Task 3

P I)

$$\begin{aligned} \text{minimize} \quad & Cx \\ \text{subject to} \quad & Ax = b \\ & x \geqq 0 \end{aligned}$$

where:

$$C = \begin{bmatrix} -1 & -2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

P II) Basis $\mathcal{B}_1 = \{2, 5, 6\}$

P III) (1) Reduced cost matrix and R (via MATLAB):

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

(2)

$$\begin{aligned} \text{maximize} \quad & \lambda_{\min} \\ \text{subject to} \quad & \lambda^T r_j = 0 \\ & \lambda^T R \geqq 0 \\ & \lambda_{\min} \leq \lambda_i \quad \forall \lambda_i \in \lambda \\ & \lambda, \lambda_{\min} > 0 \end{aligned}$$

This ensures a direction of travel that maintains optimality and is in the direction of an alternative optimum.

(3) Columns r_1, r_2 are eligible to enter the basis. r_3 returns an infeasible solution. This corresponds to x_1, x_3 being efficient entering variables.

(4) Feasible bases and solutions (via MATLAB):

$$\mathcal{B}_1 = \{2, 5, 6\}$$
$$\bar{x}_1 = [0, 1, 0, 0, 1, 5]^T$$

$$\mathcal{B}_2 = \{1, 5, 6\}$$
$$\bar{x}_2 = [1, 0, 0, 0, 2, 3]^T$$

$$\mathcal{B}_3 = \{2, 3, 5\}$$
$$\bar{x}_3 = [0, 1, 5, 0, 1, 0]^T$$