Introduction to quantum chaos

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Computer modeling of physical phenomena

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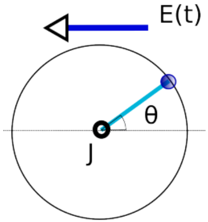
Plan prezentacji

Introduction

Physical motivation and model

Considered system

$$H(t) = \frac{J^2}{2I_0} + K\frac{I_0}{\tau_0} cos\theta \sum_n \delta(t - n\tau_0)$$



Dimensionless quantities

$$x = \theta \quad p = \frac{\tau_0}{I_0} J \quad \tilde{t} = t/\tau_0 \quad \tilde{H} = H\frac{\tau_0^2}{I_0}$$

Equations of motion

Dimensionless Hamiltonian

$$\tilde{H} = \frac{p^2}{2} + K\ cos\theta \sum_n \delta(t-n)$$

Equations of motion

$$\dot{p} = -\frac{\partial \tilde{H}}{\partial \theta}, \qquad \dot{q} = \frac{\partial \tilde{H}}{\partial p}$$

$$\begin{cases} p_{n+1} = p_n + K\ sin(x_n) \\ x_{n+1} = x_n + K\ sin(p_{n+1}) \end{cases}$$

- Where both p and q are periodic, i.e $x, p \in [0, 2\pi)$
- Each (p_n, x_n) have a meaning of phase space coordinates just before the kick.

Quantumness

Canonical quantization

$$[\theta, J] = i\hbar, \quad [x, p] = i\hbar_{eff} = i\hbar\tau_0/I_0$$

- What happens when $\hbar_{eff} \to 0$?

Single kick evolution

$$\psi(t) = F(t)\psi(0)$$

$$F(t) = \exp\left\{-\frac{i\hat{p}^2 t}{2\hbar_{eff}} - \frac{iK\cos(x)t}{\hbar_{eff}}\right\}$$

Quantization as an effect of periodicity

- x periodic, hence $p = \hbar_{eff}\, m$
- Periodicity of momentum part of $F(t)$, give us $p = \frac{2\pi}{M}m$, $x = \frac{2\pi}{M}n$, where both $m, n = 0, \ldots, M-1$ independently.

Quantum map

Fourier transform

$$\tilde{\psi}_m = \frac{1}{\sqrt{M}} \sum_n e^{-i\frac{2\pi}{M}mn} \psi_n$$

One step evolution

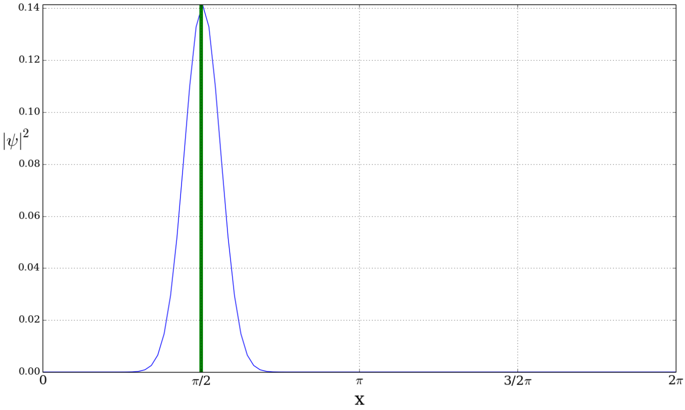
$$\bar{\psi}'_n = \frac{1}{M} \sum_{m,n} e^{i\frac{2\pi}{M}mn'} e^{-i\frac{m^2\pi}{M}} \underbrace{e^{-i\frac{2\pi}{M}mn}\psi_n}_{\tilde{\psi}_m}$$

Periodic initial Gaussian

$$\psi_0(n) = \tilde{N} e^{i\frac{2\pi m_0}{M}n} \sum_l exp[-\frac{\pi}{M}(n - n_0 + M \cdot l)^2]$$

- Centered at $\frac{2\pi}{M}(n_0, m_0)$

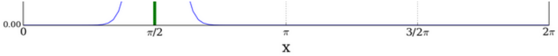
Gaussian



Implementation

Notebok online

Ipython Notebook



Excersise 2

```
In [12]: def plot_gauss_axis(data, axis, space = 'x', color = 'r'):
             """
             :param data: gaussian data np.array
             :param space: phasespace coordintate, momentum(mom)/position(pos)
             :return:
             """
             plt.rc('text', usetex=False)
             plt.rc('font', family='serif')

             nodes = len(data)

             xlabel = "x"
             if space == 'p':
                 data = sc.fft(data)/np.sqrt(nodes)
                 xlabel = "p"

             data_mod2 = np.abs(data)**2
             x0 = (np.argmax(data_mod2))/float(nodes) * 2*pi

             axis.set_xticks([])
             axis.set_yticks([])
             axis.plot(np.linspace(0, 2*pi, nodes), data_mod2, color=color)
             #axis.plot([x0, x0], [0, 1], linewidth = 1)

             axis.set_xlim(0, 2*pi)
             axis.set_ylim(0, np.max(data_mod2))
             plt.grid()
             return axis
```

```
In [19]: times = np.arange(50)

         Kn = 7
```

Crucial function - evolution

```python
def step(psi0, K = 0):
    '''
    :param psi0: Initial wave function np.array()
    :param K: Driven force coupling
    :return: Wave function after one step
    '''
    nodes = len(psi0)
    n = np.arange(nodes)
    P = np.exp(1j * (pi*n/nodes) * n)
    V = np.exp(-1j * nodes * K / 2/pi *
               np.cos(2 * pi * n/nodes))
    F = sc.fft(P * V * psi0) / np.sqrt(nodes)
    return (P*F)
```

Results

Results

In order of occurrences

- Free evolution of gaussian with $K = 0$
- Evolution of gaussian with $K = 2.1$
- Quantum versus classical: evolution of "slow" particle with initial conditions $(x_0, p_0) = (\pi/2, \pi/3)$
- Quantum versus classical: evolution of "quick" particle with initial conditions $(x_0, p_0) = (\pi/2, 2\pi/3)$
- The latter in momentum space.

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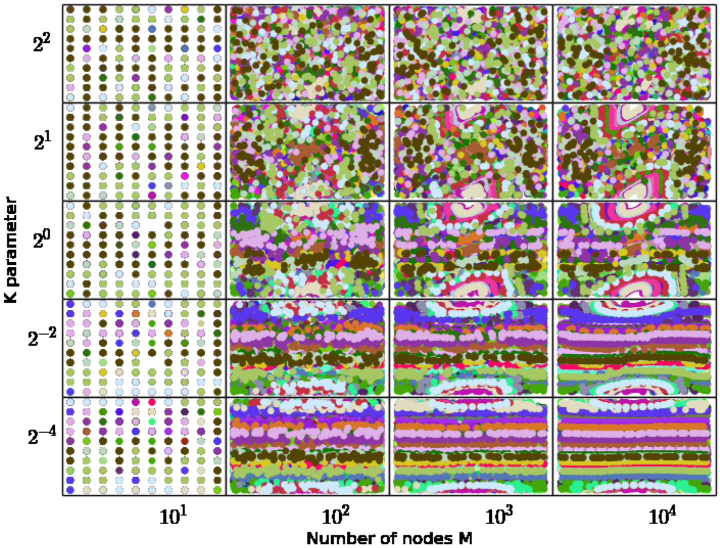
Phase space



Summary

Conclusions

Remarks

- Quantum behavior requires limited phase space,
- Classical behavior emerges as a quasi - continuum limit of QM ($\hbar_{eff} \longrightarrow 0$),
- The same conditions hold for quantum map - classical map correspondence,
- Driving force is responsible for superposition state,
- Butterfly effect is present.

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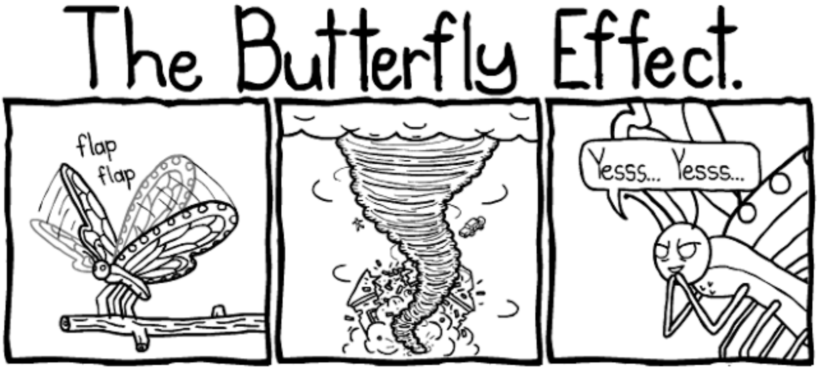

Thank you for your attention!