Question 1

Ordering: shortest weighted processing time first.

Proof:
Let $i(1)$ be the first job processed, $i(2)$ the second etc
With shortest weighted processing time we have:

$$w_1 p_{i(1)} \leq w_2 p_{i(2)} \leq \cdots \leq w_n p_{i(n)}$$

Now consider some order with

$$w_k p_{i(k)} \geq w_{k+1} p_{i(k+1)} \tag{1}$$

Let job $i(k-1)$ be completed at time t, then:

$$\begin{aligned}
C_{i(k)} &= t + p_{i(k)} \\
C_{i(k+1)} &= t + p_{i(k)} + p_{i(k+1)} \\
\sum_{i}^{n} w_i C_i &= \sum_{\substack{j=1 \\ j \neq k, j \neq k+1}}^{n} w_j C_{i(j)} + w_k C_{i(k)} + w_{k+1} C_{i(k+1)}
\end{aligned}$$

The objective function becomes:

$$z_1 = \sum_{\substack{j=1 \\ j \neq k, j \neq k+1}}^{n} w_j C_{i(j)} + w_k(t + p_{i(k)}) + w_{k+1}(t + p_{i(k)} + p_{i(k+1)})$$

Now swap job order of $i(k), i(k+1)$:

$$z_2 = \sum_{\substack{j=1 \\ j \neq k, j \neq k+1}}^{n} w_j C_{i(j)} + w_{k+1}(t + p_{i(k+1)}) + w_k(t + p_{i(k)} + p_{i(k+1)})$$

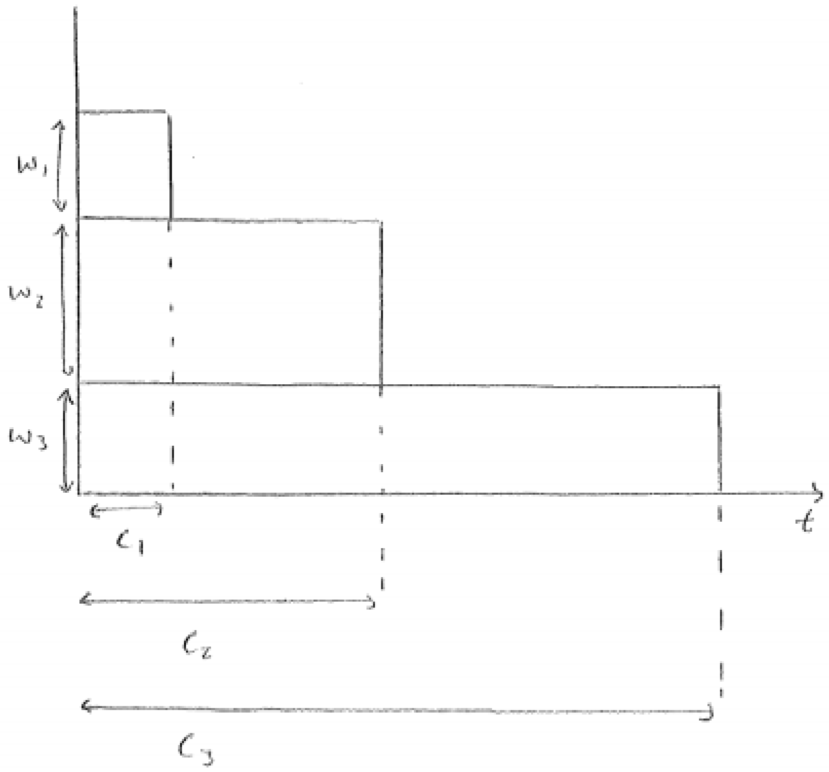
The change in objective function becomes:

$$\begin{aligned}
z_2 - z_1 &= w_{k+1}(t + p_{i(k+1)}) + w_k(t + p_{i(k)} + p_{i(k+1)}) - w_k(t + p_{i(k)}) - w_{k+1}(t + p_{i(k)} + p_{i(k+1)}) \\
&= w_k p_{i(k+1)} - w_{k+1} p_{i(k)} \\
&= w_{k+1} p_{i(k+1)} \left[\frac{w_k}{w_{k+1}} - \frac{p_{i(k)}}{p_{i(k+1)}} \right]
\end{aligned}$$

From (1), knowing weights and processing times are positive, we can state:

$$\frac{p_{i(k+1)}}{p_{i(k)}} \geq \frac{w_k}{w_{k+1}}$$

Therefore $z_2 - z_1 \leq 0$, meaning switching to shortest weighted processing time order can only improve or maintain the solution. This proves shortest weighted processing time order is optimal.

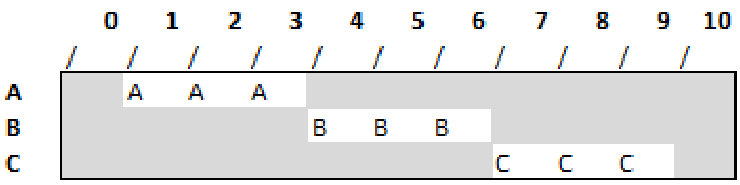

w1
w2
w3
t
c1
c2
c3

Question 2

a)

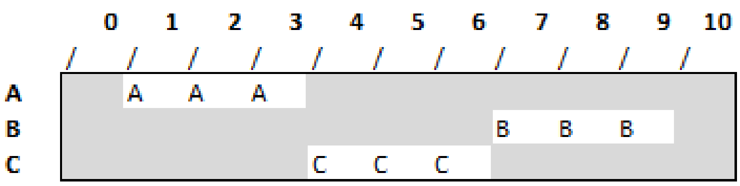
Jobs	A	B	C
p_j	3	3	3
d_j	0	0	0
r_j	1	2	3

Optimal solution via rule



L_max = 10

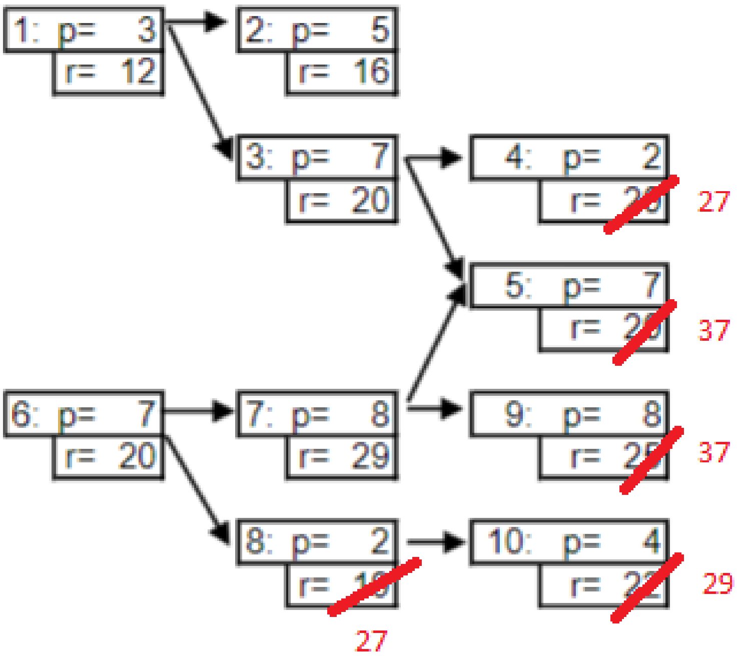
Optimal solution alternative



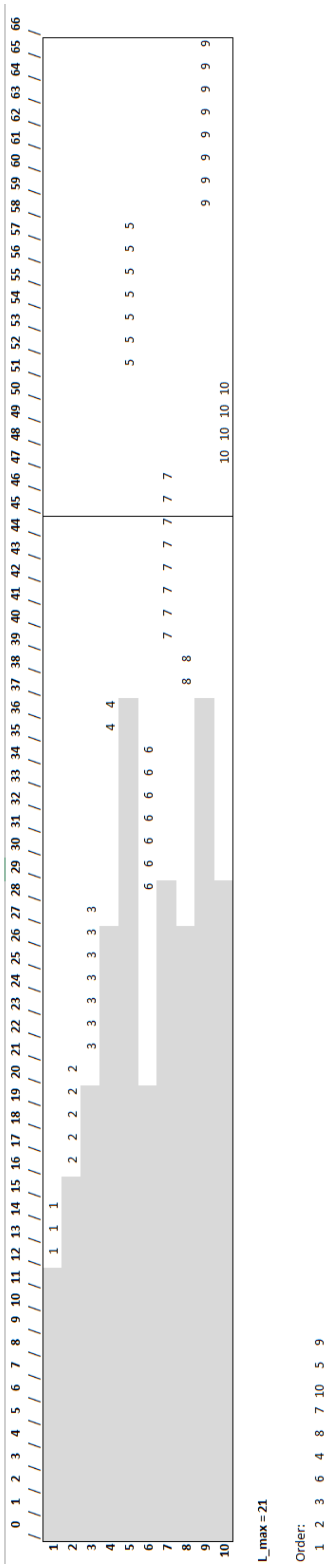
L_max = 10

b) $1/r_j/C_{\max}$

Question 3



Maximum lateness = 21
Order: 1, 2, 3, 6, 4, 8, 7, 10, 5, 9


L_max = 21
Order:
1 2 3 6 4 8 7 10 5 9