MTH4033 HW3 (Fall 2022)

Professor Youngjoon Hong — Due Date: Oct. 19 (3:00 pm)

Problem 1 *(Theory) Show that the following scheme*

$$\begin{aligned}
\tilde{v}_m^{n+1} &= v_m^n - a\lambda(v_{m+1}^n - v_m^n) + kf_m^n, \\
v_m^{n+1} &= \frac{1}{2}(v_m^n + \tilde{v}_m^{n+1} - a\lambda(\tilde{v}_m^{n+1} - \tilde{v}_{m-1}^{n+1}) + kf_m^{n+1})
\end{aligned}$$

is a second-order accurate scheme for the one-way wave equation.

Problem 2 *(Numerics) Complete the following table to implement Lax-Wendroff and Lax-Friedrichs schemes to the homogeneous one-way wave equation with* $a = 1$ *and* $x \in (-2, 5)$*. Use* $\lambda = 0.8$ *and the initial condition* u_0*:*

$$u(0, x) = \begin{cases} \cos^2(\pi x), & |x| \le 1/2, \\ 0, & \textit{otherwise}, \end{cases}$$

with the final time $T = 1.0$*. You can simply put a zero boundary condition. Note that the numerical error can be measured by*

$$\textit{Error}(t_n) = \|u(t_n, \cdot) - v^n\|_h = \left(h \sum_m |u(t_n, x_m) - v_m^n|^2 \right)^{1/2}.$$

	Lax-Wendroff		Lax-Friedrichs	
h	Error	Order	Error	Order
1/10				
1/20				
1/40				
1/80				