Log-Normal Evaluation

- Derive values of f(x), g(x), K

```
Clear["p", "k", "μ", "σ", "x", "τ", "K", "f", "g"]
Remove["p", "f", "α", "k"]
$Assumptions = σ > 0 && σ ∈ Reals && μ  > 0 && μ ∈ Reals && x ∈ Reals && x > 0;
p = PDF[LogNormalDistribution[Log[μ], σ], x]
f = k/2 FullSimplify[D[Log[p], x]]
α = Mean[TransformedDistribution[x f, x ≈ LogNormalDistribution[Log[μ], σ]]] / Variance[TransformedDistribution[x , x ≈ LogNormalDistribution[Log[μ], σ]]]

K = k /. Solve[α == -1/τ, k][[1]]
f = f /. k → K
```

$$\begin{cases} \frac{e^{-\frac{(\mathrm{Log}[x]-\mathrm{Log}[\mu])^2}{2\sigma^2}}}{\sqrt{2\pi}\, x\, \sigma} & x > 0 \\ 0 & \mathrm{True} \end{cases}$$

$$-\frac{k\left(\sigma^2 + \mathrm{Log}[x] - \mathrm{Log}[\mu]\right)}{2\, x\, \sigma^2}$$

$$-\frac{e^{-\sigma^2}\, k}{2\left(-1 + e^{\sigma^2}\right)\mu^2}$$

$$\frac{2\, e^{\sigma^2}\left(-1 + e^{\sigma^2}\right)\mu^2}{\tau}$$

$$-\frac{e^{\sigma^2}\left(-1 + e^{\sigma^2}\right)\mu^2\left(\sigma^2 + \mathrm{Log}[x] - \mathrm{Log}[\mu]\right)}{x\, \sigma^2\, \tau}$$

```
ClearAll["f[t]", "g[t]", "x", "y", "f", "g", "μ", "σ", "τ"];
μ = 1;
σ = .2;
τ = .2;
tSample = .01;
```

$$K = \frac{2\, e^{\sigma^2}\left(-1 + e^{\sigma^2}\right)\mu^2}{\tau};$$

$$f[t] = -\frac{\sigma^2 + \text{Log}[y[t]] - \text{Log}[\mu]}{y[t]\,\sigma^2};$$

```
g[t] = Sqrt[K];
```

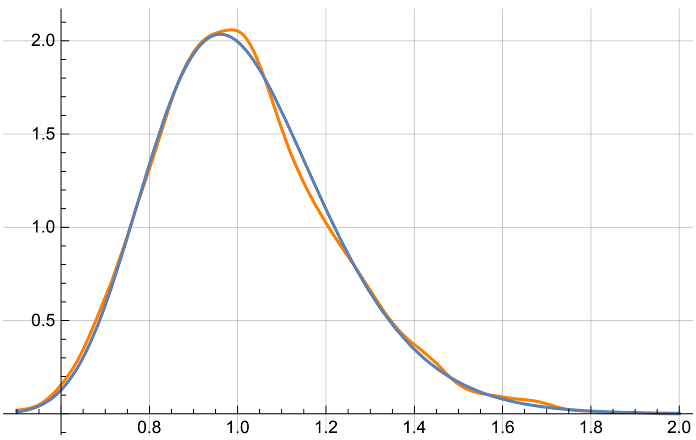
$$\text{proc} = \text{StratonovichProcess}\left[dy[t] == \frac{K}{2} f[t]\, dt + g[t]\, dw[t],\ y[t],\ \{y, \mu\},\ t,\ w \approx \text{WienerProcess}[]\right]$$

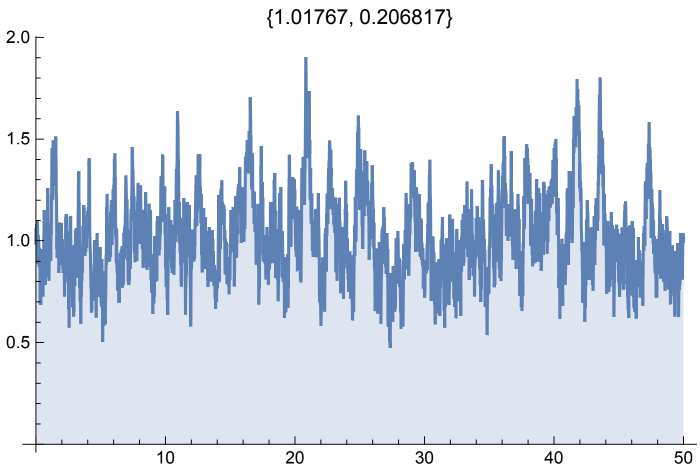
```
sample = RandomFunction[proc, {0., 50, tSample}]
```

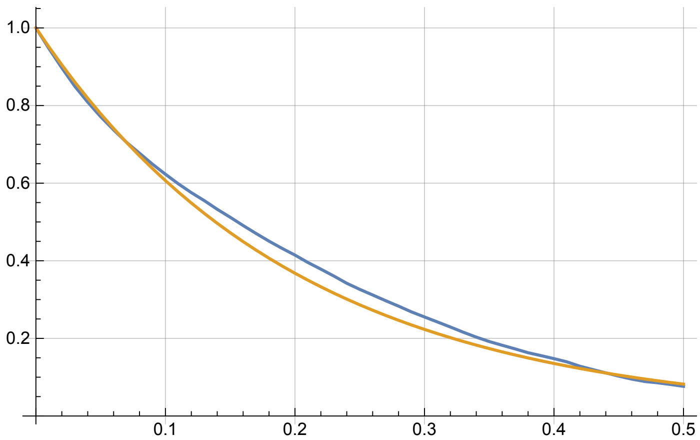
$$\text{ItoProcess}\left[\left\{\left\{0. - \frac{5.30954\,(0.04 + \text{Log}[y[t]])}{y[t]}\right\}, \{\{0.651738\}\}, y[t]\right\}, \{\{y\}, \{1\}\}, \{t, 0\}\right]$$

```
TemporalData[ Time: 0. to 50.  Data points: 5001  Paths: 1 ]
```

```
(*SmoothHistogram[sample,GridLines→Automatic]*)
Show[
 {Plot[PDF[SmoothKernelDistribution[sample], x], {x, .5, 2}, PlotStyle → Orange],
  Plot[PDF[LogNormalDistribution[Log[μ], σ], x], {x, .5, 2}]},
 GridLines -> Automatic]
ListLinePlot[sample, Filling → Axis,
 PlotLabel → {Mean[sample], StandardDeviation[sample]}]
ListLinePlot[
 {Table[{x * tSample,
    CovarianceFunction[sample, x] / CovarianceFunction[sample, 0]}, {x, 0, 50}],
  Table[{x * tSample, Exp[-x * tSample / τ]}, {x, 0, 50}]},
 PlotRange → Full, GridLines → Automatic]
```




{1.01767, 0.206817}
2.0
1.5
1.0
0.5
10
20
30
40
50


1.0
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
0.5