MTH3033 HW3 (Fall 2022)

Professor Youngjoon Hong Due Date: October. 07 (11:59 pm)

Write a python code for Problem 1 and 2. For credits, submit your fully working code to my TA at dltmdgus520@g.skku.edu. The title of your email should contain student number, your name, course number; e.g., "2022160301 Heung-min Son MTH3033HW3".

Problem 1 *Consider the integral*

$$\int_0^1 \ln(1+x^2)\,dx,$$

and note that the error bound for the Simpson's rule is

$$E_N^S(f) = \left| \int_a^b f(x)\,dx - S_N(f) \right| \le \frac{(b-a)^5}{180N^4} K_4 \tag{1}$$

where $\left| f^{(4)}(x) \right| \le K_4$ *for all* $x \in [a,b]$. *Using Simpson's rule:*
(i) Find the theoretically smallest value N *which satisfies* (1).
(ii) Find the smallest value N *such that* $E_N^S(f) \le 10^{-8}$ *by repeatedly computing* $N = 2, 3, 4, ...$

Problem 2 *Write a function called 'odeMid' which implements the following midpoint method:*

$$\begin{aligned} k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + h/2, y_n + k_1 h/2) \\ y_{n+1} &= y_n + k_2 h \end{aligned}$$

To check your code, consider the equation

$$y' = y\cos(t)\ , \quad y(0) = 1.$$

The equation is separable and we solve using separation of variables:

$$y(t) = e^{\sin(t)}$$

Plot the approximation by the midpoint method with step size $h = 0.2$ and plot the exact solution on the interval $0 \leq t \leq 2\pi$.