Version

- python 3.10.4
- NumPy 1.23.1
- matplotlib 3.5.2
- pandas 1.4.4

```
In [ ]:  import numpy as np
         import matplotlib.pyplot as plt
         import pandas as pd
```

```
In [ ]:  def euler_method(f, g, t0, tN, y0, h):
             N = int((tN - t0)/h)
             grid = np.linspace(t0, tN, N+1)
             ygrid = np.zeros_like(grid)
             ygrid[0] = y0
             iter = np.arange(0, N+1)
             for i in iter:
                 try:
                     slope = f(grid[i], ygrid[i])
                     ygrid[i+1] = ygrid[i] + slope * h
                 except:
                     break
             exact = g(grid)
             df = pd.DataFrame({"Iteration": iter, "t": grid, "Euler Approx":ygrid, "Exact":
             return df, grid, ygrid, exact
```

Problem 1

$$y' = \frac{1}{y^2}, y(0) = 1$$

exact solution $y = \sqrt[3]{1 + 3t}$

```
In [ ]:  f = lambda t, y: 1/(y**2)
         g = lambda t:(1+3*t)**(1/3)
         plt.figure(figsize=(5,5))
         u, grid, ygrid, exact = euler_method(f,g, 0, 1, 1, 0.1)
         plt.plot(grid, ygrid, label="Euler Approximation", c="C1")
         plt.plot(grid, exact, label="Exact Solution $y=\sqrt[3]{1+3t}$", c="C2", linestyle=
         plt.legend()
         plt.show()
         u.set_index("Iteration")
```
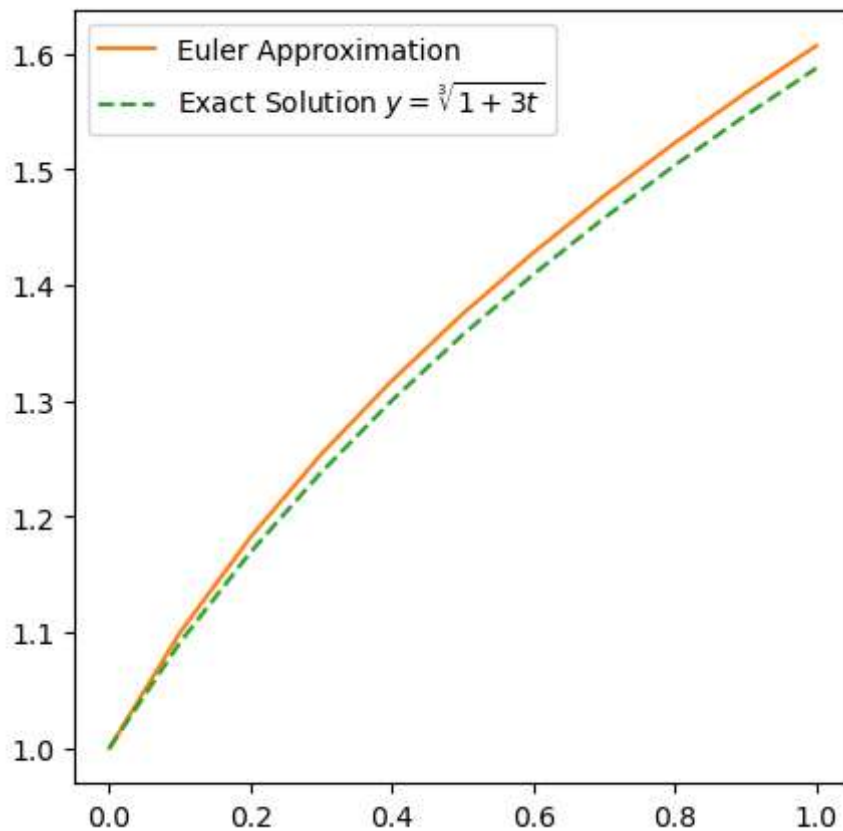
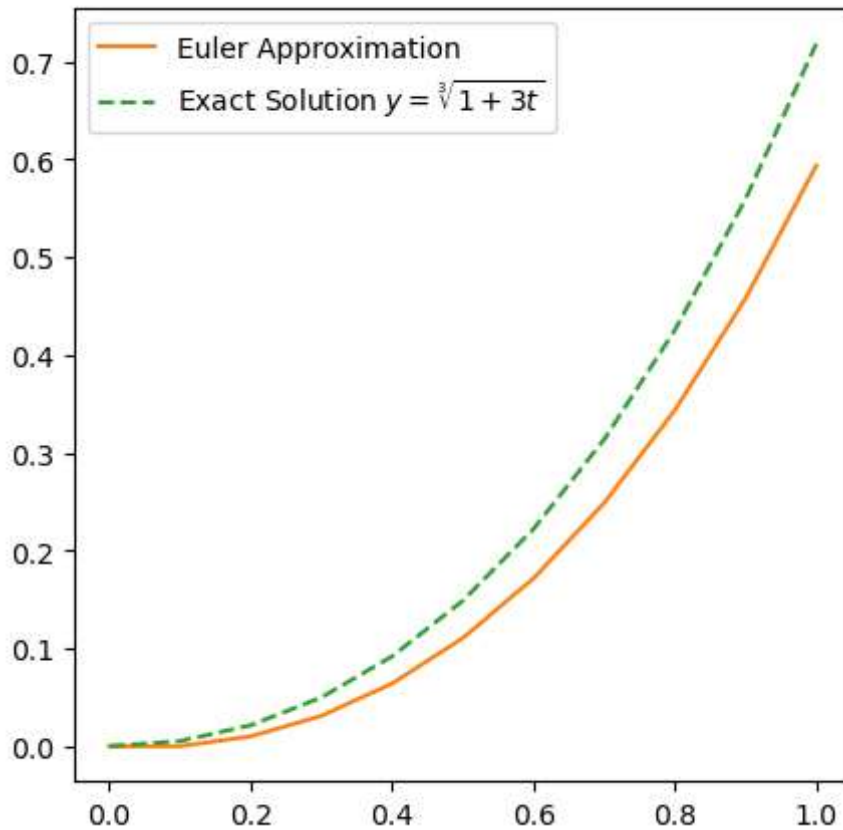| Iteration	t	Euler Approx	Exact	Error
0	0.0	1.000000	1.000000	0.000000
1	0.1	1.100000	1.091393	0.008607
2	0.2	1.182645	1.169607	0.013038
3	0.3	1.254142	1.238562	0.015580
4	0.4	1.317720	1.300591	0.017129
5	0.5	1.375311	1.357209	0.018102
6	0.6	1.428180	1.409460	0.018720
7	0.7	1.477207	1.458100	0.019107
8	0.8	1.523033	1.503695	0.019339
9	0.9	1.566143	1.546680	0.019463
10	1.0	1.606913	1.587401	0.019512

Problem 2

$$y' = t + y, y(0) = 0$$

$$\text{exact solution} \quad y = -(t+1) + 2e^{-t}$$

In [ ]:
```
f = lambda t, y : t+y ; f0 = 0
g = lambda t : -(t+1) + np.exp(t)
```

h = 0.1

```python
plt.figure(figsize=(5,5))
u, grid, ygrid, exact = euler_method(f,g, 0, 1, 0, 0.1)
plt.plot(grid, ygrid, label="Euler Approximation", c="C1")
plt.plot(grid, exact, label="Exact Solution $y=\\sqrt[3]{1+3t}$", c="C2", linestyle=
plt.legend()
plt.show()
u.set_index("Iteration")
```


Out[ ]:

Iteration	t	Euler Approx	Exact	Error
0	0.0	0.000000	0.000000	0.000000
1	0.1	0.000000	0.005171	-0.005171
2	0.2	0.010000	0.021403	-0.011403
3	0.3	0.031000	0.049859	-0.018859
4	0.4	0.064100	0.091825	-0.027725
5	0.5	0.110510	0.148721	-0.038211
6	0.6	0.171561	0.222119	-0.050558
7	0.7	0.248717	0.313753	-0.065036
8	0.8	0.343589	0.425541	-0.081952
9	0.9	0.457948	0.559603	-0.101655
10	1.0	0.593742	0.718282	-0.124539

h = 0.05

```
In [ ]: plt.figure(figsize=(5,5))
        u, grid, ygrid, exact = euler_method(f,g, 0, 1, 0, 0.05)
        plt.plot(grid, ygrid, label="Euler Approximation", c="C1")
        plt.plot(grid, exact, label="Exact Solution $y=₩sqrt[3]{1+3t}$", c="C2", linestyle=
        plt.legend()
        plt.show()
        u.set_index("Iteration")
```
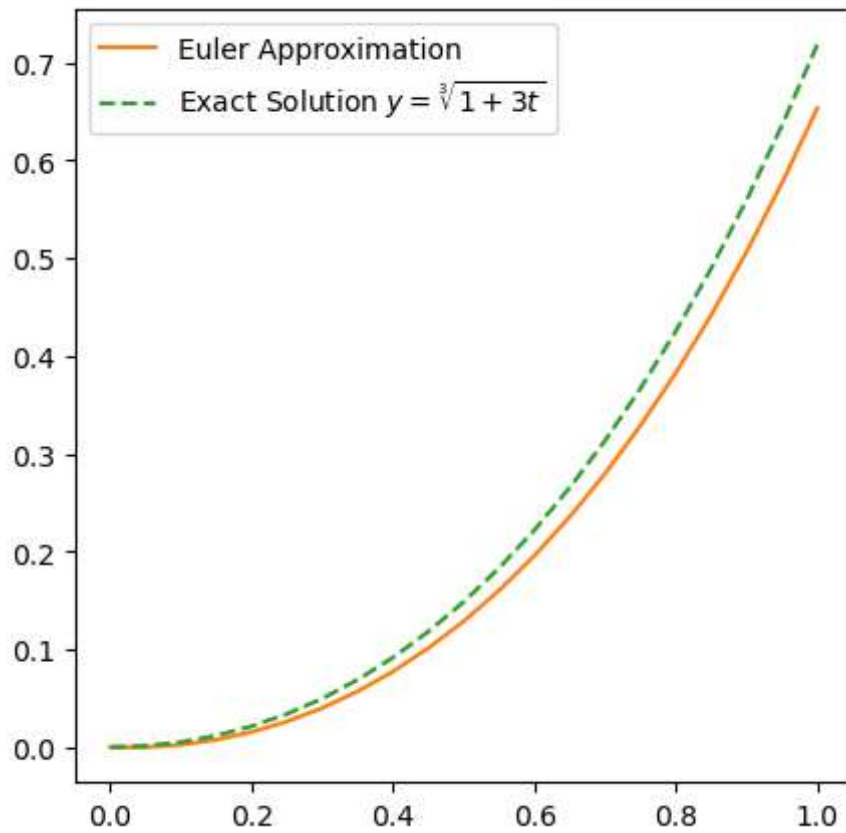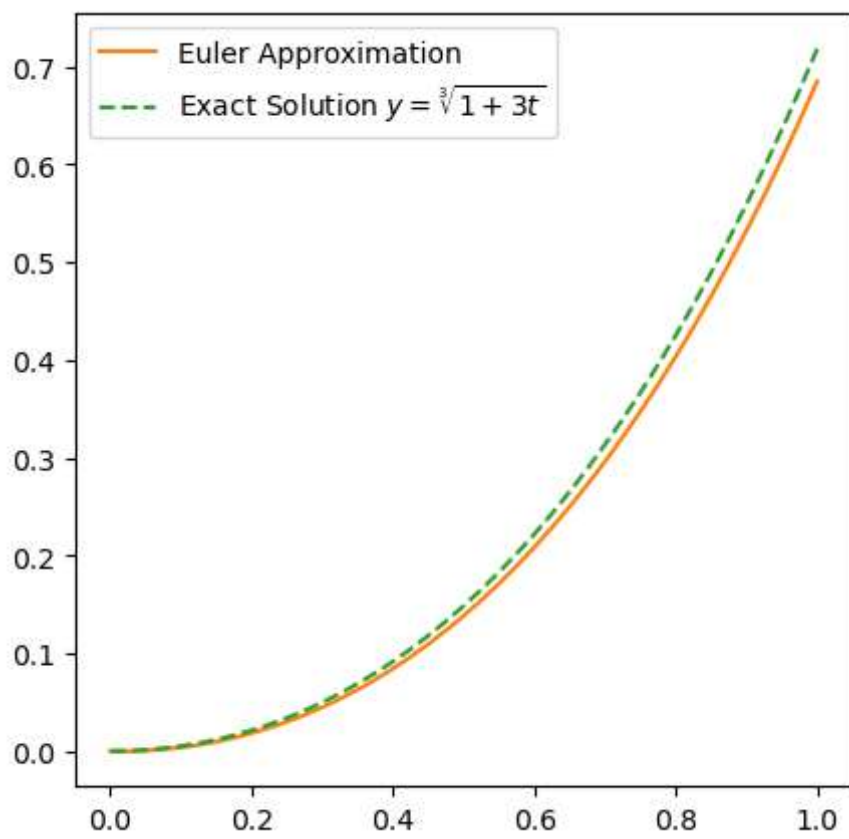
Iteration	t	Euler Approx	Exact	Error
0	0.00	0.000000	0.000000	0.000000
1	0.05	0.000000	0.001271	-0.001271
2	0.10	0.002500	0.005171	-0.002671
3	0.15	0.007625	0.011834	-0.004209
4	0.20	0.015506	0.021403	-0.005897
5	0.25	0.026282	0.034025	-0.007744
6	0.30	0.040096	0.049859	-0.009763
7	0.35	0.057100	0.069068	-0.011967
8	0.40	0.077455	0.091825	-0.014369
9	0.45	0.101328	0.118312	-0.016984
10	0.50	0.128895	0.148721	-0.019827
11	0.55	0.160339	0.183253	-0.022914
12	0.60	0.195856	0.222119	-0.026262
13	0.65	0.235649	0.265541	-0.029892
14	0.70	0.279932	0.313753	-0.033821
15	0.75	0.328928	0.367000	-0.038072
16	0.80	0.382875	0.425541	-0.042666
17	0.85	0.442018	0.489647	-0.047629
18	0.90	0.506619	0.559603	-0.052984
19	0.95	0.576950	0.635710	-0.058759
20	1.00	0.653298	0.718282	-0.064984

h = 0.025

```python
plt.figure(figsize=(5,5))
u, grid, ygrid, exact = euler_method(f,g, 0, 1, 0, 0.025)
plt.plot(grid, ygrid, label="Euler Approximation", c="C1")
plt.plot(grid, exact, label="Exact Solution $y=\sqrt[3]{1+3t}$", c="C2", linestyle=
plt.legend()
plt.show()
u.set_index("Iteration")
```

Out[ ]:

Iteration	t	Euler Approx	Exact	Error
0	0.000	0.000000	0.000000	0.000000
1	0.025	0.000000	0.000315	-0.000315
2	0.050	0.000625	0.001271	-0.000646
3	0.075	0.001891	0.002884	-0.000994
4	0.100	0.003813	0.005171	-0.001358
5	0.125	0.006408	0.008148	-0.001740
6	0.150	0.009693	0.011834	-0.002141
7	0.175	0.013686	0.016246	-0.002560
8	0.200	0.018403	0.021403	-0.003000
9	0.225	0.023863	0.027323	-0.003460
10	0.250	0.030085	0.034025	-0.003941
11	0.275	0.037087	0.041531	-0.004444
12	0.300	0.044889	0.049859	-0.004970
13	0.325	0.053511	0.059031	-0.005520
14	0.350	0.062974	0.069068	-0.006094
15	0.375	0.073298	0.079991	-0.006693
16	0.400	0.084506	0.091825	-0.007319
17	0.425	0.096618	0.104590	-0.007972
18	0.450	0.109659	0.118312	-0.008653
19	0.475	0.123650	0.133014	-0.009364
20	0.500	0.138616	0.148721	-0.010105
21	0.525	0.154582	0.165459	-0.010877
22	0.550	0.171571	0.183253	-0.011682
23	0.575	0.189611	0.202131	-0.012520
24	0.600	0.208726	0.222119	-0.013393
25	0.625	0.228944	0.243246	-0.014302
26	0.650	0.250293	0.265541	-0.015248
27	0.675	0.272800	0.289033	-0.016233
28	0.700	0.296495	0.313753	-0.017258
29	0.725	0.321407	0.339731	-0.018324
30	0.750	0.347568	0.367000	-0.019432
31	0.775	0.375007	0.395592	-0.020585
32	0.800	0.403757	0.425541	-0.021784
33	0.825	0.433851	0.456881	-0.023030
34	0.850	0.465322	0.489647	-0.024325

Iteration	t	Euler Approx	Exact	Error
35	0.875	0.498205	0.523875	-0.025670
36	0.900	0.532535	0.559603	-0.027068
37	0.925	0.568349	0.596868	-0.028520
38	0.950	0.605682	0.635710	-0.030027
39	0.975	0.644574	0.676167	-0.031593
40	1.000	0.685064	0.718282	-0.033218