
CS 361 – Homework 6

Total possible points: 60

1. (15 points) Use a **set notation** to **define** the language generated by the following grammar

$$R \rightarrow TT|U$$

$$T \rightarrow OT|T0| \#$$

$$U \rightarrow 0U00| \#$$

Consider the following strings. If they are in $L(G)$, derive them.

a) 00#0000

b) ϵ

- a. $R \rightarrow U \rightarrow 0U00 \rightarrow 00U0000 \rightarrow 00\#0000$
 b. They are not in $L(G)$

2. (15 points) Let $A = \{0^n 1^m 0^n 1^m \mid m, n > 0 \text{ and } m \neq n\}$. If A is a context free language, then build the corresponding PDA or CFG. If not, prove that A is not a context-free language using the pumping lemma.

Lets assume that A is context-free and let m be the critical length of the pumping lemma.

Let's pick any string $s = 0^m 1^{m+1} 0^m 1^{m+1}$, where $s \in L$ with length $|s| = m + m + 1 + m + m + 1 = 4m + 2$. s may be divided in 5 parts like $s = uvxyz$, where $|vy| \leq m$.

Case 1 – vxy is in the first 0^m , where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^{m+k} 1^{m+1} 0^m 1^{m+1}$. It is not in A because the first 0 is not same with the second 0. So, it is a contradiction!

Case 2 - vxy is in the second 0^m , where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^m 1^{m+1} 0^{m+k} 1^{m+1}$. It is not in A because the first 0's is not same with the second 0's. So, it is a contradiction!

Case 3 - vxy is in the first 1^{m+1} , where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^m 1^{m+1+k} 0^m 1^{m+1}$. It is not in A because the first 1's is not same with the second 1's. So, it is a contradiction!

Case 4 - vxy is in the second 1^{m+1} , where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^m 1^{m+1} 0^m 1^{m+1+k}$. It is not in A because the first 1's is not same with the second 1's. So, it is a contradiction!

Case 5 - vxy is in the first $0^m 1^{m+1}$, where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^{m+k} 1^{m+1+k} 0^m 1^{m+1}$. It is not in A because the first 0 is not

same with the second 0, and the second 1's is not same with the second 1's. So, it is a contradiction!

Case 6 - vxy is on the middle of $1^{m+1}0^m$, where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^m 1^{m+1+k} 0^{m+k} 1^{m+1}$. It is not in A because the first 0's is not same with the second 0's, and the second 1's is not same with the second 1's. So, it is a contradiction!

Case 7 - vxy is in the second $0^m 1^{m+1}$, where $p \geq k \geq 1$

: when $i = 2$, $uv^2xy^2z = 0^m 1^{m+1} 0^{m+k} 1^{m+1+k}$. It is not in A because the first 0's is not same with the second 0's, and the first 1's is not same with the second 1's. So, it is a contradiction!

Above, it showed that all cases are not including in A. So, my assumption was wrong, and A is not context-free.

3. (15 points) Let $B = \{a\#b\#c \mid a, b, c \text{ are sequences of } 1\text{'s}; |c| = |a| + |b|; |a| \geq 0; \text{ and } |b| > 0\}$. If B is a context free language, then build the corresponding CFG. If not, use the pumping lemma to show that B is not a context-free language.

$S \rightarrow aAc \mid bBc$
 $A \rightarrow aAc \mid \#B \mid \epsilon$
 $B \rightarrow bBc \mid \# \mid \epsilon$

4. (15 points) Let $C = \{a^n b^m c^{2n} \mid m, n > 0\}$. If C is a context free language, then build the corresponding PDA. If not, use the pumping lemma to show that C is not a context-free language.

C is a context-free language

$S \rightarrow aScc \mid aBcc$
 $B \rightarrow bB \mid b$

