CS 361– Homework 1

Total possible points: 30

1. (17 points) Let A ={(x), (y), ()} and B ={(x), (y)}
   a. Is A a subset of B?
      No
   b. Is B a subset of A?
      Yes
   c. What is $A \cup B$?
      {(x),(y),()}
   d. What is $A \cap B$?
      {(x),(y)}
   e. What is $A \times B$?
      {((x),(x)), ((x),(y)), ((y),(x)), ((y),(y)), ((),(x)), ((),(y))}
   f. What is the power set of A?
      { {}, {(x)}, {(y)}, {()}, {(x),(y)}, {(),(x)}, {(y),()}, {(),(y),(x)} }
   g. What is $\overline{A \cap B}$?
      {()}

2. (5 points) Find the error in the following proof that 2 =1.
   Consider equation $a = b$. Multiply both sides of a to obtain $a^2 = ab$. Substract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$ to get $(a + b) = b$. Finally, let a and b equal to 1, which shows that 2 = 1
   Both sides cannot be divided by (a-b) since in our assumptions a=b and therefore (a-b) = 0, and the division by zero is undefined.

3. (8 points) Let *w* be a string over an alphabet Σ. **Prove** that $(w^i)^R = (w^R)^i$, where *R* is the string's reverse operation and $i \geq 0$ is the string's repetition operation.
   Without loss of generality let pick a string w = (s_1, s_2, …, s_{n-1}, s_n), where n < ∞, since string are finite sequences. In our proof we show that lhs and the rhs of the equality produce the same string, i.e., at each index in both strings have the same symbol.
   First the left hand side:
   Let $w^i = s_1^1, s_2^1, \ldots, s_{n-1}^1, s_n^1, s_1^2, s_2^2, \ldots, s_{n-1}^2, s_n^2, \ldots, s_1^i, s_2^i, \ldots, s_{n-1}^i, s_n^i$, where a superscript indicates the repetition.
   Then $(w^i)^R = s_n^i, s_{n-1}^i, \ldots, s_2^i, s_1^i, \ldots, s_n^2, s_{n-1}^2, .., s_2^2, s_1^2, s_n^1, s_{n-1}^1, \ldots, s_2^1, s_1^1$
   Second the right hand side
   $w^R = s_n, s_{n-1}, \ldots, s_2, s_1$
   Then $(w^R)^i = s_n^1, s_{n-1}^1, \ldots, s_2^1, s_1^1, s_n^2, s_{n-1}^2, \ldots, s_2^2, s_1^2, \ldots, s_n^i, s_{n-1}^i, \ldots, s_2^i, s_1^i$
   The symbols of the lhs and rhs string differ in the superscripts associated with each symbol. However, $s_m^k = s_m^j$, since the superscript does not change the symbol but merely indicates how tha symbol was created, i.e., the first symbols $s_n^i = s_n^1$ are the same only s_n^i was created on ith repetition and s_n^1 is the "original" one. However those two symbols are the same. If we remove the superscript from both strings then we end up with the identical strings.