Software Testing, Quality Assurance & Maintenance—Lecture 9

Patrick Lam
University of Waterloo

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Part I

Intro: Grammar-Based Fuzzing

Types of Fuzzing

Goal: generate many test cases automatically.

Mutation-based fuzzing: generate new inputs automatically, by modifying known inputs.

Grammar-based fuzzing: generate new inputs automatically, using a grammar.

Example: Regular expression

```
^4[0-9]{12}(?:[0-9]{3})?$
```

Uses:

- check if a number is valid
- generate numbers of the right shape

(But: also must satisfy checksum rules!)

Checksum rules for credit card numbers

Luhn algorithm:

calculate a checksum from all-but-last digits;
the last digit must match the checksum.

Can’t specify checksum rules as regexp or context-free grammars.

Example (standard): expressions

Context-free grammars (CFGs):

```
<start>   ::= <expr>
<expr>    ::= <term> + <expr> | <term> - <expr> | <term>
<term>    ::= <term> * <factor> | <term> / <factor>
          |   <factor>
<factor>  ::= +<factor> | -<factor> | (<expr>) | <integer>
          |   <integer>.<integer>
<integer> ::= <digit><integer> | <digit>
<digit>   ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Can also recognize and generate strings for CFGs.

Generating from grammars

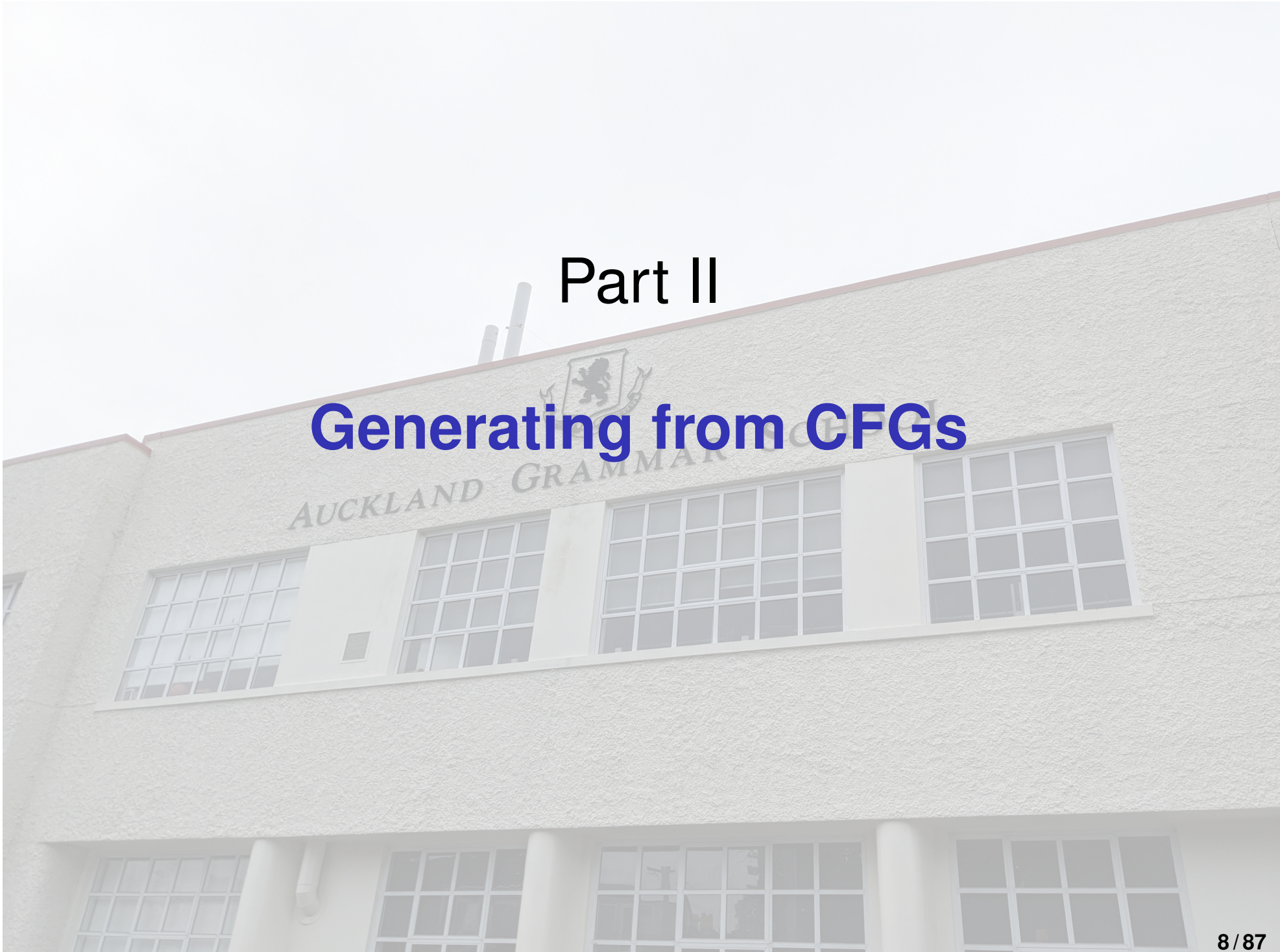
What we're doing in this lecture:

generating (trees and) strings
from grammars.

These are more interesting than random sequences of characters;
they test behaviour beyond input validation.

Aim: create grammars that specify all legal inputs.

Other uses: configs, APIs, GUIs, etc.



Part II

Generating from CFGs

Expressiveness

```
<!DOCTYPE html>
<html lang="en" dir="ltr">
<head>
<meta charset="utf-8">
<meta name="generator" content="Piwigo (aka PWG), see piwigo.org">


<meta name="description" content="Home">

<title>plam gallery</title>
<link rel="shortcut icon" type="image/x-icon" href="themes/default/icon/favicon.ico">

<link rel="start" title="Home" href="/" >
<link rel="search" title="Search" href="search.php" >


<link rel="canonical" href="/">


    <!--[if lt IE 7]>
        <link rel="stylesheet" type="text/css" href="themes/default/fix-ie5-ie6.css">
    <![endif]-->
    <!--[if IE 7]>
        <link rel="stylesheet" type="text/css" href="themes/default/fix-ie7.css">
    <![endif]-->


    <!--[if lt IE 8]>
        <link rel="stylesheet" type="text/css" href="themes/elegant/fix-ie7.css">
    <![endif]-->


<!-- BEGIN get_combined -->
<link rel="stylesheet" type="text/css" href="_data/combined/1ylcc4n.css">


<!-- END get_combined -->

<!--[if lt IE 7]>
<script type="text/javascript" src="themes/default/js/pngfix.js"></script>
```

Regexps can’t count (so no HTML parsers).
CFGs can’t implement Luhn algorithm.

Generating Inputs

How to generate inputs from this grammar?

```
<start> ::= <digit><digit>
<digit> ::= 0 | 1 | 2 | 3 | 4
          |   5 | 6 | 7 | 8 | 9
```

(Could be expressed as a regexp).

Note: `<xxx>` represents nonterminal `xxx`.

Generating Inputs: double-digits

- `<start>` → `<digit><digit>`
- visit first `<digit>`, have 10 choices;
  choose randomly, say `1`;
  replace `<digit>` by `1`.
- visit second `<digit>`,
  choose say `7` randomly;
  replace `<digit>` by `7`.
- generated input: `17`

Generating Inputs: back to expr

```
<start>    ::= <expr>
<expr>     ::= <term> + <expr> | <term> - <expr> | <term>
<term>     ::= <term> * <factor> | <term> / <factor>
           |   <factor>
<factor>   ::= +<factor> | -<factor> | (<expr>) | <integer>
           |   <integer>.<integer>
<integer>  ::= <digit><integer> | <digit>
<digit>    ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Generating Inputs: expr

- <start> → <expr>
- visit <expr>; three <expr> alternatives,
  randomly choose <term> + <expr>,
  replace <expr> by our choice.
- visit <term>; also three <term> alternatives,
  randomly choose <factor>.
- visit <factor>;
  randomly choose <integer> of the 5 alternatives.
- visit <integer>; randomly choose <digit>;
- visit <digit>;
  randomly choose terminal `4` and generate it.
- continue with next nonterminal `expr` ...

Could generate, for instance, `4 + 22 * 5.3`, or many other expressions.

Coding Things Up: Grammars in Python

Just use Python data structures.

Grammar = mapping from an alternative's LHS to its RHS.
Here is a single-production grammar.

```
DIGIT_GRAMMAR = {
  "<start>":
    ["0", "1", "2", "3", "4", "5", "6", "7", "8", "9"]
}
```

Nonterminals in `<brackets>`.
Rest of the string interpreted as terminals.

Python Type Hint for Grammars

```
from typing import Dict, List
type Expansion = str
type Grammar = Dict[str, List[Expansion]]
```

Expression Grammar in Python

```
EXPR_GRAMMAR: Grammar = {
    "<start>":
        ["<expr>"],
    "<expr>":
        ["<term> + <expr>", "<term> - <expr>", "<term>"],
    "<term>":
        ["<factor> * <term>",
         "<factor> / <term>", "<factor>"],
    "<factor>":
        ["+<factor>", "-<factor>",
         "(<expr>)",
         "<integer>.<integer>", "<integer>"],
    "<integer>":
        ["<digit><integer>", "<digit>"],
    "<digit>":
        ["0", "1", "2", "3", "4", "5", "6", "7", "8", "9"]
  }
```

Playing with Grammars

```
>>> EXPR_GRAMMAR["<digit>"]
['0', '1', '2', '3', '4', '5', '6', '7', '8', '9']
>>> "<integer>" in EXPR_GRAMMAR
True
```

Conventions and Helpers

Must always have start symbol <start>:

```
START_SYMBOL = "<start>"
```

Because non-terminals always in <brackets>:

```
import re
RE_NONTERMINAL = re.compile(r'(<[^<> ]*>)')
def nonterminals(expansion):
    if isinstance(expansion, tuple):
        # can be a tuple, use first element
        expansion = expansion[0]

    return RE_NONTERMINAL.findall(expansion)

def is_nonterminal(s):
    return RE_NONTERMINAL.match(s)
```

Using these Utilities

```
>>> nonterminals("<term> * <factor>")
["<term>", "<factor>"]
>>> is_nonterminal("<symbol-1>")
<re.Match object; span=(0, 10), match='<symbol-1>'>
>>> is_nonterminal("<symbol-1")
>>>
```

A Simple Grammar Fuzzer

```
def simple_grammar_fuzzer(grammar: Grammar,
                          start_symbol: str = START_SYMBOL,
                          max_nonterminals: int = 10,
                          max_expansion_trials: int = 100,
                          log: bool = False) -> str:
 """Produce a string from `grammar`.
    `start_symbol`: use a start symbol other than `<start>` (
                                        default).
    `max_nonterminals`: the maximum number of nonterminals
      still left for expansion
    `max_expansion_trials`: maximum # of attempts to produce a
                                        string
    `log`: print expansion progress if True"""
```

It's all strings: implementation

```
term = start_symbol
expansion_trials = 0
while len(nonterminals(term)) > 0:
    symbol_to_expand = random.choice(nonterminals(term))
    expansions = grammar[symbol_to_expand]
    expansion = random.choice(expansions)
    new_term = term.replace(symbol_to_expand, expansion, 1)

    if len(nonterminals(new_term)) < max_nonterminals:
        term = new_term
        expansion_trials = 0
    else:
        expansion_trials += 1
        if expansion_trials >= max_expansion_trials:
            raise ExpansionError("Cannot expand " + repr(term))

return term
```

Comments on Simple Grammar Fuzzer

- takes a string, finds a nonterminal, replaces with an alternative.
- it's all string replacement.
- tree manipulations would be better; stay tuned.

Termination for Simple Grammar Fuzzer

Let's look at this more closely:

```
    if len(nonterminals(new_term)) < max_nonterminals:
        term = new_term
        expansion_trials = 0
    else:
        expansion_trials += 1
        if expansion_trials >= max_expansion_trials:
            raise ExpansionError("Cannot expand " + repr(term))

  return term
```

If too many nonterminals after substitution:
give up on generated string, re-choose.

Grammar: CGI

```
CGI_GRAMMAR: Grammar = {
    "<start>":
        ["<string>"],
    "<string>":
        ["<letter>", "<letter><string>"],
    "<letter>":
        ["<plus>", "<percent>", "<other>"],
    "<plus>":
        ["+"],
    "<percent>":
        ["%<hexdigit><hexdigit>"],
    "<hexdigit>":
        ["0", "1", "2", "3", "4", "5", "6", "7",
            "8", "9", "a", "b", "c", "d", "e", "f"],
    "<other>":  # Actually, could be _all_ letters
        ["0", "1", "2", "3", "4", "5", "a", "b", "c", "d", "e",
                                            "-", "_"],
}
```

Generating CGI strings

```
>>> for i in range(10):
...     print(simple_grammar_fuzzer(grammar=CGI_GRAMMAR,
                                    max_nonterminals=10))
...
%e5
+
+3
_1a
e+
%625%ee
%db%df%5d
+44
%4b
+%b8+2
```

Grammar: URL

```
URL_GRAMMAR: Grammar = {
    "<start>": ["<url>"],
    "<url>": ["<scheme>://<authority><path><query>"],
    "<scheme>": ["http", "https", "ftp", "ftps"],
    "<authority>":
        ["<host>", "<host>:<port>",
         "<userinfo>@<host>", "<userinfo>@<host>:<port>"],
    "<host>":  # Just a few
        ["patricklam.ca", "www.google.com", "fuzzingbook.com"],
    "<port>": ["80", "8080", "<nat>"],
    "<nat>": ["<digit>", "<digit><digit>"],
    "<digit>":
        ["0", "1", "2", "3", "4", "5", "6", "7", "8", "9"],
    "<userinfo>":  # Just one
        ["user:password"],
    "<path>":  # Just a few
        ["", "/", "/<id>"],
    "<id>": ["abc", "def", "x<digit><digit>"],
    "<query>": ["", "?<params>"],
    "<params>": ["<param>", "<param>&<params>"],
    "<param>": ["<id>=<id>", "<id>=<nat>"],
}
```

Generating URLs

```
>>> for i in range(10):
...     print(simple_grammar_fuzzer(grammar=URL_GRAMMAR,
                                    max_nonterminals=10))
...
https://user:password@www.google.com/abc
http://user:password@fuzzingbook.com/
http://user:password@patricklam.ca/def?x97=60
ftp://user:password@fuzzingbook.com/x60?abc=def
https://patricklam.ca/?x84=31&x95=x12
ftp://www.google.com:1/abc
ftp://user:password@fuzzingbook.com:80/x40?def=6&x12=abc
ftp://user:password@www.google.com
http://user:password@fuzzingbook.com/def?x35=1
ftp://user:password@www.google.com/abc
```

This isn't all URLs, but we do get better coverage of our chosen subset.
Other examples possible, e.g. book titles (see *Fuzzing Book*).

On Validity

If you have a grammar,
    you can generate strings belonging to the grammar
    (as we've seen).

These strings, by definition, always satisfy the grammar.

They may not be valid inputs to a program.

- e.g. port number between 1024 and 2048;
- e.g. checksum digit satisfies Luhn's algorithm.

Can attach constraints to grammars to generate more-valid inputs, or weight some alternatives more heavily (not discussed today).

Grammars & Mutation Seeds

So far: only produce syntactically valid inputs.

What about *invalid* inputs?

One answer: mutation.

Our `MutationFuzzer` accepts seeds to start from.

Generating CGI strings

```
number_of_seeds = 10
seeds = [
    simple_grammar_fuzzer(
        grammar=URL_GRAMMAR,
        max_nonterminals=10) for i in range(number_of_seeds)]
seeds
m = MutationFuzzer(seeds)
[m.fuzz() for i in range(20)]
```

Use the grammar fuzzer to produce seeds, then the mutation fuzzer to mutate them.

EBNF

Extended Backus-Naur form; is syntactic sugar:

- `<symbol>`?: `<symbol>` can occur 0 or 1 times;
- `<symbol>`+: `<symbol>` can occur 1 or more times;
- `<symbol>`*: `<symbol>` can occur 0 or more times;
- parentheses can be used with these shortcuts, e.g. (`<s1><s2>`)+

Instead of

```
  "<identifier>": ["<idchar>", "<identifier><idchar>"],
```

just write

```
  "<identifier>": ["<idchar>+"],
```

Function `convert_ebnf_grammar()` (in `code/L09/ebnf.py`) translates EBNF to BNF.

Opts (for future expansion)

```
"<expr>":
        [("<term> + <expr>", opts(min_depth=10)),
         ("<term> - <expr>", opts(max_depth=2)),
          "<term>"]
```

and there are functions like `opts()`, `exp_string()`, `exp_opt()`, `exp_opts()`, `set_opts()` in `code/L09/opts.py`.

Grammar Utilities

found in `code/L09/grammars.py`:

`trim_grammar()`: remove unneeded expansions;

`is_valid_grammar()`: validity checks, e.g. no unreachable symbols, etc

Applications of Grammars in Testing

- Earliest known: Burkhardt, 1967: “Generating test programs from syntax.”
- CSmith: grammar-based fuzzing; they work hard to only generate valid C programs; pseudo-oracle comparing GCC and LLVM.
- EMI: fuzz dead code, observe mis-compilations.
- LangFuzz: grammar-based fuzzing on test suites.
- Grammarinator: open-source grammar fuzzer in Python.
- Domato: fuzzes Document Object Model inputs.

Part III

Efficient Grammar Fuzzing

Strings. . . really?

Easier to specify grammars, sure, but. . .

A Simple Problem

```
expr_grammar = convert_ebnf_grammar(EXPR_EBNF_GRAMMAR)
with ExpectTimeout(1):
  simple_grammar_fuzzer(grammar=expr_grammar,
                                    max_nonterminals=3)
```

does indeed raise a `TimeoutError`.

(see: `L09/simple_grammar_fuzzer_problem.py`)

(((

```
((((((((((((((((((((((((((((((((((((((((((((((((((((((
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(((((((((Traceback (most recent call last):
  File "/home/plam/courses/stqam-2026-working-notes/no
    simple_grammar_fuzzer(grammar=expr_grammar,
         max_nonterminals=3, log=True)
```

))) Why?

Rule for `<factor>`:

```
'<factor>':
  ['<sign-1><factor>', '(<expr>)', '<integer><symbol-1>']
```

Termination for Simple Grammar Fuzzer

Let's look at this more closely:

```
    if len(nonterminals(new_term)) < max_nonterminals:
        term = new_term
        expansion_trials = 0
    else:
        expansion_trials += 1
        if expansion_trials >= max_expansion_trials:
            raise ExpansionError("Cannot expand " + repr(term))

  return term
```

If too many nonterminals after substitution:
give up on generated string, re-choose.

`simple_grammar_fuzzer` fails

`simple_grammar_fuzzer()` throws away generated strings with $>$ 3 nonterminals.

```
'<factor>':
  ['<sign-1><factor>', '(<expr>)', '<integer><symbol-1>']
```

So, only allowed alternative for `<factor>` is `'(<expr>)'`...

...other alternatives add a non-terminal, so the fuzzer won't.

Not enough control surfaces

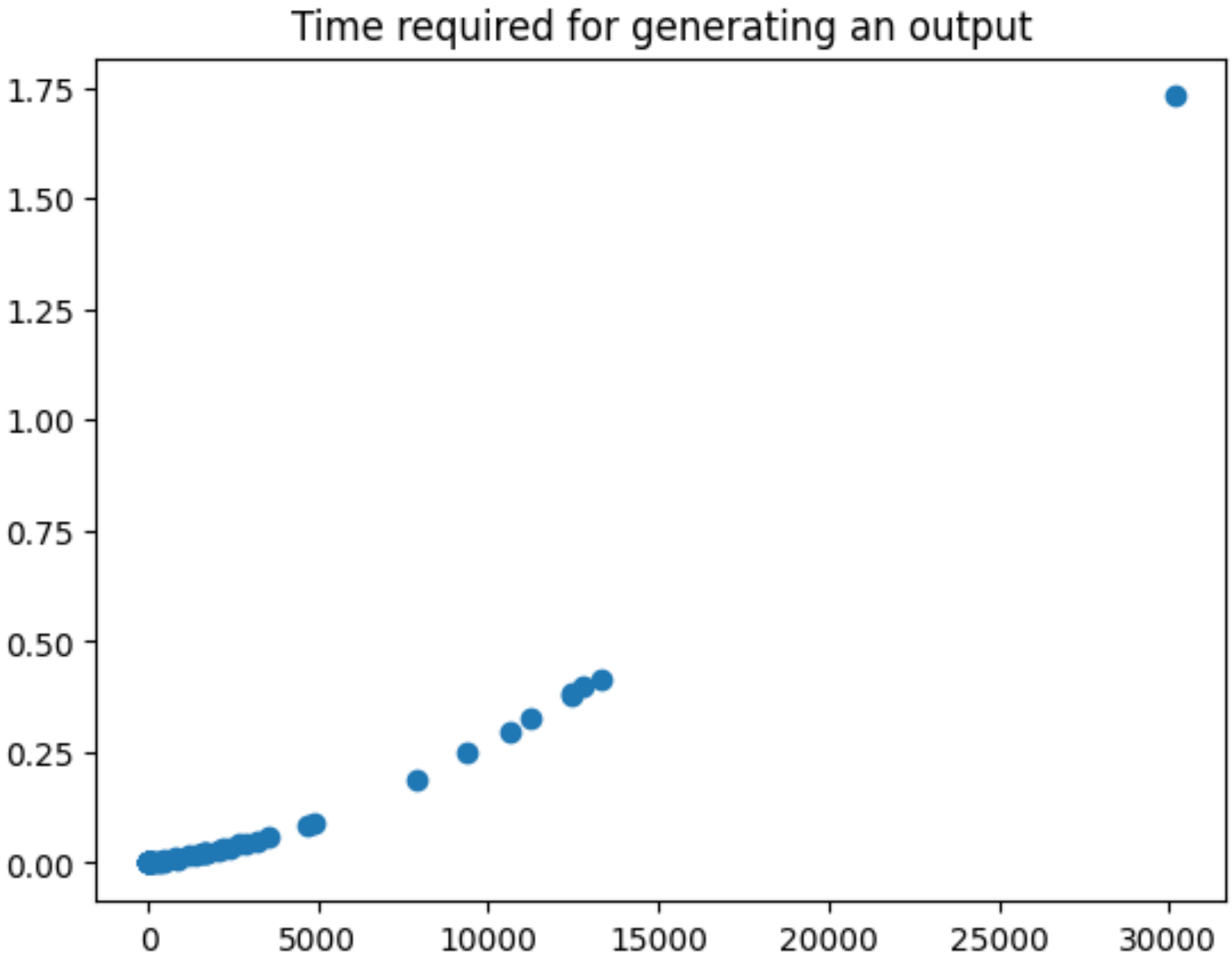
Any grammar-based approach can create arbitrarily long strings.

But `simple_grammar_fuzzer()` is too depth-first; we want more breadth.

The `max_nonterminals` control is too coarse.

Too slow

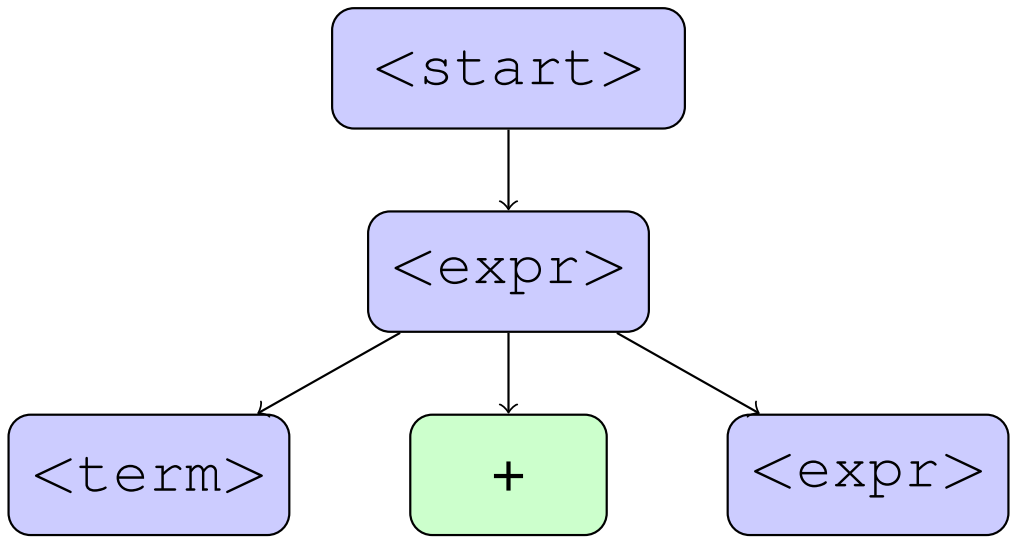
Also, we don’t want to manipulate strings.



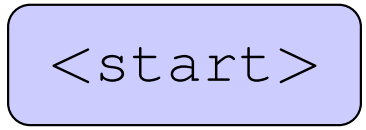

Better controls?


Trees?

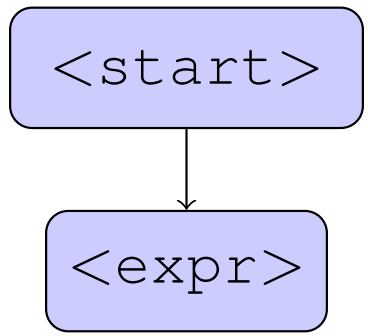
Derivation Trees!



Starting at the start

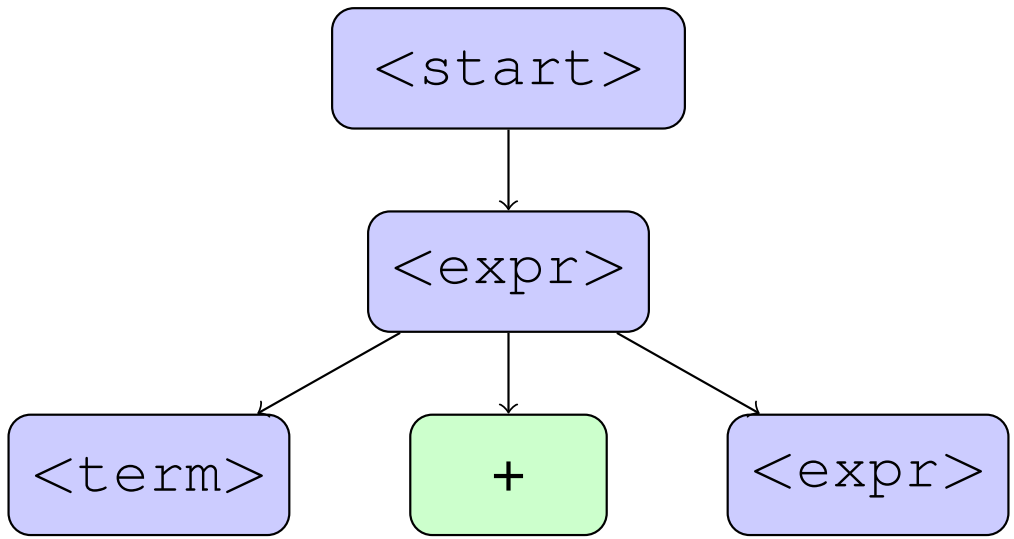


Derivation: <start> → <expr>

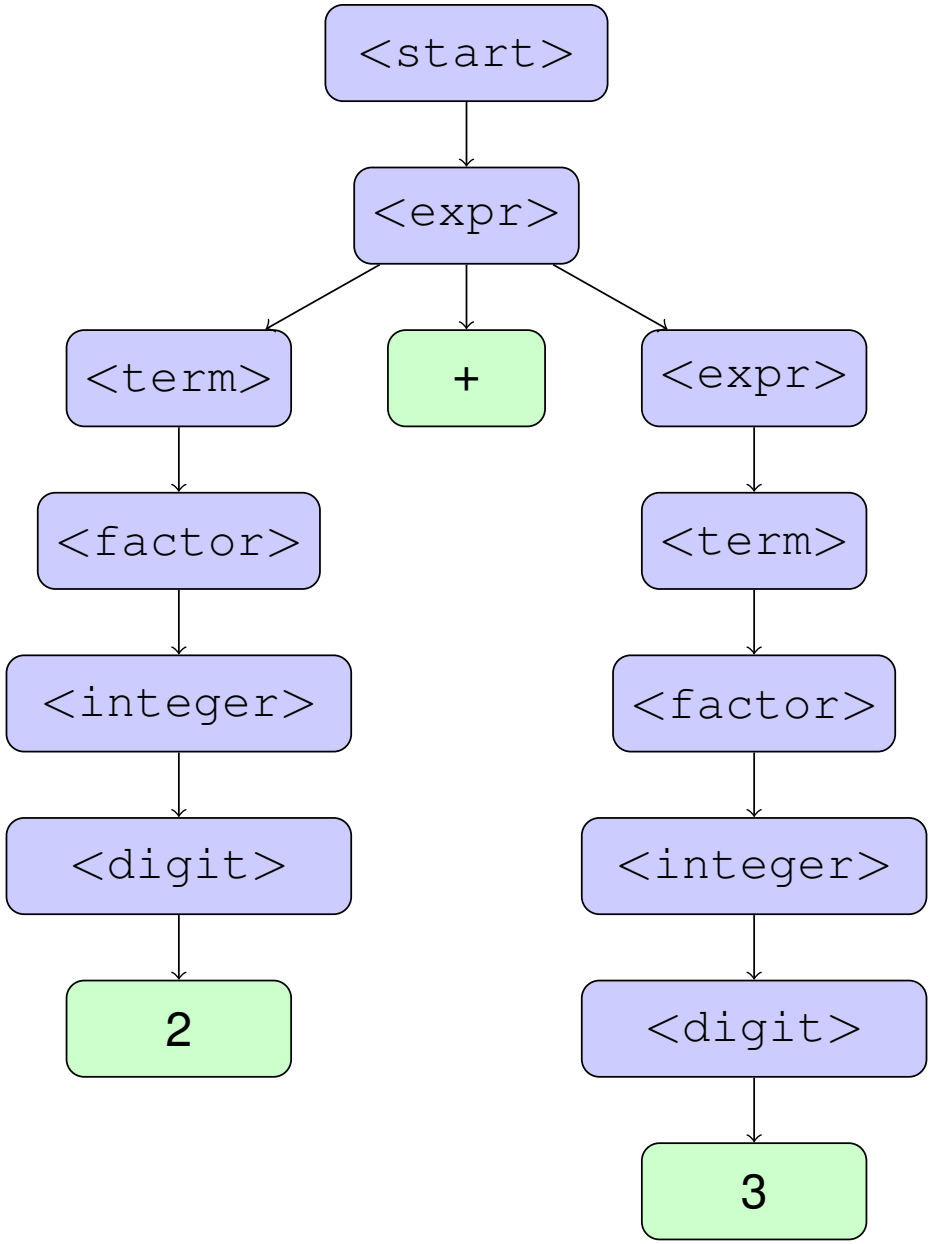


Search for nonterminal *S* without children;
here, <start>. Replace with its child <expr>.

Derivation: <expr> → <term> + <expr>



Down to terminals



Derivation Trees in Python

Representing derivation trees

```
(SYMBOL_NAME, CHILDREN)
```

SYMBOL_NAME: e.g. `<start>`, `+`
(again, nonterminals in `<brackets>`)

CHILDREN: the subtree at that node

```
DerivationTree = Tuple[str, Optional[List[Any]]]
```

Example Derivation Tree

```
derivation_tree: DerivationTree = ("<start>",
                        [("<expr>",
                          [("<expr>", None),
                           (" + ", []),
                              ("<term>", None)]
                          )])
```

Allowed CHILDREN

- the subtree
- None: this is a nonterminal still to be expanded
- []: this node is a terminal

GrammarFuzzer Implementation

```
class GrammarFuzzer(Fuzzer):
"""Produce strings from grammars efficiently,
       using derivation trees."""

  def __init__(self,
               grammar: Grammar,
               start_symbol: str = START_SYMBOL,
               min_nonterminals: int = 0,
               max_nonterminals: int = 10,
               disp: bool = False,
               log: Union[bool, int] = False)
               -> None:
      # ...
```

For now, only use `grammar` and `start_symbol`.

Partial check for grammar validity

```
    def check_grammar(self) -> None:
      assert self.start_symbol in self.grammar
      # recall: e.g. every nonterminal is reachable etc
      assert is_valid_grammar(
          self.grammar,
          start_symbol=self.start_symbol,
          supported_opts=self.supported_opts())
```

Tree manipulations

Better than string manipulations.

We want to expand nonterminals (selectively).

Alternatives in grammars still expressed as strings, e.g.

```
"<expr>":
        [("<term> + <expr>"),
         ("<term> - <expr>"),
          "<term>"]
```

Strings to trees

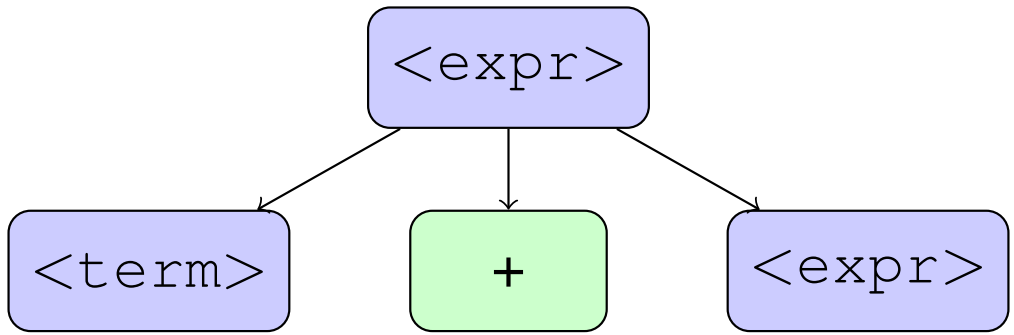
```
def expansion_to_children(expansion: Expansion) ->
            List[DerivationTree]:
  expansion = exp_string(expansion)
  assert isinstance(expansion, str)

  if expansion == "":
      return [("", [])]

  strings = re.split(RE_NONTERMINAL, expansion)
  return [(s, None) if is_nonterminal(s) else (s, [])
          for s in strings if len(s) > 0]

>>> expansion_to_children([("<term> + <expr>")])
[('<term>', None), (' + ', []), ('<expr>', None)]
```

Expanding a node



Key operation: expand nonterminal into one of its alternatives; here we have chosen +, but could also be - or a term.

We replaced <expr> with e.g. <term> + <expr>.

Randomly choosing an alternative

Same choice strategy as `simple_grammar_fuzzer`.

```
def choose_node_expansion(self, node: DerivationTree,
    children_alternatives: List[List[DerivationTree]]) -> int:
  return random.randrange(0, len(children_alternatives))
```

Random expansion implementation

For a subtree with non-expanded non-terminal at root (`children == None`):

- expand non-terminal into one of its alternatives (randomly chosen),
- return the tree.

```
def expand_node_randomly(self, node: DerivationTree) ->
                                    DerivationTree:
  (symbol, children) = node
  assert children is None

  # Fetch & parse possible expansions from grammar...
  expansions = self.grammar[symbol]
  children_alternatives: List[List[DerivationTree]] = [
      self.expansion_to_children(expansion) for expansion in
                                          expansions
  ]
```

Random expansion implementation 2

```
    # ... and select a random expansion
    index = self.choose_node_expansion(node,
                                       children_alternatives)
    chosen_children = children_alternatives[index]

    # Return with new children
    return (symbol, chosen_children)
```

`expand_node_*` return a fresh `DerivationTree`, consistent with functional programming style.

Randomly is for now our only strategy; we will later redefine `expand_node` to use other strategies.

```
  def expand_node(self, node: DerivationTree) ->
                                      DerivationTree:
      return self.expand_node_randomly(node)
```

expand_node in action

```
>>> f = GrammarFuzzer(EXPR_GRAMMAR, log=True)

>>> print("Before expand_node_randomly():")
>>> expr_tree = ("<integer>", None)
>>> print (expr_tree)
('<integer>', None)

>>> expr_tree = f.expand_node_randomly(expr_tree)
>>> print("After expand_node_randomly():")
>>> print (expr_tree)
('<integer>', [('<digit>', None), ('<integer>', None)])
```

or also possible:

```
>>> print (expr_tree)
('<integer>', [('<digit>', None)])
```

`expand_node_*` return a fresh `DerivationTree`, consistent with functional programming style.

Randomly is for now our only strategy; we will later redefine `expand_node` to use other strategies.

Picking a node to expand

`expand_node()`: return expanded current node.

`expand_tree_once()`: modify the tree; pick some node and call `expand_node()` on that.

Specifically: if root is unexpanded, expand that. Otherwise, find a random descendant and expand that.

NB: uses the current definition of `self.expand_node`, which can change.

expand_tree_once() example

```
>>> f.expand_node = f.expand_node_randomly
>>> derivation_tree = ("<start>",
                       [("<expr>",
                         [("<expr>", None), (" + ", []),
                          ("<term>", None)])])
>>> print (derivation_tree)
('<start>', [('<expr>', [('<expr>', None), (' + ', []),
              ('<term>', None)])])
>>> f.expand_tree_once(derivation_tree)
Expanding <term> randomly
>>> f.expand_tree_once(derivation_tree)
Expanding <expr> randomly
>>> print (derivation_tree)
('<start>', [('<expr>', [('<expr>', [('<term>', None),
   (' + ', []), ('<expr>', None)]), (' + ', []),
    ('<term>', [('<factor>', None), (' / ', []),
    ('<term>', None)])])])
```

Idea 1: Closing the Tree

credit: Sean Luke, 2000.

How to close the tree?

Once our tree is big enough,
choose expansions that increase tree size least.

e.g. `<factor>` could be
`<integer>(.<integer>)?`,
which could be `<integer>`,
which could be a single `<digit>`.

(Other alternatives are higher-cost).

Choosing the minimum-cost expansion

Traverse the tree & sum the minima.

```
def symbol_cost(self, symbol: str, seen: Set[str] = set()) ->
                                    Union[int, float]:
    expansions = self.grammar[symbol]
    return min(self.expansion_cost(e, seen | {symbol}) for e in
                                        expansions)

def expansion_cost(self, expansion: Expansion,
                   seen: Set[str] = set()) -> Union[int, float]
                                                    :
    symbols = nonterminals(expansion)
    if len(symbols) == 0:
        return 1

    if any(s in seen for s in symbols):
        return float('inf')

    return sum(self.symbol_cost(s, seen) for s in symbols) + 1
```

Minimum-cost expansion commentary

```
def symbol_cost(self, symbol: str, seen: Set[str] = set()) ->
                                        Union[int, float]:
```

is the minimum cost of the children of `symbol`.

```
def expansion_cost(self, expansion: Expansion,
        seen: Set[str] = set()) -> Union[int, float]:
```

if `expansion` has no nonterminals, return 1;
if any nonterminal in `expansion` is in `seen`, return inf;
else, return 1 + $\sum$ `symbol_cost`(children).

Minimum-cost expansion examples

```
>>> f = GrammarFuzzer(EXPR_GRAMMAR)
>>> f.symbol_cost("<digit>")
1
>>> f.symbol_cost("<expr>")
5
```

The min cost of <expr> is 5, through <term>, <factor>, <integer>, <digit>, and 1.

Beyond random: parametrizing by algorithm

```
def expand_node_by_cost(self, node: DerivationTree,
                        choose: Callable = min)
    -> DerivationTree:
```

This functional-style method:

- collects min cost of alternatives;
- asks `choose` to choose a desired cost;
- collects all alternatives with chosen cost;
- uses `choose_node_expansion()` to choose an alternative (default: random);
- returns chosen alternative.

Using `min` as choice algorithm

```
def expand_node_min_cost(self, node: DerivationTree) ->
                                        DerivationTree:
    return self.expand_node_by_cost(node, min)
```

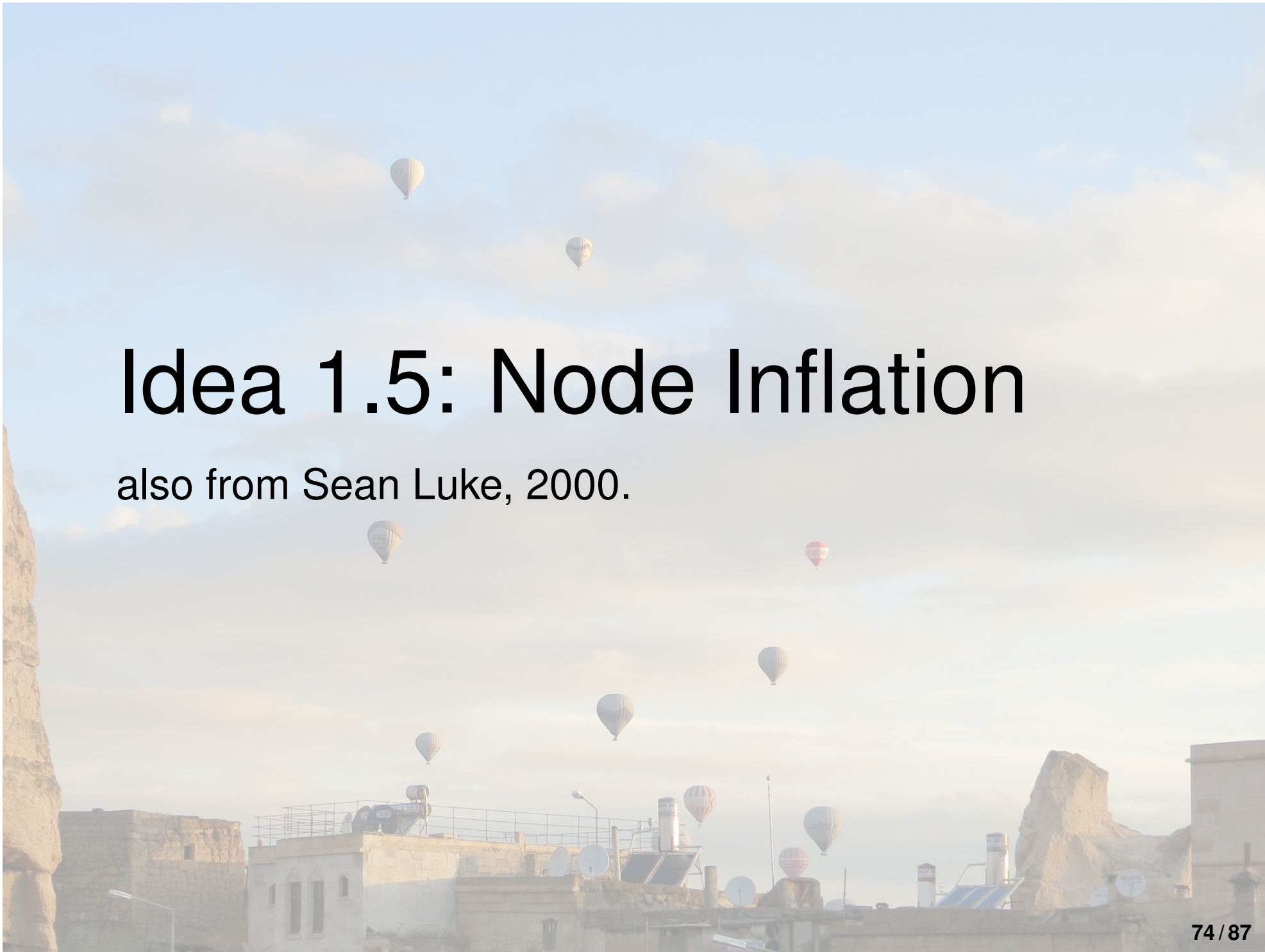
and using it:

```
>>> f.expand_node = f.expand_node_min_cost
>>> while f.any_possible_expansions(derivation_tree):
     derivation_tree = f.expand_tree_once(
                                          derivation_tree)
Expanding <term> at minimum cost
Expanding <factor> at minimum cost
Expanding <term> at minimum cost
... etc ...
>>> print (all_terminals(derivation_tree))
5 + 5 + 6 / 1
```

Why closing works

- `expand_node_min_cost()` does not increase the number of symbols,
- so all open expansions are eventually closed.

I think this is a property of the grammar as well as the implementation.



Idea 1.5: Node Inflation

also from Sean Luke, 2000.

Node Inflation

Want to make sure that our inputs are big enough.

Instead of min-cost expansion, find the max cost:

```
def expand_node_max_cost(self, node: DerivationTree) ->
                                    DerivationTree:
    return self.expand_node_by_cost(node, max)
```

Experiencing Inflation

and we can run it:

```
>>> derivation_tree = ("<start>",
                       [("<expr>", [("<expr>", None),
                                    (" + ", []),
                                    ("<term>", None)]
                       )])
>>> f = GrammarFuzzer(EXPR_GRAMMAR, log=True)
>>> f.expand_node = f.expand_node_max_cost

>>> # do this a number of times
>>> if f.any_possible_expansions(derivation_tree):
        derivation_tree = f.expand_tree_once(derivation_tree)
  ...
>>> print (all_terminals(derivation_tree))
<expr> + +<factor> / <factor> / <term>
```

Combining Ideas: max, random, min

Applying a strategy

```
def expand_tree_with_strategy(self, tree: DerivationTree,
                              expand_node_method: Callable,
                              limit: Optional[int] = None):
    """Expand tree using `expand_node_method`
    until number of possible expansions reaches `limit`."""
    self.expand_node = expand_node_method
    while ((limit is None
            or self.possible_expansions(tree) < limit)
           and self.any_possible_expansions(tree)):
        tree = self.expand_tree_once(tree)
    return tree
```

max, then random, then min

```
def expand_tree(self, tree: DerivationTree) -> DerivationTree:
    """Expand `tree` in a three-phase strategy until all
                                        expansions are complete."""
    tree = self.expand_tree_with_strategy(
        tree, self.expand_node_max_cost, self.min_nonterminals)
    tree = self.expand_tree_with_strategy(
        tree, self.expand_node_randomly, self.max_nonterminals)
    tree = self.expand_tree_with_strategy(
        tree, self.expand_node_min_cost)

    assert self.possible_expansions(tree) == 0
    return tree
```

Creating a tree

```
>>> derivation_tree = ("<start>",
                       [("<expr>", [("<expr>", None),
                                    (" + ", []),
                                    ("<term>", None)]
                       )])
>>> f = GrammarFuzzer(
          EXPR_GRAMMAR,
          min_nonterminals=3,
          max_nonterminals=5,
          log=True)
>>> f.expand_tree(derivation_tree)
... (max x1, randomly x2, minimum x20) ...
Tree: 3 * 5 - 1 + 8 * 9
```


Just a fuzzer

Abstract everything

```
class GrammarFuzzer(Fuzzer):
    def fuzz_tree(self) -> DerivationTree:
        """Produce a derivation tree from the grammar."""
        tree = self.init_tree()

        # Expand all nonterminals
        tree = self.expand_tree(tree)
        return tree

    def fuzz(self) -> str:
        """Produce a string from the grammar."""
        self.derivation_tree = self.fuzz_tree()
        return all_terminals(self.derivation_tree)
```

Easy Interface

Sensible defaults make it easy to use:

```
>>> f = GrammarFuzzer(EXPR_GRAMMAR)
>>> print (f.fuzz())
+1 / 6
>>> print (f.fuzz())
6.4 * (3 * 0 / 3) - -91 * 5 / 8 * 8
>>> g = GrammarFuzzer(URL_GRAMMAR)
>>> print (g.fuzz())
ftps://fuzzingbook.com
>>> h = GrammarFuzzer(CGI_GRAMMAR)
>>> print (h.fuzz())
%d2
```

Might also look at the derivation tree returned from `fuzz_tree()`.

```
>>> print (h.fuzz_tree())
('<start>', [('<string>', [('<letter>', [('<other>', [('c', [])
                                                    ])])])])
```

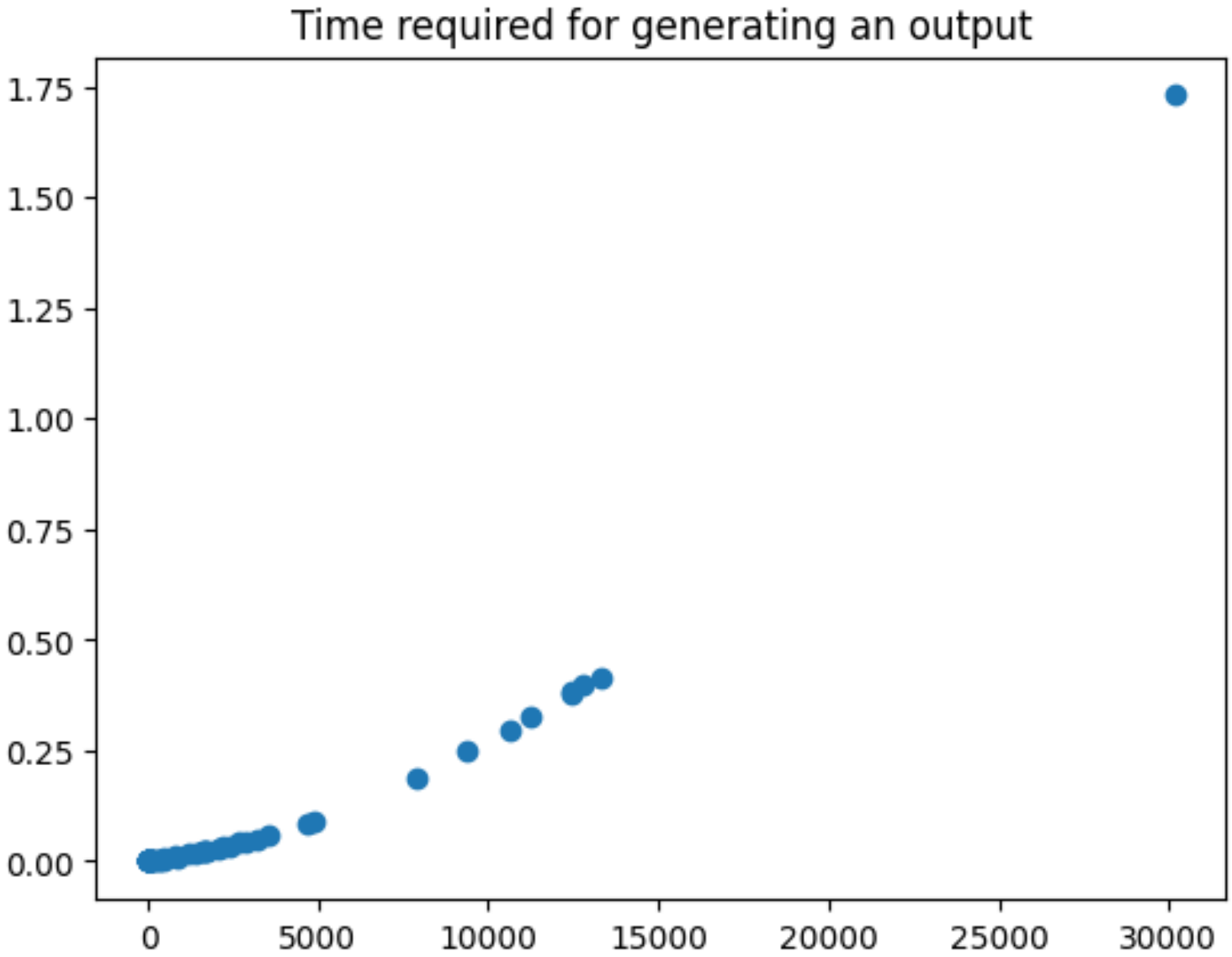
Concluding Remarks: Efficient Grammar Fuzzing

`GrammarFuzzer` much faster than `simple_grammar_fuzzer` and produces smaller inputs.

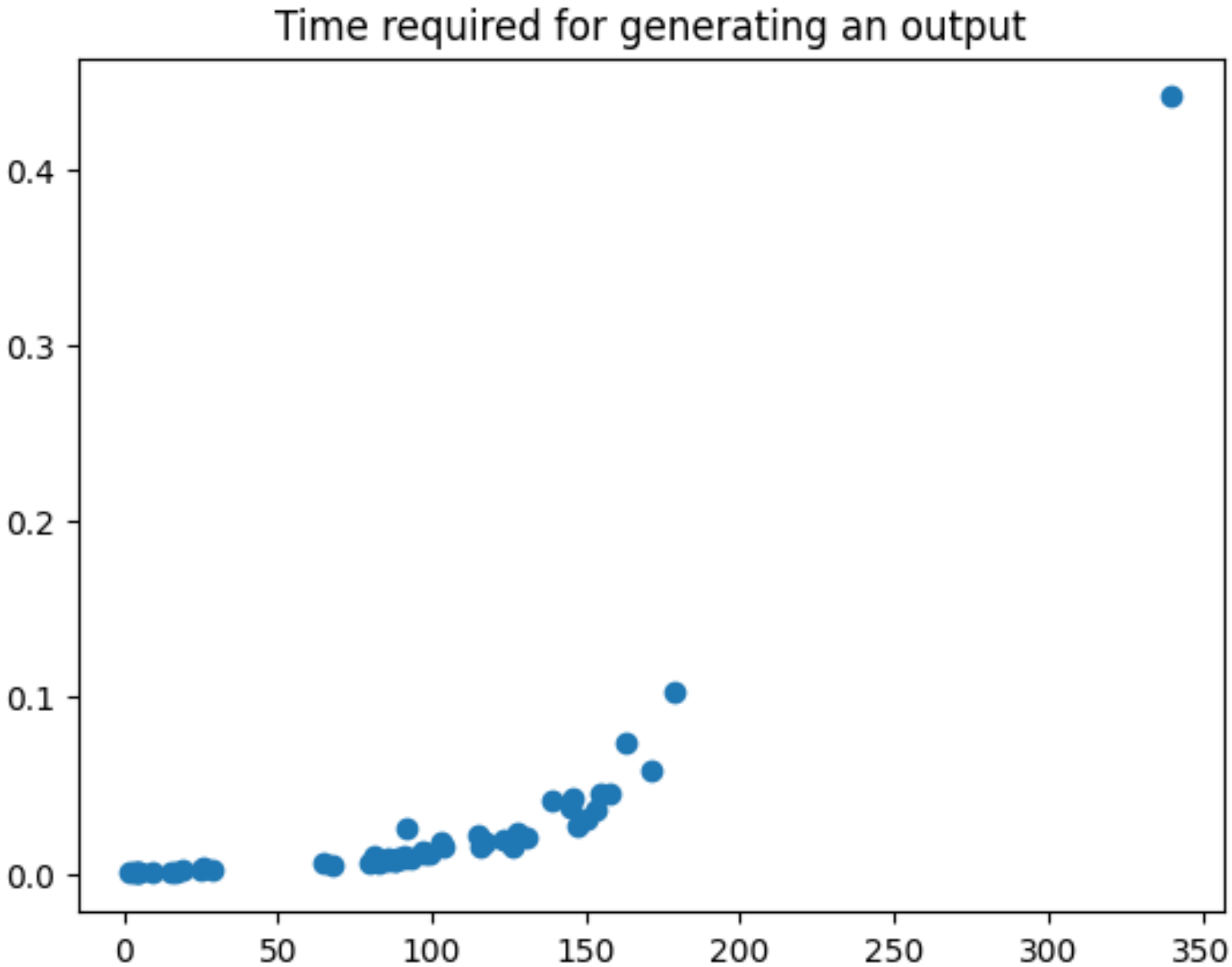
Better control over grammar production (min, max nonterminals.)

Three phases: max, random, min.

Inefficient Grammar Fuzzing Times (EXPR_GRAMMAR)



Efficient Grammar Fuzzing Times (EXPR_GRAMMAR)



And it works on EBNF, too

```
>>> expr_grammar = convert_ebnf_grammar(EXPR_EBNF_GRAMMAR)
>>> f = GrammarFuzzer(expr_grammar, max_nonterminals=10)
>>> print (f.fuzz())
97.01 * 5 * 7 - 8 * 9 - (9 + 0)
```