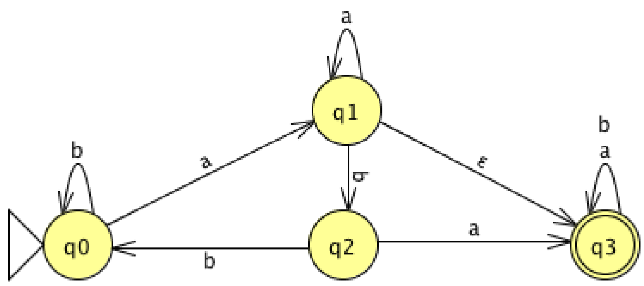
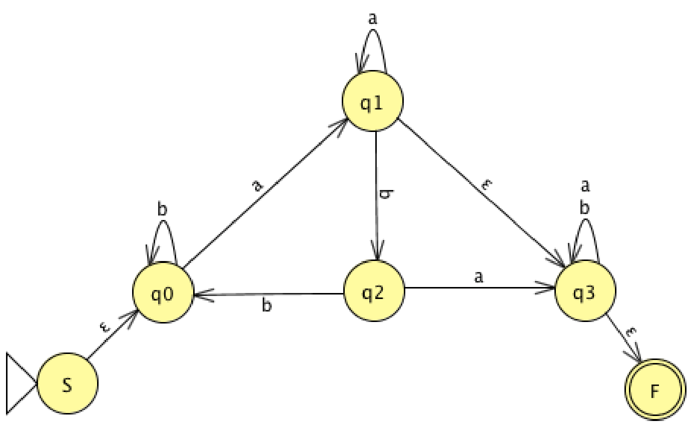
CS 361– Homework 4 – Answer key

Total possible points: 70

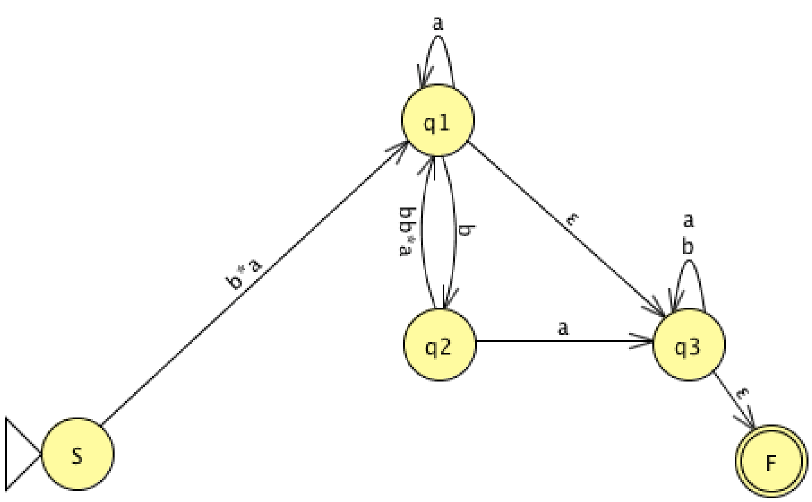
1. (10 points) Use the rules of the proof of **Lemma 1.60** to **convert** the following FA into a **regular expression**. (Remove states in *numerical order* and show all your *intermediate steps* for full credit.)



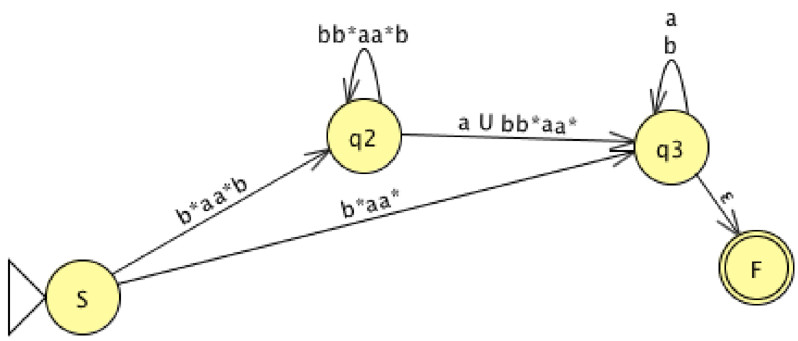
First, we create new S and F states



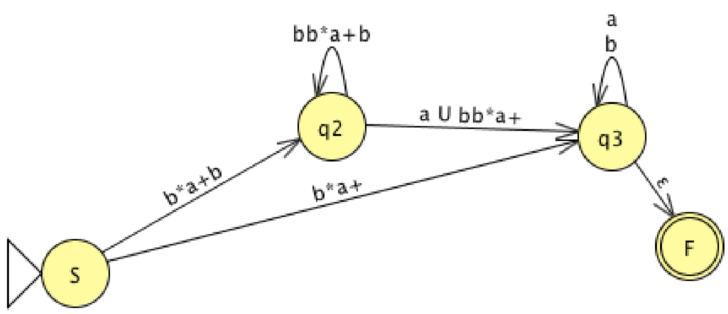
After eliminating q0



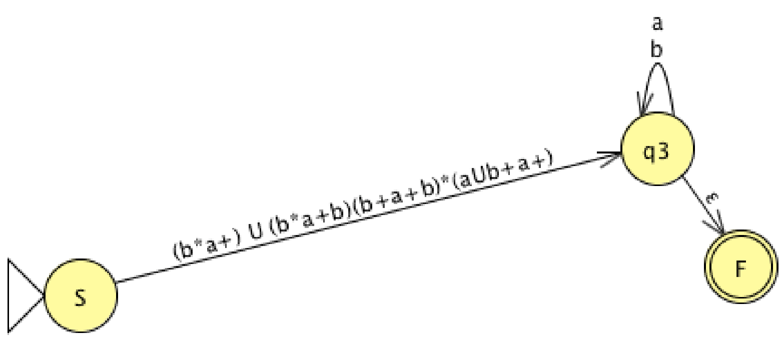
After eliminating q1



Let use a+ for aa*



After eliminating q2 and using b+ for bb*



After removing q3:

$((b^*a^+)\ U\ (b^*a^+b)(b^+a^+b)^*(aUb^+a^+))(aUb)^*$

2. (10 points) Describe the **error**[1] in the following "proof" that 0*1* is not a regular language: "The proof is by contradiction. Assume that 0*1* is regular. Let *p* be the pumping length for 0*1* given by the pumping lemma. Choose *s* to be the string 0^p1^p. We know that *s* is a member of 0*1*, but we know from the proof of $B=\{0^n1^n \mid n \geq 0\}$ not being regular[2] that *s* cannot be pumped. Thus we have a contradiction. So 0*1* is not regular."

   Even though any decomposition of either pumped up or pumped down strings of $s=0^p1^p$ are not in B, they are in 0*1*.

[1] An error must exist in this proof because 0*1* is regular.

[2] This proof is both in the textbook and lecture notes for the class.

3. (15 points) Using the pumping lemma for regular languages show that the following language is not regular

$$L_1 = \{a^m b^{m+3} \mid m > 0\}$$

   a. For the purpose of contradiction, we assume that L_1 is regular
   b. Let p be the pumping length.
   c. Consider string $s = a^p b^{p+3}$. The string is in L_1 and its length is greater or equal to p.
   d. Then according to the pumping lemma s can be divided into three parts satisfying $|y| > 0$ and $|xy| \geq p$, where for any possible decomposition y's symbols will be all *a*s. For any $|x| = j$ and $|y| = k$ we can represent all possible decompositions as $s = xyz = a^j a^k a^{p-k-j} b^{p+3}$, that is $x = a^j$, $y = a^k$ and $z = a^{p-k-j} b^{p+3}$.
   e. Now let i = 0, thus the pumped down string xy^0z becomes $a^j a^{p-k-j} b^{p+3} = a^{p-k} b^{p+3}$. Since $1 \leq k \leq p$ it will never has a decomposition with k = 0 value, thus $a^{p-k} b^{p+1} \notin L_1$.
   f. So there is a contradiction. Therefore, the assumption is wrong, L_1 is not regular

4. (15 points) Using the pumping lemma for regular languages show that the following language is not regular

$$L_2 = \{wcw \mid w \text{ is over } \{a, b, c\}^*\}.$$

For example if w = cab then string cabccab $\in L_2$.

   a. For the purpose of contradiction, we assume that L_2 is regular
   b. Let p be the pumping length.
   c. Consider string $s = a^p c a^p$. The string is in L_2 and its length is greater or equal to p.
   d. Then according to the pumping lemma s can be divided into three parts satisfying $|y| > 0$ and $|xy| \geq p$, where for any possible decomposition y's symbols will be all *a*s before symbol c . For any $|x| = j$ and $|y| = k$ we can represent all possible decompositions as $s = xyz = a^j a^k a^{p-k-j} c a^p$, that is $x = a^j$, $y = a^k$ and $z = a^{p-k-j} c a^p$.
   e. Not let i = 2, thus the pumped up string $xy^2z = xyyz$ becomes $a^j a^k a^k a^{p-k-j} c a^p = a^{p+k} c a^p$. Since $1 \leq k \leq p$ it will never has a decomposition with k = 0 value, thus the new string $a^{p+k} c a^{p\ 1} \notin L_2$.
   f. So there is a contradiction. Therefore, the assumption is wrong, L_2 is not regular

5. (20 points) Let $\Sigma = \{c, d\}$ and $L_3 = \{ c^n d^m \mid n < m \text{ and } n, m \geq 0 \text{ and } 1 < n < 3\}$. If L_3 is regular, build the corresponding FA that accepts the languages. If not, demonstrate that by using the pumping lemma proof.

   We can simplify the properties of L_3 and rewrite it as follows: $L_3 = \{c^2 d^{2+m} \mid m > 0\}$, which now allows us to determine that the language is regular: two c symbols followed by at least 3 *d* symbols. Below is an NFA that recognizes L_3.