

$$\mathbf{F} = kq_1q_2$$

Potential energy of 2 charge system

$$U = kq_1q_2$$

Derivation (Imp)

let charge q_1 and q_2 be at a distance n

Electrostatic force on q_2 =

$$F_e = kq_1q_2 \quad (F_e \text{ is positive})$$

External force to move q_2 is

$$F_{\text{ext}} = (-kq_1q_2)/x^2$$

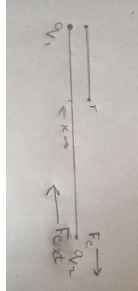
External work done to bring q_2 from infinity to r is:

$$\begin{aligned} W &= \int_{\infty}^r F_{\text{ext}} dx = \int_{\infty}^r (-kq_1q_2 \cdot dx)/x^2 \\ &= -kq_1q_2 \int_{\infty}^r dx/x^2 \\ &= \int_{\infty}^r (1/x^2) dx = \int_{\infty}^r x^{-2} dx \\ &= \int_{\infty}^r (x^{-2+1}) dx / (-2+1) \\ &= \int_{\infty}^r x^{-1} dx = x^{-1+1} / (-1+1) \\ &= x^{-1} / -1 \\ &= -1/x \end{aligned}$$

$$\begin{aligned} W &= -kq_1q_2 [-1/x]_{\infty}^r \\ &= kq_1q_2 [-1/x]_{\infty}^r \\ &= kq_1q_2 [1/r - 1/\infty] \quad [1/\infty = 0] \\ \Rightarrow W &= kq_1q_2/r \end{aligned}$$

This work is stored as potential energy of system

$$\therefore U = kq_1q_2/r$$



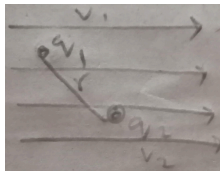
Potential energy of a charge in external E_{field}

$$U = qV$$

Potential energy of 2 charges

in external E_{field}

$$U = q_1V_1 + q_2V_2 + kq_1q_2/r$$



Potential due to dipole

Potential @ P

$$\begin{aligned} V &= V_{-q} + V_{+q} \\ &= K-q/r_1 + k+q/r_2 \\ &= Kq[-1/r_1 + 1/r_2] \Rightarrow Kq[1/r_2 - 1/r_1] \\ &= Kq[r_1 - r_2 / r_1r_2] \quad [r_1r_2 \approx r^2] \\ &= Kq[r_1 - r_2 / r^2] \\ &= Kq[2a \cos \theta] / r^2 \quad [2a = p] \\ \therefore V &= kP \cos \theta / r^2 \end{aligned}$$

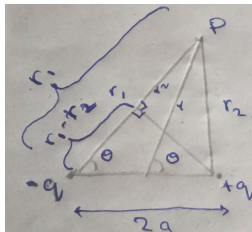
If 'p' is on equatorial line

$$\cos \theta = b/h = r_1 - r_2 / 2a$$

$$r_1 - r_2 = 2a \cos \theta$$

$$\theta = 90^\circ$$

$$V = 0 \quad [\cos \theta = 0]$$



Potential difference

$$W_{e \rightarrow p} = U$$

$$W = U_p - U_i$$

$$W = \Delta U$$

Potential energy of multiple charge system

$$U = k[(q_1q_2/r_{12}) + (q_1q_3/r_{13}) + (q_1q_4/r_{14}) + (q_2q_3/r_{23}) + (q_2q_4/r_{24}) + (q_3q_4/r_{34})]$$

imagine $u = 500J$, work done to bring charge from infinity to configuration, $W_{\infty \rightarrow \text{configuration}}$

Potential

Potential energy per unit charge = V

$$\therefore W = qV \quad (\text{from infinity to } p)$$

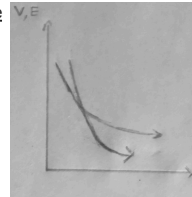
$$\therefore W = q\Delta V \quad (\text{for 2 points})$$

Electric potential due to a single charge

$$V = kq/r \quad \text{Graph of } E_{\text{field}} \text{ \& Potential due to a point charge}$$

$$V = kq/r; \quad v \propto 1/r$$

$$E = kq/r^2; \quad E \propto 1/r^2$$



Unit and DF of potential

SI unit = J/C = volt

$$\text{Dimensional formula} = [ML^2T^{-3}A^{-1}]$$

Electric potential due to a system of charges

$$V = k[q_1/r_1 + q_2/r_2 + q_3/r_3]$$

Sum of potential of potential of all charges

Capacitor

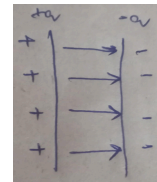
Charge storing device

Capacitance (c) is the ability to store charge

Defined as charge stored per unit potential

$$C = q/v$$

$$\text{SI unit : } e/v \text{ or Farad}$$



Parallel Plate Capacitance

$$C = q/v = q/Ed = q\epsilon_0/\sigma d$$

$$= \epsilon_0 qA/qd \quad [\sigma = q/A]$$

$$C = \epsilon_0 A/d$$

A = plate area; d = distance between plates

Effect of inserting dielectric

$$C_m = k\epsilon_0 A / d$$

$$C_m = KC_{\text{air}}$$

Parallel combination

$$C = C_1 + C_2 + C_3$$

Series combination

$$1/C = 1/c_1 + 1/c_2 + 1/c_3$$

Potential energy of a capacitor

$$W = Q^2/2C; \quad V = Q^2/2C$$

$$U = Q^2/2C$$

$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} QV$$

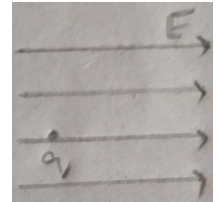
Relation between E and V

$$E = -dv/dr$$

Electrical field is the negative of potential gradient

$$V = Ed$$

If E_{field} is 0, i.e. potential is constant



Equipotential surface

A surface where electric potential is same @ all points is called an equipotential surface

• Properties

- Potential difference between 2 points = 0 [$\Delta v = 0$]
- Work done to move any charge = 0
- Two E_{q} surfaces can never intersect. (If they could, there will be 2 values of potential @ that point, which is more possible) [$W = q\Delta V = 0$]
- E_{field} must be perpendicular to E_{q} surface @ any point

Potential energy of a dipole in uniform E_{field}

Work done to rotate dipole by angle $d\theta$;

$$dw = Fdx \Rightarrow Td\theta \quad [T = \text{torque}]$$

$$W = -PE[\cos \theta_2 - \cos \theta_1]$$

$$V_{\theta_2} - V_{\theta_1} = -PE[\cos \theta_2 - \cos \theta_1]$$

$$\therefore V_{\theta} = -P \cdot E$$

Electrical energy (energy per unit volume)

$$U = \frac{1}{2} \epsilon_0 E^2 Ad$$

$$M = \frac{1}{2} \epsilon_0 E^2$$

Electrostatics of conductor

- Fields inside a conductor material = 0
- E -field is always perpendicular to conductor
- Excess charge always reside on surface of conductor
- Potential inside a conductor @ any point is same as surface
- E -field inside a cavity of a conductor is always 0, electro static shielding
- E -field @ surface of conductor = $E = \sigma / \epsilon_0$