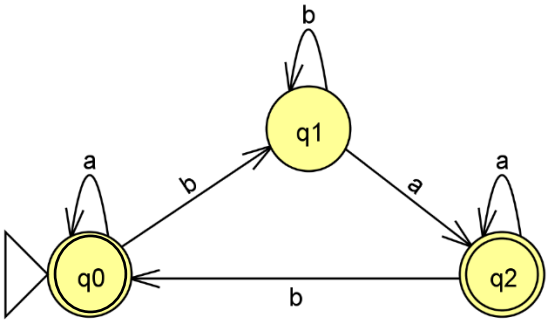
CS 361 – Homework 2

Total possible points: 65

1. (10 points) Let M_1 be the FA defined by $Q = \{q_0, q_1, q_2\}, \Sigma = \{1, 2\}, F = \{q_0, q_2\}$, and δ:

δ	*1*	*2*
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

a. Give the *state diagram* of M_1.



b. *Trace* the computations of M that process the string *abaa*, *bbbabb*, *bababa*, and *bbbaa*.

$[q_0, 1211] \vdash_{M_1} [q_0, 211] \vdash_{M_1} [q_1, 11] \vdash_{M_1} [q_2, 1] \vdash_{M_1} [q_2, \epsilon]$ (accept)

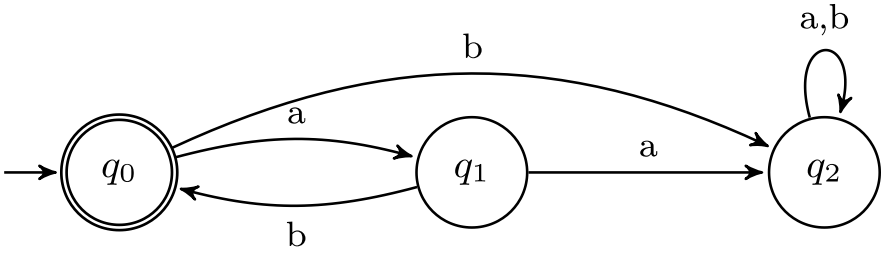
$[q_0, 222122] \vdash_{M_1} [q_1, 22122] \vdash_{M_1} [q_1, 2122] \vdash_{M_1} [q_1, 122] \vdash_{M_1} [q_2, 22] \vdash_{M_1} [q_0, 2] \vdash_{M_1} [q_1, \epsilon]$ (reject)

$[q_0, 212121] \vdash_{M_1} [q_1, 12121] \vdash_{M_1} [q_2, 2121] \vdash_{M_1} [q_0, 121] \vdash_{M_1} [q_0, 21] \vdash_{M_1} [q_1, 1] \vdash_{M_1} [q_2, \epsilon]$ (accept)

$[q_0, 22211] \vdash_{M_1} [q_1, 2211] \vdash_{M_1} [q_1, 211] \vdash_{M_1} [q_1, 11] \vdash_{M_1} [q_2, 1] \vdash_{M_1} [q_2, \epsilon]$ (accept)

c. Which of the strings in part (b) are *accepted* by M_1?
All except 222122

2. (10 points) Consider the FA M_2 given below:



   a. *Explain*, i.e., describe, what is the language recognized by M_2

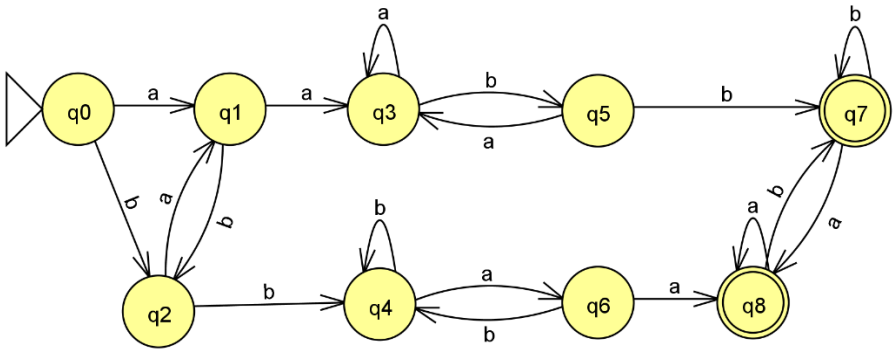
   $$L = \{ (ab)^n \mid n \geq 0 \}$$

   Substring ab repeated zero or more times.

   b. Which of the strings *baba*, *baab*, *abaaab*, ***ε*** are accepted by M_2?

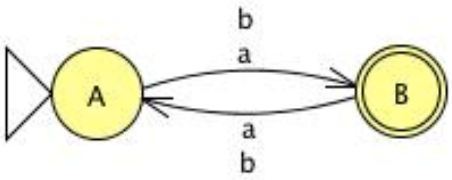
   Only the last string, ***ε***, is accepted by M_2.

3. (15 points) Build a *deterministic FA* for the following language L={*x* over {*a, b*}| *x* contains **both** substrings *aa* and *bb*}

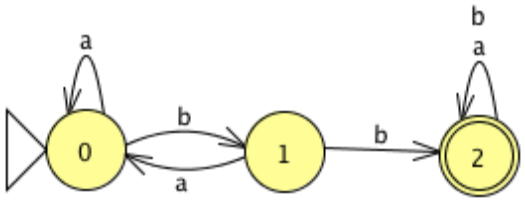


4. (15 points) Build a *deterministic FA* that accepts the set of strings of odd length over {*a, b*} that **do not** contain the substring *bb*.

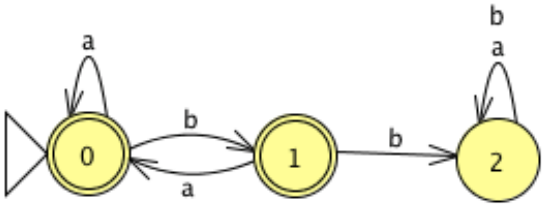
   We will do a systematic approach creating such DFA. First, let's create a DFA that accepts strings of odd length.



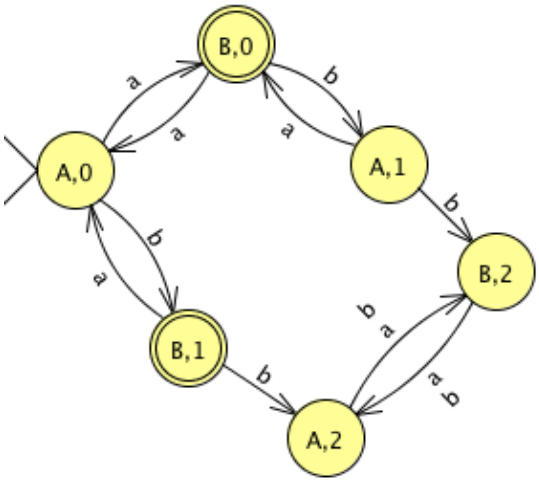
   Now let's create a DFA that accepts all string that have substring bb (it is easier to think in term of positive acceptance than negative)



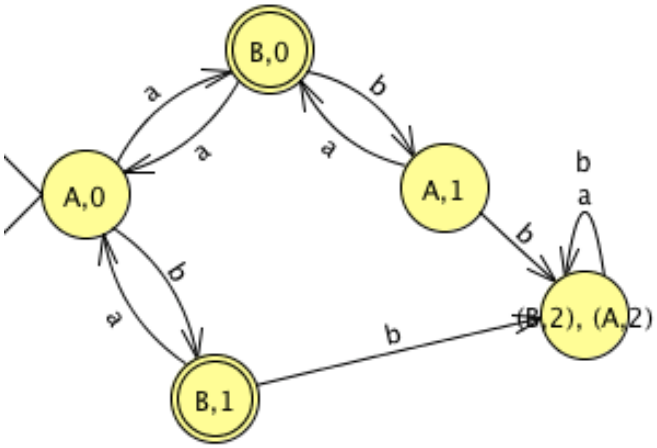
Now, let's use the negation trick to convert the above DFA to one that accepts the language that is the compliment of the above DFA.



Next, we create the cross product of the two DFAs.



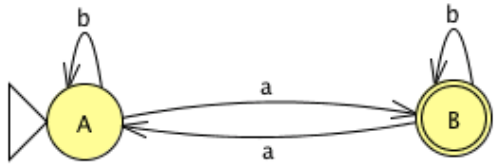
It can be simplified to:



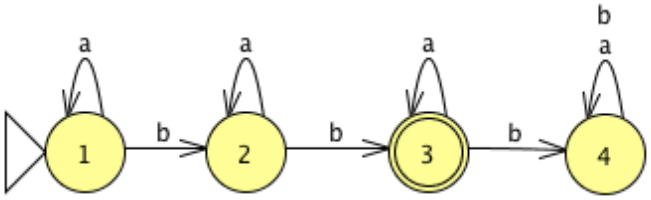
5. (15 points) Build a *deterministic FA* for the following language L={*x* over {*a, b*}| *x* contains **an odd number** of *a* symbols, or **exactly two** *b* symbols}

For this problem the simplest way is to do the cross product of two automata, but determining the final state differently: *when the resulting state contains at least one of the final states of the original DFAs*.

DFA for the first property:



DFA for the second property:



The resulting DFA