Astron 98 Final Project: Modeling the Relationship between Black Hole Radius, Mass, and Curvature of Light

1. Introduction:

This project investigates the relationship between black hole mass, Schwarzschild radii, and the curvature of light as it passes near these massive objects. Using established theoretical models and simulating data where empirical data lacks, this study aims to deepen the understanding of these enigmatic cosmic phenomena and validate general relativity in extreme conditions.

2. Methodology:

Data Generation and Sources:

Given the scarcity of direct observational data on black holes, this project primarily relies on simulated data generated through Python. Masses ranging from 5 to 50 solar masses are considered, from which the corresponding Schwarzschild radii are calculated using the formula

$$R_s = \frac{2GM}{c^2}$$

Modeling Curvature of Light:

To estimate the curvature of light, simplified gravitational lensing models are applied. These models take into account the mass and Schwarzschild radius to predict the bending angle of light, illustrating the gravitational effects exerted by black holes.

Data Analysis Approach:

- Equations Used: Alongside the Schwarzschild radius calculation, we employ a basic lensing equation derived from general relativity to simulate the curvature of light.
- Simulation Techniques: We generate random datasets for black hole masses and apply the theoretical models to predict radii and light curvature.

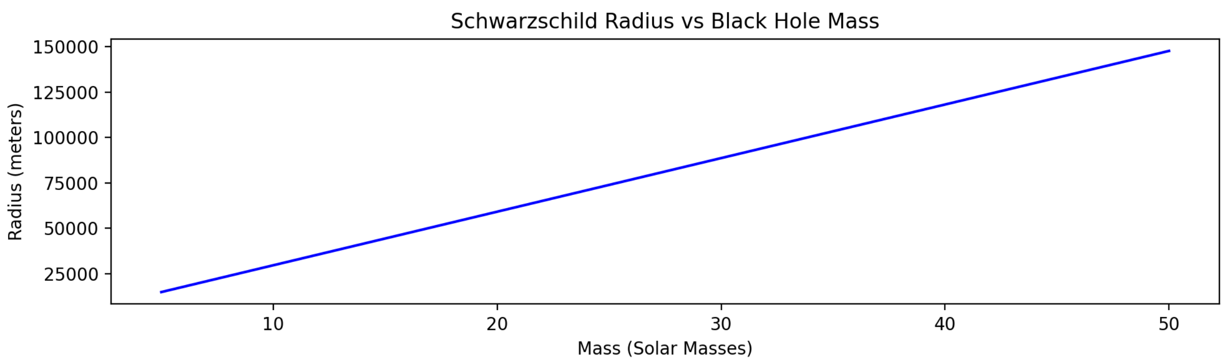
- Error Analysis: By creating synthetic datasets, we analyze the sensitivity of the lensing effect to changes in mass and radius, helping validate the robustness of our models.

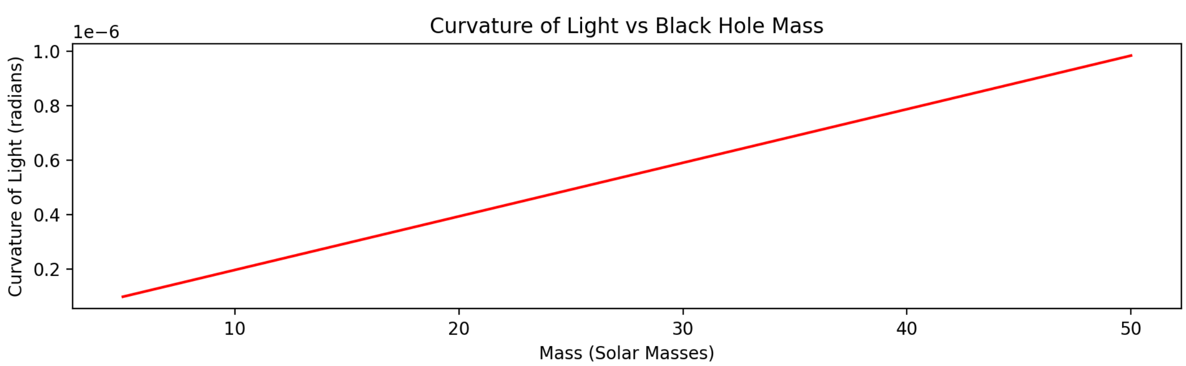
3. Results:

The simulated results reveal a clear proportional increase in both the Schwarzschild radius and the curvature of light with an increase in black hole mass. These results align well with theoretical predictions, indicating a stronger gravitational influence as the mass increases.

Visual Analysis:

The accompanying graphs illustrate the correlation between mass, radius, and curvature of light. The top graph displays the Schwarzschild radius as a function of black hole mass, while the bottom graph shows the curvature of light in radians relative to the black hole mass.





4. Challenges and Adaptations:

Data Limitations:

The primary challenge was the absence of comprehensive empirical data for black holes, which necessitated reliance on simulated datasets based on theoretical models like the Schwarzschild relation.

Complexity of Lensing Equations:

The gravitational lensing equations are inherently complex, and simplifying them for simulation without losing significant scientific integrity required careful consideration. We ensured the simulated models remained scientifically plausible by maintaining fundamental properties like the proportionality between mass and lensing effects.

Adaptation:

To address these challenges, the project focused on creating a robust simulation environment where different scenarios could be tested dynamically, enhancing the understanding of the theoretical framework's limitations and capabilities.

5. Conclusion:

This project successfully demonstrates the expected theoretical relationships in a simulated environment, supporting current models of black hole physics. It underscores the need for more detailed empirical data to further refine these models. Potential future extensions could include integrating more sophisticated lensing equations and exploring the impact of variables like spin and charge on black hole gravitational effects.

Future Directions:

Continuing this research could involve collaboration with observational astronomers to compare simulated results with new data from telescopes like the Event Horizon Telescope. Additionally, advanced statistical techniques could be employed to explore the uncertainties in current black hole mass estimates and their effects on the predicted light curvature.