CS 361 – Homework 1

Total possible points: 30

1. (17 points) Let A ={(x), (y), ()} and B ={(x), (y)}
   a. Is A a subset of B?
   b. Is B a subset of A?
   c. What is $A \cup B$?
   d. What is $A \cap B$?
   e. What is $A \times B$?
   f. What is the power set of A?
   g. What is $\overline{A \cap B}$?

a. No, A is not a subset of B
b. Yes, B is a subset of A
c. { (x), (y), () }
d. { (x), (y) }
e. { (x, x), (x, y), (y, x), (y, y), ((), x), ((), y) }
f. P(A) = { Ɵ, {(x)}, {(y)}, {()}, {(x), (y)}, {(x), ()}, {(y), ()}, {(x), (y), ()} } = 2^3=8
g. { () }

2. (5 points) Find the error in the following proof that 2 = 1.
   Consider equation $a = b$. Multiply both sides of a to obtain $a^2 = ab$. Substract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$ to get $(a + b) = b$. Finally, let a and b equal to 1, which shows that 2 = 1.

- Consider equation a = b.
- Multiply both sides of a to obtain $a^2 = ab$.
- Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$.
- Factor each side, (a + b)(a – b) = b(a – b).
- Divide each side by (a – b) : $\frac{(a+b)(a-b)}{(a-b)} = \frac{b(a-b)}{(a-b)}$. This part is the error.
- Since a = b, (a – b) is zero. Dividing by 0 is actually impossible mathematically.
- Thus, the proof is wrong.

3. (8 points) Let w be a string over an alphabet Σ. **Prove** that $(w^i)^R = (w^R)^i$, where R is the string's reverse operation and $i \geq 0$ is the string's repetition operation.

- The first proof:
- w is the string over an alphabet $\sum$, and R is the reverse string of w.

- Let w be ‘apple’ and prove that $(w^i)^R = (w^R)^i$
- At first, $(w^i)^R$: $w^i = appleappleapple$ ⋯⋯, then $(w^i)^R$ = elppaelppaelppaelppa⋯⋯.
- Secondly, $(w^R)^i$: w^R = elppa, then $(w^R)^i$ = elppaelppaelppaelppa⋯⋯.
- According the proof, $(w^i)^R = (w^R)^i$.

- The Second proof:
- $(w^i)^R = (w^R)^i$
- $w^{iR} \quad = w^{Ri}$
- $\mathrm{i} * \mathrm{R} = \mathrm{R} * \mathrm{i}$
- $(w^i)^R = (w^R)^i$