Final Year Honours Project

AUTOMATED DESIGN OPTIMISATION OF 3D-PRINTED MULTIMATERIAL STRUCTURES

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Abstract

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1 Introduction

At the current stage of additive manufacturing technologies, the ability to 3D-print multi-material objects has now been realised. Fabrication of complex, heterogeneous structures with minimal manual effort through these means provides an impetus for a shift towards a higher degree of material-driven design to fulfill functional requirements. Furthermore, the usage of spatially varying material properties within a structure can give rise to unique physical behaviour that would otherwise not be encountered by their homogeneous counterparts [8]. A wider array of functional capabilities are thereby offered, when exploiting the use of these structures. With the advent of multi-material additive manufacturing, an opportunity is presented in the fabrication of multi-material structures. Without modifications to geoemetric shape and without concerns for manufacturability - the optimisation of functional performance for a design is less constrained and can be explored more deeply.

1.1 Project Scope

The functional performance of interest that is investigated in the presented work is focused on the vibration of a fixed-free rectangular cantilever structure. More specifically, we seek to tune the natural frequencies of the structure by only modifying the internal material allocation using the materials commercially available in multi-material additive manufacturing. Uniformly distributed natural frequencies of the structure is selected as a benchmark case for a target natural frequency distribution.

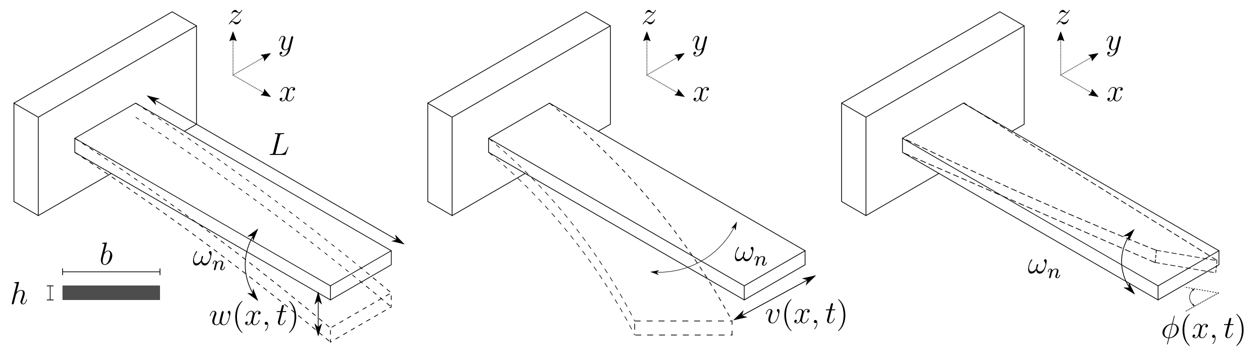
A finite element analysis (FEA) solver will be used to evaluate the natural frequencies of the structure. The multi-material cantilever is modelled using a cartesian voxel structure, in which each voxel is assigned a material. Material selection is constrained to a choice between two materials to allow for a binary decision making problem structure. The two materials chosen will have substantially different material properties, such that a wide range of possible vibrational responses can be obtained. An emphasis is placed on obtaining solutions in a computationally efficient manner. This is done through the use of an open-source eigenvalue solver for parallelisable evaluations, alongside Metaheuristic algorithms for computationally affordable methods to search the solution space.

Once sufficient numerical results have been obtained, sample specimens will be printed and experimentally validated through vibrational measurement methods such as laser doppler vibrometry. Project success will be measured in two stages. First, it will be gauged based on the efficacy of the developed optimisation framework to generate multi-material cantilever solutions that meet prescribed natural frequency response requirements within the bounds of numerical and material parameter uncertainty. The second stage will involve validating the numerical results against physical testing, and ensuring that acceptable levels of agreement are achieved.

1.2 Background

1.2.1 Structural Vibration

The elementary vibrational response of cantilever structures are typically categorised as either flexural bending modes or torsional modes. Bending modes can be further categorised as in-plane bending and out-of-plane bending (figures 1a and 1b). For the simple case of a linearly elastic, homogeneous and isotropic material with a constant cross-section - flexural bending vibrations are



(a) In-plane bending. **(b)** Out-of-plane bending. **(c)** Torsional mode.

Figure 1: Elementary vibrational responses of a cantilever. Geoemtric definitions are given in **(a)**.

governed by the Euler-Bernoulli beam equation [1]:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w(x,t)}{\partial x^2}\right) + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{1}$$

Where E is the elastic modulus, I is the beam cross-section's moment of inertia, ρ its density, A its cross-sectional area and $w(x,t)$ the transverse deflection. Fixed-free boundary conditions given by:

$$\left[w(x,t) = \frac{\partial w(x,t)}{\partial x}\right]_{x=0} = \left[\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^3 w(x,t)}{\partial x^3}\right]_{x=L} = 0 \tag{2}$$

By pursuing a harmonic analysis [6], closed-form expressions for the natural frequencies can be derived as:

$$\omega_n = \frac{\alpha_n^2}{L^2}\sqrt{\frac{EI}{\rho A}}, \qquad n = 1, 2, 3, \ldots \tag{3}$$

Where α_n are roots to the equation:

$$\cos(\alpha_n L)\cosh(\alpha_n L) = -1 \tag{4}$$

The above expressions can also be used to evaluate the out-of-plane bending responses, by correcting I to be the appropriate expression for the out-of-plane co-ordinates. It is noted however, that these expressions are valid under the assumption that the beam is long and slender (i.e. $L >> h$ for in-plane and $L >> b$ for out-of-plane).

Torsional modes are charaterised by deflection angles, ϕ as opposed to spatial deflections. The motion is governed by:

$$GK\frac{\partial^2 \phi(x,t)}{\partial x^2} - \rho I_p\frac{\partial^2 \phi(x,t)}{\partial t^2} = 0 \tag{5}$$

Where G is the shear modulus, I_p is the polar moment of inertia and K is a geoemtric function of the cross-section [2]. Warping of the cross-section along the cantilever's means it is difficult to treat equation (5) analytically for non-circular cross-sections.

It is also possible to observe coupled responses between these elementary vibrational modes. For instance, when the center of mass does not lie on the axis of rotation, torsional and flexural natural frequencies can couple [1].

1.2.2 Finite Element Analysis

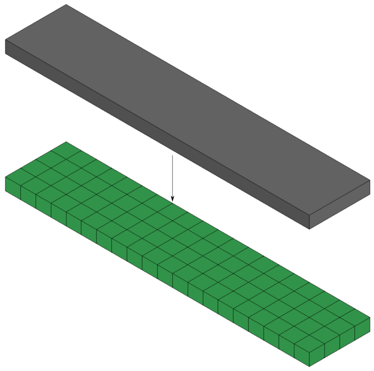


Figure 2: Illustration of a cantilever geometry (top) discretised into mesh elements (bottom).

When analytical treatment of governing equations is difficult or impossible; approximate, numerical methods can be used as an alternative approach. Finite element analysis (FEA) solvers are typically the most common class of numerical solvers used in structural problems. In such formulations, the continuum-scale differential equations are discretised into a discrete, linear system of equations [3]. Physically, this corresponds to discretising the continuous structural domain into a finite, discrete domain or *mesh* (see figure 2).Solutions to the natural frequencies of the structure are then computed as the eigenvalues of the linear system [5]. For a general 3D elastic structural problem, the equilibrium equation is given by [5]:

$$-\nabla \cdot \sigma = -\rho \frac{\partial^2 \vec{d}}{\partial t^2} \tag{6}$$

Where σ is the stress tensor and $\vec{d}$ is the displacement vector. Assuming a harmonic motion with a frequency ω defined by an eigenvector $\vec{d}$, equation (6) becomes:

$$-\nabla \cdot \sigma(\vec{d}) = \omega^2 \rho \vec{d} \tag{7}$$

In discretised form, this becomes:

$$\mathbf{K}\mathbf{u} = \omega^2 \mathbf{M}\mathbf{u} \tag{8}$$

Where $\mathbf{K}$ is called the stiffness matrix, $\mathbf{M}$ is the mass matrix and $\mathbf{u}$ is a vector representing discrete nodal values of $\vec{d}$. Equation (8) is known as the generalised eigenproblem [5]. The application of this generalised theory is advantageous in that it is possible to capture all types of modes by only discretising a single governing equation for generalised linear elastic problems in (6) (NEEDS CITATION).

1.2.3 Optimisation - Evolutionary Algorithms

TODO

1.3 Report Structure

TODO

2 Computational Modelling

This section outlines the computational model used for the evaluation of the natural frequencies of the structure. Building upon the work conducted by Waddington in previous years, usage of the commercial FEA package, Abaqus for evaluating natural frequencies was bottlenecked in time by software startup overhead. Limitations in licensing availability also meant that simulations would need to be run in an unparallelised, sequential manner. These issues have been addressed in the work presented using Elmer Multi-Physics - an open-source and lightweight FEA solver package. This allows multiple instances of the solver to be run concurrently, suited to the parallelisability of evolutionary algorithms.

The light weight infrastructure of Elmer is largely attributed to its lack of pre and post-processing utilities, where a large emphasis is instead placed on its solver capabilities [4]. The open-source Python platform Salome has been used as the pre-processing utility for geometry and mesh generation.

2.1 Computational Domain

A representative geoemetry of a typical cantilever structure is decided on *a priori* to computational modelling, and is shown in figure REF. Selection of a representative geoemetry to use for optimisation is relatively unconstrained, as there are only a few key considerations. The beam is chosen to be relatively slender such that simple beam theory can be applied for solver verification. The dimensions have also been suitably chosen such that they will fit within commercial 3D-printer printing beds.

The geometry is meshed with a uniform, hexahedral mesh grid, using the Salome meshing platform. The mesh is then exported into a format readable by Elmer for solving. To ensure a computationally efficient workflow, a single mesh grid is re-used for each Elmer solver evaluation in the optimisation procedure. Bypassing the need to recompute a mesh each iteration in the optimisation algorithm significantly reduces overhead time, and focuses the computation time towards evaluation of the natural frequencies. The process of mesh grid selection has been investigated by conducting preliminary verification studies on homogeneous beams, and comparing solve times. The results from these preliminary investigations are given in figure REF. something something parameter space n=6

- Plots to include: In-plane bending convergence plots for n.d. frequency, CPU and wall clock time for different grids

2.2 Multi-Material Structure Computation

- To accomodate a multi-material structure, we rely on using a voxelised geometry structure. This corresponds to slicing the geometry into equi-sized, cartesian blocks. Upon partitioning the geometry, the mesh is then computed.
- This mesh is then exported into a format native to Elmer.
- Elmer computes the solution through an input case file including instructions and material/-body definitions. In this file, we define the material allocations of each voxel. Generation of this case file is handled through Python scripting; where a binary vector representing a material allocation array is inputted to assign materials.

- Noteworthy and important, is that a voxel is thought of as a parcel of mesh elements. This circumvents the need to recompute the mesh after each evaluation of eigenvalues, allowing for continual re-use of the same mesh. This methodology has partly addressed the needs for a lightweight workflow, as the overhead time in case file setup is negligible. We verify this by comparing clock times and CPU times for a varying number of voxels.
- Investigations on scaling up geoemetry - natural frequencies scaled inversely with scale factor
- Plot requirements: nVoxels vs solver time

2.3 Optimisation Procedure

- A high-level description of the optimisation workflow. For a more complete description of precise implementation, see Ben's complementary report.
- GA selected as the algorithm to use: facilitates binary problem structure, metaheuristic method suited to problem with large search space and severely parallelisable algorithm.
- Objective function is given by ... Something about R^2 value and also optionally fitting a prescribed f_1 and Δf
- Parallelisation is done through running multiple instances of Elmer
- Something something generation number, population count, threshold residual error, termination time, final population of solutions

2.4 Verification

2.4.1 Mesh Sensitivity Analysis

- Mesh sensitivity analysis is conducted first using homogeneous samples, so that there exist known analytical solutions.
- Mesh sensitivity analysis is also conducted on the multi-material solutions, and estimates for numerical error are calculated through Richardson extrapolation. The methodology followed for this procedure is done as laid out by Roache. The method is reproduced in a more condensed form in the appendix.

3 Physical Testing

- This section outlines the procedure followed for experimental validation of the obtained numerical results
- Comes in multiple stages: selection of materials; material parameter estimation through homogeneous sample testing and finally multi-material beam testing. "Testing" comprised of measuring the physical vibrational response of the printed structures.

3.1 Specimen Printing

- The Prusa MKII-S MM has been used as the commercially available, MM 3D-printer. This is a filament-based printer, utilising FDM technologies to deposit material layer-by-layer to fabricate the full 3D structure. Separate materials are stored in separate stools, and are fed through to the nozzle of the printer by separate extruders. Heating of the nozzle causes the filament to begin melting through the nozzle, resulting in the depositing of material onto the printing bed.

- The Slic3r GUI is used to generate and export the 3D-printing job files as .gcode files. Gcode contains instructions for pathing, temperature, speed, etc like with CNC machines

- Two materials with substantially different material properties have been selected. Flex - a thermoplastic polyurethane (TPU) based elastomer and steelfill - a stiffer PLA plastic infused with steel particles.

- Ability to change layer as fibre orientations - opportunity to quantify and identify siginificance of anisotropy

- Key parameters to tune: printing bed temperature, volumetric flowrate, nozzle temperature and z-axis (nozzle) height from heatbed. Fine tuning required as main issues posed are: stringyness due to high temperature; delamination due to poorly printed layers, surface roughness due to poorly calibrated z-axis height.

- First, printing of homogeneous samples is conducted to obtain estimates for material parameters. 6 of each material, half of which use $\pm$ 45 and other 0/90 degree orientation angles to identify any significant effects of anisotropy.

- Moved onto multi-material prints. Five MM samples were selected to be tested based on good quality solutions as per the objective function defined. Conversion to separate stl geometries was necessary. Geoemetry generation script was once again done using Salome. Modelled as a host matrix of steelfill, with flex inclusions.

3.2 Validation

3.2.1 Experimental Testing

- Testing conducted using LDV; non-contact vibration methods by tracking the vibration of a surface.

- Specimens are tightly clamped and are excitations are applied by plucking. Data and responses are grouped by excitation methods; ooplane inplane and torsional.

- Signal is passed through a USB-audio interface pre-amp, also serving as an ADC.

- The digital signal is then processed in real-time using REW.

- REW computes spectral estimates through periodogram methods, which are extensions of FFT methods. When applying excitations to the structure, the sharp peaks in the spectral estimate will correspond to the characteristic vibrational responses of the structure.

- Welch's periodogram method is used with a Blackman-Harris-4 windowing function, as this was found to be the best method in isolating peaks from background noise.

- After measurements are collected for each sample, the spectral estimates are then averaged, grouping averages by excitation method. This data is then compared against numerical results; where material parameter uncertainty has been propagated into the numerical results.

3.3 Material Parameter Uncertainty

- Estimates for material parameters are done using the printed homogeneous samples
- ρ is estimated through physically measuring specimens and weighing - done in tandem with general quality control.
- E is estimated by fitting it to f_1 responses of homogeneous tests
- Uncertainty and quality control tables provided in appendix

4 Results and Discussion

4.1 Homogeneous Sample Testing

- Satisfactory responses obtained for steelfill - relatively stiff so easier to apply soft impulses
- Flex - difficult to do small perturbations due to low stiffness. Poor laser dispersion characteristics also meant reflective tape was necessary.
- Quality of measurements between the two materials is reflected in the peaks observed. Flex peaks less sharp than steelfill
- ± 45 degree and 0/90 degree samples showed very little difference in measured f_1 response, indicating perhaps that effects of anisotropy were negligible. Small variations in measured f_1 also resulted in relatively small uncertainties for E as a consequence

4.2 Voxel Beam Testing

- Results presented grouped by excitation method, with material parameter uncertainties as error bars
- Out of plane excitation easiest to obtain peaks consistently. This was due to higher flexural stiffness in the out-of-plane direction, so easier to obtain linear elastic excitations
- Generally very difficult to excite higher order modes reliably - paricularly mode number 6
- Can discuss level of agreement for each mode more precisely
- Ambiguity associated with peaks - can be harmonic responses or apparent frequency from beats
- Discrepancy may be associated with imperfections in fabrication
- More reliable excitation methods can be investigated - shaker table, etc

4.3 Geometric Scaling

- Work done to generalise results. Numerical simulation results showed that natural frequencies of voxelised geoemetry scaled inversely with the scale factor.
- Means that uniformity is preserved by maintaing the voxel allocations; scaling the dimensions simply scales the natural frequencies
- Only numerical results presented, has not been validated

5 Future Work

- Investigating alternatives for more reliable excitation methods. Particularly for higher order modes
- Ability to produce high quality prints consistently. Where automation has replaced ad-hoc engineering decisions in vibrational tuning, the ad-hoc process now turns to the tuning and calibration of working the 3D-printer
- Difficult to say whether anisotropy is an issue for the voxel beams
- Can obtain more better material parameter estimates for flexural stiffness with 3 point bending tests
- Torsional mode estimates not quite right - as shear modulus was not measured (because Poissons ratio was not measured). Work can be done in this regard
- Steelfill very dificult to handle due to susceptibility to brittle fracture while printing. Many other alternatives that can be investigated instead

6 Conclusions

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Appendix A

- Complete set of mesh convergence tests + methodology reproduced after Roache
- Quality control tables and corresponding supplementary graphs