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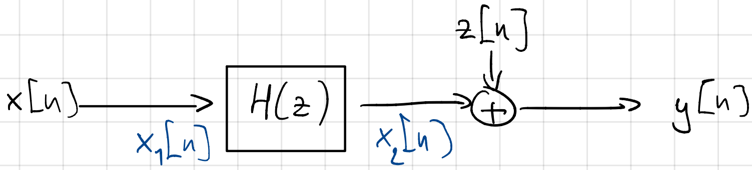
Homework 6

3.a.



Let x_1 and x_2 be the signals as in the figure:

$$x_1[n] = x[n] \quad \text{and} \quad x_2[n] = x[n] * h[n]$$

then,
$$r_y[k] = \mathbb{E}\left[\left(x_2[n] + z[n]\right)\left(x_2[n-k) + z[n-k]\right]\right]$$
$$= \mathbb{E}\left[x_2[n]\, x_2[n-k]\right] + \mathbb{E}\left[z[n]\, z[n-k]\right]$$
$$= r_{x_2}[k] + \delta[k]$$
$$= h[n] * h[-n] * r_x[k] + \delta[k]$$
$$= 3\, h[n] * h[-n] + \delta[k]$$

Hence the PSD

$$P_y(e^{j\omega}) = 3\left|H(e^{j\omega})\right|^2 + 1$$

We calculate $H(e^{j\omega})$ from $h[n]$ very easily:

$$H(e^{j\omega}) = \frac{1}{4}\left(2e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}\right)$$

we want:
$$\left|H(e^{j\omega})\right|^2 = \frac{1}{16}\left(2e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}\right)^2$$
$$= \frac{1}{16}\left(e^{-2j\omega}\right)^2\left(2e^{j\omega} + 1 + e^{-j\omega}\right)^2$$
$$= \frac{1}{16}\left(1 + 3\cos\omega + j\sin\omega\right)^2$$

$$= \frac{1}{16}\left(6 + 6\cos\omega + 4\cos 2\omega\right)$$

Going back to the expression of $P_y(e^{j\omega})$:

$$P_y(e^{j\omega}) = 3\left|H(e^{j\omega})\right|^2 + 1$$

$$= \frac{3}{16}\left(6 + 6\cos\omega + 4\cos 2\omega\right) + 1$$

$$= \frac{1}{16}\left(18 + 16 + 18\cos\omega + 12\cos 2\omega\right)$$

$$= \frac{17}{8} + \frac{9}{8}\cos\omega + \frac{3}{4}\cos 2\omega$$