1. a) Let $E = \alpha^2(\cosh^2\rho(\frac{dt}{d\sigma})^2 - (\frac{d\rho}{d\sigma})^2)$

we have $0 = \alpha^2 \int_0^{\sigma_{max}} [(-\frac{d}{d\sigma}(E^{-1/2}\frac{d\rho}{d\sigma}) - E^{-1/2}\cosh\rho\sinh\rho(\frac{dt}{d\sigma})^2)\delta\rho + \frac{d}{d\sigma}(\cosh^2\rho\frac{dt}{d\sigma})\delta t]d\sigma$

then $\frac{d^2\rho}{ds^2} + \cosh\rho\sinh\rho(\frac{dt}{ds})^2 ds = 0$

integral: $\frac{d\rho}{ds} + (\frac{dt}{ds})^2 \int \cosh\rho\sinh\rho\, ds = 0$

2. a) $\Gamma^0_{00} = 0$, $\Gamma^0_{10} = \Gamma^0_{01} = 0$, $\Gamma^0_{11} = \frac{1}{2}R_0^2(\sin 2\theta / R_0^2) = \cos\theta\sin\theta$

$\Gamma^1_{00} = 0$, $\Gamma^1_{10} = \Gamma^1_{01} = \frac{1}{2}\frac{1}{R_0^2\sin^2\theta}(R_0^2\sin 2\theta) = \frac{\cos\theta}{\sin\theta} = \cot\theta$, $\Gamma^1_{11} = 0$

b) $R_{00} = \frac{d}{d\theta}\cot\theta - \Gamma^1_{01}\Gamma^1_{10} = -\frac{1}{\sin^2\theta} - \cot^2\theta = -1 = k g_{00} = kR_0^2$

$R_{01} = R_{10} = 0$

$R_{11} = \cos 2\theta - \Gamma^0_{11}\Gamma^1_{01} + \Gamma^1_{10}\Gamma^0_{11} - \Gamma^1_{10}\Gamma^0_{11} = 2\cos^2\theta - 1 - \cos^2\theta = -\sin^2\theta = k g_{11} = kR_0^2\sin^2\theta$

$k = -R_0^2$

3. Since metric compatible $\Rightarrow [\nabla_\mu, \nabla_\nu]g_{ab} = R_{\mu\nu}{}^c{}_a g_{cb} + R_{\mu\nu}{}^c{}_b g_{ac} = 0$

$\Rightarrow R_{\mu\nu ab} + R_{\mu\nu ba} = 0 \Rightarrow R_{\mu\nu ab} = -R_{\mu\nu ba}$

4. $\frac{\dot{a}}{a^2} + \frac{k}{a^2} = 0$, $\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = 0 \Rightarrow \frac{\ddot{a}}{a} = 0 \quad a = C_1 t + C_2$

$\Rightarrow C_1 + k = 0 \Rightarrow \begin{cases} k=0: a = C \\ k=-1: a = \frac{1}{t} + C \end{cases} \quad C \in \mathbb{R}$

5. a) $1 = g_{00}u_0^2 + 0 + 0 + 0 \Rightarrow u_0 = \sqrt{1/g_{00}} = (g_{00})^{-1/2} = (1 - \frac{2m}{r_0})^{-1/2}$

b) $a^2 = \sum_{\mu=0}^{3} g_{\mu\mu}(a^\mu)^2$

notice g_{11}: $g_{11} = -(1 - \frac{2m}{r_0})^{-1} = -(1 - \frac{2m}{2m})^{-1} = -\infty \Rightarrow a^2 \to -\infty$

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