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HW 05: PROTOSTARS

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1. INITIAL PHASE OF PROTOSTAR EVOLUTION

1.1. *Method 1*

The relation used to calculate the radius of a protostar (R_{ps}) is provided in equation 1. Only the species of H and H_2 are probed, therefore higher order additions including species such as Helium are omitted. The individual stellar parameters used to calculate R_{ps} are provided in table 1. The equation was rewritten in equation 2 to yield R_{ps}. Plugging in the parameters from the table resulted in a protostar radius of $\boxed{\cancel{7.1854 \times 10^{11}\, m}}$ or approximately $\boxed{\cancel{1033\, R_\odot}}$.

$$GM\left(\frac{1}{R_{ps}} - \frac{1}{R_1}\right) \approx \frac{1}{m_H}\left(\frac{\epsilon_d}{2} + \epsilon_i\right) + R_2 \tag{1}$$

$$R_{ps} = m_H GM\left(\frac{2}{\epsilon_d} + \frac{1}{\epsilon_i}\right) \tag{2}$$

1.2. *Method 2*

The second attempt to find the protostar radius purely utilizes the gravitational energy in equation 4. The total of dissociation and ionization energy, E_{total} was found using equation 3, and the result was 3.0199×10^{46} erg. E_{total} in this case corresponds to the gravitational potential energy, $-2U_{total}$. The resulting radius of the protostar using method 2 was $\boxed{8.7436 \times 10^{10}\, m}$ or $\boxed{125.72\, R_\odot}$. If the density is uniform a 3/5 approximation can be applied for the gravitational potential energy, and multiplying the factor to the radii, results in a protostar radius of $\boxed{75.43\, R_\odot}$.

$$E_{total} = \frac{M}{2m_H}\epsilon_d + \frac{M}{m_H}\epsilon_i \tag{3}$$

$$R = \left|\frac{GM}{E_{total}}\right| \tag{4}$$

Table 1. Protostar parameters.

Quantity	Symbol	Value	Units [cgs units]
Gravitational Constant	G	6.6743×10^{-8}	$cm^3\ g^{-1}\ s^{-2}$
Boltzmann's Constant	k_B	1.3807×10^{-16}	$cm^2\ g\ s^{-2}\ K^{-1}$
Protostar Mass	M	1.989×10^{33}	g
Protostar Radius	R	6.955×10^{10}	cm
H_2 Molecule Dissociation Energy	ϵ_d	7.20979×10^{-12}	erg

Table 1 continued on next page

Table 1 *(continued)*

Quantity	Symbol	Value	Units [cgs units]
H Atom Ionization Energy	ϵ_i	2.17896×10^{-11}	erg
H Atom Mass	m_H	1.6726×10^{-24}	g

NOTE—Physical and Astronomical constants. Note that the protostar mass and radius are assumed to be the mass and radius of the Sun, respectively.

2. REACHING QUASI-HYDROSTATIC EQUILIBRIUM

To calculate the average internal temperature ($\bar{T}$), the Viral theorem is invoked in equation 5. It is imperative to note that since we are concerned with ionized hydrogen, μ is 1/2, therefore the denominator would be 6 for this case. In this instance, $\bar{T}$ is dependent on the protostar mass (M) in the numerator. The resulting average internal temperature is $\boxed{\cancel{7460.66\,K}}$. Using the radius from method 2, the average internal temperature of the protostar was $\boxed{6.131 \times 10^4\,K}$.

$$\bar{T} = \frac{\mu m_H G M}{3 k_B R_{ps}} \tag{5}$$