

# Computational Linguistics

## 6. Formal Grammars and Syntactic Parsing

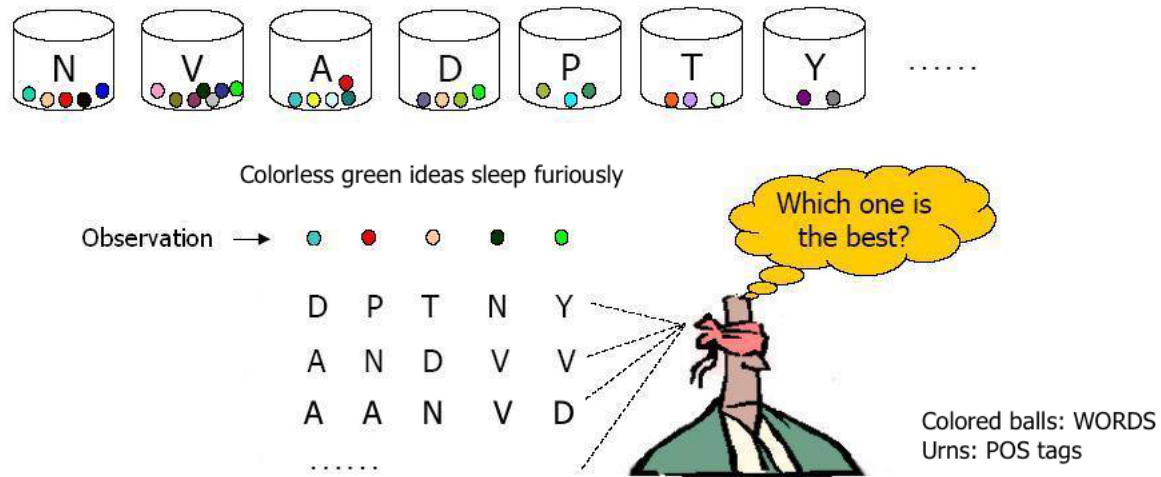
Xiaojing Bai

Tsinghua University

<https://bxjthu.github.io/CompLing>

# Recap: HMM POS tagging as a **decoding** task

Given as **input** an HMM  $\lambda = (A, B)$   
and a sequence of observations  
 $O = o_1 o_2 \cdots o_T$ , **find** the most probable  
sequence of states  $Q = q_1 q_2 q_3 \cdots q_T$ .



Hidden Markov models are characterized by three fundamental problems:

- Problem 1: Likelihood
- **Problem 2: Decoding**
- Problem 3: Learning

## Recap: The basic equation of HMM tagging

The most probable tag sequence given the observation sequence of n words  $w_1^n$ :

$$\hat{t}_1^n = \underset{t_1^n}{\operatorname{argmax}} P(t_1^n | w_1^n)$$

$\hat{t}_1^n$  means 'the estimate of the sequence of n tags'

$\underset{x}{\operatorname{argmax}} P(x)$  means 'the x such that P(x) is maximized'

# Recap: The basic equation of HMM tagging

The most probable tag sequence given the observation sequence of  $n$  words  $w_1^n$ :

$$\hat{t}_1^n = \operatorname{argmax}_{t_1^n} P(w_1^n | t_1^n) P(t_1^n) \approx \operatorname{argmax}_{t_1^n} \prod_{i=1}^n P(w_i | t_i) P(t_i | t_{i-1})$$

Two simplifying assumptions and the estimates

$$P(w_i | t_i) = \frac{\text{Frequency of } w_i \text{ tagged as } t_i \text{ in the training corpus}}{\text{Frequency of } t_i \text{ in the training corpus}}$$

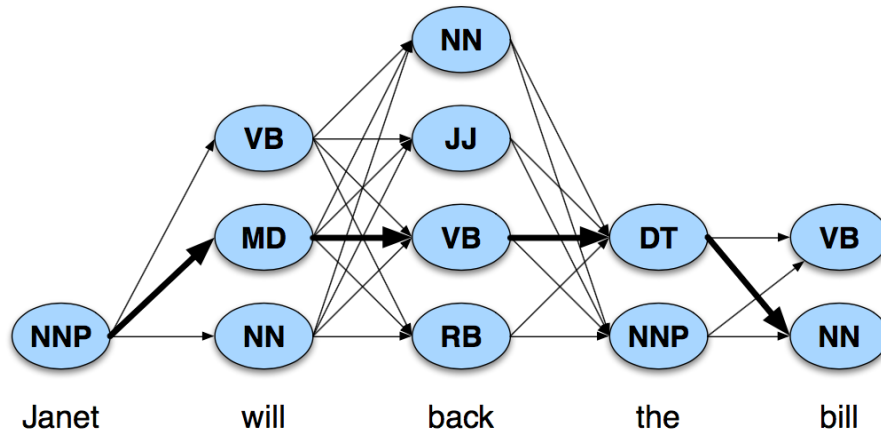
$$P(t_i | t_{i-1}) = \frac{\text{Frequency of } t_i \text{ after } t_{i-1} \text{ in the training corpus}}{\text{Frequency of } t_{i-1} \text{ in the training corpus}}$$

## At the end of this session you will

- understand the rationale behind dynamic programming;
- know how the Viterbi algorithm works for POS tagging;
- know how to describe a language using regular and context-free grammars;
- know how top-down parsing and bottom-up parsing work;
- understand the strengths and weaknesses of these two parsing algorithms;
- understand how language models and parsing algorithms work together;
- learn to analyze sentence structure with NLTK.

# POS tagging: an example

E.g. Janet will back the bill



Janet/NNP will/MD back/VB the/DT bill/NN

## The HMM defined by two tables

- Transition probabilities between pos tags
- Likelihoods of words given tags

	NNP	MD	VB	JJ	NN	RB	DT
<s>	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

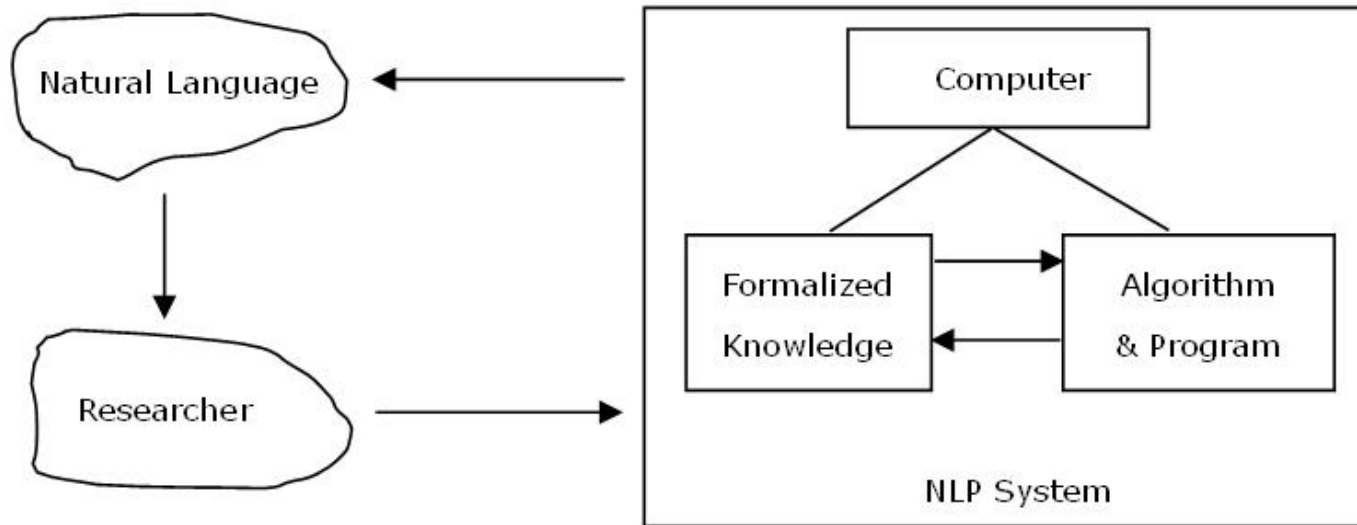
**Figure 10.5** The A transition probabilities  $P(t_i|t_{i-1})$  computed from the WSJ corpus without smoothing. Rows are labeled with the conditioning event; thus  $P(VB|MD)$  is 0.7968.

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0.000097	0
NN	0	0.000200	0.000223	0.000006	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

**Figure 10.6** Observation likelihoods  $B$  computed from the WSJ corpus without smoothing.

# Models and algorithms

## How NLP systems work?



- **A language model:**  
a formal description of linguistic knowledge to show how a language works
- **An algorithm:**  
a series of steps in a computer program to solve a problem in an efficient way

# Viterbi algorithm

- Andrew Viterbi, 1967
- A dynamic programming algorithm
- Maximize the probabilities
- Find the most likely sequence of tags
- Algorithm: exponential complexity

With a tagset of  $N$  tags, for a sequence of  $M$  words,  
there are, in the worse case,  $N^M$  possible paths.

- Efficiency:  $N^M$  vs.  $N^2 \times M$



# Rationale behind dynamic programming

Aka: dynamic optimization

- Solve a complex problem by breaking it down into a collection of simpler subproblems
- Solve each of those subproblems just once and store their solutions
- Look up the previously computed solution the next time the same subproblem occurs
- Thereby save computation time at the expense of a hopefully modest expenditure in storage space

Memoization: storing solutions to subproblems instead of recomputing them

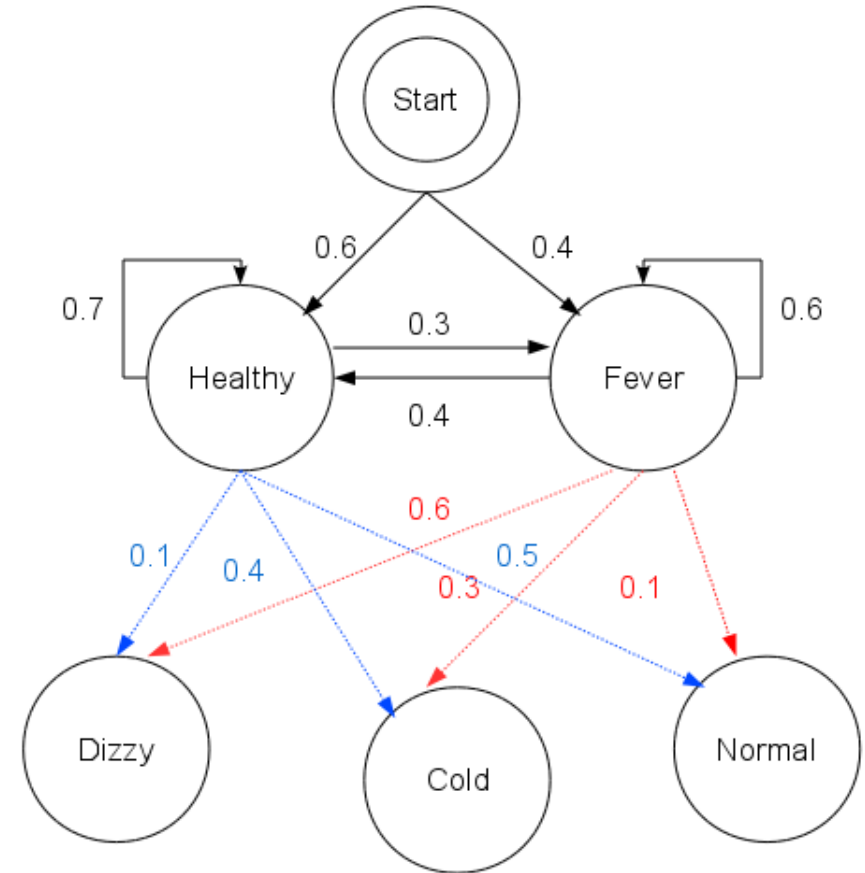
# Viterbi algorithm: an example

Consider a village where all villagers are either **healthy** or have a **fever** and only the village doctor can determine whether each has a fever. The doctor diagnoses fever by asking patients how they feel. The villagers may only answer that they feel **normal**, **dizzy**, or **cold**.

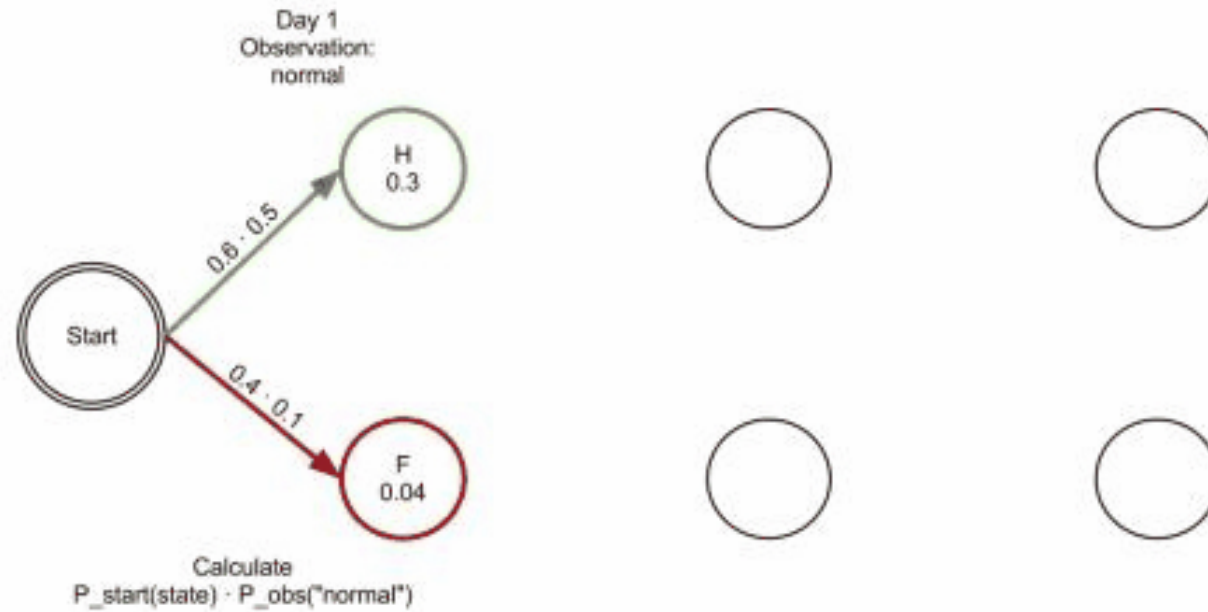
Python representation of the given HMM:

```
obs = ('normal', 'cold', 'dizzy')
states = ('Healthy', 'Fever')
start_p = {'Healthy': 0.6, 'Fever': 0.4}
trans_p = {
    'Healthy' : {'Healthy': 0.7, 'Fever': 0.3},
    'Fever' : {'Healthy': 0.4, 'Fever': 0.6}
}
emit_p = {
    'Healthy' : {'normal': 0.5, 'cold': 0.4, 'dizzy': 0.1},
    'Fever' : {'normal': 0.1, 'cold': 0.3, 'dizzy': 0.6}
}
```

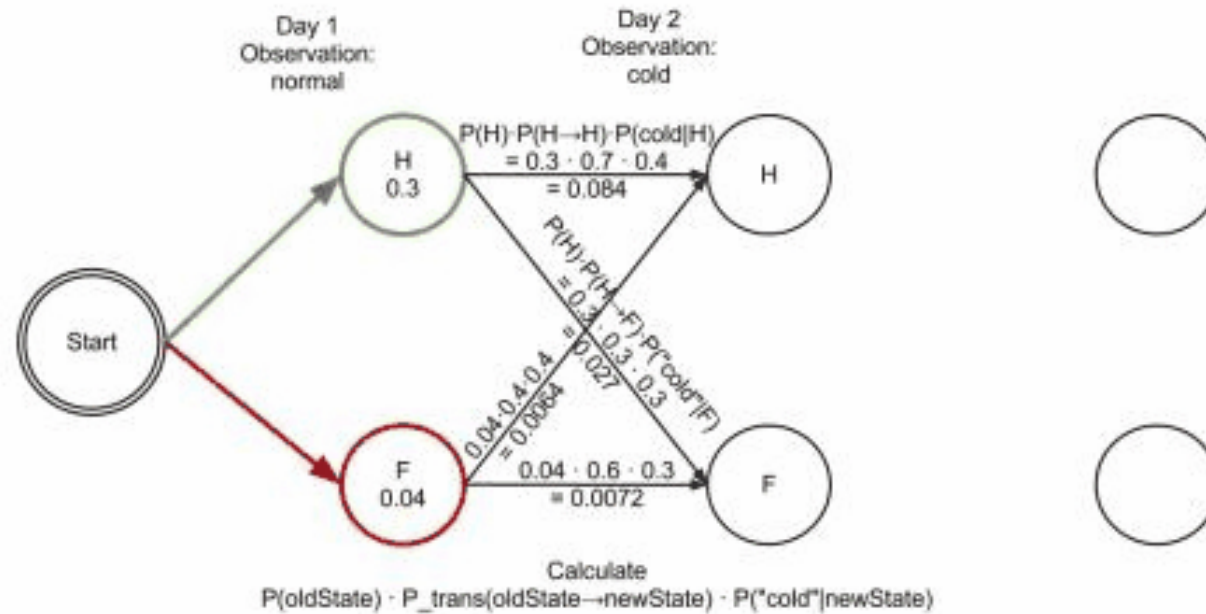
Graphical representation of the given HMM



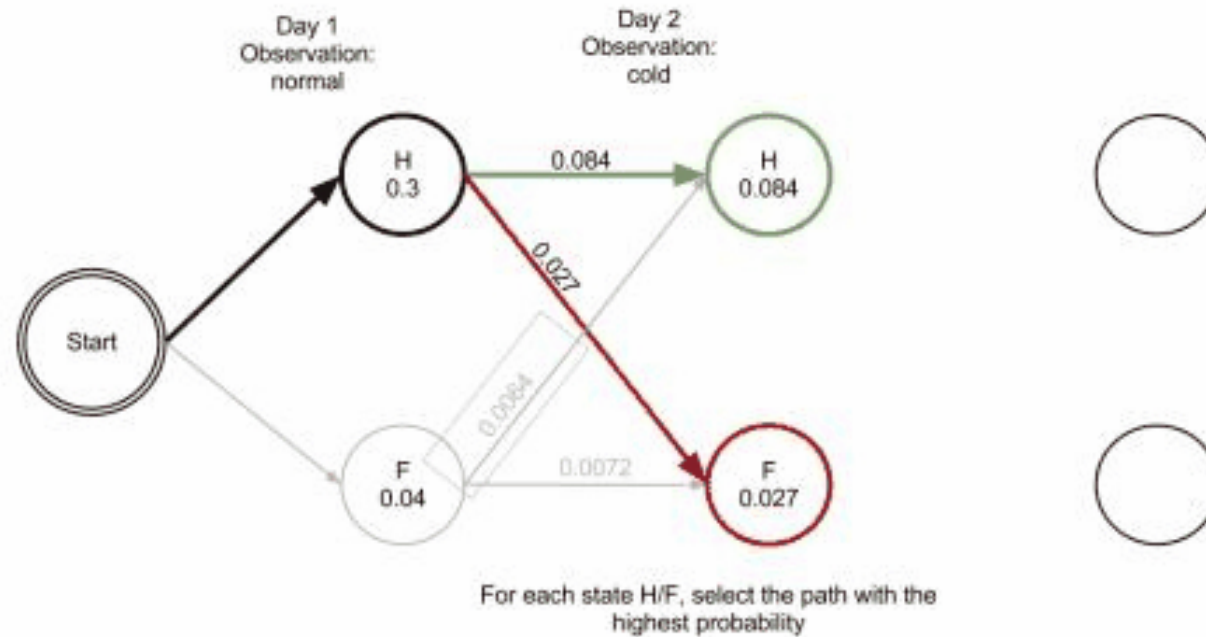
# Viterbi algorithm: an example



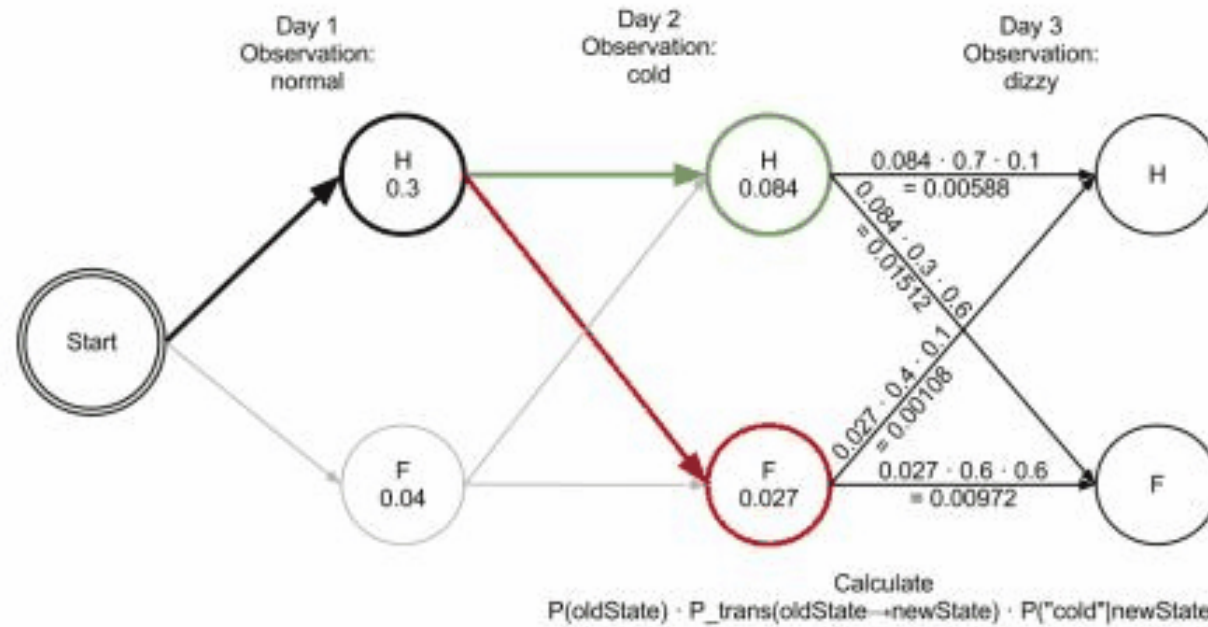
# Viterbi algorithm: an example



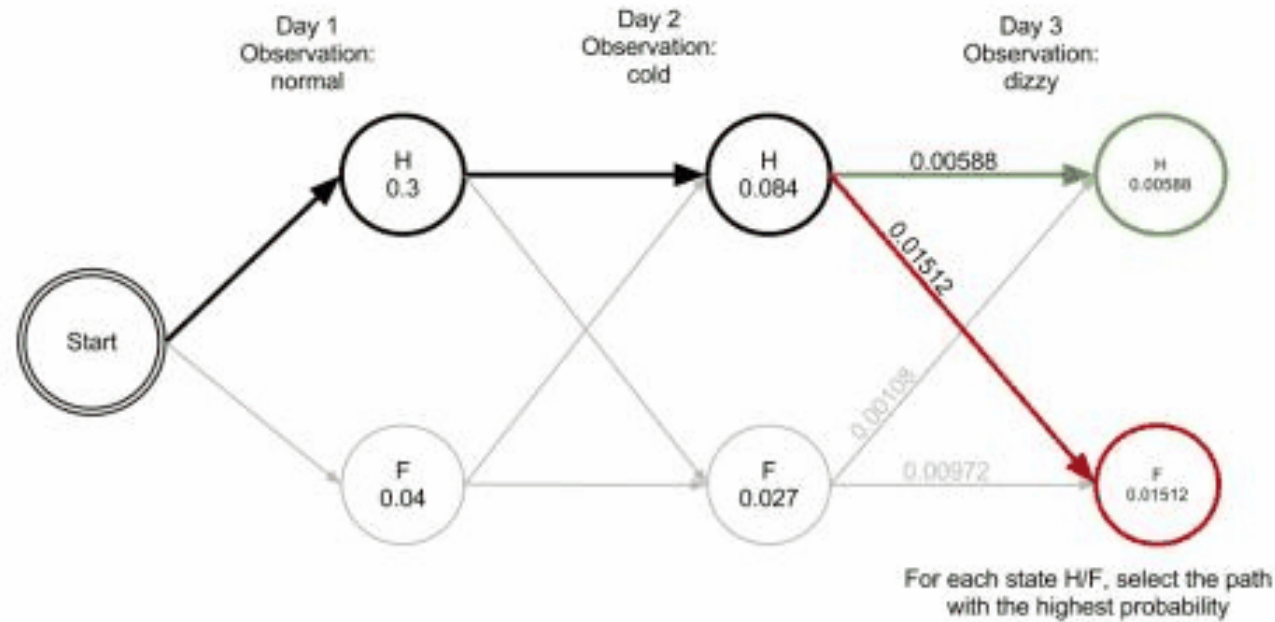
# Viterbi algorithm: an example

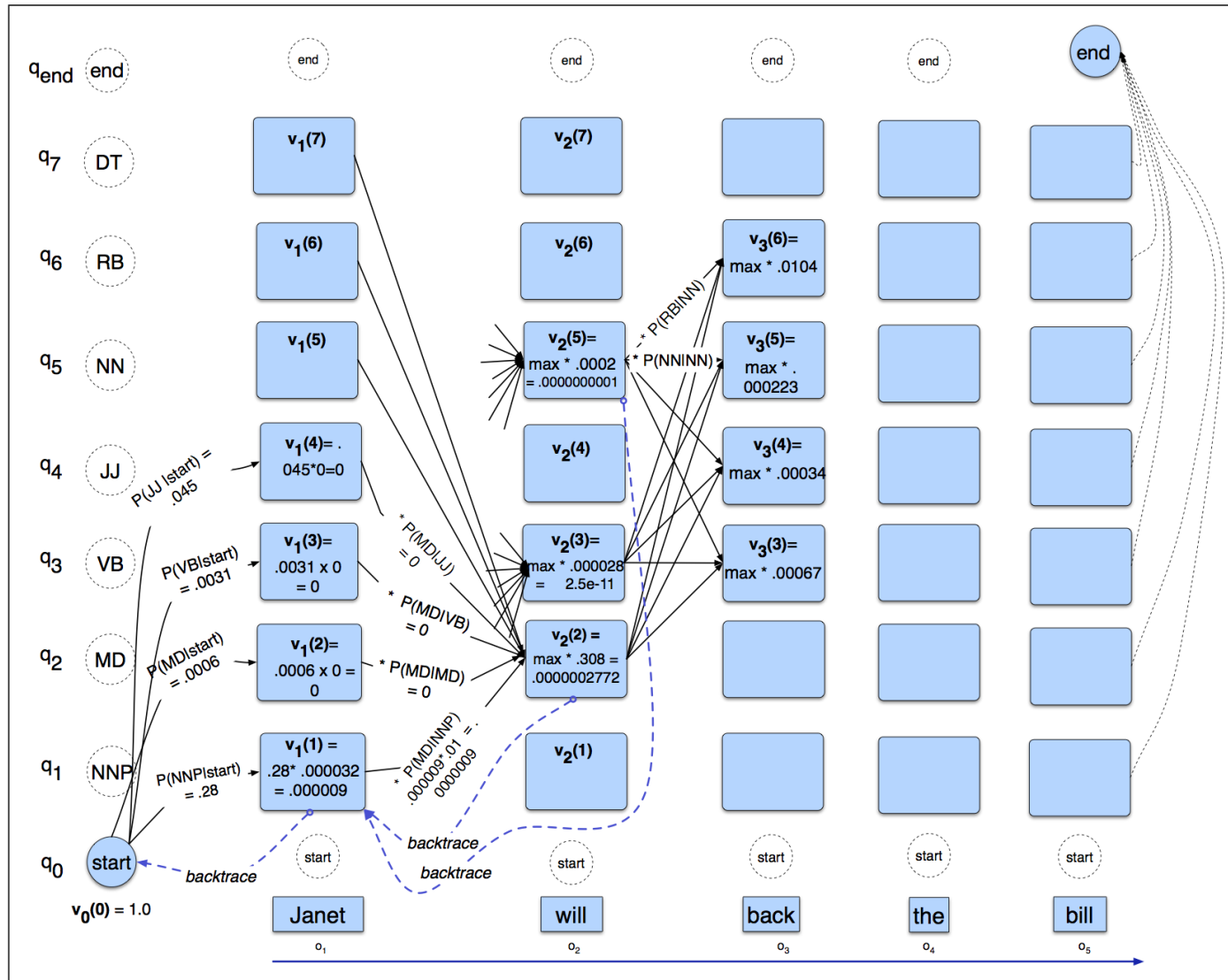


# Viterbi algorithm: an example



# Viterbi algorithm: an example





- The first few entries in the individual state columns for the Viterbi algorithm.
- Each cell keeps the probability of the best path so far and a pointer to the previous cell along that path.

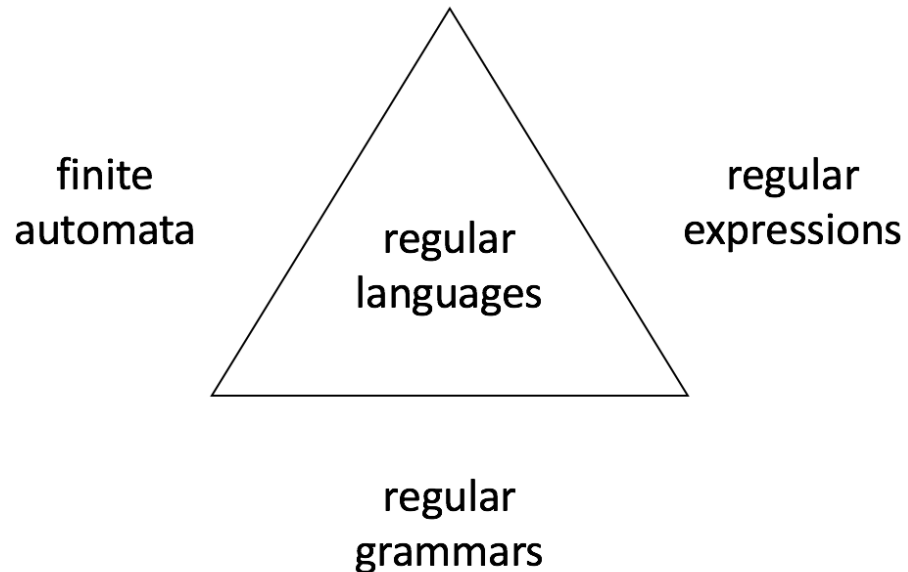


# Syntax

- The way words are arranged together
- Syntactic notions
  - Regular expressions: representing the sequences of words
  - N-grams: computing probabilities for the sequences of words
  - Parts-of-speech: acting as a kind of equivalence class for words
  - Models of syntax and grammar
    - Regular grammars
    - Context-free grammars
    - ...

# Representing an FSA as a grammar

Three equivalent ways of describing regular languages



- The Chomsky hierarchy
- Natural language and its complexity
- Formal models and formal languages
- Power of formal models: complexity of the phenomena they can describe

# A formal grammar for the sheep talk

starting symbol =  $S$

non-terminals =  $\{S, A, B, C\}$

terminals =  $\{b, a, !\}$

Rewrite rules or production rules

$S \rightarrow bA$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aC$

$C \rightarrow !$

/baa+!/  

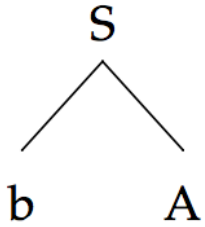

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{b, a, !\}$

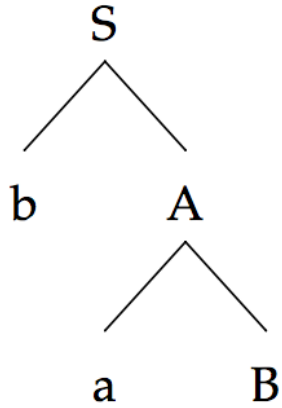
$q_0 = q_0$

$F = \{q_4\}$

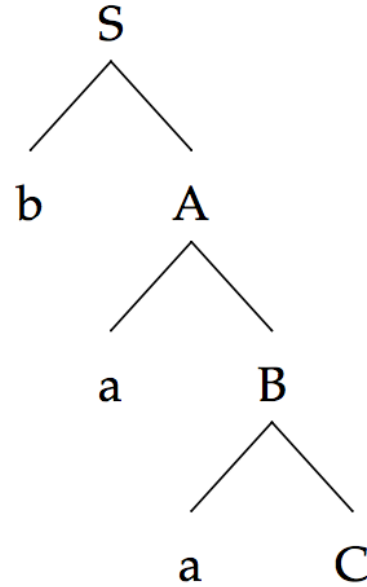
# Derivation of the sheep talk



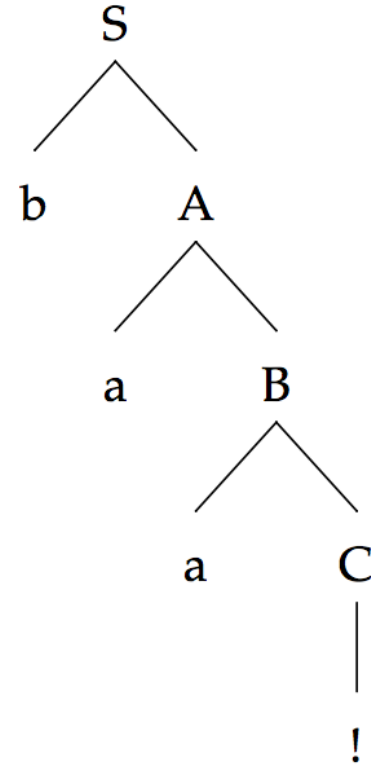
$$S \rightarrow bA$$



$$A \rightarrow aB$$



$$B \rightarrow aC$$



$$C \rightarrow !$$

# Derivation of the sheep talk

starting symbol =  $S$

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Rewrite rules or production rules

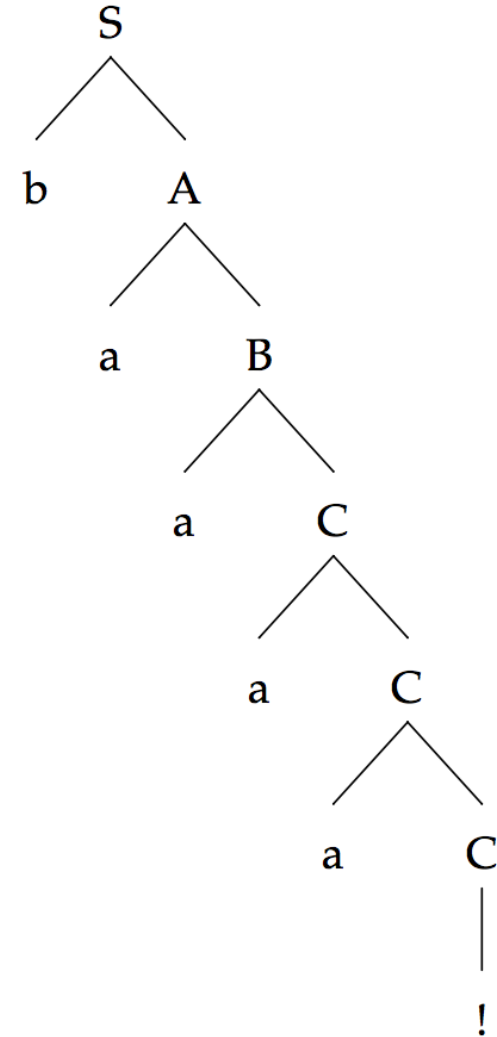
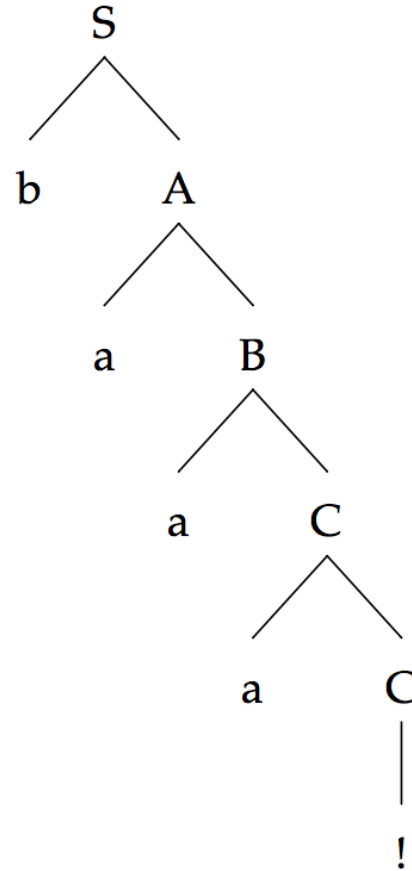
$S \rightarrow bA$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aC$

$C \rightarrow !$

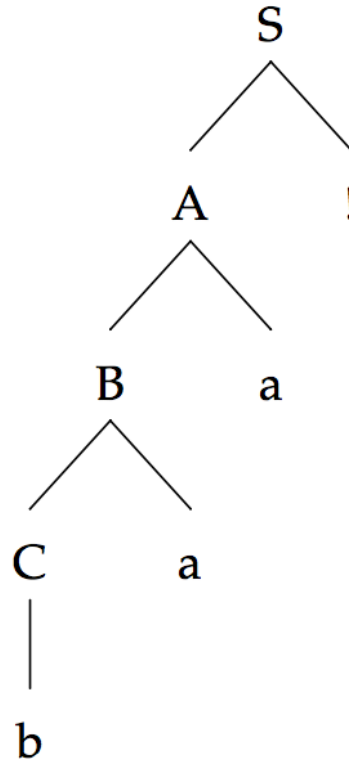


# Rewrite rules of a regular grammar

Left-branching structures

$X \rightarrow Ya$   
 $X \rightarrow a$

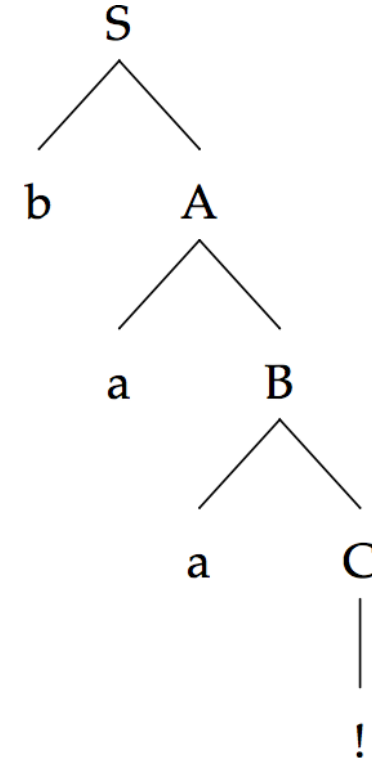
$S \rightarrow A!$   
 $A \rightarrow Ba$   
 $B \rightarrow Ca$   
 $C \rightarrow Ca$   
 $C \rightarrow b$



Right-branching structures

$X \rightarrow aY$   
 $X \rightarrow a$

$S \rightarrow bA$   
 $A \rightarrow aB$   
 $B \rightarrow aC$   
 $C \rightarrow aC$   
 $C \rightarrow !$



# Rewrite rules of a regular grammar

## Strong equivalence vs. weak equivalence

- Two grammars are **strongly equivalent** if they generate the same set of strings **and** if they assign the same phrase structure to each sentence (allowing merely for renaming of the non-terminal symbols).
- Two grammars are **weakly equivalent** if they generate the same set of strings **but** do not assign the same phrase structure to each sentence.

# Regular grammars for natural language: problems

- Redundancy
- Centre Embedding

E.g.

The students **the police arrested** complained.

The luggage **that the passengers checked** arrived.

The luggage **that the passengers that the storm delayed checked** arrived.



# Context-free grammars for natural language

- Less restrictive and hence more powerful
- Aka: phrase-structure grammars
- Equivalent to Backus-Naur Form (BNF)
- Backbone of many formal models of the syntax of natural language
- Applications
  - syntactic parsing, semantic interpretation etc.
  - grammar checking, semantic interpretation, dialogue understanding, machine translation etc.
- Computationally tractable

# Context-free grammars: the formal definition

$S$ : a designated start symbol;

$\Sigma$ : a set of terminal symbols;

$N$ : a set of non-terminal symbols;

$R$ : a set of rewrite rules of the form  $A \rightarrow \beta$

where  $A$  is a non-terminal

and  $\beta$  is a string of elements from the infinite set  $(\Sigma \cup N)^*$ .

Or most commonly written as **Chomsky Normal Form (CNF)**:

$A \rightarrow BC$ , or,  $A \rightarrow a$ , where,  $A, B, C \in N$ , and,  $a \in \Sigma$ .

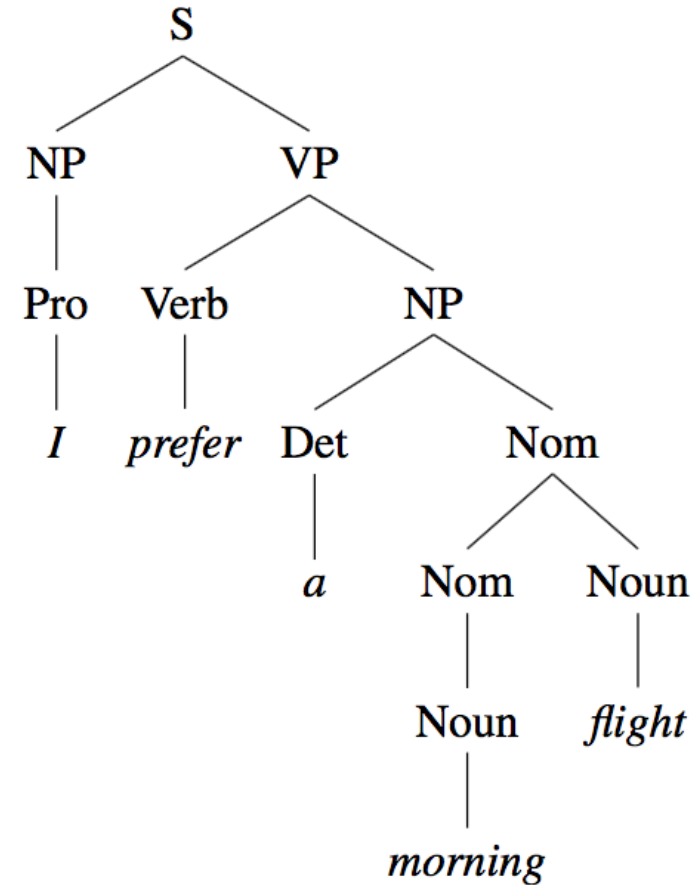
# Context-free grammars: an example

E.g. I prefer a morning flight

Bracketed notation:

[S [NP [Pro I]] [VP [V prefer] [NP [Det a]  
[Nom [N morning] [Nom [N flight]]]]]]]

Parse tree:



# Context-free grammars: an example

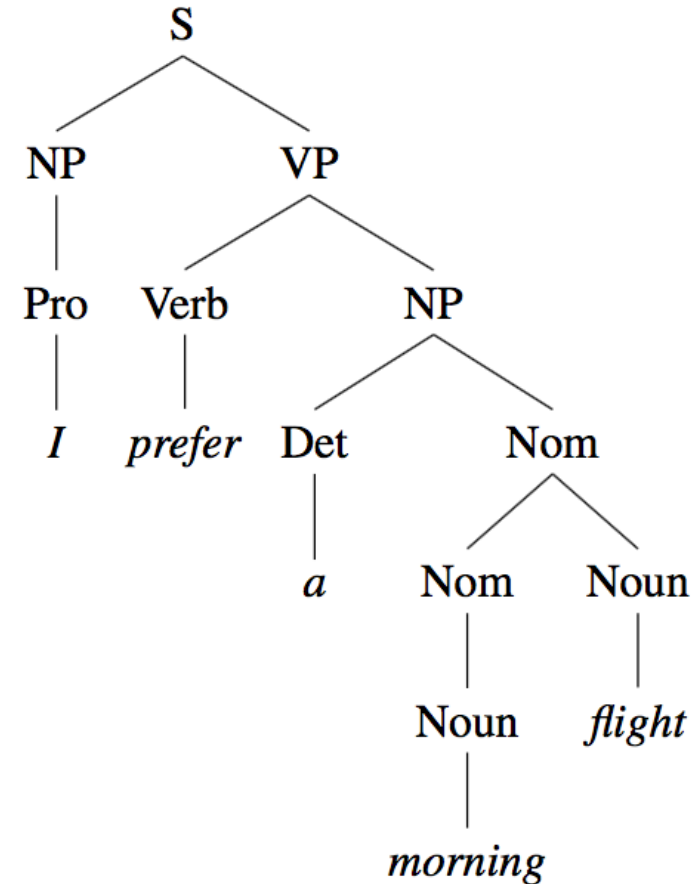
<i>Noun</i>	→	<i>flights</i>   <i>breeze</i>   <i>trip</i>   <i>morning</i>
<i>Verb</i>	→	<i>is</i>   <i>prefer</i>   <i>like</i>   <i>need</i>   <i>want</i>   <i>fly</i>
<i>Adjective</i>	→	<i>cheapest</i>   <i>non-stop</i>   <i>first</i>   <i>latest</i>   <i>other</i>   <i>direct</i>
<i>Pronoun</i>	→	<i>me</i>   <i>I</i>   <i>you</i>   <i>it</i>
<i>Proper-Noun</i>	→	<i>Alaska</i>   <i>Baltimore</i>   <i>Los Angeles</i>   <i>Chicago</i>   <i>United</i>   <i>American</i>
<i>Determiner</i>	→	<i>the</i>   <i>a</i>   <i>an</i>   <i>this</i>   <i>these</i>   <i>that</i>
<i>Preposition</i>	→	<i>from</i>   <i>to</i>   <i>on</i>   <i>near</i>
<i>Conjunction</i>	→	<i>and</i>   <i>or</i>   <i>but</i>

**Figure 11.2** The lexicon for  $\mathcal{L}_0$ .

Grammar Rules	Examples
$S \rightarrow NP VP$	I + want a morning flight
$NP \rightarrow Pronoun$	I
$NP \rightarrow Proper-Noun$	Los Angeles
$NP \rightarrow Det Nominal$	a + flight
$Nominal \rightarrow Nominal Noun$	morning + flight
$Nominal \rightarrow Noun$	flights
$VP \rightarrow Verb$	do
$VP \rightarrow Verb NP$	want + a flight
$VP \rightarrow Verb NP PP$	leave + Boston + in the morning
$VP \rightarrow Verb PP$	leaving + on Thursday
$PP \rightarrow Preposition NP$	from + Los Angeles

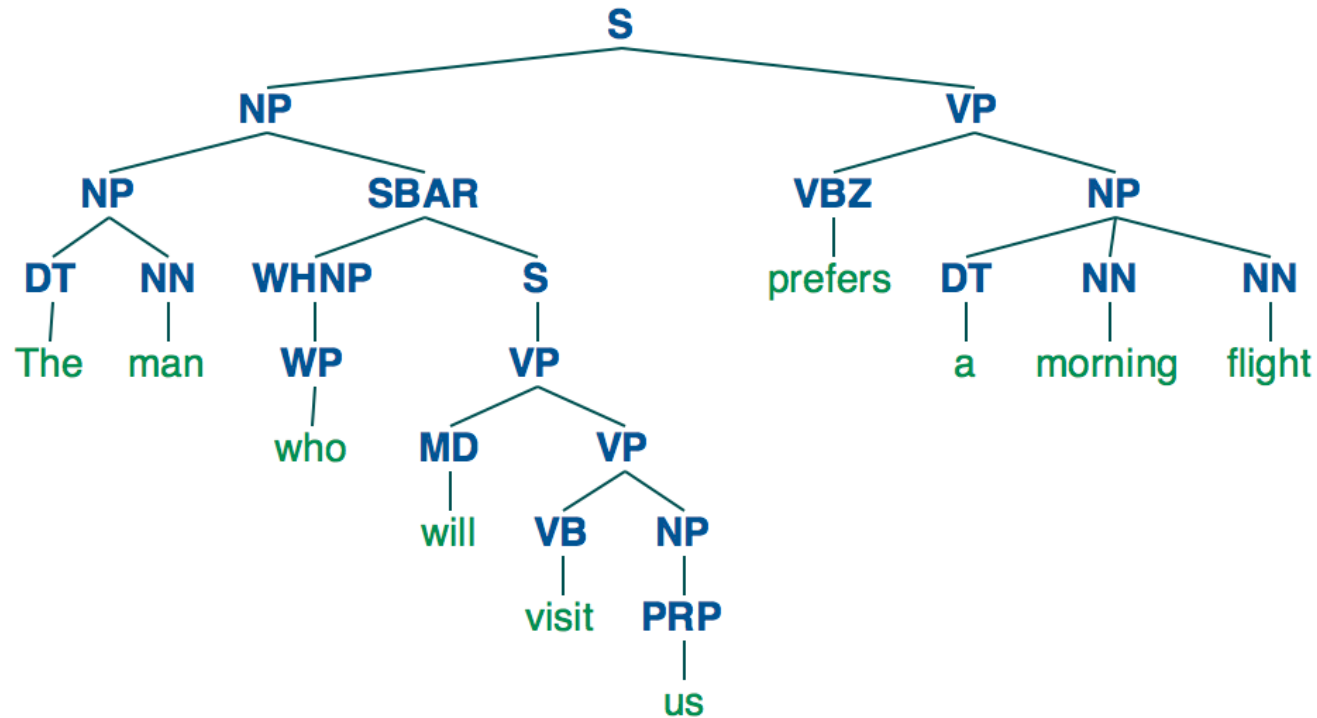
**Figure 11.3** The grammar for  $\mathcal{L}_0$ , with example phrases for each rule.

Parse tree:



# Context-free grammars: another example

E.g. The man who will visit us prefers a morning flight



# What are formal grammars used for?

- Generating sentences
- Recognizing grammatical and ungrammatical sentences
- Parsing sentences

# Syntactic parsing with context-free grammars

- **Parsing:** recognizing an input string and assigning a structure to the string
- **Syntactic parsing:** recognizing a sentence and assigning a syntactic structure to the sentence
  - **A context-free grammar:** a declarative formalism of how words combine to form larger linguistic units (phrases and sentences)
  - **A parsing algorithm:** employing the grammar to produce parse trees for a given sentence

# Parsing as a search problem

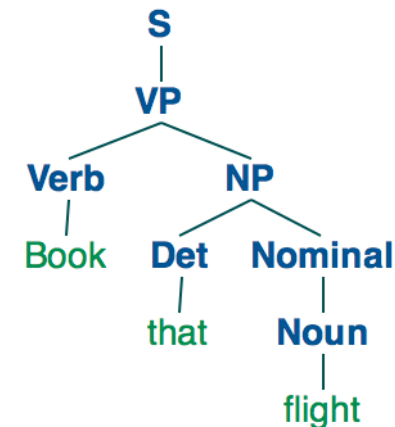
**Goal:** to find all the trees whose root is the start symbol *S* and which cover exactly the words in the input (leaves)

E.g. *Book that flight*

**Constraint 1:** Whatever else is true of the final parse tree, its leaves must be three words from the input: *book*, *that*, and *flight*.

**Constraint 2:** Whatever else is true of the final parse tree, it must have one root: the start symbol *S*.

Grammar	Lexicon
$S \rightarrow NP VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux NP VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper-Noun$	$Proper-Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal Noun$	
$Nominal \rightarrow Nominal PP$	
$VP \rightarrow Verb$	
$VP \rightarrow Verb NP$	
$VP \rightarrow Verb NP PP$	
$VP \rightarrow Verb PP$	
$VP \rightarrow VP PP$	
$PP \rightarrow Preposition NP$	





# Top-down parsing

Start with the root and keep employing rules whenever possible to expand the tree .

With a context-free grammar, the top-down algorithm inspects the left-hand side of the rules to look for matches.

*a*

# Bottom-up parsing

Start with the leaves and keep employing rules whenever possible to combine the tree pieces.

With a context-free grammar, the bottom-up algorithm inspects the right-hand side of the rules to look for matches.

*c*

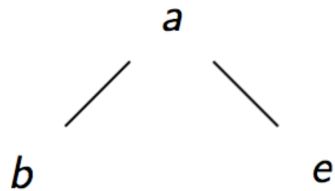
*d*

*f*

# Top-down parsing

Start with the root and keep employing rules whenever possible to expand the tree .

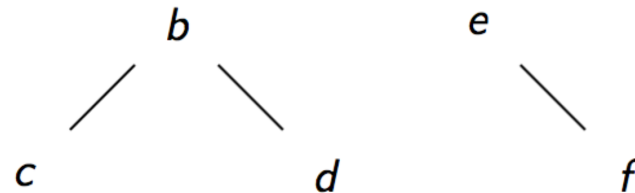
With a context-free grammar, the top-down algorithm inspects the left-hand side of the rules to look for matches.



# Bottom-up parsing

Start with the leaves and keep employing rules whenever possible to combine the tree pieces.

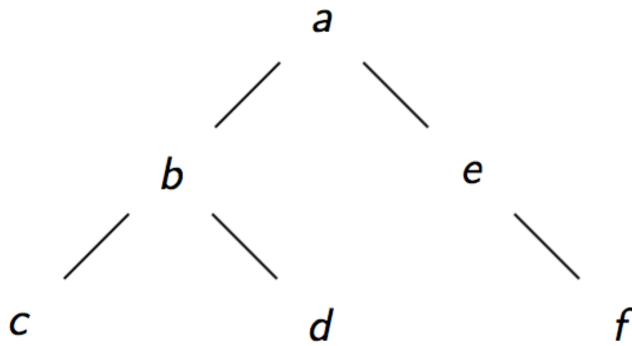
With a context-free grammar, the bottom-up algorithm inspects the right-hand side of the rules to look for matches.



# Top-down parsing

Start with the root and keep employing rules whenever possible to expand the tree .

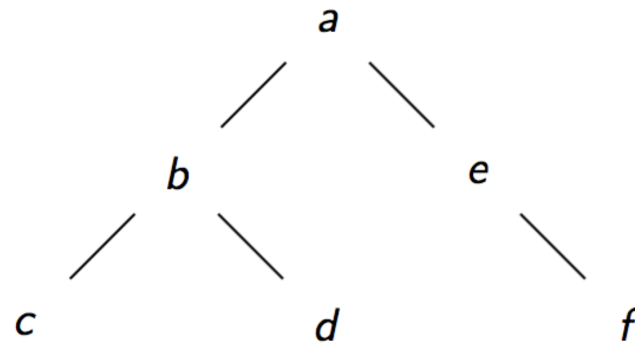
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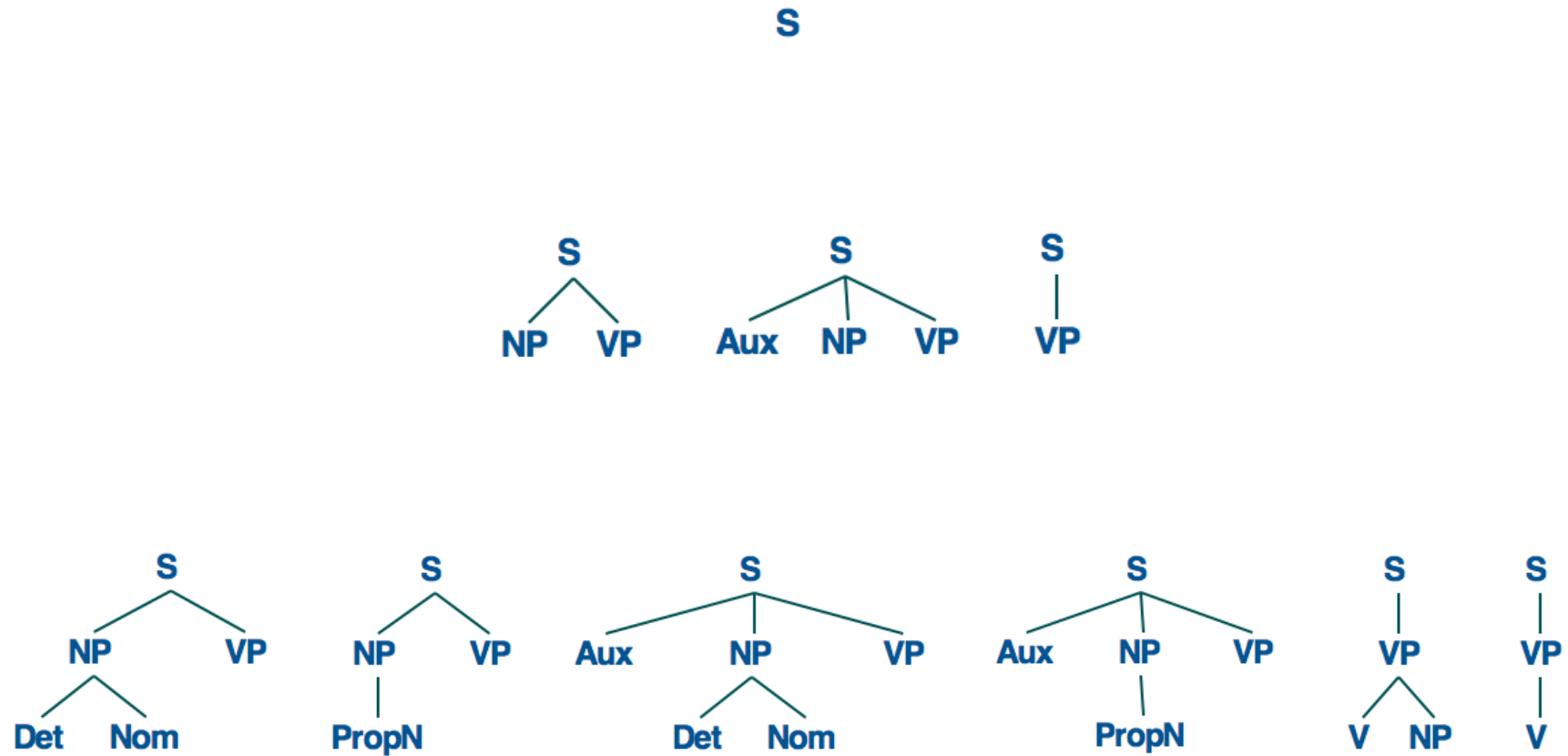
# Bottom-up parsing

Start with the leaves and keep employing rules whenever possible to combine the tree pieces.

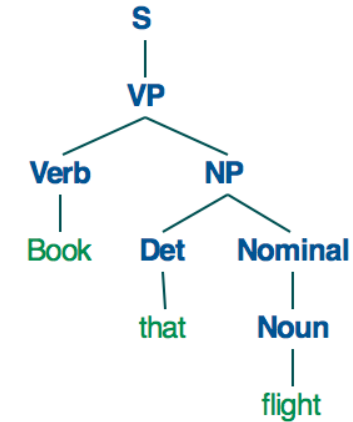
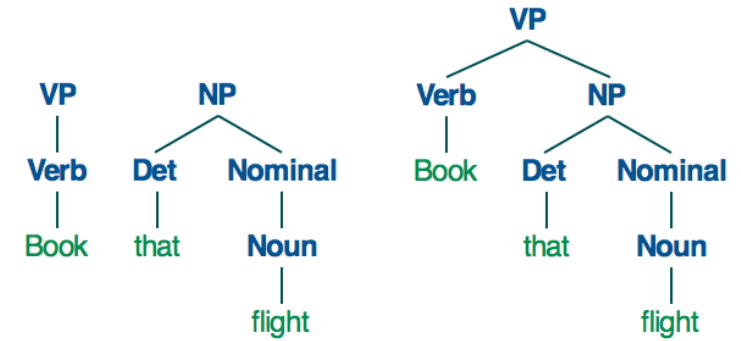
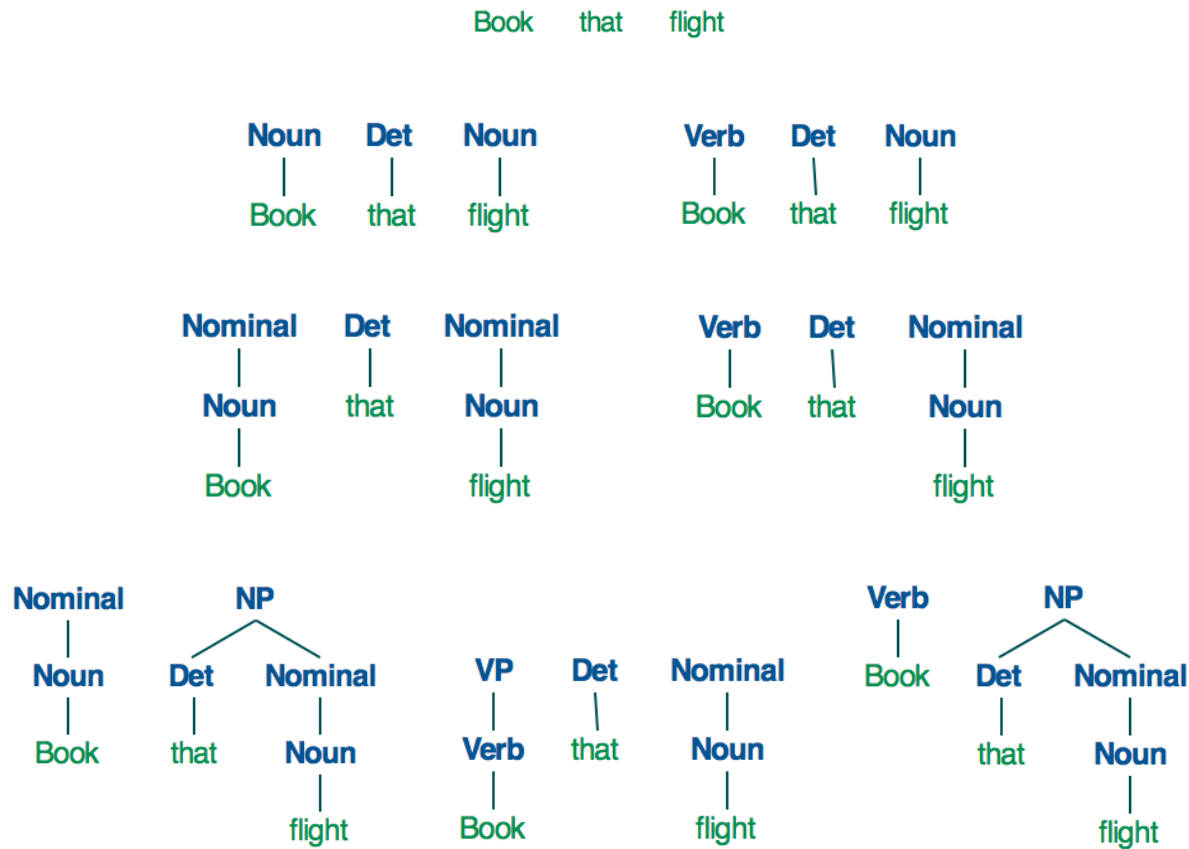
With a context-free grammar, the bottom-up algorithm inspects the right-hand side of the rules to look for matches.



# Top-down parsing: an example



## Bottom-up parsing: an example



# Strengths and weaknesses

## The top-down algorithm

- Does not explore trees that cannot result in an S, nor subtrees that cannot find a place in some S-rooted trees.
- Does waste time on trees that are not consistent with the input.

## The bottom-up algorithm

- Does not explore trees that are not consistent with the input.
- Does waste time on subtrees that cannot lead to an S or fit in with any of their neighbors.

# Models and algorithms

- Language models: **correct** parsing
  - Lexical-functional grammar
  - Head-driven phrase structure grammar
  - Link grammar
  - Dependency grammar
  - Probabilistic context-free grammar ...
- Parsing algorithms: **efficient** parsing
  - CYK parsing (Cocke-Younger-Kasami algorithm, Chomsky Normal Form)
  - Earley parsing
  - Chart parsing
  - Left-corner parsing
  - ATN parsing ...

# Practical 6: analyzing sentence structure with NLTK

<http://www.nltk.org/book/ch08.html>

- Define a simple grammar to parse a simple sentence
- Play with the recursive descent parser application
- Parse with CFG (RecursiveDescentParser, ShiftReduceParser)
- Write your own grammars in a text file
- Draw a parse tree
- Develop a grammar by using the Treebank corpus
- Use [the Stanford Parser](#) to parse sentences and draw trees



## At the end of this session you will

- understand the rationale behind dynamic programming;
- know how the Viterbi algorithm works for POS tagging;
- know how to describe a language using regular and context-free grammars;
- know how top-down parsing and bottom-up parsing work;
- understand the strengths and weaknesses of these two parsing algorithms;
- understand how language models and parsing algorithms work together;
- learn to analyze sentence structure with NLTK.

# Homework

- Read/Review (Quiz 5 on Nov. 7, 2018)
  - [J+M 8](#) (8.1-8.4; 8.7)
  - [J+M 10](#)
- Read
  - [J+M 11](#)
- Practice
  - <http://www.nltk.org/book/ch08.html>
- Practical 5 assignment - submit your codes on GradeScope (DDL: Nov. 5)

## Next session

Statistical Parsing and Dependency Parsing