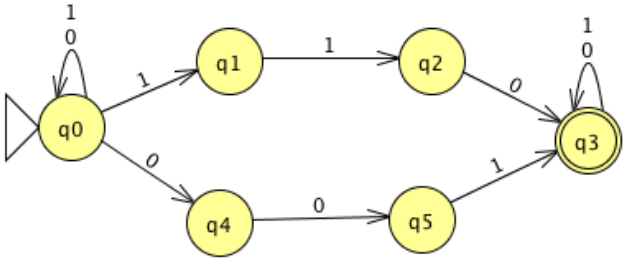
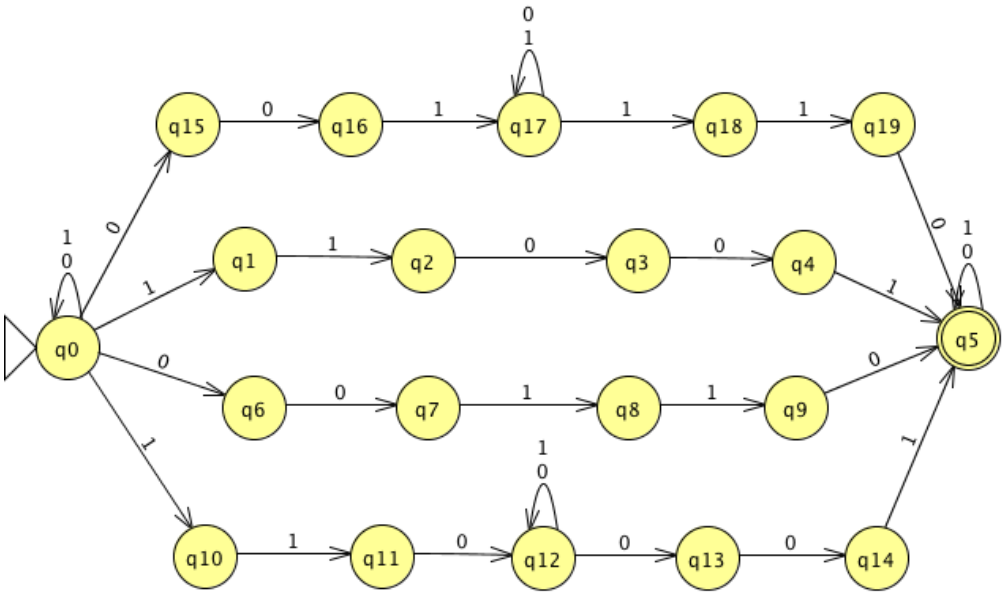
CS 361– Homework 3 – Answer Key

Total possible points: 60

1. (15 points) Design an **NFA** that recognizes the language L_1 = {*x* over {0, 1}| *x* contains substring 110 or x contains substring 001}.

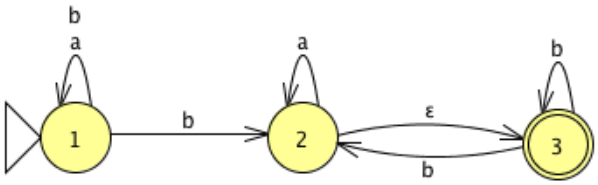


2. (15 points) Construct an **NFA with no more than 20 states** that recognizes language L_2 = {*x* over {0, 1}| *x* contains both substring 110 and substring 001}.



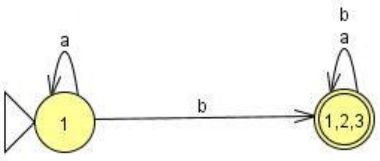
The top chain of states accepts string where 001 substring is followed by 110 substring.
The bottom chain of states accepts string where 110 substring is followed by 001 substring.
The middle chains of states accept those strings that substrings 001 and 110 overlap, that is, they have either 11001 and 00110 substrings.

3. (10 points) Use **Theorem 1.39**, which we discussed in class, to **convert** the following **NFA M** into an equivalent **deterministic FA M'**.

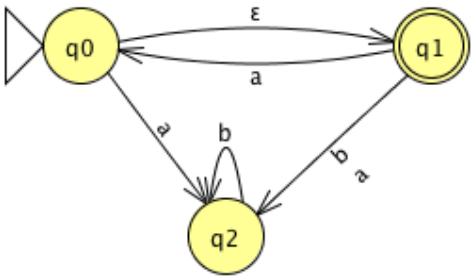


	a	b
1	{1}	{1}, {2}, {3}
2	{2}, {3}	{2}, {3}
3	{ }	{2}, {3}
{1}	{1}	{1, 2, 3}
{1, 2, 3}	{1, 2, 3}	{1, 2, 3}

Remember that:
Start state: Start of DFA = E(Start of NFA) = E(1) = 1
Final states: Any DFA state that contains a state that is final on NFA = anything with 3



4. (10 points) Use **Theorem 1.39**, which we discussed in class, to **convert** the following **NFA M** into an equivalent **deterministic FA M'**.

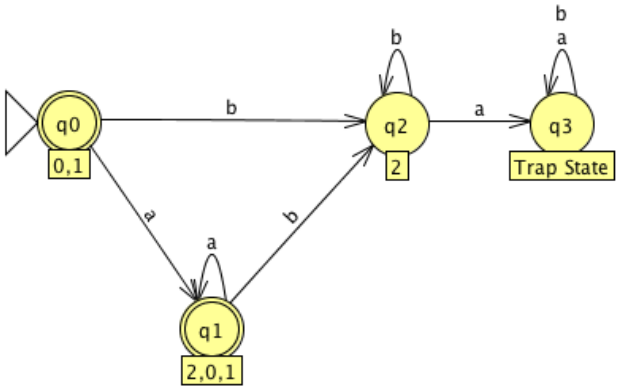


The start state of NFA is
$E(\{q0\}) = \{q0,q1\}$

Now we calculate the transition on a and b from the start state:

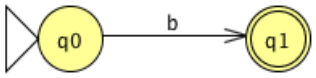
$\delta_{DFA}(\{q0,q1\},a) = E(\delta_{NFA}(q0,a)) \cup E(\delta_{NFA}(q1,a)) = E(\{q2\}) \cup E(\{q0,q2\}) = \{q2\} \cup \{q0,q1,q2\} = \{q0,q1,q2\}$
$\delta_{DFA}(\{q0,q1\},b) = E(\delta_{NFA}(q0,b)) \cup E(\delta_{NFA}(q1,b)) = E(\{\}) \cup E(\{q2\}) = \{\} \cup \{q2\} = \{q2\}$
$\delta_{DFA}(\{q0,q1,q2\},a) = E(\delta_{NFA}(q0,a)) \cup E(\delta_{NFA}(q1,a)) \cup E(\delta_{NFA}(q2,a)) = E(\{q2\}) \cup E(\{q0,q2\}) \cup E(\{\}) = \{q2\} \cup \{q0,q1,q2\} = \{q0,q1,q2\}$
$\delta_{DFA}(\{q0,q1,q2\},b) = E(\delta_{NFA}(q0,b)) \cup E(\delta_{NFA}(q1,b)) \cup E(\delta_{NFA}(q2,b)) = E(\{\}) \cup E(\{q2\}) \cup E(\{q2\}) = \{q2\} \cup \{q2\} = \{q2\}$
$\delta_{DFA}\{q2,a\} = E(\delta_{NFA}(q2,a)) = E(\{\}) = \{\}$
$\delta_{DFA}\{q2,b\} = E(\delta_{NFA}(q2,b)) = E(\{q2\}) = q2$
$\delta_{DFA}(\{\},a) = \{\}$ and $\delta_{DFA}(\{\},b) = \{\}$

The resulting machine:

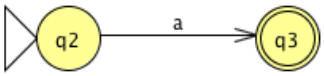


5. (10 points) Construct an **nondeterministic FA** that accepts the language described by the following regular expression: $(baUa^{+})^{*}b$ (For full credit show all your *intermediate steps*)

NFA for b

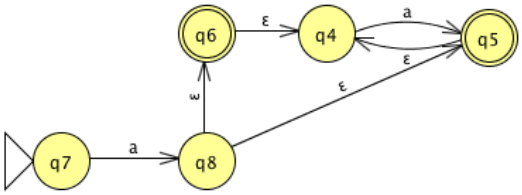


NFA for a

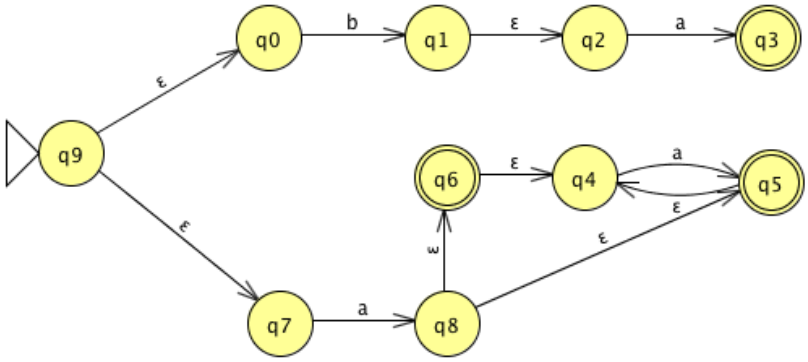


NFA for ba

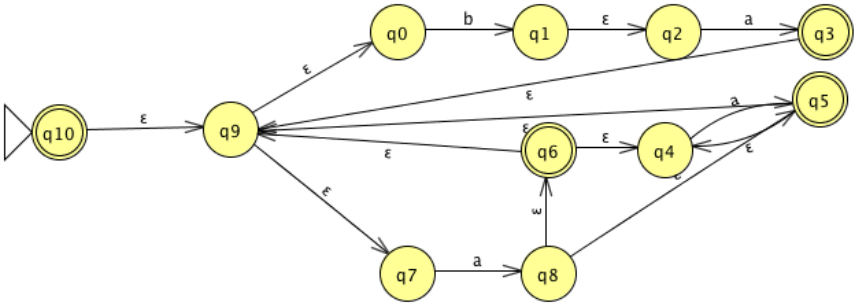
NFA for a^{+}



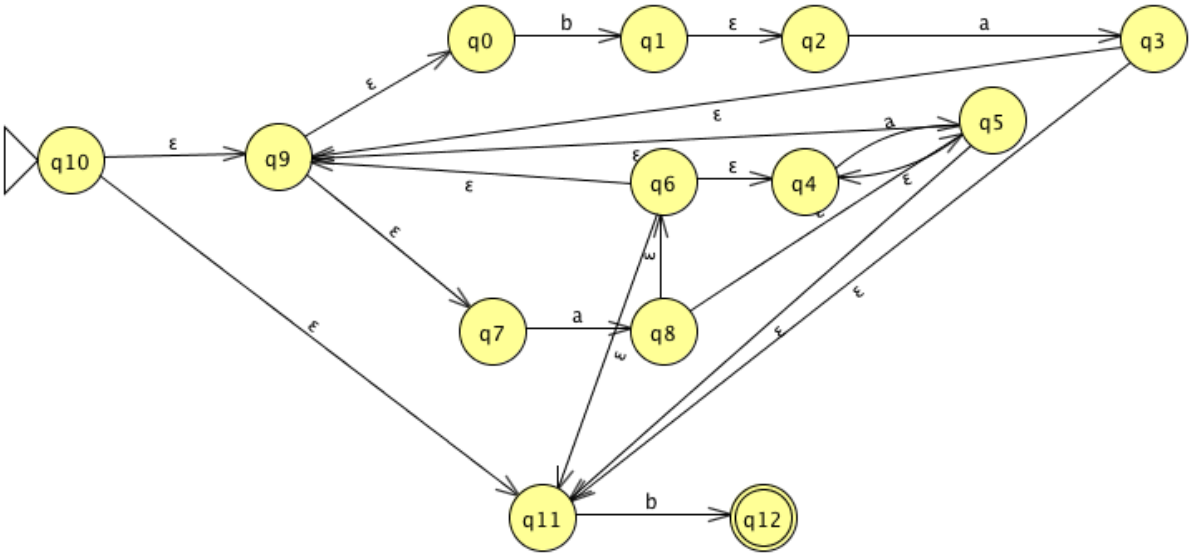
NFA for ba U a^{+}



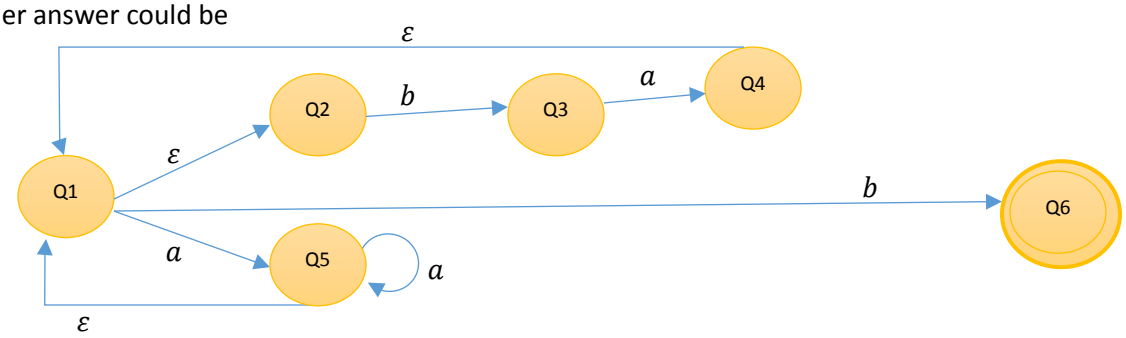
NFA for (ba U a^{+})*



NFA for (ba U a^{+})*b



A simpler answer could be



Please give full credit as long as at least *the most important* of the intermediate steps are shown (e.g, q1 to Q4, Q1 to Q5, and connection from Q1 to Q6)