

HW 03: CHARACTERIZING A LENS

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1. INTRODUCTION

This study focuses on solidifying basic optics concepts while utilizing the thin-lens formula, provided in equation 1. A source "at infinity" will produce a focal point where the rays converge passing through a thin convex lens. This effect can be emulated at shorter distances, but the incoming light rays will focus the point at a longer focal length than the infinite rays due to the steeper angle [Tipler \(2004\)](#). The focal length is defined as the distance between the point where all light rays converge and are brought to a focus and the center of the lens. [Young \(2004\)](#).

The thin-lens formula uses an object and image length and allows for the generation of the apparatus lens focal length. An important property of a thin lens is that it allows for neglecting lens thickness, which greatly simplifies the optics calculations. The application of the thin-lens equation can be extrapolated from the optical apparatus, which is detailed in figure 1 [Chromey \(2011\)](#). The equation shows the relation of the inverse object distance (d_o) added to the inverse image distance (d_i) results in the inverse focal length (f). The focal length is defined as the distance from the center of the lens and the point in which light rays are focused.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad (1)$$

The studied apparatus consisted of a light source, convex lens, an optical rail, and a screen.

The light-gathering capability of the lens is commonly expressed by photographers in terms of the ratio f/D , called the f-number [Young \(2004\)](#). The f-number (N) of an optical system, shown in equation 2, is defined as the focal length divided by the diameter (D) [Smith \(2005\)](#).

$$N = \frac{f}{D} \quad (2)$$

The plate scale is described as the number of degrees corresponding to centimeters at the focal plane of an optical system [Perlmutter \(2010\)](#). The equation to derive the plate scale is expressed in equation 3. The plate scale (S_p) is defined as the angle formed by light source

and incident ray multiplied by the focal length.

$$S_p = f\theta \quad (3)$$

2. APPARATUS

The optical setup, shown in figure 1, included a screen, a convex lens, and a light source. The lens diameter, measured by a ruler, was (7.6 ± 0.1) cm. These three components were installed on an optical track with precise measurement markings from 0 cm-100 cm with mounts that allowed for position manipulations along the track. A module is described as a component, the mount that holds up the component, and the base that connects the upper portion to the optical track. The optical track had measurement increment markings of 1 mm. Object distances were defined from the front face of the screen to the lens center. Additionally, the image distance consisted of the distance between the lens center to the face of the light source. Both distance measurements were completed when the image was deemed to be in focus.

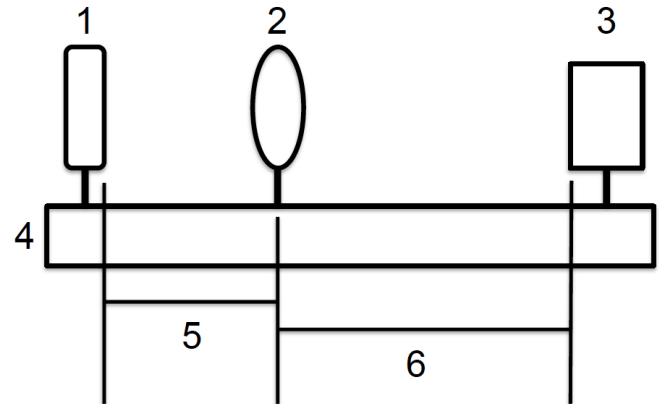


Figure 1. A simple schematic of the optical apparatus. 1) Screen 2) Convex Lens 3) Light Source 4) Optical Track 5) Distance to Image (d_i) 6) Distance to Object (d_o)

3. PROCEDURE AND OBSERVED QUANTITIES

Object and image distances were generated by summing both the raw module position measurements and respective offsets, provided in tables 1 and 2. Image distances consisted of the difference between the lens and screen measurements with offsets. Conversely, the object distance was the difference between the light source and the lens with offsets.

The raw position measurements of distance between the three modules, in table 2, were obtained using the precise tick marks on the optical track. The front edge of each mount was considered to be the position. It is imperative to note that the image distance spans from the front edge of the screen to the center of the lens. Additionally, the object distance starts from the center of the lens to the front edge of the light source.

Table 1. Offset measurements for the screen, lens, and light source modules.

Module	Offset (cm)
Screen	3.0±0.1
Lens	2.8±0.15
Light Source	1.3±0.1

Table 2. Module measurements on the optical track. The light source is kept fixed, and only the screen and lens distances are modified.

Screen (cm)	Lens (cm)	Light Source (cm)
1.0±0.05	36.1±0.05	96.0±0.05
2.0±0.05	37.9±0.05	96.0±0.05
3.0±0.05	39.1±0.05	96.0±0.05
4.0±0.05	41.1±0.05	96.0±0.05
5.0±0.05	43.2±0.05	96.0±0.05
6.0±0.05	45.8±0.05	96.0±0.05
7.0±0.05	48.2±0.05	96.0±0.05
7.5±0.05	50.8±0.05	96.0±0.05

The study was conducted by manipulating the distances of both the screen and lens along the optical track. Each distance manipulation was recorded for both the screen and lens. The light source is kept fixed since a focused point was attainable purely by moving the other two latter modules. Initially three measurements were taken, but eventually eight different distances were measured for better statistics. The projection was considered focused when the three circular projections were clearly visible without much smearing. This definition of "focus" is highly relative and adds error around the distances that yielded the image. The

focused point was determined by only one constant observer to limit various definitions of a "focused" image.

The experimental focal distance were generated from equation 4, which was derived from the thin-lens equation, and the input distances in table 3. Each experimental focal length, included in table 4, was determined from the respective rows in table 3.

$$f = \left(\frac{1}{d_{img}} + \frac{1}{d_{obj}} \right)^{-1} \quad (4)$$

Table 3. Derived object and image distances from the respective module measurements.

Object Distance (cm)	Image Distance (cm)
58.4±0.02	34.9±0.02
56.6±0.02	35.7±0.02
55.4±0.02	35.9±0.02
53.4±0.02	36.9±0.02
51.3±0.02	38.0±0.02
48.7±0.02	39.6±0.02
46.3±0.02	41.0±0.02
43.7±0.02	43.1±0.02

3.1. Measurements

Each module required offset measurements since the components weren't perfectly aligned with the edge of the track markings. Offset measurements were collected using a meter stick, measuring from 0 cm-100 cm, and using the edge of the mount to edge of the actual component of each individual module.

The equipment errors arise from the optical track and the meter stick. The measurements regarding the modules on the optical track were measured with the straight edge mount along the track tick marks. The edge of each mount was measured to the nearest millimeter, resulting in measurement uncertainty of 0.05 cm. Measurements using the meter stick for offsets, included in table 1, were done differently by placing the module mount on a flat surface. The mount holding a component wasn't perfectly perpendicular to the table surface, so the discrepancy must be accounted for. Positions closer to a certain millimeter tick mark weren't easily determined, therefore the measurement error of the offsets were instead ±0.1 cm. It must be noted that the error relating to the lens is ±0.15 cm since the position is in the lens center, which was less determined with the same error measurement method. The lens center was difficult to determine primarily due to the eye measurement and no reference edge to accurately determine a measurement. Diameter of the lens was determined by measuring the length using a meter stick. The meter stick was held parallel to

the lens center plane, and the uncertainty was largest due to the curvature of the lens. Uncertainty of the lens diameter was then set to ± 0.15 cm.

4. RESULTS

All eight object/image pairs were input into the equation and the resulting focal distances, shown in table 4. The resulting values were around the class sample of approximately 21 cm, which included the focal lengths calculated by other groups.

Table 4. Thin-lens focal length results for each respective set of image and object distances.

Measurement	Focal Length (cm)
1	21.85 \pm 0.01
2	21.89 \pm 0.01
3	21.78 \pm 0.01
4	21.82 \pm 0.01
5	21.83 \pm 0.01
6	21.84 \pm 0.01
7	21.74 \pm 0.01
8	21.70 \pm 0.01

The initial model was presented in equation 5. Each calculated focal length was added to the equation and the corresponding object length. The mean was obtained by taking the average focal length of the 8 focal length set, and was defined as the "sample fit". The uncertainty of the mean was calculated through dividing the range of focal lengths and dividing it by the square root of the population multiplied by two. This procedure provided related image lengths and were all plotted as a "sample fit", represented in figure 2 (green). For a particular focal length, the set of model image distances were computed using equation 5.

$$d_{img} = \left(\frac{1}{f} - \frac{1}{d_{obj}} \right)^{-1} \quad (5)$$

The second method to obtain focal lengths was a χ^2 (chi-squared) minimization, represented in equation 6. A range between the entire object range was sampled and a loop for the range in intervals of 0.005 cm was utilized. The outcome of the loop yielded focal lengths for the range of object distances and related chi-squared values.

$$\chi^2 = \sum \frac{\left(\left(\frac{1}{f} - \frac{1}{d_{obj}} \right)^{-1} - d_{img} \right)^2}{d_{img}} \quad (6)$$

There are a total of 4 different fit methods, tabulated in table 5, used to represent the trend exhibited by the 8

data points. Each of the four models utilized the equation 5, which solved the thin-lens equation for the image distance. It was noted that a range of The "mean fit" was generated by averaging the eight resultant focal lengths, in table 4, and inputting a range of image distances to yield a functional fit of the independent object distance. The next three fit methods required a range of focal lengths (21.69 cm-21.89 cm) and related chi-squared values (7.97-28.45). The focal length which resulted in the lowest chi-square value, 7.99908¹, was (21.79246 \pm 0.01) cm. Lastly there are two values of focal distance for the "upper and lower fit" model. According to statistics the most optimal chi-squared value is approximately the same to the amount of samples (8). Our vertex chi-squared value is below 8, therefore there exists two different focal lengths corresponding to a chi-squared value of 8. In this case the lower fit is considered to be "left" and the upper fit is defined as "right".

Error was propagated by normalization for addition and of the initial position error and the offset errors. Normalization was again used for determining the error of object and image distances when subtraction was used. Partial derivatives, provided in equation 7, were taken for the error propagation concerning the thin-lens equation. All measurements were done with the same procedure and the same measuring equipments, therefore the errors are consistently identical in all eight samples. The error of the mean was calculated with the standard deviation of the population of focal lengths.

$$s_f = \sqrt{s_x^2 \left(\frac{\partial f}{\partial x} \right)^2 + s_y^2 \left(\frac{\partial f}{\partial y} \right)^2 + \dots} \quad (7)$$

Table 5. Four fit models for the focal distance data¹ and the respective chi-squared values¹.

Fit Method	Focal Distance (cm)	Chi-Squared Value
Mean	21.807 \pm 0.03	8.4196
Lower Chi (left)	21.7885 \pm 0.01	8.0009
Upper Chi (right)	21.796 \pm 0.01	7.99908
Best Chi	21.79246 \pm 0.01	7.9710

5. DISCUSSION

The four different methods of finding the focal lengths, in table 5, yielded values that are within 0.03 cm. These errors fully agree with the range between the smallest

¹ Subsequent chi-squared and focal length values with large amounts of significant figures, are needed to represent the very small changes in both measurements that correspond to small-scale deviations.

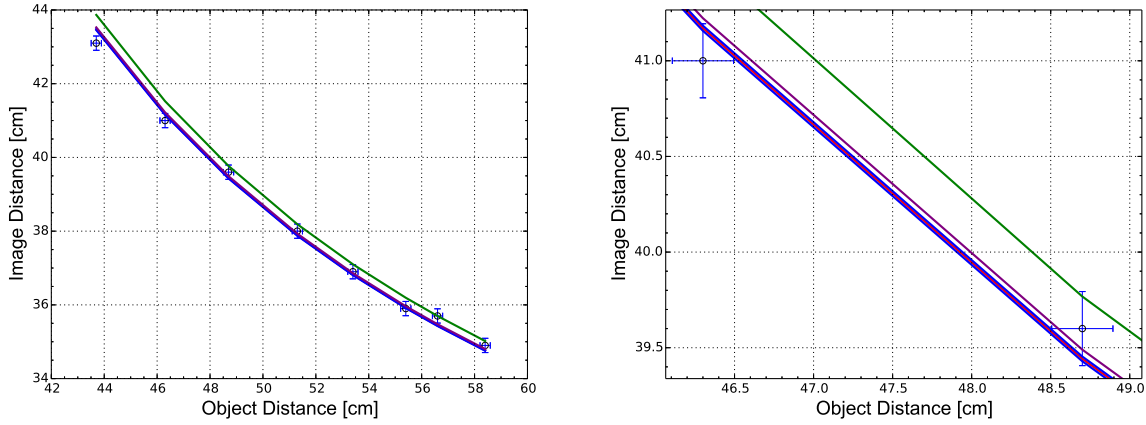


Figure 2. Object distance against image distance for eight different samples. The four different models include, a sample fit (green), mean fit (red), upper and lower fits (blue), and a best fit (purple). The left graph shows the eight individual points and the various fits. The right plot is a zoomed region of the graph, and it shows the fits that are not well seen in the full version on the left.

and largest values, 21.79 cm and 21.81 cm respectively. The best measurement of focal length is determined by the "best" chi-squared fit at (21.79 ± 0.01) cm. This "best" fit was chosen since it lies between the upper and lower chi-squared fits (blue), seen in figure 2 (right). A resulting f-number for the optical system was 2.9 using equation 2 and the "best" fit focal length result. The plate scale for a single degree, which used equation 3,

was determined as (21.79 ± 0.01) cm $^\circ$.

Errors of measurement were initially overestimated as 1.0 cm. This error was far too large and resulted in chi-squared values of approximately 4.0. Expected chi-squared values are theorized as the number of samples, which in this study is 8. The measurement errors were then adjusted, shown in the raw data tables, and the generated chi-squared values around 8.0 as theorized.

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