

PRACTICAL 9

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# CYK ALGORITHM REVISITED

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# CONVERSION TO CHOMSKY NORMAL FORM

- Requirement: grammars used with the CYK algorithm must be in Chomsky Normal Form (CNF), i.e. each rule is of the form:

$$A \rightarrow BC \text{ or } A \rightarrow w$$

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# CONVERSION TO CHOMSKY NORMAL FORM

Restriction:  $A \rightarrow B C$  or  $A \rightarrow w$

Original Grammar

$S \rightarrow VP$

$NP \rightarrow Prn$

$NP \rightarrow Det Nom$

$Nom \rightarrow N$

$Nom \rightarrow Nom PP$

$VP \rightarrow V$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$PP \rightarrow P NP$

$Det \rightarrow the \mid a$

$N \rightarrow book \mid cook \mid meal$

$V \rightarrow book \mid cook \mid prefer$

$Prn \rightarrow I \mid she \mid me$

$P \rightarrow from \mid for \mid to$

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2 kinds of offending rules:

▶  $A \rightarrow B$

▶  $A \rightarrow B C D$

(also:  $A \rightarrow w B$ )

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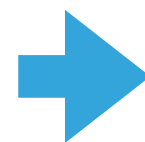
# CONVERSION TO CHOMSKY NORMAL FORM

- ▶ For the 1st kind,  $A \rightarrow B$  (unit productions), we can rewrite the right-hand side of the original rules with the right-hand side of all the non-unit production rules that they ultimately lead to:

e.g.

Original Grammar

$NP \rightarrow Prn$   
 $Prn \rightarrow I \mid she \mid me$



CNF form

$NP \rightarrow I \mid she \mid me$   
 $Prn \rightarrow I \mid she \mid me$

- ▶ flattening of the grammar

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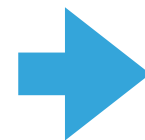
## CONVERSION TO CHOMSKY NORMAL FORM

- ▶ For the 2nd kind,  $A \rightarrow B C D$  (right-hand side length  $> 2$ ), we can replace a pair of non-terminals with a new non-terminal and introduce a new production rule:

e.g.

Original Grammar

$VP \rightarrow V NP PP$



CNF form

$VP \rightarrow X1 PP$

$X1 \rightarrow V NP$

- ▶ In the case of longer right-hand sides, we simply iterate this process until the offending rule has been replaced by rules of length 2.

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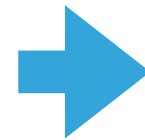
# CONVERSION TO CHOMSKY NORMAL FORM

- ▶ 3rd kind?

e.g.

Original Grammar

*INF-VP*  $\rightarrow$  *to VP*



CNF form

???

# CONVERSION TO CHOMSKY NORMAL FORM

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???

???

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$NP \rightarrow Det\ Nom$

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$Nom \rightarrow Nom\ PP$

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$VP \rightarrow V\ NP$

$VP \rightarrow X1\ PP$

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# CYK ALGORITHM

- ▶ For a sentence of length  $n$ , we will work with the upper-triangular portion of an  $(n + 1) \times (n + 1)$  matrix
- ▶ e.g.

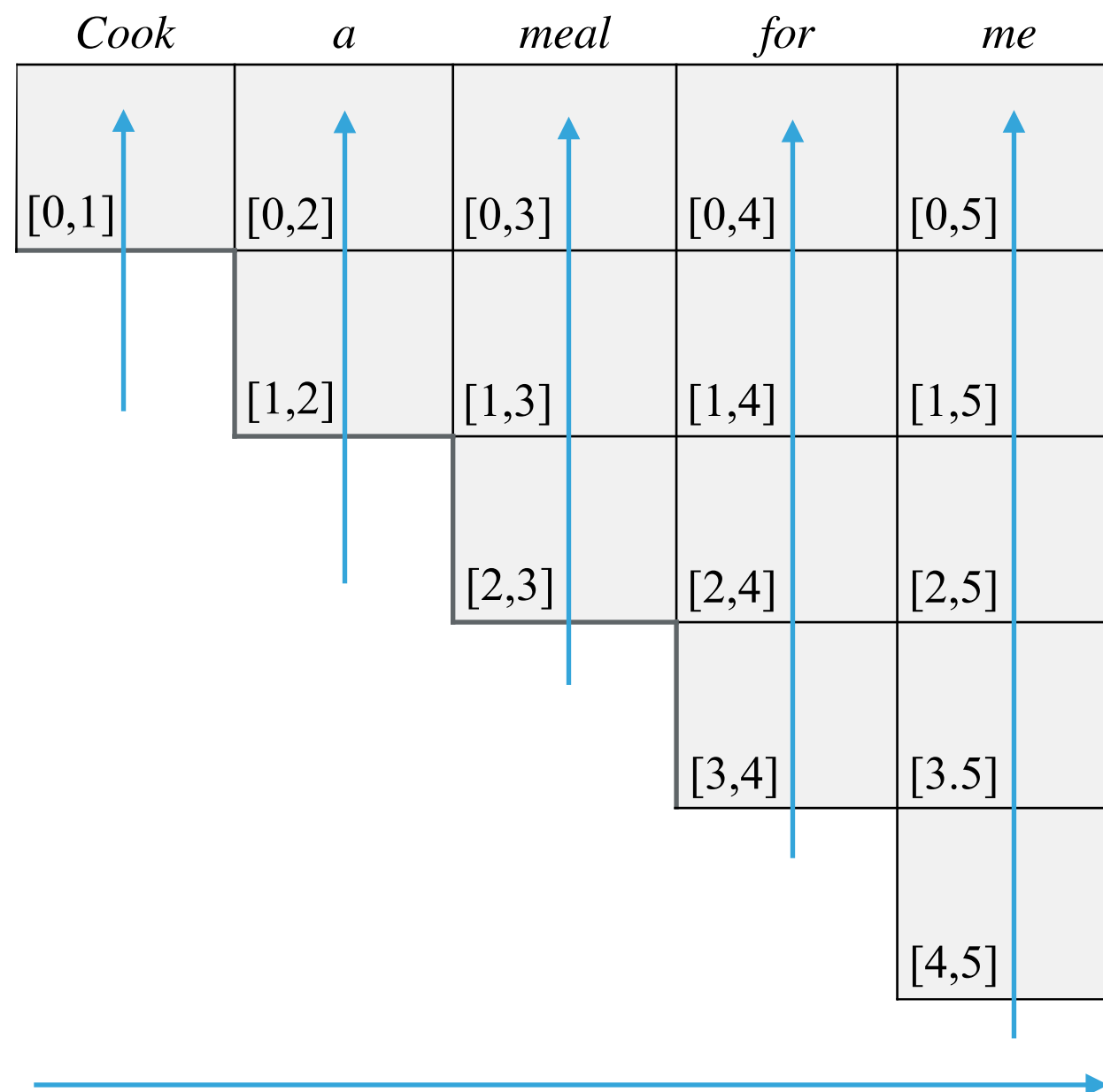
$_0$  *Cook*  $_1$  *a*  $_2$  *meal*  $_3$  *for*  $_4$  *me*  $_5$

<i>Cook</i>	<i>a</i>	<i>meal</i>	<i>for</i>	<i>me</i>	
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	→ the whole sentence
	[1,2]	[1,3]	[1,4]	[1,5]	
		[2,3]	[2,4]	[2,5]	
			[3,4]	[3,5]	
				[4,5]	

- ▶ Each cell  $[i, j]$  in this matrix contains the set of non-terminals that represent all the constituents that span positions  $i$  through  $j$  of the input

# CYK ALGORITHM

- ▶ For each cell  $[i, j]$ , there must be a position in the input,  $k$ , where the constituent represented by  $[i, j]$  can be split into two parts such that  $i < k < j$ .



# CYK ALGORITHM

- ▶ The **superdiagonal** row in the matrix contains the **parts of speech** (actually, **all possible pre-terminals**) for each input word in the input
- ▶ e.g.

<i>Cook</i>	<i>a</i>	<i>meal</i>	<i>for</i>	<i>me</i>
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]

## Our grammar

$S \rightarrow book \mid cook \mid prefer$   
 $S \rightarrow V NP$   
 $S \rightarrow X1 PP$   
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 $Nom \rightarrow Nom PP$   
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# CYK ALGORITHM

- ▶ The subsequent diagonals above that superdiagonal contain constituents that cover all the spans of increasing length in the input

<i>Cook</i>	<i>a</i>	<i>meal</i>	<i>for</i>	<i>me</i>
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- What's the result (syntactic tree) of our parsing algorithm?  
Is there any kind of ambiguities?