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## Common

```
const double eps = 1e-8;
const double pi = acos(-1.0);

int sgn(double d) {
   if (d > eps)
        return 1;
   if (d < -eps)
        return -1;
   return 0;
}

void to_normal(double& d, double l = 1.0) {
   if (d > l)
        d = l;
   if (d < -l)
        d = -l;
}</pre>
```

# Dimension of Two

## Point

```
struct point {
   double x, y;
   point(double _x = 0, double _y = 0): x(_x), y(_y) {
   }
   void input() {
       scanf("%lf%lf", &x, &y);
   }
   double len() const {
       return sqrt(x * x + y * y);
   }
   point trunc(double l) const {
       double r = l / len();
       return point(x * r, y * r);
   }
   point rotate left() const {
```

```
return point(-y, x);
   point rotate left(double ang) const {
      double c = cos(ang), s = sin(ang);
       return point(x * c - y * s, y * c + x * s);
   point rotate right() const {
       return point(y, -x);
   point rotate right(double ang) const {
      double c = cos(ang), s = sin(ang);
      return point (x * c + y * s, y * c - x * s);
};
bool operator==(const point& p1, const point& p2) {
   return sgn(p1.x - p2.x) == 0 && sgn(p1.y - p2.y) == 0;
}
bool operator<(const point& p1, const point& p2) {</pre>
   return sqn(p1.x - p2.x) == 0 ? sqn(p1.y - p2.y) < 0 : p1.x < p2.x;
}
bool operator>(const point& p1, const point& p2) {
   return sqn(p1.x - p2.x) == 0 ? sqn(p1.y - p2.y) > 0 : p1.x > p2.x;
}
point operator+(const point& p1, const point& p2) {
   return point(p1.x + p2.x, p1.y + p2.y);
point operator-(const point& p1, const point& p2) {
   return point(p1.x - p2.x, p1.y - p2.y);
double operator^(const point& p1, const point& p2) {
   return p1.x * p2.x + p1.y * p2.y;
}
double operator*(const point& p1, const point& p2) {
   return p1.x * p2.y - p1.y * p2.x;
```

```
point operator*(const point& p, double r) {
   return point(p.x * r, p.y * r);
point operator/(const point& p, double r) {
   return point(p.x / r, p.y / r);
Relationship of Point and Line Segment
double get distance(const point&p, const point& p1, const point& p2) {
   if (sgn((p2 - p1) ^ (p - p1)) <= 0)
      return (p - p1).len();
   if (sgn((p1 - p2) ^ (p - p2)) \le 0)
      return (p - p2).len();
   return abs ((p1 - p) * (p2 - p) / (p1 - p2).len());
Relationship of Line Segments
bool get intersection (const point& p1, const point& p2, const point& p3, const
point & p4, point & c) {
   double d1 = (p2 - p1) * (p3 - p1), d2 = (p2 - p1) * (p4 - p1);
   double d3 = (p4 - p3) * (p1 - p3), d4 = (p4 - p3) * (p2 - p3);
   int s1 = sgn(d1), s2 = sgn(d2), s3 = sgn(d3), s4 = sgn(d4);
   if (s1 == 0 \&\& s2 == 0 \&\& s3 == 0 \&\& s4 == 0)
      return false;
```

## Relationship of Point and Line

return s1 \* s2 <= 0 && s3 \* s4 <= 0;

- d1));

```
double get_distance(const point&p, const point& p1, const point& p2) {
   return abs((p1 - p) * (p2 - p) / (p1 - p2).len());
}
```

c = point((p3.x \* d2 - p4.x \* d1) / (d2 - d1), (p3.y \* d2 - p4.y \* d1) / (d2 - d1), (p3.y \* d2 - p4.y \* d1) / (d2 - d1), (p3.y \* d2 - p4.y \* d1) / (d2 - d1), (p3.y \* d2 - p4.y \* d1)

```
point get_perpendicular(const point& p, const point& p1, const point& p2) {
   double d = (p1 - p) * (p2 - p) / (p1 - p2).len();
   return p - (p2 - p1).rotate_left().trunc(d);
}

point get_reflection(const point& p, const point& p1, const point& p2) {
   double d = (p1 - p) * (p2 - p) / (p1 - p2).len();
   return p - (p2 - p1).rotate_left().trunc(d * 2.0);
}
```

## Relationship of Point and Polygon

```
int get_position(const point& p, const point* pol, int n) {
   double ang = 0;
   for (int i = 0; i < n; ++i) {
      point p1 = pol[i] - p, p2 = pol[(i + 1) % n] - p;
      double c = (p1 ^ p2) / (p1.len() * p2.len());
      to_normal(c);
      ang += sgn(p1 * p2) * acos(c);
   }
   ang = abs(ang);
   return ang < 0.5 * pi ? -1 : (ang < 1.5 * pi ? 0 : 1);
}</pre>
```

# Relationship of Point and Convex Polygon

```
ub = mid - 1;
}
return sgn((pol[lb + 1] - pol[lb]) * (p - pol[lb]));
}
```

# Relationship of Line and Convex Polygon

```
struct edge {
   int id;
   point v;
   double ang;
   edge() {
   edge(int id, const point& v): id(id), v(v) {
       ang = atan2(v.y, v.x);
       if (sgn(ang - pi) == 0)
          ang = -pi;
};
bool operator<(const edge& e1, const edge& e2) {</pre>
   return sgn(e1.ang - e2.ang) < 0;</pre>
edge e[max n];
point 11, 12;
void pre compute(point* pol, int n) {
   for (int i = 0; i < n; ++i) {</pre>
       pol[n + i] = pol[i];
       e[i] = edge(i, pol[i + 1] - pol[i]);
   sort(e, e + n);
bool is less(const point& p1, const point& p2) {
   return sgn((11 - p1) * (12 - p1) - (11 - p2) * (12 - p2)) < 0;
bool get intersection (const point* pol, int n, const point& p1, const point& p2,
point& c1, point& c2) {
```

```
int p_1 = e[(lower_bound(e, e + n, edge(-1, p1 - p2)) - e) % n].id;
int p_r = e[(lower_bound(e, e + n, edge(-1, p2 - p1)) - e) % n].id;
if (sgn((p2 - p1) * (pol[p_1] - p1)) * sgn((p2 - p1) * (pol[p_r] - p1)) >= 0)
    return false;
l1 = p2, l2 = p1;
int k1 = (lower_bound(pol + p_1, pol + (p_r < p_1 ? p_r + n : p_r) + 1, p1,
is_less) - pol) % n;
l1 = p1, l2 = p2;
int k2 = (lower_bound(pol + p_r, pol + (p_1 < p_r ? p_1 + n : p_1) + 1, p2,
is_less) - pol) % n;
c1 = get_intersection(p1, p2, pol[k1], pol[(k1 + n - 1) % n]);
c2 = get_intersection(p1, p2, pol[k2], pol[(k2 + n - 1) % n]);
return true;
}</pre>
```

### Circle

```
struct circle {
   point c;
   double r;
   circle() {
   circle(const point& c, double r): c(c), r(r) {
   void input() {
      c.input();
      scanf("%lf", &r);
   double area() const {
      return pi * r * r;
   int get intersection(const point& p1, const point& p2, point& c1, point& c2)
const {
      double d = (p1 - c) * (p2 - c) / (p1 - p2).len();
      if (sgn(abs(d) - r) >= 0)
          return 0;
      point pp = c - (p2 - p1).rotate left().trunc(d);
      double l = sqrt(r * r - d * d);
      c1 = pp - (p2 - p1).trunc(1);
      c2 = pp + (p2 - p1).trunc(1);
      int res = 0;
```

```
res |= (sgn((p1 - c1) ^ (p2 - c1)) <= 0 ? 1 : 0) << 0;
                res |= (sgn((p1 - c2) ^ (p2 - c2)) <= 0 ? 1 : 0) << 1;
                return res;
        bool get intersection(const circle& cir, point& c1, point& c2) const {
                double d = (c - cir.c).len();
                if (sgn(d - (r + cir.r)) >= 0 || sgn(d - abs(r - cir.r)) <= 0)
                       return false;
               double p = (d + r + cir.r) / 2.0;
                double h = sqrt(abs(p * (p - d) * (p - r) * (p - cir.r))) * 2.0 / d;
               point pp = c + (cir.c - c).trunc((r * r + d * d - cir.r * cir.r) / (2.0 * ci
d));
                c1 = pp - (cir.c - c).rotate left().trunc(h);
                c2 = pp + (cir.c - c).rotate left().trunc(h);
                return true;
        bool get tangency points(const point& p, point& t1, point& t2) const {
               double d = (p - c).len();
               if (sgn(d - r) \ll 0)
                       return false;
               point pp = c + (p - c).trunc(r * r / d);
               double h = sqrt(abs(r * r - (r * r * r * r) / (d * d)));
                t1 = pp - (p - c).rotate left().trunc(h);
                t2 = pp + (p - c).rotate left().trunc(h);
               return true;
        vector<pair<point, point> > get tangency points(const circle& cir) const {
               vector<pair<point, point> > t;
               double d = (c - cir.c).len();
               if (sgn(d - abs(cir.r - r)) \le 0)
               double l = sqrt(abs(d * d - (cir.r - r) * (cir.r - r)));
               double h1 = r * 1 / d, h2 = cir.r * 1 / d;
               point p = (r > cir.r ? cir.c - c : c - cir.c);
               point pp1 = c + p.trunc(sqrt(abs(r * r - h1 * h1))), pp2 = cir.c +
p.trunc(sqrt(abs(cir.r * cir.r - h2 * h2)));
                t.push back(make pair(pp1 + p.rotate left().trunc(h1), pp2 +
p.rotate left().trunc(h2)));
                t.push_back(make_pair(pp1 - p.rotate_left().trunc(h1), pp2 -
p.rotate left().trunc(h2)));
               if (sgn(d - (r + cir.r)) \le 0)
                       return t;
```

```
double d1 = d * r / (r + cir.r), d2 = d * cir.r / (r + cir.r);
                point pp3 = c + (cir.c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r * r / d1), pp4 = cir.c + (c - c).trunc(r
cir.c).trunc(cir.r * cir.r / d2);
                double h3 = sqrt(abs(r * r - (r * r * r * r) / (d1 * d1))), h4 = sqrt(abs(cir.r)
* cir.r - (cir.r * cir.r * cir.r * cir.r) / (d2 * d2)));
                t.push back(make pair(pp3 + (cir.c - c).rotate left().trunc(h3), pp4 + (c
- cir.c).rotate left().trunc(h4)));
                t.push back(make pair(pp3 - (cir.c - c).rotate left().trunc(h3), pp4 - (c
- cir.c).rotate left().trunc(h4)));
                return t;
        double get intersection area(const point& p1, const point& p2) const {
                point v1 = (p1 - c), v2 = (p2 - c);
                double d1 = v1.len(), d2 = v2.len();
                point c1, c2;
                int s = get intersection(p1, p2, c1, c2);
                if (s == 0) {
                        if (sgn(d1 - r) > 0 \&\& sgn(d2 - r) > 0) {
                                 double t = (v1 ^ v2) / (d1 * d2);
                                 to normal(t);
                                 return r * r * acos(t) / 2.0;
                        return abs(v1 * v2 / 2.0);
                if (s == 1) {
                        point k = c1 - c;
                        double t = (v1 ^ k) / (d1 * k.len());
                        to normal(t);
                        return abs(v2 * k / 2.0) + r * r * acos(t) / 2.0;
                if (s == 2) {
                        point k = c2 - c;
                        double t = (v2 ^ k) / (d2 * k.len());
                        to normal(t);
                        return abs(v1 * k / 2.0) + r * r * acos(t) / 2.0;
                point k1 = c1 - c, k2 = c2 - c;
                double t1 = (v1 ^ k1) / (d1 * k1.len());
                to normal(t1);
                double t2 = (v2 ^ k2) / (d2 * k2.len());
                to normal(t2);
                return abs(k1 * k2 / 2.0) + r * r * (acos(t1) + acos(t2)) / 2.0;
```

```
double get_intersection_area(const circle& cir) const {
    double d = (c - cir.c).len();
    if (sgn(d - (r + cir.r)) >= 0)
        return 0;
    if (sgn(d - abs(r - cir.r)) <= 0)
        return min(area(), cir.area());
    double c1 = (r * r + d * d - cir.r * cir.r) / (2.0 * r * d);
    double c2 = (cir.r * cir.r + d * d - r * r) / (2.0 * cir.r * d);
    to_normal(c1);
    to_normal(c2);
    double p = (r + cir.r + d) / 2.0;
    double s = sqrt(p * (p - r) * (p - cir.r) * (p - d));
    return acos(c1) * r * r + acos(c2) * cir.r * cir.r - s * 2.0;
}
};</pre>
```

### Convex Hull

```
int dn, hd[max n], un, hu[max n];
 void get convex hull(point* p, int n, point* pol, int& m) {
                  sort(p, p + n);
                  dn = un = 2;
                 hd[0] = hu[0] = 0;
                hd[1] = hu[1] = 1;
                  for (int i = 2; i < n; ++i) {
                                  for (; dn > 1 &  sgn((p[hd[dn - 1]] - p[hd[dn - 2]]) * (p[i] - p[hd[dn - 2])) * (p[i] - p[hd[d
1|||) <= 0; --dn|;
                                  for (; un > 1 && sgn((p[hu[un - 1]] - p[hu[un - 2]]) * (p[i] - p[hu[un - 2])) *
1|||) >= 0; --un);
                                 hd[dn++] = hu[un++] = i;
                 m = 0:
                 for (int i = 0; i < dn - 1; ++i)
                                  pol[m++] = p[hd[i]];
                  for (int i = un - 1; i > 0; --i)
                                  pol[m++] = p[hu[i]];
}
```

# Half-plane Intersection (O(n2))

```
void get intersection(point* pol1, int n1, const point& p1, const point& p2, point*
pol2, int& n2) {
   n2 = 0;
   if (n1 == 0)
       return;
   point v = p2 - p1;
   int last s = sgn(v * (pol1[n1 - 1] - p1));
   for (int i = 0; i < n1; ++i) {</pre>
       int s = sgn(v * (pol1[i] - p1));
       if (s == 0) {
          pol2[n2++] = pol1[i];
       } else if (s < 0) {</pre>
          if (last s > 0)
              pol2[n2++] = qet intersection(p1, p2, i == 0 ? pol1[n1 - 1] : pol1[i]
- 1], pol1[i]);
       } else if (s > 0) {
          if (last s < 0)
              pol2[n2++] = get intersection(p1, p2, i == 0 ? pol1[n1 - 1] : pol1[i]
- 1], pol1[i]);
          pol2[n2++] = pol1[i];
       last s = s;
   }
}
```

# Half-plane Intersection (O(n \* lg(n)))

```
struct half_plane {
  point p1, p2;
  double ang;
  half_plane() {
  }
  half_plane(const point& _p1, const point& _p2): p1(_p1), p2(_p2) {
    ang = atan2(p2.y - p1.y, p2.x - p1.x);
    if (sgn(ang - pi) == 0)
        ang = -pi;
  }
  int get_position(const point& p) const {
```

```
return sqn((p2 - p1) * (p - p1));
};
bool operator<(const half plane& pl1, const half plane& pl2) {</pre>
   return sgn(pl1.ang - pl2.ang) == 0 ? pl1.get position(pl2.pl) < 0 : pl1.ang</pre>
< pl2.ang;
double operator^(const half plane& pl1, const half plane& pl2) {
   return (pl1.p2 - pl1.p1) ^ (pl2.p2 - pl2.p1);
double operator*(const half plane& pl1, const half plane& pl2) {
   return (pl1.p2 - pl1.p1) * (pl2.p2 - pl2.p1);
point get intersection(const half plane& pl1, const half plane& pl2) {
   double d1 = (pl1.p2 - pl1.p1) * (pl2.p1 - pl1.p1), d2 = (pl1.p2 - pl1.p1) *
(pl2.p2 - pl1.p1);
   return point ((pl2.pl.x * d2 - pl2.p2.x * d1) / (d2 - d1), (pl2.pl.y * d2 - pl2.p2.y
* d1) / (d2 - d1));
void get intersection(const half plane* pl, int n, point* pol, int& m) {
   m = 0;
   deque<int> deq1;
   deque<point> deq2;
   deq1.push back(0);
   deq1.push back(1);
   deq2.push back(get intersection(pl[0], pl[1]));
   for (int i = 2; i < n; ++i) {</pre>
       while (!deq2.empty() && pl[i].get position(deq2.back()) <= 0) {</pre>
          if (sqn(pl[deq1.size() - 2]) * pl[i]) \le 0 \&& sqn(pl[deq1.back())]
* pl[i]) >= 0
              return:
          degl.pop back();
          deg2.pop back();
       while (!deq2.empty() && pl[i].get position(deq2.front()) <= 0) {</pre>
          deq1.pop front();
          deq2.pop front();
```

```
}
    deq2.push_back(get_intersection(pl[deq1.back()], pl[i]));
    deq1.push_back(i);
    while (deq2.size() > 1 && pl[deq1.front()].get_position(deq2.back()) <= 0)

{
        deq1.pop_back();
        deq2.pop_back();
    }
    while (deq2.size() > 1 && pl[deq1.back()].get_position(deq2.front()) <= 0)

{
        deq1.pop_front();
        deq2.pop_front();
        deq2.pop_front();
    }

m = deq2.size();
    copy(deq2.begin(), deq2.end(), pol);
    pol[m++] = get_intersection(pl[deq1.front()], pl[deq1.back()]);
}
</pre>
```

### Diameter of a Set of Points

```
double get_max_distance(point* p, int n, point* pol, int& m) {
    get_convex_hull(p, n, pol, m);
    double dis = 0;
    for (int i = 0, j = dn - 1; i < m; ++i) {
        dis = max(dis, (pol[j] - pol[i]).len());
        while (sgn((pol[(i + 1) % m] - pol[i]) * (pol[(j + 1) % m] - pol[j])) > 0)
    {
        j = (j + 1) % m;
        dis = max(dis, (pol[j] - pol[i]).len());
     }
    return dis;
}
```

### Dimension of Three

### Point

```
struct point {
   double x, y, z;
   point(double x = 0, double y = 0, double z = 0): x(x), y(y), z(z) {
   void input() {
       scanf("%lf%lf%lf", &x, &y, &z);
   double len() const {
       return sqrt(x * x + y * y + z * z);
   point trunc(double 1) const {
       double r = 1 / len();
       return point(x * r, y * r, z * r);
};
bool operator==(const point& p1, const point& p2) {
   return sgn(p1.x - p2.x) == 0 \&\& sgn(p1.y - p2.y) == 0 \&\& sgn(p1.z - p2.z) == 0
0;
bool operator<(const point& p1, const point& p2) {</pre>
   return sgn(p1.x - p2.x) == 0 ? (sgn(p1.y - p2.y) == 0 ? sgn(p1.z - p2.z) < 0 :
p1.y < p2.y) : p1.x < p2.x;
bool operator>(const point& p1, const point& p2) {
   return sgn(p1.x - p2.x) == 0 ? (sgn(p1.y - p2.y) == 0 ? sgn(p1.z - p2.z) > 0 :
p1.y > p2.y) : p1.x > p2.x;
point operator+(const point& p1, const point& p2) {
   return point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z);
point operator-(const point& p1, const point& p2) {
   return point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z);
double operator^(const point& p1, const point& p2) {
   return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z;
```

```
point operator*(const point& p1, const point& p2) {
    return point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x - p1.x * p2.z, p1.x * p2.y
    - p1.y * p2.x);
}

point operator*(const point& p, double r) {
    return point(p.x * r, p.y * r, p.z * r);
}

point operator/(const point& p, double r) {
    return point(p.x / r, p.y / r, p.z / r);
}
```

## Relationship of Point and Line Segment

```
double get_distance(const point& p, const point& p1, const point& p2) {
  if (sgn((p2 - p1) ^ (p - p1)) <= 0)
    return (p - p1).len();
  if (sgn((p1 - p2) ^ (p - p2)) <= 0)
    return (p - p2).len();
  return abs(((p1 - p) * (p2 - p)).len() / (p1 - p2).len());
}</pre>
```

## Relationship of Point and Line

```
double get_distance(const point& p, const point& p1, const point& p2) {
    return abs(((p1 - p) * (p2 - p)).len() / (p1 - p2).len());
}

point get_perpendicular(const point& p, const point& p1, const point& p2) {
    point v = (p1 - p) * (p2 - p);
    double d = v.len() / (p1 - p2).len();
    return p - (v * (p2 - p1)).trunc(d);
}

point get_reflection(const point& p, const point& p1, const point& p2) {
    point v = (p1 - p) * (p2 - p);
    double d = v.len() / (p1 - p2).len();
    return p - (v * (p2 - p1)).trunc(d * 2.0);
```

# Relationship of Point and Plane

```
double get_distance(const point& p, const point& p1, const point& p2, const point&
p3) {
    point v = (p2 - p1) * (p3 - p1);
    return abs((v ^ (p - p1)) / v.len());
}

point get_perpendicular(const point& p, const point& p1, const point& p2, const point& p3) {
    point v = (p2 - p1) * (p3 - p1);
        double d = (v ^ (p - p1)) / v.len();
        return p - v.trunc(d);
}

point get_reflection(const point& p, const point& p1, const point& p2, const point& p3) {
    point v = (p2 - p1) * (p3 - p1);
        double d = (v ^ (p - p1)) / v.len();
        return p - v.trunc(d * 2.0);
}
```

## Relationship of Line Segment and Plane

```
bool get_intersection(const point& p1, const point& p2, const point& p11, const
point& p12, const point& p13, point& c) {
   point v = (p12 - p11) * (p13 - p11);
   double d1 = v ^ (p1 - p11), d2 = v ^ (p2 - p11);
   int s1 = sgn(d1), s2 = sgn(d2);
   if (s1 == 0 && s2 == 0)
        return false;
   c = point((p1.x * d2 - p2.x * d1) / (d2 - d1), (p1.y * d2 - p2.y * d1) / (d2 - d1), (p1.z * d2 - p2.z * d1) / (d2 - d1));
   return s1 * s2 <= 0;
}</pre>
```

### Convex Hull

```
struct face {
   int a, b, c;
   face(int a = 0, int b = 0, int c = 0): a(a), b(b), c(c) {
};
const int max n = 300 + 10, max f = max n * 2;
int n1, n2, pos[max n] [max n];
face buf1[max f], buf2[max f], *p1, *p2;
int get position (const point& p, const point& p1, const point& p2, const point&
p3) {
   return sgn((p2 - p1) * (p3 - p1) ^ (p - p1));
void check(int k, int a, int b, int s) {
   if (pos[b][a] == 0) {
       pos[a][b] = s;
       return;
   if (pos[b][a] != s)
       p2[n2++] = (s < 0 ? face(k, b, a) : face(k, a, b));
   pos[b][a] = 0;
}
void get convex hull(point* p, int n, face* pol, int& m) {
   for (int i = 1; i < n; ++i) {</pre>
       if (p[i] != p[0]) {
          swap(p[i], p[1]);
          break;
   for (int i = 2; i < n; ++i) {</pre>
       if (sgn(((p[0] - p[i]) * (p[1] - p[i])).len()) != 0) {
          swap(p[i], p[2]);
          break;
   for (int i = 3; i < n; ++i) {</pre>
```

```
if (get position(p[i], p[0], p[1], p[2]) != 0) {
       swap(p[i], p[3]);
      break;
p1 = buf1;
p2 = buf2;
n1 = n2 = 0;
for (int i = 0; i < n; ++i)
   fill(pos[i], pos[i] + n, 0);
p1[n1++] = face(0, 1, 2);
p1[n1++] = face(2, 1, 0);
for (int i = 3; i < n; ++i) {
   n2 = 0;
   for (int j = 0; j < n1; ++j) {
      int s = get_position(p[i], p[p1[j].a], p[p1[j].b], p[p1[j].c]);
      if (s == 0)
          s = -1;
      if (s \ll 0)
          p2[n2++] = p1[j];
      check(i, p1[j].a, p1[j].b, s);
      check(i, p1[j].b, p1[j].c, s);
       check(i, p1[j].c, p1[j].a, s);
   swap(p1, p2);
   swap(n1, n2);
m = n1;
copy(p1, p1 + n1, pol);
```