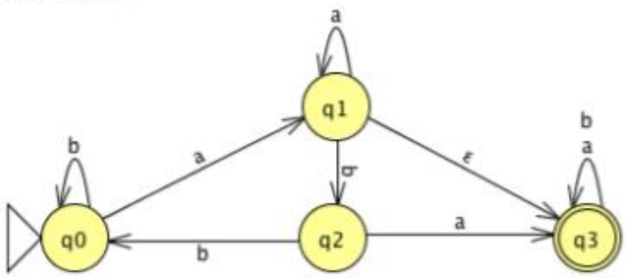
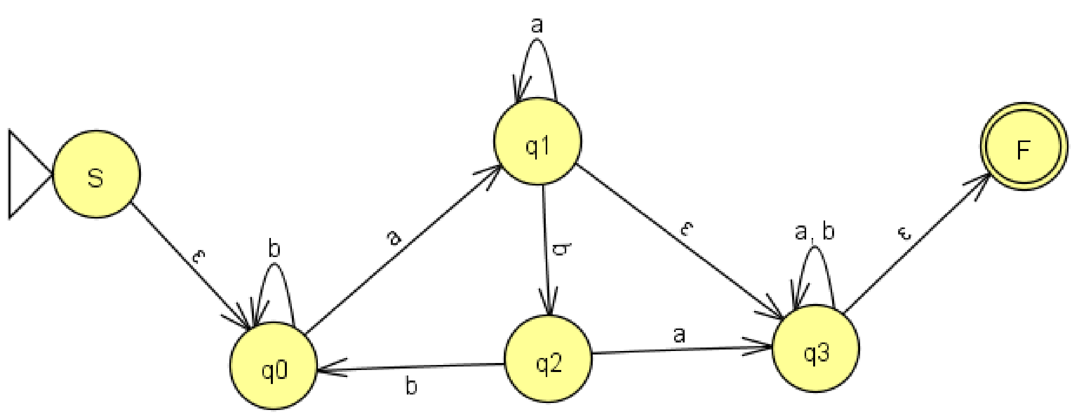
CS 361– Homework 4
Total possible points: 75

1. (15 points) Use the rules of the proof of **Lemma 1.60** (or the method we used in class) to **convert** the following FA into a **regular expression**. (Remove states in *numerical order* and show all your *intermediate steps* for full credit.)

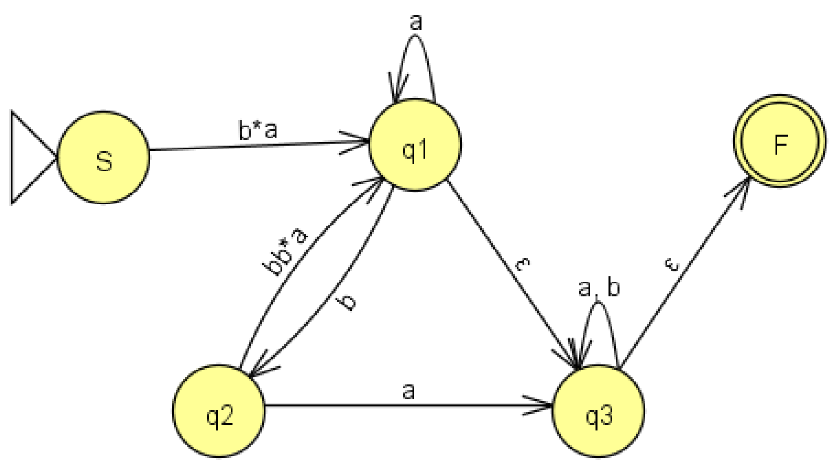




q0

$$S \xrightarrow{\varepsilon b^{*} a} q1$$

$$q2 \xrightarrow{b b^{*} a} q1$$



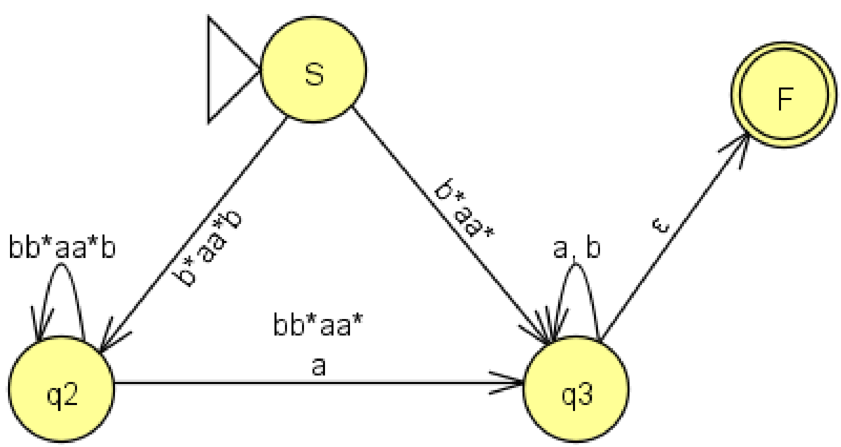
q1

$$S \xrightarrow{b^*aa^*b} q2$$

$$S \xrightarrow{b^*aa^*\varepsilon} q3$$

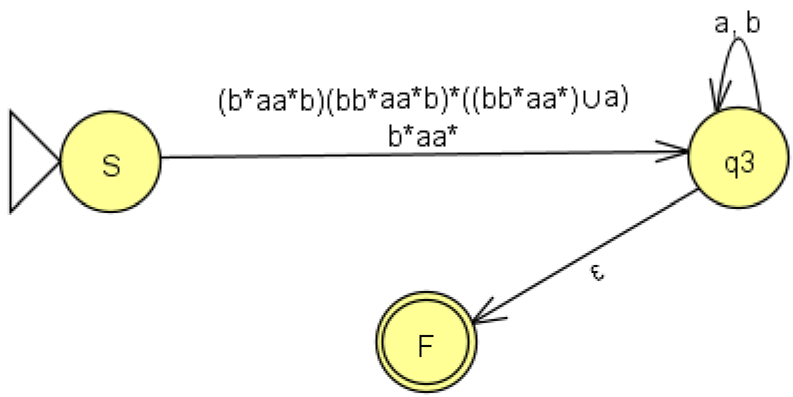
$$q2 \xrightarrow{bb^*aa^*b} q2$$

$$q2 \xrightarrow{bb^*aa^*\varepsilon} q3$$



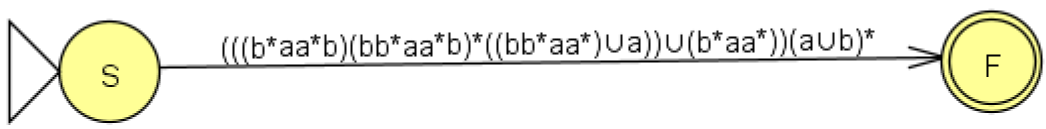
q2

$$S \xrightarrow{(b^*aa^*b)(bb^*aa^*b)^*((bb^*aa^*)\cup a)} q3$$



q3

$$S \xrightarrow{(((b^*aa^*b)(bb^*aa^*b)^*((bb^*aa^*)\cup a))\cup(b^*aa^*))(a\cup b)^*} q3$$



Regular Expression : $(((b^*aa^*b)(bb^*aa^*b)^*((bb^*aa^*)\cup a))\cup(b^*aa^*))(a\cup b)^*$

2. (10 points) Describe the **error**[1] in the following "proof" that 0*1* is not a regular language: "The proof is by contradiction. Assume that 0*1* is regular. Let p be the pumping length for 0*1* given by the pumping lemma. Choose s to be the string 0^p1^p. We know that s is a member of 0*1*, but we know from the proof of B={0^n1^n| n ≥ 0} not being regular[2] that s cannot be pumped. Thus we have a contradiction. So 0*1* is not regular."

The sentence 'we know that s is a member of 0^*1^*, but we know from the proof of B={0^n1^n | n>= 0} not being regular2 that s cannot be pumped' is wrong. Because it can be pumped but it is just not a regular language.

3. (15 points) Using the pumping lemma for regular languages show that the following language is not regular

$$L_1 = \{a^m b^{m+3} \mid m > 0\}$$

Assume L_1 is regular, p is the pumping length.
Let s = $a^p b^{p+3}$, |s| = 2p+3 ≥ p
Then s may be divided into three parts (s = xyz) satisfying the conditions of the Pumping Lemma:
|xy|≤ p, |y|>0, and $xy^iz \epsilon L$
Let x = a, y = a, z = $a^{p-2}b^{p+3}$
|y| = 1 > 0
|xy| = 2 ≤ p
Let i = 2
x = a, $y^2 = aa,\ z = a^{p-2}b^{p+3}$
$a^{p+1}b^{p+3} \nsubseteq L_1$
# of a doesn't match the # of b.
Because $a^{p+1}b^{p+3} \nsubseteq L_1$, it is not the regular. Our initial assumption was incorrect.

4. (15 points) Using the pumping lemma for regular languages show that the following language is not regular

$$L_2 = \{wcw \mid w \text{ is over } \{a, b, c\}^*\}.$$

For example if w = cab then string cabccab ∈ L_2.

Assume L_1 is regular, p is the pumping length.
Let s = $a^p bcca^p bc$, |s| = 2p+5 ≥ p
Then s may be divided into three parts (s = xyz) satisfying the conditions of the Pumping Lemma:
|xy|≤ p, |y|>0, and $xy^iz \epsilon L$
Let x = a^p, y = b, z = $cca^p bc$
|y| = 1 > 0
|xy| = p+1 ≤ p
Let i = 2
x = a^p, $y^2 = bb,\ z = cca^p bc$

a^pbbcc$a^p bc \nsubseteq L_1$

(a^pbb)c($ca^p bc$), (a^pbb) is not equal to ($ca^p bc$) but they should be same

Because (a^pbb) is not equal to ($ca^p bc$) , it is not the regular.

Our initial assumption was incorrect.

5. (20 points) Let $\Sigma = \{c, d\}$ and $L_3 = \{c^n d^m \mid n < m;\ n, m \geq 0;\ 1 < n < 3\}$. If L_3 is regular, build the corresponding FA that accepts the languages. If not, demonstrate that by using the pumping lemma proof.