The Corporate Default Probability model in the Barclays Capital POINT platform (POINT CDP)

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We describe our second generation default prediction methodology that delivers forward-looking estimates of individual default probabilities of US corporations. Daily updates of these estimates are available through POINT. The POINT CDP model is a proprietary hybrid approach that incorporates the output of an advanced option-based structural model with additional predictive information in a dynamic nonlinear reduced-form hazard rate model. These default probabilities take into account the specific circumstances of the firm, equity market performance, the state of the economy, and certain industry effects. They are constructed to give a dynamic view on a firm's credit condition as implied by the equity market and other relevant sources of information. The model performs very well over the different phases of the business cycle, with robust performance in a highly diverse universe of firms.

1. Introduction

We continuously strive to improve the capabilities of the models in POINT (Barclays Capital global, multi-asset class, portfolio analysis platform). The POINT Corporate Default Probability (POINT CDP) model is our second generation default prediction model. It builds on the methodology of the Hybrid Default Probability (HDP) model in POINT, and the Corporate Default Probability (CDP) on Barclays Capital Live. The new model includes the best features of the two existing ones, as well as additional improvements.

Our methodology uses market information, fundamental accounting data, macroeconomic data, and historical default events to provide accurate and reliable estimates for default probabilities. One important feature that we improved upon is the rigorous treatment of accounting information. In addition to processing the as-reported accounting information with improved filters to increase data quality, we analyze details and footnotes of financial reports to reconstitute balance sheet and income statement items for more realistic views of the firms' economic standings. To optimally model the dynamics of the aggregate default rate in the economy, we separately forecast the level of default rates and the relative ranking of distress of individual firms. This ensures that we capture sharp movements in system-wide distress during economic downturns.

The POINT CDP model has been developed for the purpose of portfolio risk modeling. However, it can be employed for a wide range of applications, including portfolio selection, ratings arbitrage, and cross-market trade evaluations. We illustrate the relationship between credit spreads and default risk by documenting the correlations between issuer default probabilities and CDS spreads.

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2. The model

Models of corporate default risk can generally be divided into structural and reduced-form models. The former estimates default probabilities from the firm's asset process (and default barrier), based on variations of the Merton (1974) model. The latter is agnostic about the actual economic processes underlying corporate defaults, and estimates these probabilities by an assumed statistical relationship between defaults and variables deemed relevant for default prediction. The POINT CDP methodology incorporates a structural model into a broader reduced-form econometric approach. In doing so, we seek to avoid the rigidity of a structural model while retaining some of the advantages of the logic underlying this approach. We directly model the default intensities of individual firms by a discrete time hazard rate process, while including information based on a structural model.

The structural model we use is an extension of the Merton (1974) structural framework, where the firm's asset value is modelled explicitly and equity is equivalent to a call option on the asset. Our extensions include the use of a more realistic option pricing model, rigorous treatment of accounting information, advanced econometric techniques for the calibration of the model to historical default events and default rate forecasts. Details of our model are summarized below.

The structural framework

In the structural model framework, the default probability of a firm is the probability that its asset value falls below a certain threshold (default boundary) within a specified period. Our model carefully extracts information on different classes of liabilities from the firm's financial reports to construct a term structure of the default boundary. We perform rigorous adjustments to the accounting information, taking into account off-balance sheet assets and liabilities such as operating leases and pension funds, which can have a significant impact on corporate default risks in certain industries. We describe the details of these adjustments in Section 3.

We use the term structure of default boundary and the market value of equity in an option pricing framework to estimate the firm's asset value and its volatility. Specifically, we model equity as a down-and-out call option on the firm's assets, as the firm may default at any time prior to its debt maturity.

The default probability depends on the distance-to-default (DD_t), which is defined as:

$$DD_t = \frac{\ln\left(\frac{V}{DP_t}\right) + \left(\mu - \frac{\sigma_V^2}{2}\right)t}{\sigma_V \sqrt{t}},$$

Where V is the market-implied asset value, σ_V is the asset volatility, and DP_t is the default point at time t. DD_t quantifies how far the asset value is from the default boundary and is a key input in our econometric model that is used to estimate the default probability.

The econometric model

Before discussing the choice of additional predictive variables, we briefly outline the econometric technique in our model. There are different ways of statistically modeling the default probability of a firm. The methodology we use is a discrete time hazard rate approach where we allow for individual nonlinear effects of the predictive variables on the default probabilities:

$$p_{i,t} = P\left(I_{i,t+1} = 1 \middle| I_{i,t} = 0, X_{i,t}\right) = \frac{1}{1 + e^{-\alpha - \beta \cdot f(X_{i,t})}}$$

The hazard rate that applies over a fixed period to firm *i* is defined through a *logit* function, which is based on exogenous firm-specific variables $X_{i,t}$. In this approach, default can occur at any time within the interval (t, t+1] with an intensity determined by the predictive variables. The model allows default probabilities to be updated at different frequency levels. A more detailed discussion of the estimation procedure is included in Appendix A.

Our research shows that the nonlinear effects of predictive variables on default probabilities are very important. For example, the probability of default decreases as the interest coverage ratio increases, but the sensitivity of this relationship rises as the coverage ratio falls. We examine these nonlinear relationships through univariate and multivariate non-parametric techniques. We ensure that the functional form of the relationship agrees with economic intuition in each case[1].

Once we have decided on our modeling approach, the quality of the model is determined by the choice of the specific predictive variables $X_{i,t}$. The most significant predictive variable is the distance-to-default (DD_t) from the structural framework described above. Even though studies have shown that DD_t provides a very useful statistic in predicting default, it is not a sufficient statistic. We find that other variables contain significant additional information. Therefore, in addition to distance-to-default, we explicitly consider other information such as a firm's size and interest coverage ratio. Tests reveal that these variables are highly significant even in the presence of distance-to-default. Finally, because of the different nature of the balance sheet information of financial institutions, we allow the predictive variables to enter the model differently for this particular industry[2]. For example, banks use leverage as part of their operating activities, so their distance-to-default and interest coverage ratio will not be comparable with industrial firms with similar default risks.

Our model is calibrated to historical default events to give baseline estimates for corporate default probabilities. We extensively studied a large sample of data that includes over 10,600 firms and 1,334 default events (our definition of default includes missed or delayed interest payments, bankruptcy filing, and distressed exchanges[3]).

Aggregate default rate forecast

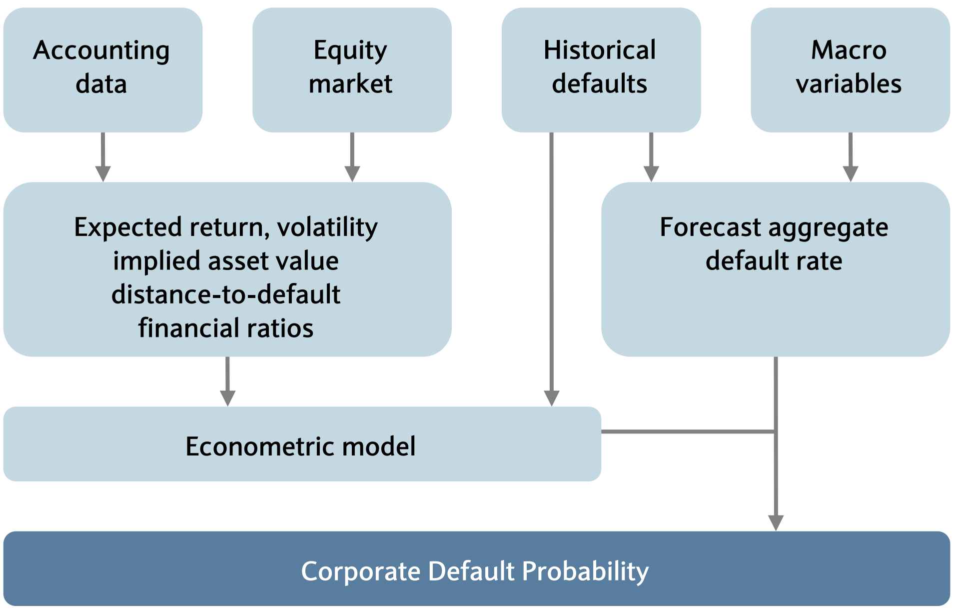
In our previous work on default prediction (see Silva and Staal (2007)), we illustrated the strong relationship between the default rate and the state of the economy. We incorporated this stylized fact by jointly predicting individual default rates and the aggregate default rate in a single model. So far, we have deliberately omitted macro-economic information from the POINT CDP model. We still stand by the importance of the business cycle in predicting defaults, but in our second generation model, we choose to separate the prediction of the relative magnitude of individual default probabilities, and the overall expected default rate in the economy. In this two-step approach, we first focus on the ability to accurately rank contemporaneous credit risks. Macro-economic variables are then used in a second step to forecast the aggregate default rate. By separating the problem of forecasting the aggregate default rate and ranking the firms on relative distress risk, we can estimate both of these

[1] A monotonic relationship between the default rate and individual explanatory variables is enforced. While nonmonotonic relationships conceivably exist (e.g. for growth-related variables), this is done to guard against overfitting while being consistent with the first order effects that economic reasoning suggests.
[2] We follow the GICS definition of "financials" sector, which includes banks, diversified financials, insurance, and real estate.
[3] See Keenan et al. (2000) for discussion on distressed exchanges.

components more cleanly, particularly in times where the overall economy, the equity market, and credit markets experience sharp movements. We discuss the aggregate default rate forecasting model in more detail in Section 4.

The final step of our methodology transforms the baseline estimates of the default risk of individual firms, so that the average probability matches our forecast for the US aggregate high-yield default rate.

Figure 1: Schematics of the POINT CDP model



Source: Barclays Capital

3. Accounting treatments

One of the unique features of the POINT CDP model is the way we reconstitute financial information of the firms from details and footnotes of their financial reports. To make a more realistic assessment of a firm's credit risk, one should also recognize assets and liabilities that are not on the balance sheet. For instance, recognizing these off-balance sheet liabilities can have considerable impact on financial leverage, which is one of the key determinants of a firm's default risk. Analyses that omit items such as operating leases and underfunded pension plans can result in significant understatement of financial leverage, and hence the default probabilities for some firms. Our study shows that these adjustments are particularly significant in the Industrials, consumer discretionary, consumer staples, and information technology sectors, where financial leverage computed without appropriate adjustments can understate the true leverage by as much as 60%.

Below are the key accounting adjustments in our model.

- Capitalization of leases: Capitalized value of off-balance sheet operating leases is added to Net Property Plant and Equipment (NPPE) and long-term debt. The implied interest component is reclassified as interest expense. This adjustment can have a significant impact on leverage and interest coverage ratio for firms in certain industries such as retail and transportation.

- Pension/OPEB: Net pension liability is recognized as long-term debt. Operating expenses are restated to include only the pension/OPEB service cost. This adjustment may cause long-term debt to increase/decrease for firms with under/overfunded pensions. In the case of overfunded pension funds, we reduce the liability by only a fraction of the surplus amount because of the excise tax imposed on early dissolution of the funds and other restrictions. This effect will be significant for firms with large defined benefit pension plans.
- LIFO/FIFO: Inventory is restated using the “first in, first out” (FIFO) rule. Then, deferred tax liabilities, common equity, and the cost of goods sold (COGS) are adjusted accordingly. The primary objective of this adjustment is to make the interest coverage ratio comparable across the firms.

Examples below provide further evidence of the importance of our accounting adjustments in estimating corporate default probabilities.

Example 1: Starbucks Corporation

We illustrate how off-balance sheet liabilities may affect the default probability of an individual firm. Consider Starbucks Corporation (SBUX), which had a BBB rating from S&P in December 2008. SBUX operates over 7,400 retail stores globally (as of 2008 fiscal year-end). Since the majority of the stores are leased rather than owned, SBUX has a significant amount of financial obligations not reported on its balance sheet. In December 2008, SBUX reported a total liability of $2.9bn with $550mn long-term debt. After making the appropriate adjustments, we estimate that SBUX’s total liability should be $7.3bn with $4.9bn long-term debt. With a market capitalization of $6.9bn, this corresponded to an increase in the market leverage from 30% to 51%. Consequently, *CDP* for SBUX with appropriate adjustments was 0.56%, compared with 0.10% without the adjustments. These were equivalent to BBB- and A ratings, respectively.

Example 2: Specialty retailers

In the previous example, we illustrated how *CDP* of individual firms can be affected by the accounting adjustments. The next example highlights the importance of these adjustments when we use *CDP* to compare default risk across the firms.

Suppose we wish to evaluate the relative default risks of firms in the Specialty Retail[4] industry by dividing them into deciles according to the ranking of their *CDP*s. Failing to recognize off-balance sheet liabilities can lead to an inaccurate and unfair comparison between the firms. For instance, home improvement retailers tend to have lower lease obligations than other specialty retailers; the average lease obligation for this sub-industry in 2008 was only 10% of the total adjusted liabilities[5,] compared with the average of 25% for the entire specialty retail industry. Consequently, home improvement retailers may inaccurately appear to be riskier than other retailers when the off-balance sheet liabilities are not recognized. For example, at the end of March 2009, Home Depot (HD) and Sherwin-Williams (SHW) moved up from the 6th to the 3rd decile and the 4th and the 2nd decile, respectively, after we made the appropriate accounting adjustments.

Apparel (softline) retailers, on the other hand, tend to have much higher lease obligations than other sub-industry. The average lease obligation for these apparel retailers in 2008 was as high as 35%. Among those with the highest percentage of lease obligations were Shoe Carnival (SCVL) and Cache (CACH), whose percentage of lease obligations were above 45% of their total adjusted liabilities. At the end of March 2009, SCVL and CACH moved down

[4] According to Standard and Poor’s Global Industry Classification Standard (GICS).
[5] Total adjusted liability excludes deferred taxes and minority interest.

from the 4th to the 7th decile and the 7th and the 9th decile, respectively, after we made the appropriate accounting adjustments. The full list of firms and their *CDP*s used in this example can be found in Appendix C.

As illustrated above, incorporating appropriate accounting adjustments can have a positive or negative effect on default probabilities. Therefore, to make a fair comparison of credit risks across the firms, it is essential that we properly account for the differences in off-balance sheet liabilities as well.

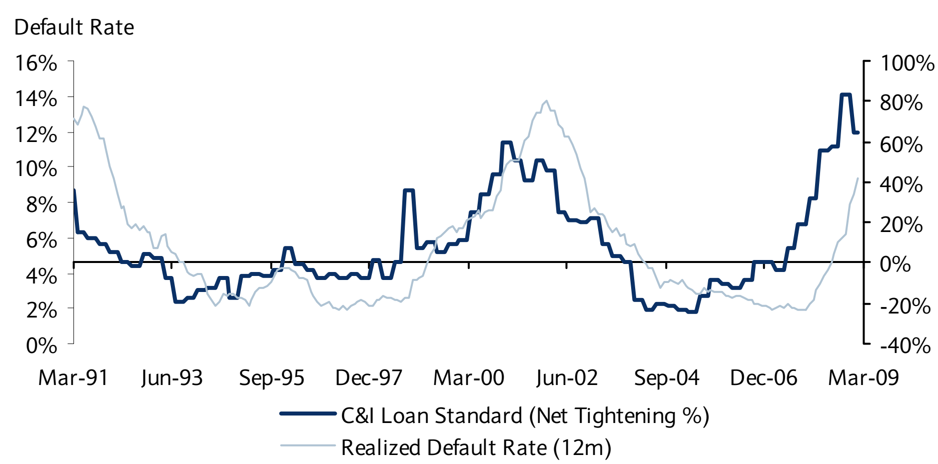
4. Aggregate default rate forecast

Our first generation default prediction models incorporate macro-economic variables into the econometric model to capture business cycle effects on the default rate. The objective was to produce default probability estimates that are unbiased and consistent with the realized default rate over the business cycles. The POINT CDP model takes a slightly different approach where we forecast the aggregate default rate separately, and use non-linear transformation to convert baseline default probabilities to *CDP*s. The separation between estimating baseline probabilities at a point in time (ranking firms) and adjusting aggregate level default rates dynamically provides a robust framework that allows us to update our aggregate forecast independently from the relative distress of individual firms.

There are several macro-economic variables that lead the aggregate default rate. For example, as discussed in Erlandsson and Rennison (2008), there is a strong relationship between loan standard tightening and aggregate high-yield default rate. Figure 2 shows the lead-lag relationship between the net percentage tightening of loan standards for commercial and industrial companies from the Federal Reserves' Senior Loan Officer Opinion Survey and the 12-month trailing default rate.

Our forecasting model employs several macro-economic variables, including the observed loan standard tightening, to predict the high-yield default rate over the next 12 months.

Figure 2: Net tightening of C&I loan standards and high-yield default rate



Source: Moody's Investors Service, Federal Reserve

5. Performance

We measure the performance of the POINT CDP model by analyzing the following characteristics:

1. Power to distinguish between healthy and distressed firms
2. Accuracy of the probability
3. Timeliness of the prediction
4. Correlation with CDS spreads

In the predictive power test, we show that our model has significant power in distinguishing healthy firms from distressed ones. The accuracy test shows that levels of our probability estimates are consistent with realized default rates in each category. The timely prediction test shows that our model provides early warnings several months prior to defaults. Finally, the correlation between probability estimates and CDS spreads shows that the POINT CDP model is consistent with the market's perception of the firms' default risks.

5.1. Predictive power test

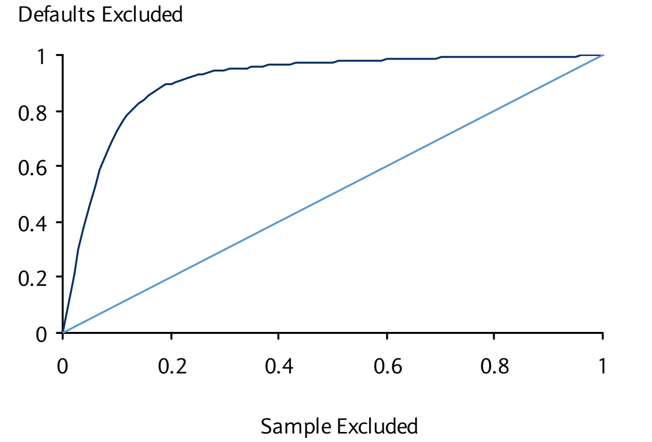
The predictive power of a default probability model is characterized by its ability to distinguish healthy firms from distressed ones. The most commonly used technique to measure predictive power is the Cumulative Accuracy Profile (CAP) and its summary statistics, the Accuracy Ratio (AR). We construct a CAP curve for the POINT CDP model as follows. First, we order the firms in our coverage from highest to lowest *CDP*. Then, as we remove them from our sample, we record the fraction of defaulted firms that have been removed, and plot it against the fraction of the entire sample that have been removed.[6] Figure 3 shows the CAP curve of the POINT CDP model. The AR is the area between the CAP curve and the 45-degree line, which represents the performance of a random model. Ideally, as we move down the list of firms in the reverse order of their *CDP*, we want to remove all the defaulted ones as quickly as possible. Therefore, a good model would have a high AR and a steep CAP curve that rises sharply from 0 to 1. The POINT CDP model has an AR of 0.81, which compares very well with other available models over the same sample of firms and time period.

5.2. Accuracy of the CDP model

The accuracy of the POINT CDP model is measured by how close predicted default rates are to actual realized default rates. To determine the level of accuracy, we divide all firm-month observations into a finite number of groups with similar values of *CDP*. For each group, we compare the realized default rate, which is the fraction of observations that defaulted within the following 12 months, with the predicted default rate, which is the group's average value of *CDP*. Figure 4 plots predicted versus realized default rates for each group. The scatter plot shows that the POINT CDP model is accurate in predicting the economy-wide default rate, as the data points do not deviate far from the 45-degree line.

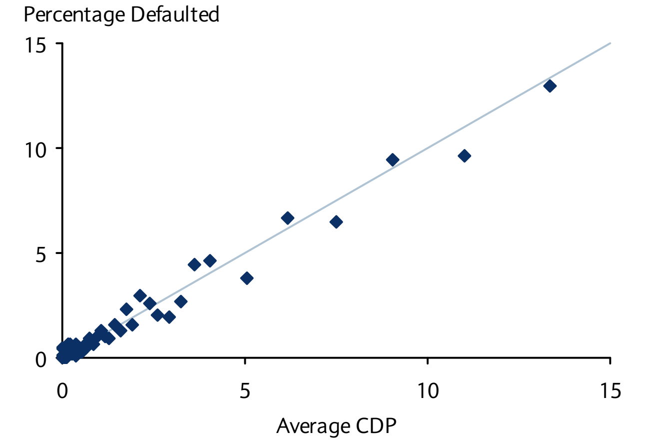
[6] See Sobehart, Keenan, Stein (2000) and Engelmann, Hayden, Tasche (2003) for discussion on Cumulative Accuracy Profile.

Figure 3: CAP test (1997 – 2008)



Source: Moody's Investors Service, Barclays Capital

Figure 4: Accuracy test (1997 – 2008)

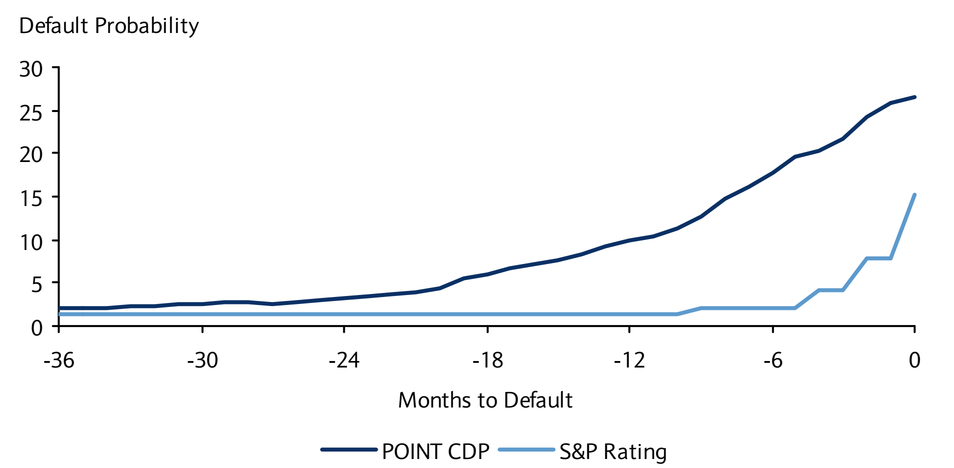


Source: Moody's Investors Service, Barclays Capital

5.3. Timely prediction

We compute the median value of *CDP* of the defaulted firms at the end of each month, for up to 36 months prior to their default dates. Figure 5 shows that the median *CDP* exceeds 6%, which is equivalent to a CCC+ rating, as early as 18 months, and 17%, which is equivalent to a CCC- rating, as early as six months prior to default. We also plot default probabilities implied by median S&P ratings over the same period leading to defaults. The median-implied rating decreases from B+ to B 9 months prior to default, and does not reach CCC+ until as little as two months prior to default. In Figure 5, we imply the default probabilities by using historical median of *CDP* for each rating category as shown in Figure 7. This shows that the POINT CDP model provides timely warnings that firms are approaching severe financial distress well ahead of rating downgrades and actual default events.

Figure 5: Median CDP default month approaches (1997 – 2008)



Source: Standard & Poor's, Barclays Capital

5.4. Correlation with single-name CDS spreads

The market's view of corporate default risk can be observed in the CDS market. The CDS spread level of a firm is driven by its default risk, as well as other factors such as liquidity and market risk premium. Since default risk is a major determinant of CDS spreads, we expect our default probability estimates to be strongly correlated with the CDS spread levels. The rank correlation between estimates from the POINT CDP model and CDS spreads ranges from 61% for utilities sector to 83% for information technology sector, and is as high as 75% across all sectors. Our default probability estimates display a very strong relationship to the level of the firms' CDS spreads.

Figure 6: Rank correlations (1997-2008)

Sector	Rank correlation
Energy	0.713
Materials	0.720
Industrials	0.736
Consumer discretionary	0.740
Consumer staples	0.711
health care	0.678
Information Technology	0.825
Telecommunication services	0.730
Utilities	0.613
All sectors	0.748

Source: Markit Partners, Barclays Capital

6. Spread-implied default probabilities

A large number of firms that issue debt (especially in the high-yield universe) have no publicly traded equity outstanding. Since our POINT CDP model extensively uses equity data as an input, the methodology discussed in the previous sections cannot be directly applied to this large universe of private firms. To address this issue, we developed an intuitive extension of the POINT CDP model to produce spread-implied default probabilities.

We start with the basic observation that default risk is a determinant of the spread at which a firm's debt instrument trades. While we can infer risk-neutral default probabilities from observed prices and a pricing model, our aim here is to construct an empirical relationship between observed spreads and actual default probabilities. In other words, we try to uncover empirically what actual default probability would lead to the observed spread. To do so, we use the risk-neutral default probabilities implicit in spreads as a source of information about actual default probabilities. This is achieved by constructing a mapping between a representative spread on a public firm's debt and its estimated *CDP*. This mapping is constructed using public, non-financial firms, and extrapolated to produce default probabilities for other firms. The assumption is that the relationship between spreads and actual default probabilities is similar for all firms. In particular, we start with the assumption that there exists a (time-varying) function $f_t(.)$ that maps the OAS of a senior unsecured bond with a time-to-maturity of one year to one-year default probabilities[7]:

[7] Of course, such a bond might not exist for a particular issuer. In that case, we construct the mapping based on the next shortest maturity available, under the assumption of a flat term structure of default probabilities.

$$p_{i,t} = f_t(OAS_{i,t})$$

Then, using the POINT CDP's default probabilities and current OAS levels for public firms, we estimate $f_t(.)$:

$$CDP_{i,t} = \hat{f}(OAS_{i,t})$$

Finally, we extrapolate the estimated function to the OAS of private firms to infer the spread-implied default probabilities:

$$\hat{p}_{i,t}^{impliedCDP} = \hat{f}(OAS_{i,t}^{private})$$

The calibration to public firm probabilities is updated monthly to capture the possibility of the relationship between spreads and default probabilities changing (eg, due to time-varying risk premia). As suggested by the relatively high correlation between our default probabilities and CDS spreads, we find a good statistical fit between spreads and default probabilities. We use this relationship to provide default probabilities for default risk calculations on portfolios containing firms for which firm-specific equity or accounting information is not available. Although it may seem natural to extract default information from corporate spreads, it is not a generally applicable solution because of the relative lack of liquidity of corporate spread instruments for a large cross-section of firms.

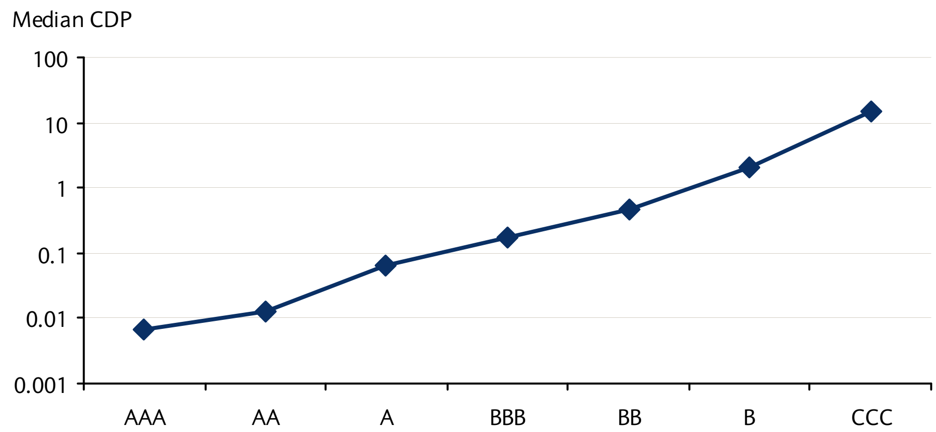
7. POINT CDP-implied ratings

Many investors and credit analysts may not be familiar with default probability as a measure of credit risks. For this reason, we translate *CDP* into a more familiar metric of credit ratings.

The CDP-implied ratings are inferred from a lookup table that we construct monthly by establishing a relationship between the default probabilities and the issuers credit ratings. Historically, we observe a log-linear relationship between the median *CDP* within each rating category and the rating itself. We use this information to compute the midpoints for each of the broad rating categories, and perform geometric interpolation to get the midpoints for the fine rating categories and the breakpoints between them.

Figure 8 compares the distribution of firms across the rating categories, as rated by S&P and our implied-rating model at the end of March 2009. We find that our implied-rating model disperses the firms across different ratings more so than S&P. As high as 80% of the firms were rated between BBB to B by S&P, and only 5% were rated CCC or below. Our POINT-CDP model provides more differentiability among firms even in the credit rating metric. Our model assigned ratings between BBB and B to only 64% of the firms, and CCC or below to 12% of the firms.

Figure 7: Log-linear relationship between median CDP and ratings (1997 – 2008)



Source: Standard & Poor's, Barclays Capital

Figure 8: Distribution of ratings (as of March 31, 2009)

Rating	S&P	CDP-Implied
AAA	1%	3%
AA	2%	6%
A	13%	14%
BBB	29%	20%
BB	27%	28%
B	23%	16%
CCC	5%	12%

Source: Standard & Poor's, Barclays Capital

8. Conclusion

In this paper, we introduce the POINT Corporate Default Probability (POINT CDP) model. This is our second generation default probability model. The model incorporates information from an option-based structural model with other firm-specific information, as well as indicators of the market and macro-economic environment. The model produces forward-looking default probabilities that provide good separation between good and bad credits, timely signals of impending defaults, and accurately measures the average default level in the relevant universe over the macroeconomic credit cycle.

The POINT CDP model is integrated into the risk models of the Barclays Capital POINT portfolio platform. The default probabilities for the liquid publicly traded US equity universe are updated daily. Coverage for private firms is provided by using spread information on credit instruments to infer default probabilities that are calibrated to the public firm model.

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A. Probability Estimation Methodology

We briefly discuss the technical details of the estimation of a simplified version of the discrete time hazard rate model described in this article[8]. This simplified version differs slightly in the definition of the default indicator variable used above. This discrepancy arises because of the difference in the prediction horizon and the sampling frequency of the data on which the estimation process is based. We take a fully parametric maximum likelihood approach. A bootstrap procedure was used to verify the asymptotic results.

Suppose we have a sample of sequential observations $I_{i,t}$ on firms i =1,…, n, for each of the times t_i,…,T_i that it enters the sample. The dependent variable $I_{i,Ti}$ equals one if the observation corresponds to the last observation for a firm that defaults within the time span covered by the sample. A firm that does not default in the time span covered by the sample or a firm that exits the sample for other reasons contributes observations equal to zero.

We define the discrete time conditional hazard rate process as:

$$P_{i,t} = P\left(I_{i,t+1} = 1 \mid I_{i,t} = 0, \left\{X_{i,t}\right\}\right)$$

We assume that default events across firms are independent in any single period given the information sets generated by the predictive variables $\{X_{i,t}\}$. This conditional independence is necessary for tractability and is a standard assumption in hazard rate modeling.

Conditioning on previous events in the time series of observations on a firm, we can write:

$$P\left(I_{i,T_i} = 1\right) = P_{i,T_i-1} \prod_{l=t_i-1}^{T_i-2} \left(1 - P_{i,l}\right)$$

$$P\left(I_{i,T_1} = 0\right) = \prod_{l=t_i-1}^{T_i-1} \left(1 - P_{i,l}\right)$$

And the likelihood of observing the data for an individual firm is:

$$L\left(\left\{I_{i,t}\right\}_{t=t_i}^{T_i} \mid \left\{X_{i,t}\right\}\right) = \left(P_{i,T_i-1} \prod_{l=t_i-1}^{T_i-2} \left(1 - P_{i,l}\right)\right)^{I_{T_i}} \left(\prod_{l=t_i-1}^{T_i-1} \left(1 - P_{i,l}\right)\right)^{1-I_{T_i}}$$

Using the conditional independence across firms it is straightforward to derive the log-likelihood function:

$$\log L\left(\left\{\left(I_{i,t}\right)_{t=t_i}^{T_i}\right\}_{i=1}^{n} \mid \left\{X_{i,t}\right\}\right) = \sum_{i=1}^{n} \sum_{t=t_i}^{T_i-1} \left(I_{i,t+1} - I_{i,t}\right) \log\left(\frac{P_{i,t}}{1 - P_{i,t}}\right) + \sum_{i=1}^{n} \sum_{t=t_i-1}^{T_i-1} \log\left(1 - P_{i,t}\right)$$

This log-likelihood function is identical to the log-likelihood for a dichotomous dependent variable regression model, which implies that all the standard options for maximization and parameter estimation are available. We use a logistic model for the discrete time hazard rate:

$$P_{i,t} = \frac{1}{1 + e^{-\alpha - \beta f(X_{i,t})}} .$$

[8] See Shumway (2001) and Chava and Jarrow (2004) for some additional details relating to this statistical methodology.

B. Lookup table for implied credit ratings

Below is the lookup table used to determine implied credit ratings. It shows a mapping between *CDP* and S&P rating as of March 31, 2009.

	CDP (%)	
Rating	Lower bound	Upper bound
AAA	0.0000	0.0093
AA+	0.0093	0.0152
AA	0.0152	0.0235
AA-	0.0235	0.0405
A+	0.0405	0.0700
A	0.0700	0.1140
A-	0.1140	0.1754
BBB+	0.1754	0.2699
BBB	0.2699	0.4142
BBB-	0.4142	0.6338
BB+	0.6338	0.9700
BB	0.9700	1.6221
BB-	1.6221	2.9643
B+	2.9643	5.4171
B	5.4171	9.4593
B-	9.4593	15.7837
CCC+	15.7837	26.3366
CCC	26.3366	34.7156
CCC-	34.7156	36.1498
CC	36.1498	38.4130
C	38.4130	100.0000

Source: Standard & Poor's, Barclays Capital

C. CDP and Ranking of Specialty Retailers

The table below shows *CDPs* and corresponding deciles for firms in the Specialty Retail industry used in Example 2 of Section 3.

Without adjustments		With adjustments		Decile	Without adjustments		With adjustments		Decile
Ticker	CDP	Ticker	CDP		Ticker	CDP	Ticker	CDP	
AEO	0.0001	ROST	0.0019	1	HD	0.0315	HGG	0.8900	6
ARO	0.0001	URBN	0.0029	1	LOW	0.0340	JCG	0.9100	6
BBBY	0.0001	CTR	0.0050	1	DKS	0.0499	RCII	0.9379	6
BEBE	0.0001	ARO	0.0063	1	GME	0.0592	BKS	1.0189	6
BKE	0.0001	HOTT	0.0074	1	FL	0.0703	CAB	1.0192	6
CTR	0.0001	AEO	0.0109	1	CRMT	0.0879	FL	1.0298	6
HOTT	0.0001	TIF	0.0109	1	DSW	0.0934	DBRN	1.0757	6
ROST	0.0001	BEBE	0.0121	1	HGG	0.1057	LTD	1.2505	6
URBN	0.0001	GYMB	0.0150	1	PLCE	0.1072	MW	1.2631	6
ANF	0.0001	ANF	0.0185	2	BBY	0.1215	AN	1.2786	7
CBK	0.0001	SHW	0.0212	2	CACH	0.1333	WTSLA	1.5078	7
CHIC	0.0001	SYMS	0.0245	2	ULTA	0.1558	MDS	1.6834	7
CHS	0.0001	ZUMZ	0.0260	2	RCII	0.1779	RSH	1.7698	7
CONN	0.0001	CONN	0.0263	2	SSI	0.1799	SSI	1.7779	7
GYMB	0.0001	BKE	0.0298	2	NWY	0.2272	SCVL	2.1130	7
LL	0.0001	CHS	0.0334	2	HVT	0.2432	ULTA	2.2160	7
SYMS	0.0001	RSC	0.0389	2	MDS	0.2512	DKS	2.2390	7
TIF	0.0001	SYX	0.0389	2	RSH	0.2871	JAS	2.2574	7
ZUMZ	0.0001	FINL	0.0395	2	GCO	0.3526	PLCE	2.2809	7
BBW	0.0001	AZO	0.0486	3	JAS	0.5216	PAG	2.3680	8
FINL	0.0001	CHIC	0.0573	3	CAB	0.5820	SBH	2.7368	8
GPS	0.0001	CBK	0.0649	3	LTD	0.6153	BGFV	3.4664	8
PSUN	0.0001	LL	0.0661	3	BGFV	0.6913	DSW	3.4747	8
RSC	0.0001	MNRO	0.0719	3	SIG	1.2820	GCO	3.5747	8
SYX	0.0001	BBBY	0.0760	3	AN	1.4213	SIG	3.7468	8
JOSB	0.0003	TJX	0.0801	3	ANN	2.2026	PSS	4.5571	8
TJX	0.0006	BBW	0.0819	3	SBH	2.8620	NWY	4.9351	8
HIBB	0.0007	HD	0.0873	3	PAG	2.9236	GPI	5.0860	8
RNT	0.0015	LOW	0.1054	4	BWS	3.8722	ANN	5.6748	8
MNRO	0.0017	RNT	0.1258	4	TWB	4.4545	ABG	8.5978	9
WSM	0.0019	PSUN	0.1359	4	PSS	4.9369	PBY	10.4292	9
GES	0.0022	GPS	0.1575	4	GPI	8.7042	CACH	10.7304	9
CTRN	0.0027	SPLS	0.1577	4	ABG	9.0403	BWS	11.0492	9
SHW	0.0046	ORLY	0.1602	4	SAH	19.2160	OMX	19.3826	9
ORLY	0.0047	AAP	0.1852	4	CMRG	19.7508	TWB	20.5458	9
SCVL	0.0056	PETM	0.2660	4	BBI	22.1325	SMRT	20.6565	9
AZO	0.0069	GES	0.2793	4	OMX	22.2166	TLB	24.2258	9
BKS	0.0100	KMX	0.2856	4	PBY	22.9610	SAH	24.5410	9
TSCO	0.0118	HVT	0.4080	5	SMRT	23.7121	CHRS	24.5744	10
DBRN	0.0124	TSCO	0.4315	5	CWTR	25.7335	CMRG	24.7449	10
KMX	0.0134	GME	0.4420	5	CHRS	40.0000	CWTR	25.1010	10
AAP	0.0136	HIBB	0.4484	5	TLB	40.0000	ODP	25.8049	10
MW	0.0170	JOSB	0.5527	5	HZO	40.0000	BBI	28.9528	10
WTSLA	0.0172	CRMT	0.5684	5	ODP	40.0000	BGP	40.0000	10
SPLS	0.0208	BBY	0.5936	5	ZLC	40.0000	PIR	40.0000	10
JCG	0.0284	WSM	0.7440	5	PIR	40.0000	HZO	40.0000	10
PETM	0.0287	CTRN	0.7470	5	BGP	40.0000	ZLC	40.0000	10

*As of March 31, 2009. Source: Barclays Capital

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