

Problem 1 (Theory) Show that the BTCS (backward-time central-space) scheme is consistent with equation $u_t + au_x = 0$ and is unconditionally stable.

Problem 2 (Theory) Show that the box scheme

$$\frac{1}{2k} [(v_m^{n+1} + v_{m+1}^{n+1}) - (v_m^n + v_{m+1}^n)] + \frac{a}{2h} [(v_{m+1}^{n+1} - v_m^{n+1}) + (v_{m+1}^n - v_m^n)] = 0$$

is consistent with the one-way wave equation $u_t + au_x = 0$. Describe numerical stability with respect to λ for this scheme.