Homework #2 (due date: November 29)

This homework has a total of 3 questions on 2 pages, each of which has sub-questions. The homework will be collected at the beginning of class on the due date. Late homework will not be accepted. For problems requiring coding, please attach a copy of the code at the end of the work.

1. (Computer Problem) Consider the initial value problem

$$\begin{cases} y' = 6y - 6y^2, \ 0 \le t \le 20 \\ y(0) = 1/2 \end{cases}$$

(a) Which of the equilibrium solutions are approached by the approximate solution of a numerical method?

(b) Apply Euler's method. For what approximate range of h can Euler be used successfully to converge to the equilibrium?

(c) Apply Backward Euler given by $y_{i+1} = y_i + hf_{i+1}$, using Newton's method as a solver. *Plot approximate solutions given by Backward Euler, and by Euler with an excessive step size.*

2. Consider the family of linear multistep methods

$$y_{n+2} - 2ay_{n+1} + (2a-1)y_n = h\left[af_{n+2} + (2-3a)f_{n+1}\right],$$

where a is a parameter.

(a) What are its first and second characteristic polynomials?

(b) When is the method consistent?

(c) Under what conditions is it zero-stable?

(d) When is the method convergent?

(e) What is its order? What is the erro constant?

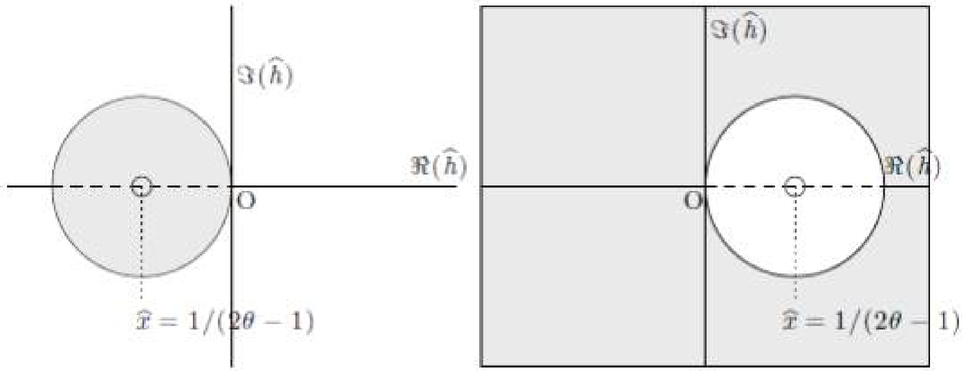
(f) Are there any members of the family that are A_0-stable?

(A numerical method is said to be $\underline{A_0\text{-stable}}$ if its interval of absolute stability includes the entire left real axis)

3. Determine the region of absolute stability of the θ-method

$$y_{n+1} - y_n = h(\theta f_{n+1} + (1-\theta) f_n).$$

Let $\hat{h} = \lambda h$. By writing $\hat{h} = \hat{x} + i\,\hat{y}$ show that this corresponds to the exterior of the circle $\hat{x}^2 + \dfrac{2}{2\theta - 1}\hat{x} + \hat{y}^2 = 0$ if $\dfrac{1}{2} < \theta \le 1$ and to the interior of the circle if $0 \le \theta < \dfrac{1}{2}$. See the figure below. What happens at $\theta = \dfrac{1}{2}$?



The region of absolute stability for the θ-method (shaded) and the interval of absolute stability (broken line). Left: $0 \le \theta < \dfrac{1}{2}$; right: $\dfrac{1}{2} < \theta \le 1$.