Relativistic Kinematics

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This study aimed to understand relativistic kinematics while utilizing detector simulations in both the lab frame, and the center of mass. Two programs used in this study include the Relativistic Kinematics Program (RELKIN), and the Detector Simulation Program (DETSIM).

Chapter 3

Question 3.4

$$\sqrt{s_{p\rightarrow e}} = \sqrt{(m_p + m_e)^2 + 2m_e(E_p - m_p)} \tag{1}$$

$$\sqrt{s_{e\rightarrow p}} = \sqrt{(m_p + m_e)^2 + 2m_p(E_e - m_e)} \tag{2}$$

The subscripts in equations 1&2, p&e, correlate to proton and electron, respectively. The subscripts on the CM (Center of Mass) energies ($\sqrt{s}$) correspond to the two situations situations either involving a proton colliding into the initial electron (p→ e) and an electron colliding with a stationary proton (e→ p). The CM energy of the electron colliding into a target proton was higher in energy by an order of magnitude. $\sqrt{s_{p\rightarrow e}}$ was $\boxed{9.44 \times 10^{-1}\,\text{GeV}}$, and $\sqrt{s_{e\rightarrow p}}$ was $\boxed{4.43\,\text{GeV}}$.

The advantage of using a collider (beam-beam) rather than a stationary target (beam-target) is the total 4-momentum for both frames. Converse to the beam-target, which has a total 3-momentum equaling zero, the beam-beam contains doubled 4-momentum. The (beam-target) and (beam-beam) collisions yielded energies of $\boxed{0.303\,\text{GeV}}$ and $\boxed{90.0\,\text{GeV}}$, respectively. This trait allows for higher energy collisions compared to a stationary hit.

Question 3.5

The threshold energy for producing K^0 mesons plus Λ^0 in pion-proton collisions is $\boxed{885.7\,\text{MeV}}$. No other reactions could produce K^0 mesons plus Λ^0 other than the pion-proton collision through the RELKIN program. To produce pions, a q-value of $\boxed{140.0\,\text{MeV}}$ and momentum of $\boxed{794.5\,\text{MeV}}$ was required.

$$\frac{s_{p\rightarrow Be} - (m_p + m_{Be})^2}{2m_{Be}} + m_p = E_p \tag{3}$$

The mass of Beryllium (Be) used in equation 3, was 1.497×10^{-26} kg *or 8.364 GeV. The threshold energy mentioned before was used for $s_{p\rightarrow Be}$, and the constants for a proton and Be. The beam energy required for a (p → Be) collision is −4.181 GeV.*[1]

$$\frac{s_{p\rightarrow p} - (m_p + m_p)^2}{2m_p} + m_p = E_p \tag{4}$$

Equation 4 represents the (p→ p) collision.[2] Following the prior procedures, the beam energy required was found to be −0.04663 GeV.

The electron and positron energy required was $\boxed{1776.9\,\text{MeV}/c^2}$.

Question 3.8

$$\tan(\theta) = \frac{\sin(\theta^*)}{\gamma(\cos(\theta^*) + \beta/\beta^*)} \tag{5}$$

$$\theta_{max} = \frac{E^*_\nu}{E_\nu} \tag{6}$$

The maximum muon Lab angle was found to be $\boxed{135°}$ for a momentum of 1000 MeV with 1000 iterations. It is imperative to note that the maximum muon Lab angle changes with respect to different momentum parameters. A test was conducted with a momentum of 500 MeV with 1000 iterations, and the muon Lab angle was $\boxed{150°}$. The neutrino also has a maximum Lab angle represented in equation 6. The equation was derived from equation 5, using the relationships for β.

Question 3.12

The data generated for the decaying π^0 of momentum 5 GeV were plotted on figure 1. The decay was collected for 1000 iterations, yielding the gamma ray opening angle and the characteristic shape of the data.

$$M = 2\sqrt{E_1E_2}\sin\frac{\theta}{2} \tag{7}$$

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[1] This section is for reference, the corrected ($p \rightarrow p$) is extrapolated upon

[2] Directions were not read as they state, "you may assume that the target is a proton".

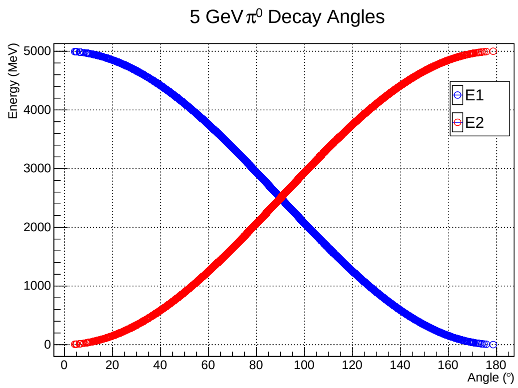

Figure 1. Decay angles of π^0 for 5 GeV using 1000 iterations. E1 and E2 represent gamma ray 1 and gamma ray 2, respectively.

Equation 7 was used to find the π^0 mass for each of the 1000 iterations at 5 GeV. The π^0 mass taken with the average value for 1000 iterations was $\boxed{3.327\,\mathrm{GeV}}$.

Purely with only the energies and angles, the decay point is not obtainable. Extra parameters to more accurately determine the decay point of π^0 include, detector distance, coordinate system, additional lengths, and time intervals.

Chapter 4

Table I. Sample table of particle parameters measured by DETSIM. This sample includes an energy resolution of 10% and 0 cm spatial resolution.

$\gamma 1$ x_1	$\gamma 1$ y_1	$\gamma 1$ E_1 (MeV)	$\gamma 2$ x_2	$\gamma 2$ y_2	$\gamma 2$ E_2 (MeV)
60.61	-7.92	2618.12	-73.88	9.66	2222.89
-19.23	-21.18	4261.38	0	0	0
46.22	3.44	3412.53	-98.01	-7.29	1653.78

Question 4.8

Three trials were conducted at 20 m to measure the accuracy of the detector, with 10 iterations each ([6/10],[4/10],[6/10]). The decay point should be located at approximately $\boxed{20\,m}$ for a ≈53.3 percent accuracy in detecting decay photons from 5 GeV/c π^0 mesons.

To determine the mass of the π^0 another trial was conducted, but this time with 100 iterations. The accuracy was 52% [52/100]. Using equation 7, the mass determined from the angle and energy outputs was $\boxed{0.135\,GeV}$. The angles were derived using the x and y points and arctan trigonometry with the 20 m distance.

This detector will be able to reconstruct the mass of the decaying π^0 with a 0.047% efficiency.

Question 4.9

The momentum magnitude of the π^0 is $\boxed{4998.18\,\mathrm{MeV}}$ with a direction in the z-direction with a z-offset of $\boxed{3.31^\circ}$.

The decay point can be deduced by knowing the known mass of the π^0, but not through deduction with the prior given parameters. The DETSIM program provides the x and y coordinates for the detector "hit" locations as well as the z-directional offsets, so the directionality can be determined. With the directionality and the magnitude in the detector, trigonometry can be used to traceback the particle to the point of decay.

Question 4.11

The nomenclature for this section will be defined as (energy (%)/spatial (cm)) resolution for more intuitive analysis. The following runs were run with 50 iterations each.

For a 5 percent determination of the mass of a decaying particle, the data determined a high energy resolution of $\boxed{20\,\%}$ and a spatial resolution of $\boxed{2\,cm}$ are both needed. The accuracy and the relatively low particle deviation from (0/0) shows that in table II.

Table II. Accuracies and masses of the DETSIM detector.

Resolution Parameters (GeV/cm)	Accuracy (%)	Decaying Particle Mass (GeV)	Deviation from (0/0) (%)
(0/0)	52	0.1350±0.3674	0.00
(10/0)	42	0.1349±0.3673	0.04
(20/0)	60	0.1337±0.3656	0.97
(0/1)	46	0.1349±0.3673	0.06
(10/1)	58	0.1352±0.3677	0.14
(20/1)	56	0.1373±0.3706	1.73
(10/2)	60	0.1353±0.3678	0.22
(20/2)	68	0.1351±0.3675	0.06

Acknowledgments

We would like to thank Group 1 for providing data for question 4.9.

[1] `http://www.phys.hawaii.edu/~shige/phys481L/Particle.txt`

[2] `http://www.phys.hawaii.edu/~shige/phys481L/relkin.pdf`

[3] `http://www.phys.hawaii.edu/~shige/phys481L/detsim.pdf`