Professor Youngjoon Hong

Due Date: October. 30 (11:59 pm)

Write a python code for Problem 1. For credits, submit your fully working code to my TA at dltmdgus520@g.skku.edu. The title of your email should contain student number, your name, course number; e.g., "2022160301 Heung-min Son MTH3033HW4".

Problem 1 For the function

$$f(x) = \frac{x^2}{1 + x^4}, -1 \le x \le 1$$

- a) (2pts) Using an equispaced set of 10 nodes, generate the Lagrange interpolating polynomial to f(x). Generate a plot which shows how the error in your approximation varies over the interval.
- b) (2pts) Using an equispaced set of 20 nodes, generate the Lagrange interpolating polynomial to f(x). Generate a plot which shows how the error in your approximation varies over the interval.
- c) (2pts) At what number of equispaced nodes does your Lagrange interpolation approximation break down?
- d) (4pts) Does using Chebyshev points help resolve the issues you saw in c)? Provide examples to verify your claim.

For the error plot, please refer the following sample code: see page 2.

```
def interp_plotter(xnodes,fnodes,xquery,ftrue):
    finterp = lagran_interp(xnodes,fnodes,xquery)
    plt.plot(xquery,np.ma.log10(np.abs(ftrue-finterp)))
    plt.xlabel("$x$")
    plt.ylabel("$\log_{10}\|Error\|\$")

f1 = lambda x: x**2./(1.+x**4.)
xquery = np.linspace(-1.,1.,int(1e3)+1)
ftrue = f1(xquery)

xnodes = xquery[::100]
fnodes = ftrue[::100]
interp_plotter(xnodes,fnodes,xquery,ftrue)
```