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Homework 2

$$x = [-1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1]^T$$

The DFT coeficients are given by

$$X_K = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}nK}$$

We notice that x can be decomposed into δ signals:

$$x = -\delta[n] - \delta[n-1] - \delta[n-4] - \delta[n-5]$$
$$+ \delta[n-2] + \delta[n-3] + \delta[n-6] + \delta[n-7]$$

For which we know the Fourier Transforms and linearity with respect to addition.

$$X[0] = 0$$

$$X[1] = -1 - e^{-j\frac{2\pi}{8}} - e^{-j\pi} - e^{-j\frac{5\pi}{4}}$$
$$+ e^{-j\frac{\pi}{2}} + e^{-j\frac{3\pi}{4}} + e^{-j\frac{3\pi}{2}} + e^{-j\frac{7\pi}{4}}$$
$$= -\cancel{1} - \left(\cancel{\frac{\sqrt{2}}{2}} - j\cancel{\frac{\sqrt{2}}{2}}\right) + \cancel{1} - \left(-\cancel{\frac{\sqrt{2}}{2}} + j\cancel{\frac{\sqrt{2}}{2}}\right)$$
$$+ (0 - \cancel{j}) + \left(-\cancel{\frac{\sqrt{2}}{2}} - j\cancel{\frac{\sqrt{2}}{2}}\right) + (0 + \cancel{j}) + \left(\cancel{\frac{\sqrt{2}}{2}} + j\cancel{\frac{\sqrt{2}}{2}}\right)$$
$$= 0$$

$$X[2] = -1 - e^{-j\frac{\pi}{2}} - e^{-j2\pi} - e^{-j\frac{5\pi}{2}}$$

$$+ e^{-j\pi} + e^{-j\frac{3\pi}{2}} + e^{-j3\pi} + e^{-j\frac{7}{2}\pi}$$

$$= -1 \quad -1 \quad -1 \quad +j \quad -1 \quad +j + j = 4(-1+j)$$

$$X[3] = -1 - e^{-j\frac{3\pi}{4}} - e^{-j3\pi} - e^{-j\frac{7\pi}{2}} + e^{-j\frac{3\pi}{2}} + e^{-j\frac{9}{4}\pi} + e^{-j9\pi} + e^{-j\frac{21}{4}\pi}$$

$$= -1 - \left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + 1 - j + j + \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) - 1 + \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$

$$= 0$$

$$X[4] = \sum_{n=0}^{7} x[n]\, e^{-j\frac{\pi}{4}4n} = \sum_{n=0}^{7} x[n]\, e^{-j\pi n}$$

$$= -1\cdot(-1)^0 \quad -1(-1)^1 + 1\cdot(-1)^2 + 1(-1)^3 - 1(-1)^4 - 1(-1)^5$$

$$+1(-1)^6 + 1(-1)^7$$

$$= 0$$

$$X[5] = \sum_{n=0}^{7} x[n]\, e^{-j\frac{5}{4}\pi n}$$

$$= -1 - 1\cdot\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) - j + \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) - 1(-1) - 1\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + j$$

$$+\left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$

$$= 0$$

$$X[6] = \sum_{n=0}^{7} x[n]\, e^{-j\frac{3\pi}{2}n}$$

$$= -1 - 1\cdot(j) + 1(-1) + 1(-j)$$

$$-1(1) - 1(j) + 1(-1) + 1(-j)$$

$$= -4 - 4j = 4(-1-j)$$

$$X[7] = \sum_{n=0}^{7} x[n]\, e^{-j\frac{7\pi}{4}n}$$

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$$= -1 - 1\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) + 1(j) + 1\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$$

$$-1(-1) - 1\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + 1(-j) + 1\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$

$$= 0$$

We have the following coeficients

$$X[n] = [0 \quad 0 \quad 4(-1+j) \quad 0 \quad 4(-1-j) \quad 0\,]^T$$