

# Introduction to Machine Learning

Christos Dimitrakakis

September 16, 2024

# Outline

## The problems of Machine Learning (1 week)

- Introduction

## Estimation

- Answering a scientific problem

- Pandas and dataframes

- Single variable models

- Two variable models

## Statistics, validation and model selection

## Course summary

- Course Contents

## Reading for this week

- Reading

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# Machine Learning And Data Mining

## The nuts and bolts

- ▶ Models
- ▶ Algorithms
- ▶ Theory
- ▶ Practice

## Workflow

- ▶ Scientific question
- ▶ Formalisation of the problem
- ▶ Data collection
- ▶ Analysis and model selection

## Types of statistics / machine learning problems

- ▶ Classification
- ▶ Regression
- ▶ Density estimation

# Machine learning

## Data Collection

- ▶ Downloading a clean dataset from a **repository**
- ▶ **Scraping** data from the web
- ▶ Conducting a **survey**
- ▶ Performing **experiments**, and obtaining measurements.

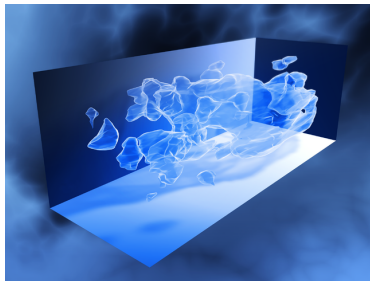
## Modelling

- ▶ Simple: the bias of a coin
- ▶ Complex: a language model.
- ▶ The model depends on the data and the problem

## Algorithms and Decision Making

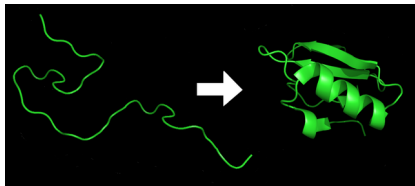
- ▶ We want to use models to make decisions.
- ▶ Decisions are made every step of the way.
- ▶ Both humans and algorithms can make decisions.

# The main problems in machine learning and statistics



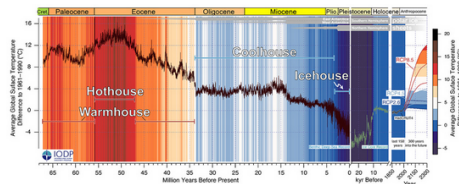
Matter

Dark

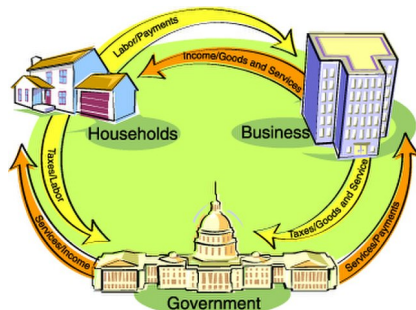


Protein Folding

Christos Dimitrakakis



Climate Modelling



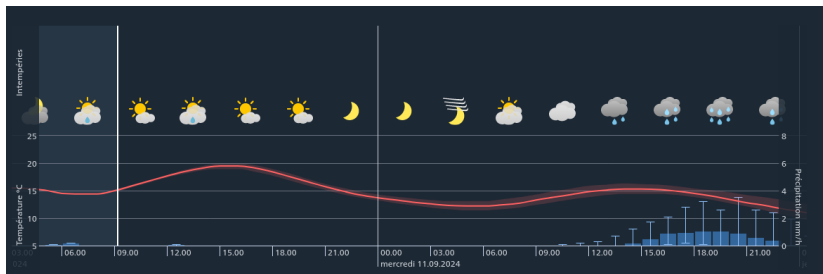
Economic Policy

Introduction to Machine Learning

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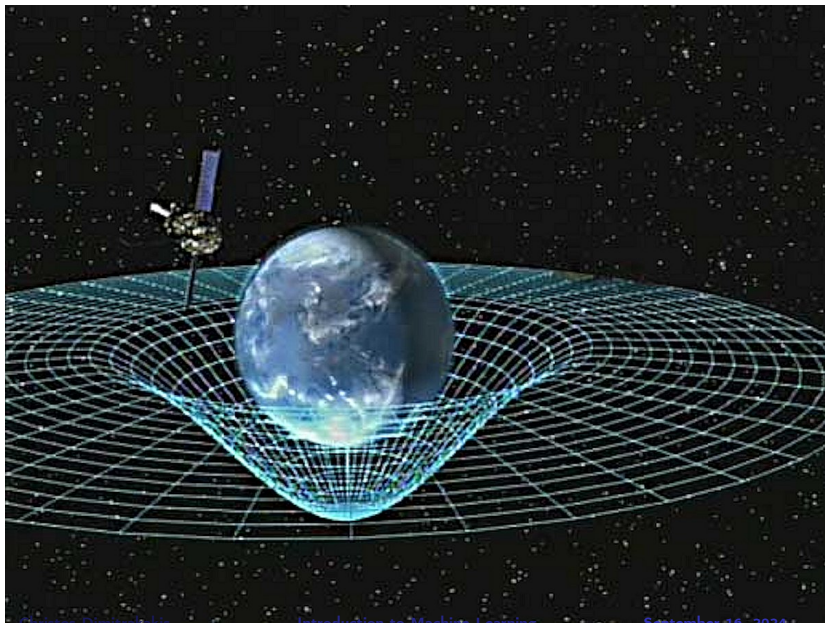
6 / 43

# Prediction



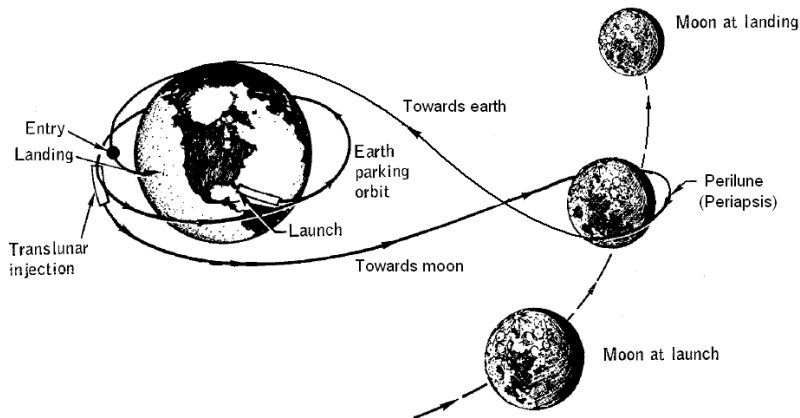
- ▶ Will it rain tomorrow?
- ▶ How much will bitcoin be worth next year?
- ▶ When is the next solar eclipse?

# Inference





# Decision Making



./fig/artemis.gif

- ▶ What data should I collect?
- ▶ Which model should I use?
- ▶ Should I fold, call, or raise in my poker game?
- ▶ How can I get a spaceship to the moon and back?

# The need to learn from data

## Problem definition

- ▶ What problem do we need to solve?
- ▶ How can we formalise it?
- ▶ What properties of the problem can we learn from data?

## Data collection

- ▶ **Why** do we need data?
- ▶ **What** data do we need?
- ▶ How **much** data do we want?
- ▶ **How** will we collect the data?

## Modelling and decision making

- ▶ How will we **compute** something useful?
- ▶ How can we use the model to make **decisions**?

# Course Material

## Moodle

- ▶ Assignments and project
- ▶ Additional reading material
- ▶ Asking questions

Course Github <https://github.com/olethrosdc/machine-learning-neuch/tree/main/BSc>

- ▶ .org files for notes, PDF for slides
- ▶ source code for examples

## Course literature

- ▶ An Introduction to Statistical Learning with Python  
[https://hastie.su.domains/ISLP/ISLP\\_website.pdf.download.html](https://hastie.su.domains/ISLP/ISLP_website.pdf.download.html)

book chapters will be mentioned in the course

# Assignment, teaching and questions

## Assignments and project

- ▶ Individual Weekly assignments in the first half
- ▶ Group project in the second half
- ▶ Project presentation
- ▶ No exam.

## Other questions

- ▶ Use Moodle for technical/administrative questions: That way everybody gets the same information.
- ▶ Use email for personal problems or extra help, if the moodle is not enough.
- ▶ Complicated questions can be answered at the next lecture

## Office hours

- ▶ Fridays 13:00-14:00: book with an email to avoid clashes.
- ▶ Email me for an appointment outside those hours.

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# Problem definition

- ▶ Example: Health, weight and height

## Example (Health questions regarding height and weight)

- ▶ What is a normal height and weight?
- ▶ How are they related to health?
- ▶ What variables affect height and weight?

## Define a research question

Find a **non-sensitive** variable that we can easily measure via a survey, e.g. related to sleep, smoking, exercise, food, politics, sports, hobbies etc.

- ▶ Discuss in small groups and post suggestions
- ▶ We then vote for what to measure

# Data collection

Think about **which variables** we need to collect to answer our **research question**.

## Necessary variables

The variables we need to know about

- ▶ Weight
- ▶ Height
- ▶ Dependent: (health/vote/opinion/salary)

## Auxiliary variables

Measurable factors related to the variables of interest

## Possible confounders

Hidden factors that might affect variables

# Class data and variables

- ▶ The class enters their data into the excel file.



- ▶ Pay attention to the variables we wish to measure

## Privacy

- ▶ Is the use of a pseudonym sufficient to hide your identity?



# Variables

The class data looks like this

First Name	Gender	Height	Weight	Age	Nationality	Smoking
Lee	M	170	80	20	Chinese	10
Fatemeh	F	150	65	25	Turkey	0
Ali	Male	174	82	19	Turkish	0
Joan	N	5'11	180	21	American	4

- ▶  $\mathbf{X}$ : Everybody's data
- ▶  $\mathbf{x}_t$ : The  $t$ -th person's data
- ▶  $x_{t,k}$ : The  $k$ -th feature of the  $t$ -th person.
- ▶  $\mathbf{x}_k$ : Everybody's  $k$ -th feature

## Raw versus neat data

- ▶ Neat data:  $\mathbf{x}_t \in \mathbb{R}^n$
- ▶ Raw data: web pages, handwritten text, graphs, data packets, with missing/incorrect values, etc

# Types of learning problems

## Unsupervised learning (unconditional estimation)

- ▶ Predict the **gender** of an unknown individual.
- ▶ Predict the **height**.
- ▶ Predict the **height and weight**?

## Supervised learning problems (conditional estimation)

- ▶ Classification: Can we predict gender from height/weight?
- ▶ Regression: Can we predict weight from height and gender?
- ▶ In both cases we predict **output** variables from **input** variables

## Variables

- ▶ **Input** variables: aka features, predictors, independent variables
- ▶ **Output** variables: aka response, dependent variables, labels, or targets.
- ▶ The input/output dichotomy only exists in **some prediction problems**.

# Python pandas for data wrangling

## Reading class data

```
import pandas as pd
X = pd.read_excel("data/class.xlsx")
X["First_Name"]
```

- ▶ Array columns correspond to features
- ▶ Columns can be accessed through namesx

## Summarising class data

```
X.hist()
import matplotlib.pyplot as plt
plt.show()
```

# Pandas and DataFrames

- ▶ Data in pandas is stored in a **DataFrame**
- ▶ DataFrame is **not the same** as a numpy array.

## Core libraries

```
import pandas as pd
import numpy as np
```

## Series: A sequence of values

```
# From numpy array:
s = pd.Series(np.random.randn(3), index=["a", "b", "c"])
# From dict:
d = {"a": 1, "b": 0, "c": 2}
s = pd.Series(d)
# accessing elemets
s.iloc[2] #element 2
s.iloc[1:2] #elements 1,2
s.array # gets the array object
s.to_numpy() # gets the underlying numpy array
```

# DataFrames

## Constructing from a numpy array

```
data = np.random.uniform(size = [3,2])
df = pd.DataFrame(data, index=["John", "Ali", "Sumi"],
                  columns=["X1", "X2"])
```

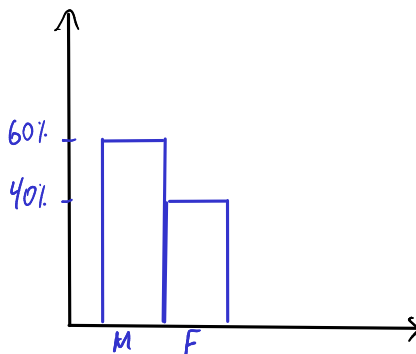
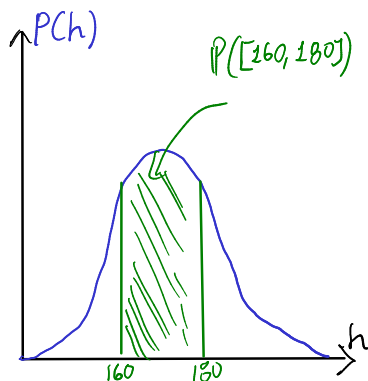
## Constructing from a dictionary

```
d = { "one": pd.Series([1, 2], index=["a", "b"]),
      "two": pd.Series([1, 2, 3], index=["a", "b", "c"])
df = pd.DataFrame(d)
```

## Access

```
X["First_Name"] # get a column
X.loc[2] # get a row
X.at[2, "First_Name"] # row 2, column 'first name'
X.loc[2].at["First_Name"] # row 2, element 'first name' of
X.iat[2,0] # row 2, column 0
```

## Modelling single variables

Figure:  $x \in \mathbb{N}$ Figure:  $x \in \mathbb{R}$

## Means using python

### Example (Calculating the mean of our class data)

*X.mean() # gives the mean of all the variables through pandas*  
`X["Height"].mean()`  
`np.mean(X["Weight"])`

- ▶ The mean here is **fixed** because we calculate it on the same data.
- ▶ If we were to **collect new data** then the answer would be different.

### Example (Calculating the mean of a random variable)

```
import numpy as np
X = np.random.gamma(170, 1, size=20)
X.mean()
np.mean(X)
```

- ▶ The mean is **random**, so we get a different answer everytime.

# One variable: expectations and distributions

## Definition (The expected value)

- $\Omega$ : random outcome space



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## Definition (The sample mean)

- ▶ **i.i.d.** variables  $x_1, \dots, x_t, \dots, x_T$ : with  $x_t = x(\omega_t)$ ,  $\omega_t \sim P$ .

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- ▶ The sample mean of  $x_1, \dots, x_T$  is

$$\frac{1}{T} \sum_{t=1}^T x_t$$

The sample mean is  $O(1/\sqrt{T})$ -close  $\mathbb{E}_P[x_t]$  with high probability.

## Reminder: expectations of random variables

### A gambling game

What are the expected winnings if you play this game?

- ▶ [a] With probability 1%, you win 100 CHF
- ▶ [b] With probability 40%, you win 20 CHF.
- ▶ [c] Otherwise, you win nothing

### Solution



## Reminder: expectations of random variables

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### Solution

- ▶ Let  $x$  be the amount won, then  $x(a) = 100$ ,  $x(b) = 20$ ,  $x(c) = 0$ .

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- ▶ Let  $x$  be the amount won, then  $x(a) = 100, x(b) = 20, x(c) = 0$ .
- ▶ We need to calculate

$$\mathbb{E}_P(x) = \sum_{\omega \in \{a,b,c\}} x(\omega)P(\omega) = x(a)P(a) + x(b)P(b) + x(c)P(c)$$

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$$\mathbb{E}_P(x) = 1 + 8 + 0 = 9.$$

# Models

## Models as summaries

- ▶ They summarise what we can see in the data
- ▶ The ultimate model of the data **is** the data

## Models as predictors

- ▶ They make predictions about things **beyond** the data
- ▶ This requires some assumptions about the **data-generating process**.

## Example models

- ▶ A numerical mean
- ▶ A linear classifier
- ▶ A linear regressor
- ▶ A deep neural network
- ▶ A Gaussian process
- ▶ A large language model

# Estimates and decisions

We always need to make decisions based on some **estimates**.

## Estimate the bias of a coin

- ▶ I give you a coin that, lands with some fixed probability on heads.
- ▶ You are allowed to experiment with the coin.
- ▶ I will pay you **1 CHF** if you guess the throw correctly
- ▶ Otherwise you pay me  **$x$  CHF**.
- ▶ How much should I ask you to **pay** for the bet to be **fair**?
- ▶ What do you need to **know** to determine this?

## Example (If the coin is fair)

- ▶ If the coin is fair, then you only have 50% probability of guessing correctly.
- ▶ If you bet  $x$  CHF, your expected return is  $x$

# The Bernoulli distribution

## Definition (Bernoulli distribution)

We say that  $x \in \{0, 1\}$  has Bernoulli distribution with parameter  $\theta$  and write

$$x \sim \text{Bernoulli}(\theta),$$

when

$$\mathbb{P}(x) = \begin{cases} \theta & x = 1 \\ 1 - \theta & x = 0. \end{cases}$$

## Example (Applications of the Bernoulli distribution)

- ▶ A biased coin flip.
- ▶ Classification errors.

## Exercise: The expected value

If  $x$  is Bernoulli with parameter  $\theta$ , then what is the expected value of

- ▶ The variable  $f(x) = x$ ?
- ▶ The variable  $g(x) = x^2$ ?

# Two-variable models

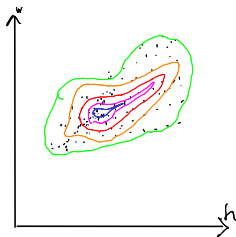


Figure:  $x \in \mathbb{R}^2$

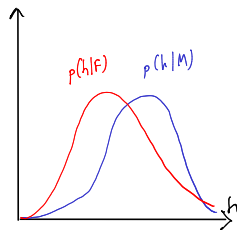
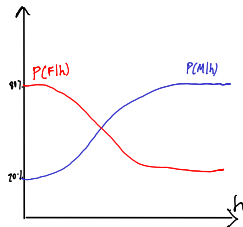
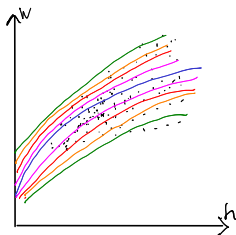


Figure:  $x \in \mathbb{N} \rightarrow y \in \mathbb{R}$





## Predicting $y$ from $x$ , discrete case.

Consider two variables,  $x, y$ . We can either care about

- ▶  $\mathbb{E}[y|x]$  the expectation of  $y$  for all  $x$ .
- ▶  $\mathbb{P}[y|x]$  the distribution of  $y$  for all  $x$ .

### Probability table for $P(x, y)$

$P(x, y)$	$y = 0$	$y = 1$
$x = 0$	54%	6%
$x = 1$	16%	24%

- ▶ How can we graph this?
- ▶ What is  $P(x)$ ?

### Conditional probability table for $P(y|x)$

$P(y   x)$	$y = 0$	$y = 1$
$x = 0$	90%	10%
$x = 1$	40%	60%

- ▶ What is  $\mathbb{E}[y | x]$ ?

# Distributions of two variables

In this setting, both  $x$  and  $y$  have a Bernoulli distribution. If we use a model whereby  $x$  is sampled first, and then  $y$ , then we can define two Bernoulli distributions. The first, for  $x$  is unconditional, while the second, for  $y$ , depends on the value of  $x$ :

$$\begin{aligned}x &\sim \text{Bernoulli}(\theta) \\ y \mid x &\sim \text{Bernoulli}(\phi_x).\end{aligned}$$

In our example,  $\phi_0 = 0.1$  and  $\phi_1 = 0.6$ .

# Homework

## Probability table for $P(x, y)$

$P(x, y)$	$y = -1$	$y = 0$	$y = 1$
$x = 0$	10%	20%	10%
$x = 1$	30%	20%	10%

Given the above table, calculate

- ▶  $P(x)$
- ▶ The conditional probability table for  $P(y|x)$ .
- ▶  $\mathbb{E}[y|x]$  for all values of  $x$ .

## Two variables: conditional expectation

### The height of different genders

The conditional expected height

$$\mathbb{E}[h \mid g = 1] = \sum_{\omega \in \Omega} h(\omega) P[\omega \mid g(\omega) = 1]$$

The empirical conditional expectation

$$\mathbb{E}[h \mid g = 1] \approx \frac{\sum_{t: g(\omega_t)=1} h(\omega_t)}{|\{t : g(\omega_t) = 1\}|}$$

### Python implementation

## Two variables: conditional expectation

### The height of different genders

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### Python implementation

```
h[g==1] / sum(g==1)
## alternative
import numpy as np
np.mean(h[g==1])
```

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# Populations, samples, and distributions

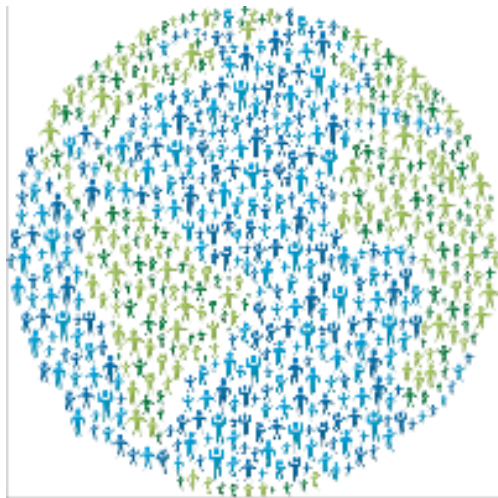


Figure: The world population



Figure: A sample

# Statistical assumptions

## Independent, Identically Distributed data

- ▶  $\omega_t \sim P$ : individuals  $\omega_t \in \Omega$  are drawn from some **distribution**  $P$
- ▶  $\mathbf{x}_t \triangleq \mathbf{x}(\omega_t)$  are some **features** of the  $t$ -th individual
- ▶ Here we are interested in properties of the **unknown** distribution  $P$ .

## Representative sample from a fixed population

- ▶ Finite population  $\Omega = \{\omega_1, \omega_2, \dots, \omega_N\}$
- ▶ A subset  $S \subset \Omega$  of size  $T < N$  is selected with a **uniform distribution**, i.e. so that

$$P(S) = T/N, \quad \forall S \subset \Omega.$$

- ▶ Here we are interested in statistics of the **unknown** population  $\Omega$ .
- ▶ We assume an underlying distribution  $P$  for convenience.
- ▶ We can try both cases essentially the same.



# Learning from data

## Unsupervised learning

- ▶ Given data  $x_1, \dots, x_T$ .
- ▶ Learn about the data-generating process.
- ▶ Example: Estimation, compression, text/image generation

## Supervised learning

- ▶ Given data  $(x_1, y_1), \dots, (x_T, y_T)$
- ▶ Learn about the relationship between  $x_t$  and  $y_t$ .
- ▶ Example: Classification, Regression

## Online learning

- ▶ Sequence prediction: At each step  $t$ , predict  $x_{t+1}$  from  $x_1, \dots, x_t$ .
- ▶ Conditional prediction: At each step  $t$ , predict  $y_{t+1}$  from  $x_1, y_1, \dots, x_t, y_t, x_{t+1}$

## Reinforcement learning

Learn to act in an **unknown** world through interaction and rewards

# Validating models

## Training data

- ▶ Calculations, optimisation
- ▶ Data exploration

## Validation data

- ▶ Fine-tuning
- ▶ Model selection

## Test data

- ▶ Performance comparison

## Simulation

- ▶ Interactive performance comparison
- ▶ White box testing

## Real-world testing

- ▶ Actual performance measurement

# Model selection

- ▶ Train/Test/Validate
- ▶ Cross-validation
- ▶ Simulation

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## Models

- ▶ k-Nearest Neighbours.
- ▶ Linear models and perceptrons.
- ▶ Multi-layer perceptrons (aka deep neural networks).
- ▶ Bayesian Networks

## Algorithms

- ▶ (Stochastic) Gradient Descent.
- ▶ Bayesian inference.

## Reproducibility

- ▶ Modelling assumptions
- ▶ Interactions and feedback

## Fairness

- ▶ Implicit biases in training data
- ▶ Fair decision rules and meritocracy

## The problems of Machine Learning (1 week)

Introduction

## Estimation

Answering a scientific problem

Pandas and dataframes

Single variable models

Two variable models

## Statistics, validation and model selection

## Course summary

Course Contents

## Reading for this week

Reading

# Reading for this week

## ISLP Chapter 1