omega t)$$

1) This time, let's assume that there is no damping, so $c = 0$.

   Assuming $x(0) = x'(0) = 0$, find $x(t)$ for $t > 0$ in terms of m, k, F_o and ω. Note: there are two cases here.

2) An external force equal to $\frac{1}{2}cos8t$ is acting on the spring mass system, where the mass is $\frac{1}{8}$ and the spring constant is 8. Find the displacement equation $(x(t))$ if the weight is started from its equilibrium position with an upward velocity of 4m/s.