



Schema Refinement and Normal Forms

Chapter 19



The Evils of Redundancy

- ❖ *Redundancy* is at the root of several problems associated with relational schemas:
 - redundant storage, insert/delete/update anomalies
- ❖ Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.
- ❖ Main refinement technique: *decomposition* (replacing ABCD with, say, AB and BCD, or ACD and ABD).
- ❖ Decomposition should be used judiciously:
 - Is there reason to decompose a relation?
 - What problems (if any) does the decomposition cause?

Functional Dependencies (FDs)



- ❖ A functional dependency $X \rightarrow Y$ holds over relation R if, for every allowable instance r of R:
 - $t1 \in r, t2 \in r, \pi_X(t1) = \pi_X(t2)$ implies $\pi_Y(t1) = \pi_Y(t2)$
 - i.e., given two tuples in r , if the X values agree, then the Y values must also agree. (X and Y are *sets* of attributes.)
- ❖ An FD is a statement about *all* allowable relations.
 - Must be identified based on semantics of application.
 - Given some allowable instance $r1$ of R, we can check if it violates some FD f , but we cannot tell if f holds over R!
- ❖ K is a candidate key for R means that $K \rightarrow R$
 - However, $K \rightarrow R$ does not require K to be *minimal*!

Example: Constraints on Entity Set



- ❖ Consider relation obtained from Hourly_Emps:
 - Hourly_Emps (ssn, name, lot, rating, hrly_wages, hrs_worked)
- ❖ Notation: We will denote this relation schema by listing the attributes: **SNLRWH**
 - This is really the *set* of attributes {S,N,L,R,W,H}.
 - Sometimes, we will refer to all attributes of a relation by using the relation name. (e.g., Hourly_Emps for SNLRWH)
- ❖ Some FDs on Hourly_Emps:
 - *ssn is the key*: $S \rightarrow \text{SNLRWH}$
 - *rating determines hrly_wages*: $R \rightarrow W$

Example (Contd.)

Wages

R	W
8	10
5	7



Hourly_Emps2

❖ Problems due to $R \rightarrow W$:

- Update anomaly: Can we change W in just the 1st tuple of SNLRWH?
- Insertion anomaly: What if we want to insert an employee and don't know the hourly wage for his rating?
- Deletion anomaly: If we delete all employees with rating 5, we lose the information about the wage for rating 5!

S	N	L	R	H
123-22-3666	Attishoo	48	8	40
231-31-5368	Smiley	22	8	30
131-24-3650	Smethurst	35	5	30
434-26-3751	Guldu	35	5	32
612-67-4134	Madayan	35	8	40

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

Will 2 smaller tables be better?



Reasoning About FDs

- ❖ Given some FDs, we can usually infer additional FDs:
 - $ssn \rightarrow did, did \rightarrow lot$ implies $ssn \rightarrow lot$
- ❖ An FD f is implied by a set of FDs F if f holds whenever all FDs in F hold.
 - $F^+ =$ closure of F is the set of all FDs that are implied by F .
- ❖ Armstrong's Axioms (X, Y, Z are sets of attributes):
 - Reflexivity: If $X \subseteq Y$, then $Y \rightarrow X$
 - Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
 - Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- ❖ These are sound and complete inference rules for FDs!

Reasoning About FDs (Contd.)



- ❖ Couple of additional rules (that follow from AA):
 - *Union*: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
 - *Decomposition*: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- ❖ Example: **Contracts**(*cid,sid,jid,did,pid,qty,value*), and:
 - C is the key: $C \rightarrow CSJDPQV$
 - Project purchases each part using single contract: $JP \rightarrow C$
 - Dept purchases at most one part from a supplier: $SD \rightarrow P$
- ❖ $JP \rightarrow C, C \rightarrow CSJDPQV$ imply $JP \rightarrow CSJDPQV$
- ❖ $SD \rightarrow P$ implies $SDJ \rightarrow JP$
- ❖ $SDJ \rightarrow JP, JP \rightarrow CSJDPQV$ imply $SDJ \rightarrow CSJDPQV$

Reasoning About FDs (Contd.)



- ❖ Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)
- ❖ Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs F . An efficient check:
 - Compute attribute closure of X (denoted X^+) wrt F :
 - Set of all attributes A such that $X \rightarrow A$ is in F^+
 - There is a linear time algorithm to compute this.
 - Check if Y is in X^+
- ❖ Does $F = \{A \rightarrow B, B \rightarrow C, C D \rightarrow E\}$ imply $A \rightarrow E$?
 - i.e, is $A \rightarrow E$ in the closure F^+ ? Equivalently, is E in A^+ ?

Normal Forms



- ❖ Returning to the issue of schema refinement, the first question to ask is whether any refinement is needed!
- ❖ If a relation is in a certain *normal form* (BCNF, 3NF etc.), it is known that certain kinds of problems are avoided/minimized. This can be used to help us decide whether decomposing the relation will help.
- ❖ Role of FDs in detecting redundancy:
 - Consider a relation R with 3 attributes, ABC.
 - **No FDs hold:** There is no redundancy here.
 - **Given $A \rightarrow B$:** Several tuples could have the same A value, and if so, they'll all have the same B value!

Boyce-Codd Normal Form (BCNF)



- ❖ Reln R with FDs F is in **BCNF** if, for all $X \rightarrow A$ in F^+
 - $A \in X$ (called a *trivial* FD), or
 - X contains a key for R.
- ❖ In other words, R is in BCNF if the only non-trivial FDs that hold over R are key constraints.
 - No dependency in R that can be predicted using FDs alone.
 - If we are shown two tuples that agree upon the X value, we cannot infer the A value in one tuple from the A value in the other.
 - If example relation is in BCNF, the 2 tuples must be identical (since X is a key).

X	Y	A
x	y1	a
x	y2	?

Third Normal Form (3NF)



- ❖ Reln R with FDs F is in **3NF** if, for all $X \rightarrow A$ in F^+
 - $A \in X$ (called a *trivial* FD), or
 - X contains a key for R, or
 - A is part of some key for R.
- ❖ *Minimality* of a key is crucial in third condition above!
- ❖ If R is in BCNF, obviously in 3NF.
- ❖ If R is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no “good” decomp, or performance considerations).
 - *Lossless-join, dependency-preserving decomposition of R into a collection of 3NF relations always possible.*



What Does 3NF Achieve?

- ❖ If 3NF violated by $X \rightarrow A$, one of the following holds:
 - X is a subset of some key K
 - We store (X, A) pairs redundantly.
 - X is not a proper subset of any key.
 - There is a chain of FDs $K \rightarrow X \rightarrow A$, which means that we cannot associate an X value with a K value unless we also associate an A value with an X value.
- ❖ **But:** even if reln is in 3NF, these problems could arise.
 - e.g., Reserves SBDC, $S \rightarrow C$, $C \rightarrow S$ is in 3NF, but for each reservation of sailor S , same (S, C) pair is stored.
- ❖ Thus, 3NF is indeed a compromise relative to BCNF.

Decomposition of a Relation Scheme



- ❖ Suppose that relation R contains attributes $A_1 \dots A_n$.
A decomposition of R consists of replacing R by two or more relations such that:
 - Each new relation scheme contains a subset of the attributes of R (and no attributes that do not appear in R), and
 - Every attribute of R appears as an attribute of one of the new relations.
- ❖ Intuitively, decomposing R means we will store instances of the relation schemes produced by the decomposition, instead of instances of R .
- ❖ E.g., Can decompose **SNLRWH** into **SNLRH** and **RW**.



Example Decomposition

- ❖ Decompositions should be used only when needed.
 - SNLRWH has FDs $S \rightarrow \text{SNLRWH}$ and $R \rightarrow W$
 - Second FD causes violation of 3NF; W values repeatedly associated with R values. Easiest way to fix this is to create a relation RW to store these associations, and to remove W from the main schema:
 - i.e., we decompose SNLRWH into SNLRH and RW
- ❖ The information to be stored consists of SNLRWH tuples. If we just store the projections of these tuples onto SNLRH and RW , are there any potential problems that we should be aware of?

Problems with Decompositions



- ❖ There are three potential problems to consider:
 - Some queries become more expensive.
 - e.g., How much did sailor Joe earn? (salary = $W \times H$)
 - Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation!
 - Fortunately, not in the SNLRWH example.
 - Checking some dependencies may require joining the instances of the decomposed relations.
 - Fortunately, not in the SNLRWH example.
- ❖ Tradeoff: Must consider these issues vs. redundancy.



Lossless Join Decompositions

- ❖ Decomposition of R into X and Y is lossless-join w.r.t. a set of FDs F if, for every instance r that satisfies F:
 - $\pi_X(r) \bowtie \pi_Y(r) = r$
- ❖ It is always true that $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$
 - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- ❖ Definition extended to decomposition into 3 or more relations in a straightforward way.
- ❖ *It is essential that all decompositions used to deal with redundancy be lossless! Avoids Problem (2).*

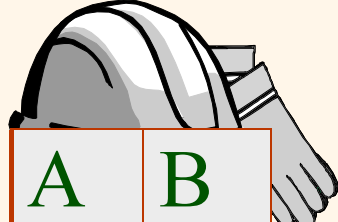
More on Lossless Join

- ❖ The decomposition of R into X and Y is **lossless-join wrt F** if and only if the closure of F contains:

- $X \cap Y \rightarrow X$, or
- $X \cap Y \rightarrow Y$

- ❖ In particular, the decomposition of R into UV and R - V is lossless-join if $U \rightarrow V$ holds over R.

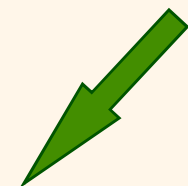
A	B	C
1	2	3
4	5	6
7	2	8

A	B
1	2
4	5
7	2

B	C
2	3
5	6
2	8

A	B	C
1	2	3
4	5	6
7	2	8
1	2	8
7	2	3



Dependency Preserving Decomposition



- ❖ Consider CSJDPQV, C is key, $JP \rightarrow C$ and $SD \rightarrow P$.
 - BCNF decomposition: CSJDQV and SDP
 - Problem: Checking $JP \rightarrow C$ requires a join!
- ❖ **Dependency preserving decomposition** (Intuitive):
 - If R is decomposed into X, Y and Z, and we enforce the FDs that hold on X, on Y and on Z, then all FDs that were given to hold on R must also hold. (*Avoids Problem (3).*)
- ❖ Projection of set of FDs F: If R is decomposed into X, ... projection of F onto X (denoted F_X) is the set of FDs $U \rightarrow V$ in F^+ (closure of F) such that U, V are in X.

Dependency Preserving Decompositions (Contd.)



- ❖ Decomposition of R into X and Y is dependency preserving if $(F_X \text{ union } F_Y)^+ = F^+$
 - i.e., if we consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+ .
- ❖ Important to consider F^+ , not F , in this definition:
 - ABC, $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$, decomposed into AB and BC.
 - Is this dependency preserving? Is $C \rightarrow A$ preserved?????
- ❖ Dependency preserving does not imply lossless join:
 - ABC, $A \rightarrow B$, decomposed into AB and BC.
- ❖ And vice-versa! (Example?)

Decomposition into BCNF



- ❖ Consider relation R with FDs F . If $X \rightarrow Y$ violates BCNF, decompose R into $R - Y$ and XY .
 - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
 - e.g., CSJDPQV, key C , $JP \rightarrow C$, $SD \rightarrow P$, $J \rightarrow S$
 - To deal with $SD \rightarrow P$, decompose into SDP , CSJDQV.
 - To deal with $J \rightarrow S$, decompose CSJDQV into JS and $CJDQV$
- ❖ In general, several dependencies may cause violation of BCNF. The order in which we “deal with” them could lead to very different sets of relations!

BCNF and Dependency Preservation



- ❖ In general, there may not be a dependency preserving decomposition into BCNF.
 - e.g., $CSZ, CS \rightarrow Z, Z \rightarrow C$
 - Can't decompose while preserving 1st FD; not in BCNF.
- ❖ Similarly, decomposition of CSJDQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. the FDs $JP \rightarrow C, SD \rightarrow P$ and $J \rightarrow S$).
 - However, it is a lossless join decomposition.
 - In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
 - JPC tuples stored only for checking FD! (*Redundancy!*)



Decomposition into 3NF

- ❖ Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- ❖ To ensure dependency preservation, one idea:
 - If $X \rightarrow Y$ is not preserved, add relation XY .
 - Problem is that XY may violate 3NF! e.g., consider the addition of CJP to 'preserve' $JP \rightarrow C$. What if we also have $J \rightarrow C$?
- ❖ **Refinement:** Instead of the given set of FDs F , use a *minimal cover for F* .



Minimal Cover for a Set of FDs

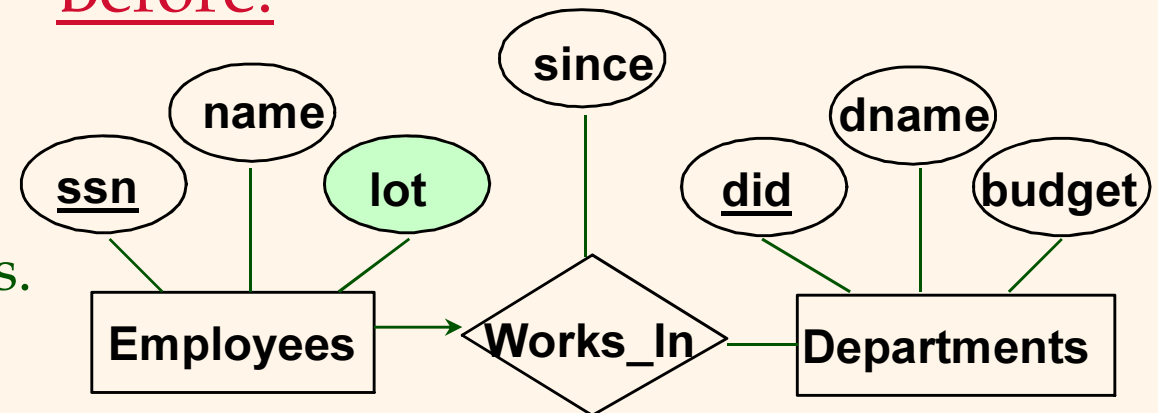
- ❖ Minimal cover G for a set of FDs F :
 - Closure of F = closure of G .
 - Right hand side of each FD in G is a single attribute.
 - If we modify G by deleting an FD or by deleting attributes from an FD in G , the closure changes.
- ❖ Intuitively, every FD in G is needed, and “*as small as possible*” in order to get the same closure as F .
- ❖ e.g., $A \rightarrow B$, $ABCD \rightarrow E$, $EF \rightarrow GH$, $ACDF \rightarrow EG$ has the following minimal cover:
 - $A \rightarrow B$, $ACD \rightarrow E$, $EF \rightarrow G$ and $EF \rightarrow H$
- ❖ M.C. \rightarrow Lossless-Join, Dep. Pres. Decomp!!! (in book)



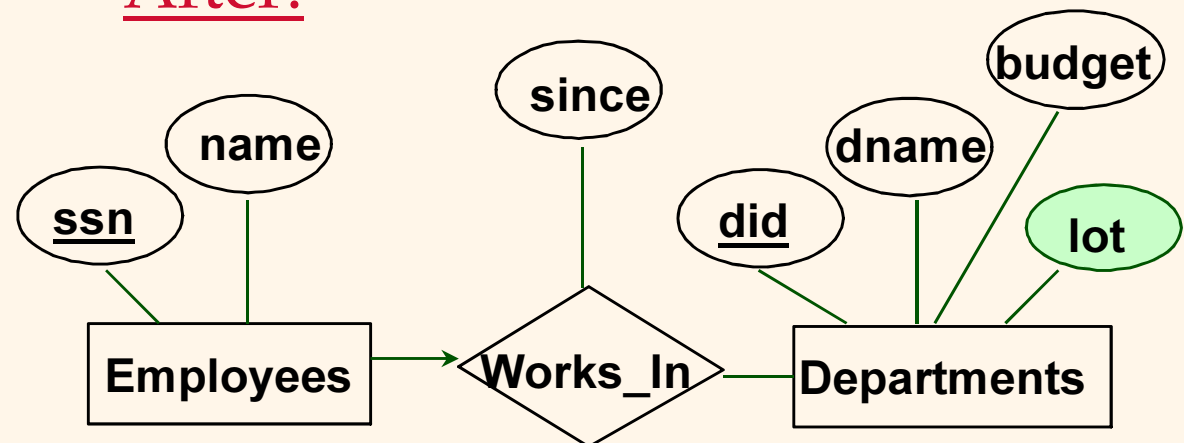
Refining an ER Diagram

- ❖ 1st diagram translated:
Workers(S,N,L,D,S)
Departments(D,M,B)
 - Lots associated with workers.
- ❖ Suppose all workers in a dept are assigned the same lot: $D \rightarrow L$
- ❖ Redundancy; fixed by:
Workers2(S,N,D,S)
Dept_Lots(D,L)
- ❖ Can fine-tune this:
Workers2(S,N,D,S)
Departments(D,M,B,L)

Before:



After:



Summary of Schema Refinement



- ❖ If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- ❖ If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
 - Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
 - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.