Binary Numbers

Not Really in Textbook / Kinda Section 2.1

Introduction

- Binary numbers are an important part of low-level computing
- Having a basic knowledge of binary numbers will be important for us to understand how data types are represented

Number Systems

- In daily usage, the number system we use is the **base ten** or **decimal** number system
  - deci~ has to do with ten
- This system has ten digits: **0**, **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, and **9**
- Any whole number can be written using only some combination of these ten digits

- However, decimal is not the only number system
  - Ancient Babylonians used a base 60 number system
- Really possible to have a number system of any base
  - This is a topic from the field of number theory

- In programming / computer science, two commonly used:
  - **Binary numbers** – base two
  - **Hexadecimal numbers** – base sixteen
  - Octal numbers (base eight) are also sometimes used

Binary Numbers

- Binary number system consists of only two digits: **0** and **1**
- Any decimal natural number can be represented as a binary number
  - Natural number = non-negative integer

Decimal	Binary	Decimal	Binary
0	**0**	**8**	**1000**
1	1	9	1001
2	**10**	10	1010
3	11	11	1011
4	**100**	12	1100
5	101	13	1101
6	110	14	1110
7	111	15	1111

- Some observations
  - Binary numbers grow in length faster than decimal numbers
  - Powers of two (0, 2, 4, 8, etc.) are all a 1 followed by only 0s
  - Numbers before a power of two (1, 3, 7, 15, etc.) are all 1s

- In programming, we're commonly dealing with a fixed number of binary digits at a time, like eight or 32
- Thus, common to put extra 0s at the front of a binary number that has less than that number of digits
  - I.e., instead of 1010, we would write 00001010
- Also might put a space between every four digits to make it easier to read
  - Kinda like how we put commas every three digits in long decimal numbers

Decimal	Binary	Decimal	Binary
0	**0000 0000**	**8**	**0000 1000**
1	0000 0001	9	0000 1001
2	**0000 0010**	10	0000 1010
3	0000 0011	11	0000 1011
4	**0000 0100**	12	0000 1100
5	0000 0101	13	0000 1101
6	0000 0110	14	0000 1110
7	0000 0111	15	0000 1111

Converting Binary to Decimal

- Each 1 in a binary number corresponds to a **power of two** (**2^n**, where n is some integer)
- For example, the 1 in **0001** corresponds to **2^0**
  - = 1
- And, the 1s in **1001** correspond to **2^3** and **2^0**, respectively
  - 8 + 1 = 9

Binary Digits	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**1**
Power of Two	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
In Decimal	128	64	32	16	8	4	2	1

- So, to convert a binary number to decimal, we need to look at its 1s
- And then we make a sum of the corresponding powers of two
- We add up that sum and it gives us the corresponding decimal number

- Consider the binary number **1001 0111**
- Its 1s correspond to: $2^7 + 2^4 + 2^2 + 2^1 + 2^0$
- $= 128 + 16 + 4 + 2 + 1$
- $= 151$

Practice

- What is 1001 in decimal?
- What is 1100 in decimal?

- Answers:
  - 1001 = **9**
    - $\mathbf{2^3} + \mathbf{2^0} = 8 + 1 = 9$
  - 1100 =**12**
    - $\mathbf{2^3} + \mathbf{2^2} = 8 + 4 = 12$

Converting Decimal to Binary

- To convert a decimal number to a binary number, we need to figure out which digits will be 1s
- To do so, we think about the biggest decimal number that is power of two that is no bigger than the decimal number
- And then repeat on the leftover part and continue on

- Consider 151
  - We start with all 0s: 0000 0000
  - Biggest number that is a power of two that is no bigger than 151 is 128, which is $\mathbf{2^7}$
- So we set the corresponding digit to 1: 1000 0000
  - 151 - 128 = 23

- What's the biggest power of two that's no bigger than 23?
- $16 = 2^4$
- So set the corresponding digit: 1001 0000

- For 23 - 16 = 7? 4 = 2^2
  - Binary so far: 1001 0100
- For 7 - 4 = 3? 2 = 2^1
  - Binary so far: 1001 0110
- For 3 - 2 = 1? 2^0 = 1
- Final answer: **1001 0111**

Practice

- What is 62 in binary?
- What is 102 in binary?

- Answers:
  - 62 = **0011 1110**
    - 62 = 32 + 16 + 8 + 4 + 2
    - 62 = $2^5 + 2^4 + 2^3 + 2^2 + 2^1$
  - 102 = **0110 0110**
    - 102 = 64 + 32 + 4 + 2
    - 102 = $2^6 + 2^5 + 2^2 + 2^1$

Adding Binary Numbers

- Adding binary numbers works much like it would for adding decimal numbers
- Add digit by digit
- May need to add extra 0s to front if both numbers do not have the same number of digits

0011 (3 in decimal)

\+

1000 (8 in decimal)

=

1011 (11 in decimal)

0011 0011 (51 in decimal)

\+

1000 1000 (136 in decimal)

=

1011 1011 (187 in decimal)

Carrying

- Sometimes we have to carry when adding decimal numbers
- Happens when adding binary numbers as well
- When it happens, we add a 1 to the next column to the left

```
   1
  0101   (5 in decimal)
+
  0001   (1 in decimal)
  ====
  0110   (6 in decimal)
```

111
0111 (7 in decimal)
+
0101 (5 in decimal)
=
1100 (12 in decimal)