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APPENDIX B

MATLAB M-FILE FUNCTIONS

M-file Name	Description	Page
`bisect`	Root location with bisection	151
`eulode`	Integration of a single ordinary differential equation with Euler's method	586
`fzerosimp`	Brent's method for root location	180
`GaussNaive`	Solving linear systems with Gauss elimination without pivoting	258
`GaussPivot`	Solving linear systems with Gauss elimination with partial pivoting	263
`GaussSeidel`	Solving linear systems with the Gauss-Seidel method	310
`goldmin`	Minimum of one-dimensional function with golden-section search	208
`incsearch`	Root location with an incremental search	144
`IterMeth`	General algorithm for iterative calculation	105
`Lagrange`	Interpolation with the Lagrange polynomial	443
`linregr`	Fitting a straight line with linear regression	372
`natspline`	Cubic spline with natural end conditions	477
`Newtint`	Interpolation with the Newton polynomial	440
`newtmult`	Root location for nonlinear systems of equations	318
`newtraph`	Root location with the Newton-Raphson method	174
`quadadapt`	Adaptive quadrature	539
`rk4sys`	Integration of system of ODEs with 4th-order RK method	602
`romberg`	Integration of a function with Romberg integration	530
`TableLook`	Table lookup with linear interpolation	458
`trap`	Integration of a function with the composite trapezoidal rule	500
`trapuneq`	Integration of unequispaced data with the trapezoidal rule	509
`Tridiag`	Solving tridiagonal linear systems	266

APPENDIX C
INTRODUCTION TO SIMULINK

Simulink® is a graphical programming environment for modeling, simulating, and analyzing dynamic systems. In short, it allows engineers and scientists to build process models by interconnecting blocks with communication lines. Thus, it provides an easy-to-use computing framework to quickly develop dynamic process models of physical systems. Along with offering a variety of numerical integration options for solving differential equations, Simulink includes built-in features for graphical output which significantly enhance visualization of a system's behavior.

As a historical footnote, back in the Pleistocene days of analog computers (aka the 1950s), you had to design information flow diagrams that showed graphically how multiple ODEs in models were interconnected with themselves and with algebraic relationships. The diagrams also showed flaws in modeling where there was information lacking or structural defects. One of the nice features of Simulink is that it also does that. It's often beneficial to see that aspect separate from numerical methods and then merge the two in MATLAB.

As was done with Chap. 2, most of this appendix has been written as a hands-on exercise. That is, you should read it while sitting in front of your computer. The most efficient way to start learning Simulink is to actually implement it on MATLAB as you proceed through the following material.

So let's get started by setting up a simple Simulink application to solve an initial value problem for a single ODE. A nice candidate is the differential equation we developed for the velocity of the free-falling bungee jumper in Chap. 1,

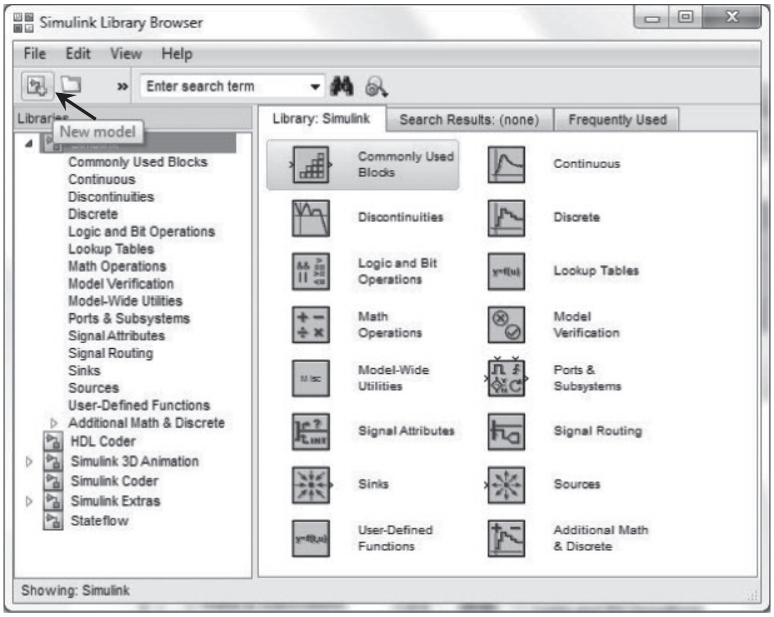
$$\frac{dv}{dt} = g - \frac{c_d}{m}v^2 \tag{C.1}$$

where v = velocity (m/s), t = time (s), $g = 9.81$ m/s^2, c_d = drag coefficient (kg/m), and m = mass (kg). As in Chap. 1 use $c_d = 0.25$ kg/m, $m = 68.1$ kg, and integrate from 0 to 12 s with an initial condition of $v = 0$.

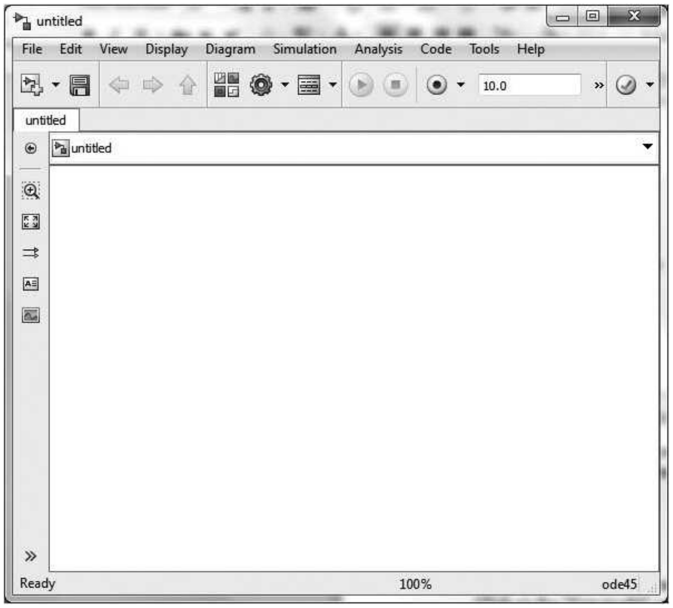
To generate the solution with Simulink, first launch MATLAB. You should eventually see the MATLAB window with an entry prompt, ≫, in the Command Window. After changing MATLAB's default directory, open the Simulink Library Browser using one of the following approaches:

- On the MATLAB toolbar, click the Simulink button (▦).
- At the MATLAB prompt, enter the `simulink` command.

The Simulink Library Browser window should appear displaying the Simulink block libraries installed on your system. Note, to keep the Library Browser above all other windows on your desktop, in the Library Browser select **View, Stay on Top.**

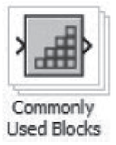


Click on the **New model** command button on the left of the toolbar, and an untitled Simulink model window should appear.

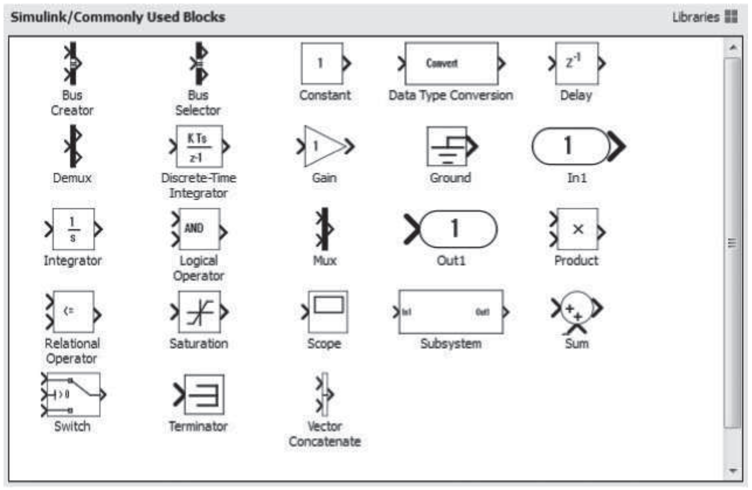

untitled
File Edit View Display Diagram Simulation Analysis Code Tools Help
10.0
untitled
Ready
100%
ode45

You will build your simulation model in the untitled window by choosing items from the Library Browser and then dragging and dropping them onto the untitled window. First, select the untitled window to activate it (clicking on the title bar is a sure way to do this) and select **Save As** from the File menu. Save the window as **Freefall** in the default directory. This file is saved automatically with an **.slx** extension. As you build your model in this window, it is a good idea to save it frequently. You can do this in three ways: the **Save** button, the **Ctrl+s** keyboard shortcut, and the menu selections, **File, Save.**

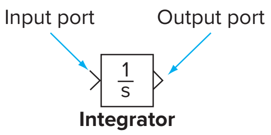
We will start by placing an integrator element (for the model's differential equation) in the **Freefall** window. To do this, you need to activate the **Library Browser** and double click on the **Commonly Used Blocks** item.



The Browser window should show something like



Examine the icon window until you see the Integrator icon.

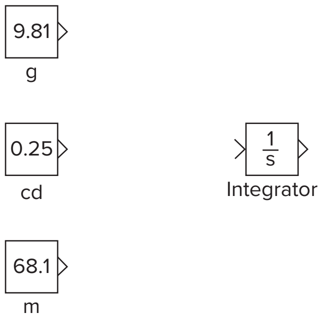


The block symbol is what will appear in your model window. Notice how the icon has both input and output ports which are used to feed values in and out of the block. The 1/s symbol represents integration in the Laplace domain. Use the mouse to drag an **Integrator** icon onto your **Freefall** window. This icon will be used to integrate the differential equations. Its input will be the differential equation [the right hand side of Eq. (C.1)] and its output will be the solution (in our example, velocity).

Next you have to build an information flow diagram that describes the differential equation and "feeds" it into the integrator. The first order of business will be to set up constant blocks to assign values to the model parameters. Drag a **Constant** icon from the **Commonly Used Blocks**[1] branch in the **Browser** window to the **Freefall** model window and place it above and to the left of the integrator.

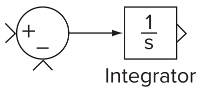
Next, click on the **Constant Label** and change it to g. Then, double click the Constant block and the Constant Block Dialogue box will be opened. Change the default value in the Constant field to 9.81 and click OK.

Now set up Constant blocks for both the drag coefficient (cd = 0.25) and mass (m = 68.1) positioned below the g block. The result should look like



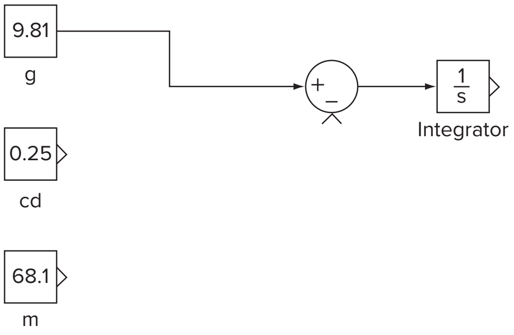
From the Math Operations Library Browser, select a Sum icon and drag it just to the left of the Integrator block. Place the mouse pointer on the output port of the sum block. Notice that the mouse pointer will change to a crosshair shape when it's on the output port. Then drag a connecting line from the output port to the Integrator block's input port. As you drag, the mouse pointer retains its crosshair shape until it is on the input port whereupon it changes to a double-lined crosshair. We have now "wired" the two blocks together with the output of the sum block feeding into the Integrator.

Notice that the sum block has two input ports into which can be fed two quantities that will be added as specified by the two positive signs inside the circular block. Recall that our differential equation consists of the difference between two quantities: $g - (c_d/m)v^2$. We therefore, have to change one of the input ports to a negative. To do this, double click on the Sum Block in order to open the Sum Block Dialogue box. Notice that the List of Signs has two plus signs (++). By changing the second to a minus sign (+−), the value entering the second input port will be subtracted from the first. After closing the Sum Block Dialogue box, the result should look like

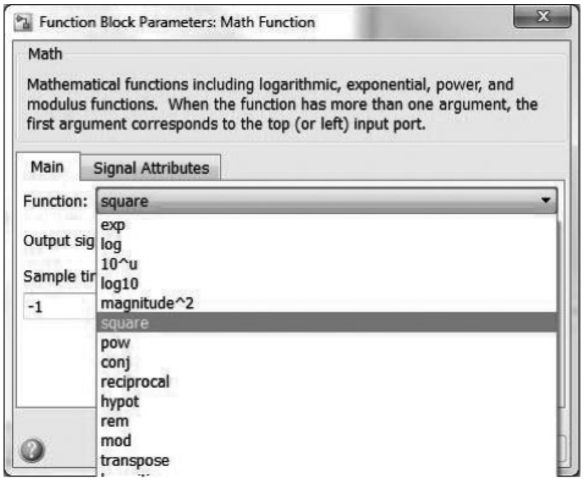


[1] All the icons in the **Commonly Used Blocks** group are available in other groups. For example, the **Constant** icon is located in the **Sources** group.

Because the first term in the differential equation is g, wire the output of the g block to the positive input port of the sum block. The system should now look like



In order to construct the second term that will be subtracted from g, we must first square the velocity. This can be done by dragging a Math Functions block from the Math Operations Library Browser and positioning it to the right and below the integrator block. Double click the Math Functions icon to open the Math Functions Dialogue box and use the pull down menu to change the Math Function to square.

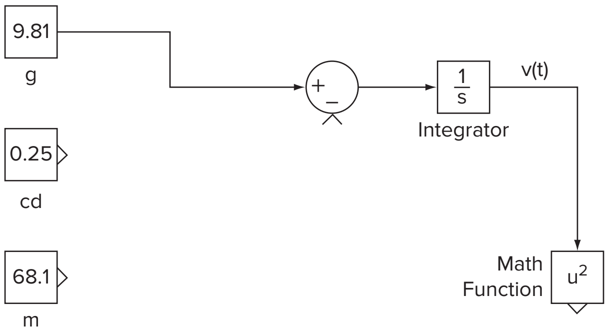

Function Block Parameters: Math Function

Math

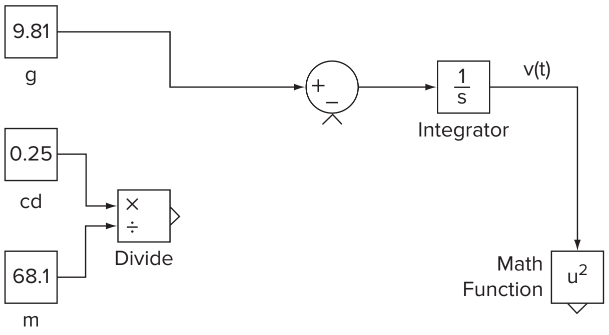
Mathematical functions including logarithmic, exponential, power, and modulus functions. When the function has more than one argument, the first argument corresponds to the top (or left) input port.

Function: square

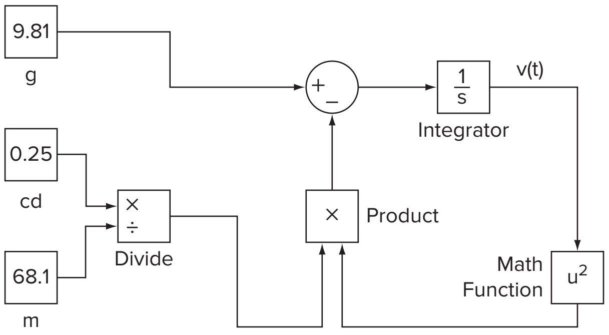
Before connecting the Integrator and Square blocks, it would be nice to rotate the latter so its input port is on top. To do this, select the Square block and then hit ctrl-r once. Then, we can wire the output port of the Integrator block to the input port of the square block. Because the output of the Integrator block is the solution of the differential equation, the output of the square block will be v^2. To make this clearer, double click the arrow connecting the Integrator and Square blocks and a text box will appear. Add the label, v(t), in this text box to indicate that the output of the Integrator block is the velocity. Note that you can label all the connecting wires in this way in order to better document the system diagram. At this point, it should look like



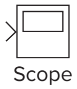
Next, drag a Divide block from the Math Operations Library Browser and position it to the right of the cd and m blocks. Notice that the Divide block has two input ports: one for the dividend (×) and one for the divisor (÷). Note that these can be switched by double clicking the Divide block to open the Divide Block Dialogue box and switching the order in the "number of inputs:" field. Wire the cd block output port to the × input port and the m block output port to the ÷ input port of the Divide block. The output port of the divide block will now carry the ratio, c_d/m.



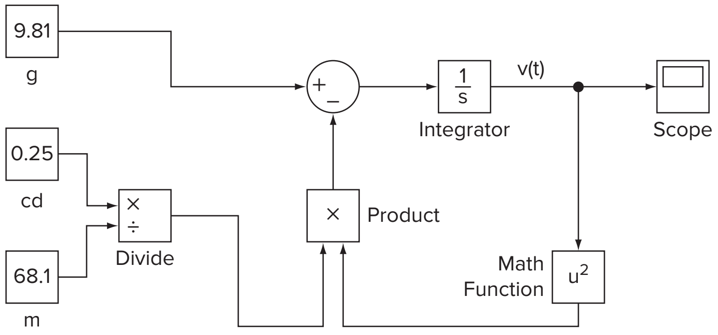
Drag a Product block from the Math Operations Library Browser and position it to the right of the divide block and just below the sum block. Use ctrl-r to rotate the product block until its output port points upward toward the sum block. Wire the divide block output port to the nearest product input port and the square Math Function block output port to the other product input port. Finally, wire the product output port to the remaining sum input port.



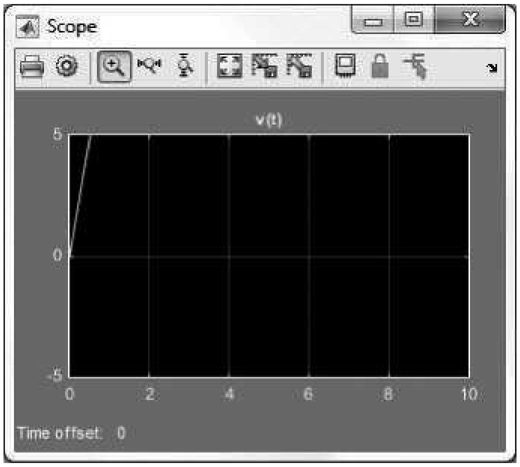
As displayed above, we have now successfully developed a Simulink program to generate the solution to this problem. At this point, we could run the program but we have not yet set up a way to display the output. For the present case, a simple way to do this employs a Scope Block



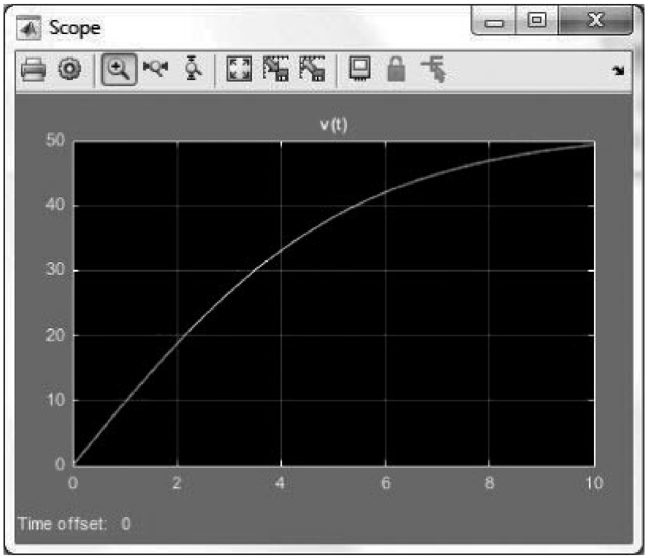
The Scope block displays signals with respect to simulation time. If the input signal is continuous, the Scope draws a point-to-point plot between major time step values. Drag a Scope block from the **Commonly Used Blocks** browser and position it to the right of the Integrator block. Position the mouse pointer on the Integrator's output wire (for the present case a nice position would be at the corner). Simultaneously hold down the control key and another line across to the Scope block's input port.



We are now ready to generate results. Before doing that, it's a good idea to save the model. Double click on the Scope block. Then click the run button, ▶. If there are any mistakes, you will have to correct them. Once your have successfully made corrections, the program should execute and the scope display should look something like



Click on the Autoscale button, ⛶, and the plot will resize to fit the entire range of results



Note that a Scope window can display multiple *y*-axes (graphs) with one graph per input port. All of the *y*-axes have a common time range on the *x*-axis. By selecting the parameter button on the graph window (⚙), you can use scope parameters to change graph features such as figure color and style and axis settings.

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