

T₄

$$\xi \sim R(0, 20) \quad p(x) = \frac{1}{\theta} \{ (0, 20) \}$$

a) \bar{x}_n - выборка

ОМ

$$x_1 = M[\xi] = \int_{-\infty}^{\infty} x p(x) dx = \int_0^{20} x \frac{1}{\theta} dx =$$

$$= \frac{3}{2} \theta = \bar{x}$$

$$\tilde{\theta} = \frac{2}{3} \bar{x}$$

1) несмещ.

$$M[\tilde{\theta}] = \frac{2}{3} M[\bar{x}] = \theta$$

2) состоят

$$D[\tilde{\theta}] = \frac{4}{9} D[\bar{x}] = \frac{4}{9} \cdot \frac{1}{n^2} \cdot n D[\xi] =$$

$$= \frac{4}{9n} \left(\frac{7}{3} \theta^2 - \frac{9}{4} \theta^2 \right) = \frac{4}{9n} \left(\frac{7}{3} \theta^2 - \frac{9}{4} \theta^2 \right) = \frac{1}{27n} \theta^2$$

$$M[\xi^2] = \int_0^{20} x^2 \frac{1}{\theta} dx = \frac{7}{3} \theta^2$$

$$D[\tilde{\theta}] = \frac{4}{9} D[\bar{x}] = \frac{4}{9} \cdot \frac{1}{n^2} \cdot n D[\xi] =$$

$$= \frac{4}{9n} \left(\frac{7}{3} \theta^2 - \frac{9}{4} \theta^2 \right) = \frac{1}{27n} \theta^2 \xrightarrow{n \rightarrow \infty} 0$$

+ несмещ.

состоят
по geom.
уравн.

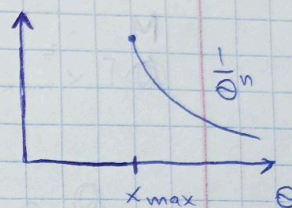
ОМП

$$L(\theta) = \prod_{i=1}^n p(x_i, \theta) = \frac{1}{\theta^n} \{ (0 \leq x_i \leq 2\theta) \}$$

$$L(\theta) \rightarrow \sup$$

$$2\theta > x_{\max}$$

$$\tilde{\theta} = \frac{x_{\max}}{2}$$



1) recursively.

$$M[\tilde{\theta}] = \frac{1}{2} M[x_{\max}]$$

$$q(y) = n(F(y))^{n-1} \cdot F'(y) =$$

$$= n \left(\frac{x-\theta}{\theta} \right)^{n-1} \frac{1}{\theta}$$

$$F(x) = \int_0^x \frac{1}{\theta} dx =$$

$$= \frac{x-\theta}{\theta} \{ (0, 2\theta) \}$$

$$M[x_{\max}] = \int_0^{2\theta} x n \left(\frac{x-\theta}{\theta} \right)^{n-1} \frac{1}{\theta} dx =$$

$$= \frac{n}{\theta} \int_0^{2\theta} (\theta + x - \theta) \left(\frac{x-\theta}{\theta} \right)^{n-1} dx =$$

$$= \frac{n}{\theta} \int_0^{2\theta} \theta \left(\frac{x-\theta}{\theta} \right)^{n-1} dx + n \int_0^{2\theta} \left(\frac{x-\theta}{\theta} \right)^{n-1} d\left(\frac{\theta-x}{\theta} \right) =$$

$$= \frac{n\theta}{n+1} \left(\frac{x-\theta}{\theta} \right)^{n+1} \Big|_0^{2\theta} + \frac{n\theta}{n} \left(\frac{x-\theta}{\theta} \right)^n \Big|_0^{2\theta} =$$

$$= \frac{n\theta}{n+1} + \theta =$$

$$= 2n \frac{\theta}{n+1} + \theta = \theta \frac{(2n+1)}{n+1} \quad \boxed{\text{curry.}}$$

$$\tilde{\theta}' = \frac{n+1}{(2n+1)} \theta = \frac{n+1}{(2n+1)} x_{\max}$$

$$D\left[\frac{n+1}{2n} x_{\max}\right] = \frac{(n+1)^2}{4n} \cdot \frac{1}{n} [x_{\max}]$$

$$M[x_{\max}^2] = \int_0^{2\theta} x^2 \frac{n}{\theta} \left(\frac{x-\theta}{\theta}\right)^{n-1} dx =$$

$$= \left[\frac{\theta x^2}{n} \frac{n}{\theta} \left(\frac{x-\theta}{\theta}\right)^{n-1} \right]_0^{2\theta} - 2 \int_0^{2\theta} \theta x \frac{n}{\theta} \left(\frac{x-\theta}{\theta}\right)^{n-1} dx =$$

$$= 4\theta^2 - \text{scribbles}$$

$$- 2 \times \theta \cdot \frac{1}{n+1} \left(\frac{x-\theta}{\theta}\right)^{n+1} \Big|_0^{2\theta} + 2 \int_0^{2\theta} \frac{\theta}{\theta} \cdot \frac{\theta^2}{(n+1)} \left(\frac{x-\theta}{\theta}\right)^{n+1} dx$$

$$= 4\theta^2 - \frac{4\theta^2}{n+1} + 2\theta^2 \frac{1}{(n+1)(n+2)} =$$

$$= \theta^2 \left(\frac{4(n+1)(n+2) - 4(n+2) + 2}{(n+1)(n+2)} \right)$$

$$D[\tilde{\theta}'] = \frac{(n+1)^2}{(2n+1)^2} \left[\theta^2 \left(\frac{4n^2 + 12n + 8 - 4n - 8 + 2}{(n+1)(n+2)} \right) \right.$$

$$\left. + \theta^2 \frac{4n^2}{(n+1)^2} \right] \xrightarrow{n \rightarrow \infty} \tilde{\theta}' \quad \boxed{\text{converges}} \\ \left(D[\theta] = \frac{n\theta^2}{(2n+1)^2(n+2)} + \text{term} \right) \quad \text{no good yet.$$

c)

d)

f

f

$$\int_0^1 \frac{1}{\theta^2} dx = \frac{1}{\theta^2}$$

$$\Delta[\tilde{\Theta}_{\text{опт}}] = \frac{1}{27n} \theta^2$$

$$\Delta[\tilde{\Theta}'_{\text{опт}}] = \frac{n}{(2n+1)^2 (n+2)} \theta^2$$

$$\frac{1}{27} \theta^2 > 0$$

$\Rightarrow \hat{\Theta}'_{\text{опт}}$ ассимпт.

эффективнее
интервал

d) ¹⁴ монотонно возрастающая непрерывная

$$f(\theta, \vec{x}_n) = ?$$

$$\xi \sim R[0, 2\theta]$$

$$\eta = \frac{\xi}{\theta} \quad \Phi(z) = P(\eta < z) =$$

$$= P\left(\frac{\xi}{\theta} < z\right) = P(\xi < z\theta) = F(z\theta) =$$

$$= (z-1) \cdot \frac{\theta}{\theta} = z-1$$

$$\eta = \frac{\xi}{\theta} \sim \text{~~R[0, 2\theta]~~} R[1, 2]$$

$$\vec{x}_n, \quad x_i \sim \text{~~R[0, 2\theta]~~} R[1, 2]$$

$$\text{~~max } \Phi(x) = \Phi(x)~~$$

$$x_{\max} \sim \psi(x) = n(x-1)^{n-1} \cdot \{ (1, 2) \}$$

$$\max \left(\frac{x_i}{\theta} \right) \sim \psi(x)$$

$$\max_i \left(\frac{\xi}{\theta} \right) = \frac{1}{\theta} \max_i (x_i) = \frac{x_{\max}}{\theta}$$

$$F(t_1) = 0,025$$

$$F(t_2) = 0,975$$

~~$$t_1 = \sqrt[n]{0,025} + 1$$~~

$$t_1 = \sqrt[n]{0,025} + 1$$

$$t_2 = \sqrt[n]{0,975} + 1$$

$$t_1 < \frac{x_{\max}}{\Theta} < t_2$$

$$\frac{x_{\max}}{t_2} < \Theta < \frac{x_{\max}}{t_1}$$

markieren

е) асимптотический доверительный интервал по ОММ

$$\tilde{\theta} = \frac{2}{3} \bar{x} = g(\tilde{\alpha}) = \frac{2}{3} \tilde{\alpha}_1$$

$$\theta = g(\alpha) = \frac{2}{3} \alpha_1$$

$$G(\alpha) = \sqrt{\frac{2}{3} (\alpha_2 - \alpha_1^2) \frac{2}{3}} = \frac{2}{3} \sqrt{\alpha_2 - \alpha_1^2}$$

$$\sqrt{n} \frac{\frac{2}{3} \tilde{\alpha}_1 - \frac{2}{3} \alpha_1}{\frac{2}{3} \sqrt{\alpha_2 - \alpha_1^2}} \sim N(0, 1)$$

$$\sqrt{n} \frac{\tilde{\theta} - \theta}{\frac{2}{3} \sqrt{\alpha_2 - \alpha_1^2}} \sim N(0, 1)$$

по лемме Сильского

$$f(\tilde{\alpha}_1, \theta) \xrightarrow{P} N(0, 1)$$

$$-1,96 < \sqrt{n} \frac{\tilde{\theta} - \theta}{\frac{2}{3} \sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}} < 1,96$$

$$-1,96 \frac{\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}}{\sqrt{n} \cdot \frac{2}{3}} + \tilde{\theta} < \theta < \frac{1,96 \sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}}{\sqrt{n} \cdot \frac{2}{3}} + \tilde{\theta}$$

Исследование $\frac{x_{\max}}{2} = \tilde{\theta}_2$ на ~~находящуюся~~ сост.

$$\forall \varepsilon > 0 \quad P(|\tilde{\theta}_2 - \theta| \geq \varepsilon) \rightarrow 0, n \rightarrow \infty$$

$$P(x_{\max}/2 \geq \theta + \varepsilon) = P(x_{\max} \geq 2\theta + 2\varepsilon) = 0$$

$$P(x_{\max}/2 \leq \theta - \varepsilon) = P(x_{\max} \leq 2\theta - 2\varepsilon) = P(2(\theta - \varepsilon)) = \left(\frac{2\theta - 2\varepsilon - \theta}{\theta}\right)^n = \left(\frac{\theta - 2\varepsilon}{\theta}\right)^n =$$

$$= \left(1 - 2\frac{\varepsilon}{\theta}\right)^n \rightarrow 0$$

$$n \rightarrow \infty$$

$$\begin{aligned} & \theta - \varepsilon > \theta - \varepsilon > \theta - \varepsilon \\ & \theta - \varepsilon > \theta - \varepsilon > \theta - \varepsilon \\ & \theta - \varepsilon > \theta - \varepsilon > \theta - \varepsilon \end{aligned}$$

$$\theta < 2\theta - 2\varepsilon < 2\theta$$

$$0 < \varepsilon \leq \frac{\theta}{2}$$

$$\begin{aligned} & P(x_{\max}/2 \leq \theta - \varepsilon) = P(x_{\max} \leq 2\theta - 2\varepsilon) \\ & \Rightarrow P(2(\theta - \varepsilon)) = 0 \Rightarrow P(2(\theta - \varepsilon)) = 0 \Rightarrow P(2(\theta - \varepsilon)) = 0 \end{aligned}$$

$$P(x_{\max}/2 \leq \theta - \varepsilon) = P(x_{\max} \leq 2(\theta - \varepsilon))$$

$$\text{Если } \varepsilon \geq \frac{\theta}{2}, \text{ то } 2(\theta - \varepsilon) \leq 0 \Rightarrow$$

$$\Rightarrow P(2(\theta - \varepsilon)) = 0 \rightarrow 0, n \rightarrow \infty$$

\Rightarrow оценка состоятельная