

## 142\_12c\_4.1

Math 142 -copyright Angela Allen, Fall 2012 4.1 Supplement: Derivatives of Powers, Exponents, and Sums f(x) = linit (when h > 0)

f(x) = (difference quotions

between (x+h f(x+h))

(x, f(x)) **<u>Derivative Notation</u>** - If y = f(x), then all represent the derivative of f at x. **Derivative Rules:**  $= \lim_{h\to 0} \frac{f(x+h)-f(x)}{x+h-x}$ 1) If f(x) = c, where c is a constant, then f'(x) = 0. (Constant Function Rule) constant 2) If f(x) = ax + b, then f(Derivative of a Linear Function) 3) If  $f(x) = x^n$ , where n is any nonzero real number, then  $f'(x) = nx^{n-1}$ . (Power Rule)  $Y = \frac{1}{2} - \frac{1}$ = lim f(x+h) - f(x) 5) If f(x) = u(x) + v(x), then f'(x) = u'(x) + v'(x). If f(x) = u(x) - v(x), then f'(x) = u'(x) - v'(x). (Sum and Example:  $y = \sqrt{7}$ . Find y' = 0 since it is constant. line at x Example:  $f(x) = x^5$ . Find  $f'(x) = 5x^4$ 1st Notation

Example:  $y = t^{-3}$ . Find  $\frac{dy}{dt} = -3 + 1 = -3 + 4$ =) Derivative of y**Example:**  $f(x) = x^5$ . Find f'(x). = 5  $\times$ Example:  $\frac{d}{dx} \frac{1}{\sqrt{x^2}} = \frac{d}{dx} \frac{1}{\sqrt{x^2$ 22 ---> 2X

**Example:** 
$$y = \sqrt[3]{w} - 3w$$
. Find  $\frac{dy}{dw}$ .

Example:  $y = \sqrt[3]{w} - 3w$ . Find  $\frac{dy}{dw}$ . Ohange terms with exponent form

@ Use pover rule to each term

$$\frac{47}{49} = \frac{3}{100} = \frac{3}{3}$$

Example: 
$$\frac{d}{dx} \frac{3x^2 + x^4}{5\sqrt{x}} = \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{\sqrt{x}} = \frac{3}{5} \cdot \frac{1}{$$

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## Applications

Example: An object moves along the y axis (marked in feet) so that its position at time t (in seconds) is

 $s(t) = t^3 - 6t^2 + 9t$ . Find derivative of 5 S(+)=+3-6++9+ a) The instantaneous velocity function v.

 $V = S'(+) = 3t^{2} - 12t + 9$   $V(5) = 3\cdot25 - 12\cdot5 + 9 = 115 - 60 + 9 = 24$ b) The velocity at t = 2 and t = 5 seconds. -) V(2)= 3.4-12-2+9=12-24+9=-3

c) The time(s) when the velocity is 0 ft/s

 $\Rightarrow$  Find t such that V(t) = 0

 $3t^2 - 12t + 9 = 0$ Example: Let  $f(x) = 4 - 6x^2 + 10$ . Find  $f(x) = 4x^3 - 12x + 6$ a) The equation of the tangent line at x = 1. Y = -9x + 9 + 5 = -9x + 13 Y = -8(7(-1))Shape point form on (1, f(x)) = -8 (1, f(x)) = -8

(Slobe DOM+ Grm ON Shope =- 8, point = (1,5)

b) Find the values of *x* where the tangent line is horizontal.

(⇒) " Where f'(x) = 0.

 $f_{N} = 4x^{3} - 12x = 0 \qquad 4x^{3} - 12x = 0$   $4x(x^{2} - 3) = 0$  4x(x - 3)(x + 3) = 0

**Example:** The total sales of a company (in millions of dollars) t months from now are given by  $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$ . Find S(4) and S'(4). Then, **interpret** both results.

S(4) = Use calculator = 120. 84 | The about of sales S'(4)=

= 60.24 after 4 months

will be 120.84 mil\$

=(1,5)

Four months now on Sales will increase by 60, 24 mils/port Math 142 -copyright Angela Allen, Fall 2012

Marginal Cost, Revenue, and Profit - If x is the number of units of a product produced in some time interval, then

- 1) total cost = C(x) and marginal cost = C'(x)
- 2) total revenue = R(x) and marginal revenue = R'(x)
- 3) total profit = P(x) = R(x) C(x) and marginal profit = R'(x) C'(x)

Example: A company that makes grills has a total weekly cost function (in dollars) of

 $C(x) = 10,000 + 90x - 0.05x^2$ , where x is the number of grills produced.

a) Find C(500) and interpret your answer.

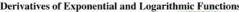
b) Find the marginal cost at a production level of 500 grills per week. Then, interpret your result.

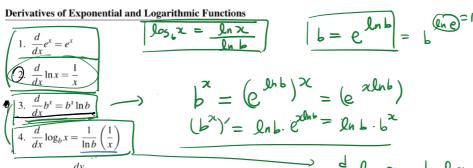
c) Approximate/estimate the cost of producing 501 grills.

$$((500) + C(500) = 42,500$ + 40$$$

e) Find the exact cost of producing the 501st grill.

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Example: Find 
$$\frac{dy}{dx}$$
 for the following. Do not simplify your answer!

a)  $y = \log_{\pi} x$   $\frac{dy}{dx} = \begin{pmatrix} b = \pi \\ eulc \ 4 \end{pmatrix}$   $\frac{dy}{dx} = \frac{1}{2\pi} \frac{1}{2\pi} \cdot \frac{1}{2\pi} \frac{1}{2\pi} \cdot \frac{1}{2\pi} \frac{1}{2\pi} = \frac{1}{2\pi} \frac{1}{2\pi} \cdot \frac{1}{2\pi} \frac{1}{2\pi} \cdot \frac{1}{2\pi} \frac{1}{$ 

d) 
$$y = x^4 - 4^x - 3\log_6 x + 7(5^x)$$
 $y = \frac{x^4 - 4^x - 3\log_6 x + 7(5^x)}{5^2 2}$ 
 $y = \frac{x^3 - 3x(2^x) - \frac{4}{11}}{x}$ 
 $\frac{dy}{dx} = 4x^3 - 4^2 \ln 4 - 3 \cdot \frac{1}{2 \ln 6} \cdot \frac{1$ 

$$= \chi^{2} - 3(2^{x}) - \frac{4}{11}x^{2} \qquad \frac{dy}{dx} = 2x - 3(2^{x} \cdot 9n2) = \frac{4}{11} \cdot (9x^{2})$$

$$= 2x - (3 \ln 2) \cdot 2^{x} + \frac{4}{11} \cdot x^{2}.$$

Note: In some cases it might be necessary to simplify a function using logarithmic properties before taking the derivative.

$$f(x) = 5 + 7 (\ln 6 - 3 \ln x)$$

$$f(x) = 5 + 7 \ln 6 - 21 \ln x$$

$$f'(x) = 0 - 21 \cdot \frac{1}{x} = -21 \cdot \frac{1}{x}$$