1. An Introduction to Social Choice

There are many methods that we will consider in an attempt to find a voting system that most accurately reflects the voter's choices. For each method, the voters must cast their ballots.

A _____ballot allows voters to choose one person for each position. The voter's opinions about the other candidates are not expressed.

A _____ballot allows voters to rank the candidates. This allows the voter to express opinions about all of the candidates, and this information can be used in the voting system.

Notation: In the notes, when we list ABC, we will assume that candidate A is the voter's first choice and candidate C is the voter's least desired choice. We will use this notation to indicate the voter's preference even if the voter is only allowed to indicate his/her first choice. In the book, these preferences will be listed vertically.

Ties can occur in any voting system, but they rarely occur when there are a large number of voters. However, it is advisable to determine a tie-breaking method prior to the vote. In November 2015, a Mississippi state representative position was determined by the drawing of straws after the vote was tied with 4589 votes for each candidate. The book will assume an odd number of voters to avoid ties.
What are some other ways to break ties?

2. Majority Rule and Condorcet's Method

Definition 2.1 (Majority Rule)**.**
Each voter votes for one candidate. The candidate with the majority of the votes wins. Majority means more than _____. For two candidates, majority rule meets all three goals for fairness. The notes and book let the reader know the entire preference list of the voters. Assume that the voter votes for his/her first preference.

Definition 2.2 (Condorcet's Method)**.**
Each voter ranks the candidates (preference list voting). Each candidate is compared to every other candidate. If one candidate wins all of his/her one-on-one contests, he/she is declared the Condorcet winner.

Example 2.3.

Assume that the following list reflects the voting preferences of all voters.

Pref. List	XYZ	XZY	YXZ	YZX	ZYX	ZXY
Voters	5	0	2	1	4	0

- Who is the majority winner?

X	
Y	
Z	

- Who is the Condorcet winner?

Pref. List	XYZ	XZY	YXZ	YZX	ZYX	ZXY
Voters	5	0	2	1	4	0

Contest	Votes for 1st contestant	Votes for 2nd contestant	Winner
X vs Y			
X vs Z			
Y vs Z			

Example 2.4.

Assume that the following list reflects the voting preferences of all voters. Who is the Condorcet winner?

Pref. List	ABC	BCA	CAB
Voters	4	4	4

Contest	Votes for 1st contestant	Votes for 2nd contestant	Winner
A vs B			
A vs C			
B vs C			

This is an example of Condorcet's _____because the voters' preferences are cyclic (the group of voters prefer A over B, B over C, and C over A). Condorcet's paradox is exhibited in the game Rock, Paper, Scissors.

3. Other Voting Systems for Three or More Candidates

Definition 3.1 (Plurality Voting)**.**

Each voter votes for one candidate. The candidate with the _____votes wins. A candidate can win a plurality vote with less than half the votes. The notes and book let the reader know the entire preference list of the voters. Assume that the voter votes for his/her first preference.

In the 2000 presidential election, George Bush won the plurality of the votes, but polling data indicates that Al Gore would have won using the Condorcet method.

Example 3.2.

Assume that the following list reflects the voting preferences of all voters.

Pref. List	XYZ	XZY	YXZ	YZX	ZYX	ZXY
Voters	5	0	2	1	4	0

Who is the plurality winner?

X	
Y	
Z	

Definition 3.3 (Manipulable voting system)**.**

A voting system is _____if there are elections in which it is to a voter's advantage to submit a ballot that misrepresents his or her true preferences.

Plurality voting is manipulable. This is often exhibited in our presidential elections by voters who support a third-party candidate but vote for a candidate from one of the two major parties.

One of the strengths of Condorcet's method is that it is not manipulable.

Definition 3.4 (Borda Count and other Rank Methods)**.**

A _____method of voting assigns points in a nonincreasing manner to the ordered candidates on each voter's preference list ballot then sums these points to arrive at a group's final ranking (more points indicate a higher ranking).

A rank method in which there are n candidates with first place votes receiving n-1 points, second place receiving n-2 points, and so on down to each last-place vote receiving 0 points is called the _____count.

Example 3.5.

Assume that the following list reflects the voting preferences of all voters.

Pref. List	XYZ	XZY	YXZ	YZX	ZYX	ZXY
Voters	5	0	2	1	4	0

Who is the plurality winner?

	1st place × _____ pts	2nd place × _____ pt	3rd place × _____ pts	Total
X				
Y				
Z				

Ranking:

Example 3.6.

A sports award uses a rank method with 5 points for first place, 3 points for second place, 2 points for third place, 1 point for 4th place, and 0 points for any place below 4th place. Rank the candidates (A, B, C, D, E, F, G) for this award.

Pref. List	ABCDEFG	CDFAEBG	ADCBEGF	BACDFGE	ACBDFEG
Voters	4	3	2	1	5

	1st place × 5 pts	2nd place × 3 pts	3rd place × 2 pts	4th place × 1 pt	Total
A B					
C					
D					
E					
F					
G					

Ranking:

Rank methods are often used for sports award rankings.

Definition 3.7 (Runoff Election)**.**

If there is no majority winner, _____is taken after eliminating one or more of the candidates. The top two candidates could be in the runoff, or the Hare System (described below) could be used.

Definition 3.8 (Hare System)**.**
If there is no majority winner, then the candidate with the fewest number of _____votes is eliminated and the results are calculated again. If there is still no majority winner, the process continues until a majority winner is found or the remaining candidates are tied.

Example 3.9.
Use the Hare system to determine a winner.

Pref. List	XYZ	XZY	YXZ	YZX	ZYX	ZXY
Voters	3	2	1	5	4	2

Candidate	Votes in 1st round	Votes in 2nd round
X		
Y		
Z		

Winner using the Hare system:

Definition 3.10 (Sequential Pairwise Voting)**.**
In sequential pairwise voting candidates are compared two at a time in a _____order known as an agenda. The winner of the pairing is compared to the next candidate on the pre-determined list. This process continues until a winner is determined.

Unfortunately, the order of the pairings can affect who wins the election.However, this process is used by our legislature as bills are presented. As amendments to the bill are proposed, the votes are either for the current version of the bill or the amended version. After that process has concluded, a final vote (yes/no) is taken on the amended version (rather than allowing a choice of no bill, original version, or amended version).

Example 3.11.

Assume that the following list reflects the voting preferences of all voters.

Pref. List	ABDC	CABD	BDCA
Voters	1	1	1

(a) Who is the Condorcet winner?

Contest	Votes for 1st contestant	Votes for 2nd contestant	Winner
A vs B			
A vs C			
A vs D			
B vs C			
B vs D			
C vs D			

Who is the sequential pairwise winner for each of the following agendas? (b) ABCD? (c) BCDA? (d) CABD? (e) DBAC?

4. Approval Voting

Each voter votes for all the candidates they find acceptable (approve of).
The candidate with the _____wins.

Example 4.1.
A family is deciding what to serve for dinner on Saturday and Sunday. Mom draws up a list and the votes are listed below. What will the family have for dinner this weekend?

	Mom	Dad	Boy	Girl 1	Girl 2	Total
Liver and Onions	Y					
Lamb Stew	Y		Y			
Fish Sticks	Y		Y	Y		
Fried Chicken		Y		Y		
Hamburgers	Y	Y	Y	Y	Y	
Spaghetti	Y	Y		Y	Y	

Example 4.2.
A group is deciding what movie to see. The votes are listed below. What movie will they go see?

	8	5	3	3	1	Total
Spectre	Y		Y		Y	
Star Wars	Y	Y				
Bridged	Y		Y		Y	
Joy		Y		Y	Y	
Sisters		Y		Y	Y	
Hunger Games			Y	Y	Y	

5. Fairness Criteria

Definition 5.1 (Majority Criterion)**.** If a candidate receives a majority of the first place votes, then that candidate should be declared the winner.

Example 5.2.
Assume that the following list reflects the voting preferences of all voters.

Pref. List	XYZ	YZX	ZYX
Voters	16	8	7

(a) Who is the majority winner?

X	
Y	
Z	

(b) Use the Borda count to determine the winner.

	1st place × _____ pts	2nd place × _____ pt	3rd place × _____ pts	Total
X				
Y				
Z				

(c)Would you consider the Borda count method as "fair" considering the majority criterion?

Definition 5.3 (Condorcet Criterion)**.** If a candidate is favored when compared one-on-one with every other candidate, then that candidate should be declared the winner.

Example 5.4.
Assume that the following list reflects the voting preferences of all voters.

(a) Who is the Condorcet winner?

Pref. List	DABC	ACBD	BCAD	CBDA	CBAD
Voters	120	100	90	80	45

Contest	Votes for 1st contestant	Votes for 2nd contestant	Winner
A vs B			
A vs C			
A vs D			
B vs C			
B vs D			
C vs D			

(b) Who is the plurality winner?

Pref. List	DABC	ACBD	BCAD	CBDA	CBAD
Voters	120	100	90	80	45

A	
B	
C	
D	

(c) Who is the majority winner?

(d) Who is the winner using the Hare method?

Candidate	Votes in 1st round	Votes in 2nd round	Votes in 3rd round
A			
B			
C			
D			

(e) Who is the winner using the Borda count?

Pref. List	DABC	ACBD	BCAD	CBDA	CBAD
Voters	120	100	90	80	45

	1st place × _____ pts	2nd place × _____ pt	3rd place × _____ pts	Total
A				
B				
C				
D				

(f) Which of the method failed to meet the Condorcet criterion for this example?

Definition 5.5 (Monotonicity Criterion)**.** This was initially described in section 9.1. Assume that candidate A won the election, but the votes were recounted. The only errors discovered were that some votes originally counted for other candidates should have been counted for candidate A. Then candidate A should still win the election. A candidate who wins a first election and then gains additional support, without losing any of the original support, should also win a second election.

Example 5.6.
The IOC was voting for the Winter Olympics to be held in Quebec (Q), Salt Lake City (L), Ostersund (T) or Sion (S). They took a straw vote (a preliminary vote that is not binding) on the first day. They took a binding vote on the second day. A total of 87 votes were cast (fictional data).

Day 1 Pref. List	TLSQ	LQTS	QSTL	TQSL	TSLQ
Voters	21	24	30	6	6

(a) Use the Hare Method to determine the winner from Day 1.

Candidate	Votes in 1st round	Votes in 2nd round	Votes in 3rd round
Q			
L			
S			
T			

(b) That night, the 12 people who preferred TQSL and TSLQ decided to move Q to the top of their list without changing any other preferences. Use the Hare Method to determine the winner from Day 2.

Day 2 Pref. List	TLSQ	LQTS	QSTL	QTSL	QTSL
Voters	21	24	30	6	6

Candidate	Votes in 1st round	Votes in 2nd round	Votes in 3rd round
Q			
L			
S			
T			

(c) In this example, did the Hare method violate the monotonicity criterion?

Definition 5.7 (Independence of Irrelevant Alternatives)**.** If a candidate would be declared the winner of an election and one or more of the non-winning candidates is removed, then the previous winner should still be declared the winner.
In other words, in order for a candidate to go from losing an election to winning an election, at least one voter should have to reverse his or her ranking of that candidate and the previous winner.

Example 5.8.
There are 3 candidates and 20 voters with preferences listed below.

Pref. List	XYZ	ZYX	YXZ
Voters	7	8	5

(a) Who would be the winner using Hare's method?

Candidate	Votes in 1st round	Votes in 2nd round
X		
Y		
Z		

(b) If candidate Z dropped out before the election and no voters changed their preference, who would win the election?

Pref. List	XYZ	ZYX	YXZ
Voters	7	8	5

Candidate	Votes
X	
Y	

Definition 5.9 (No Perfect System)**.** We have seen problems with all of the methods that we discussed. Kenneth Arrow proved that with three or more candidates, there does not (and cannot ever) exist a voting system that always produces a winner, satisfies the Pareto condition (if every voter prefers candidate X over candidate Y, then candidate Y will not be a winner), and satisfies the Irrelevant Alternatives condition that is not a dictatorship.