Log-Normal Evaluation

- Derive values of f(x), g(x), K

In[1]:=
```
Clear["p", "k", "μ", "σ", "x", "τ", "K", "f", "g"]
Remove["p", "f", "α", "k"]
$Assumptions = σ > 0 && σ ∈ Reals && μ  > 0 && μ ∈ Reals && x ∈ Reals  && x > 0;
p = PDF[LogNormalDistribution[Log[μ], σ], x]
```
$$f = \frac{k}{2}\,\texttt{FullSimplify[D[Log[p], x]]}$$
```
α = Mean[TransformedDistribution[x f, x ≈ LogNormalDistribution[Log[μ], σ]]] /
  Variance[TransformedDistribution[x , x ≈ LogNormalDistribution[Log[μ], σ]]]
```
$$K = k\ /.\ \texttt{Solve}\left[\alpha == -\frac{1}{\tau},\ k\right][[1]]$$
```
f = f /. k → K
```

Out[4]=
$$\begin{cases} \frac{e^{-\frac{(\text{Log}[x]-\text{Log}[\mu])^2}{2\sigma^2}}}{\sqrt{2\pi}\,x\,\sigma} & x > 0 \\ 0 & \text{True} \end{cases}$$

Out[5]=
$$-\frac{k\left(\sigma^2 + \text{Log}[x] - \text{Log}[\mu]\right)}{2\,x\,\sigma^2}$$

Out[6]=
$$-\frac{e^{-\sigma^2}\,k}{2\left(-1 + e^{\sigma^2}\right)\mu^2}$$

Out[7]=
$$\frac{2\,e^{\sigma^2}\left(-1 + e^{\sigma^2}\right)\mu^2}{\tau}$$

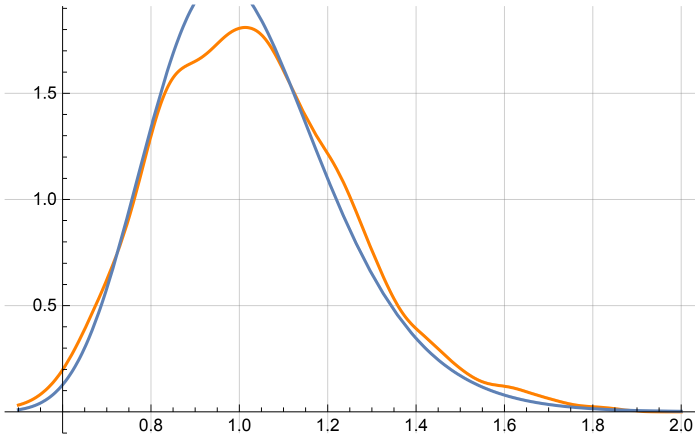
Out[8]=
$$-\frac{e^{\sigma^2}\left(-1 + e^{\sigma^2}\right)\mu^2\left(\sigma^2 + \text{Log}[x] - \text{Log}[\mu]\right)}{x\,\sigma^2\,\tau}$$

In[61]:=
```
ClearAll["f[t]", "g[t]", "x", "y", "f", "g", "μ", "σ", "τ"];
μ = 1;
σ = .2;
τ = .2;
tSample  =  .01;
```

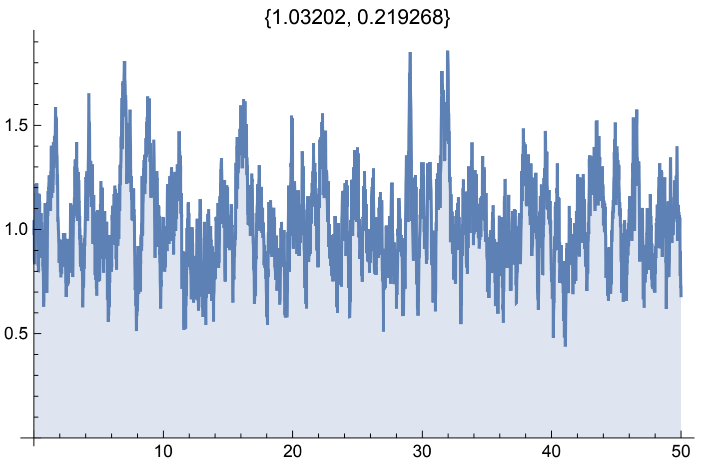
In[66]:=
$$K = \frac{2\, e^{\sigma^2} \left(-1 + e^{\sigma^2}\right) \mu^2}{\tau};$$
$$f[t] = -\frac{\sigma^2 + \mathrm{Log}[y[t]] - \mathrm{Log}[\mu]}{y[t]\, \sigma^2};$$
```
g[t] = Sqrt[K];

proc = StratonovichProcess[dy[t] == K/2 f[t] dt + g[t] dw[t], y[t], {y, μ},
   t, w ≈ WienerProcess[]];
sample = RandomFunction[proc, {0., 50, tSample}]
```

Out[70]= TemporalData[ Time: 0. to 50. Data points: 5001 Paths: 1]

```
(*Accurate results require more samples
 with significantly longer running time*)
Show[{Plot[PDF[SmoothKernelDistribution[sample], x], {x, .5, 2},
   PlotStyle → Orange], Plot[PDF[LogNormalDistribution[Log[μ], σ], x],
   {x, .5, 2}]}, GridLines -> Automatic]
ListLinePlot[sample, Filling → Axis, PlotLabel →
  {Mean[sample], StandardDeviation[sample]}]
ListLinePlot[
 {Table[{x * tSample,
    CovarianceFunction[sample, x] / CovarianceFunction[sample, 0]}, {x, 0, 50}],
  Table[{x * tSample, Exp[-x * tSample / τ]}, {x, 0, 50}]},
 PlotRange → Full, GridLines → Automatic]
```

Out[71]=


Out[72]=



Out[73]=