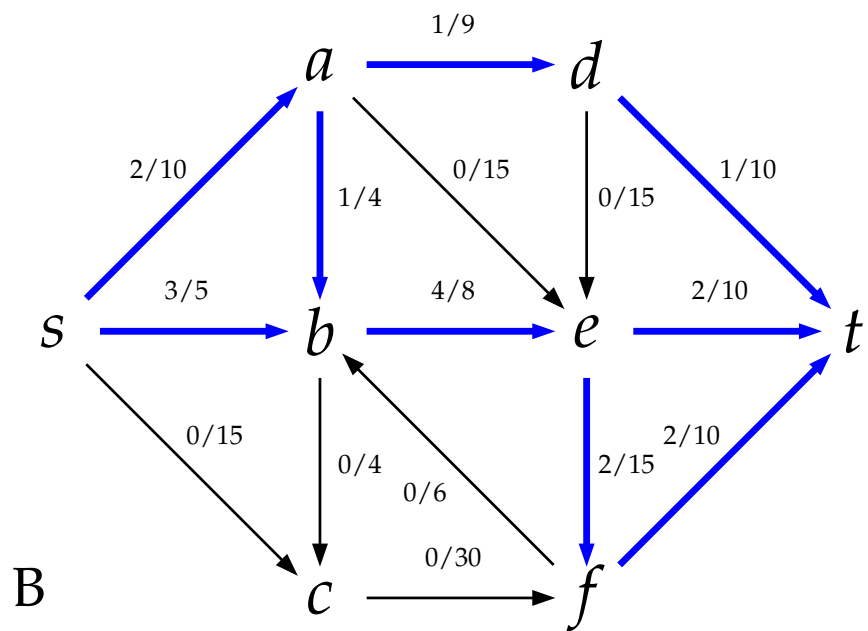
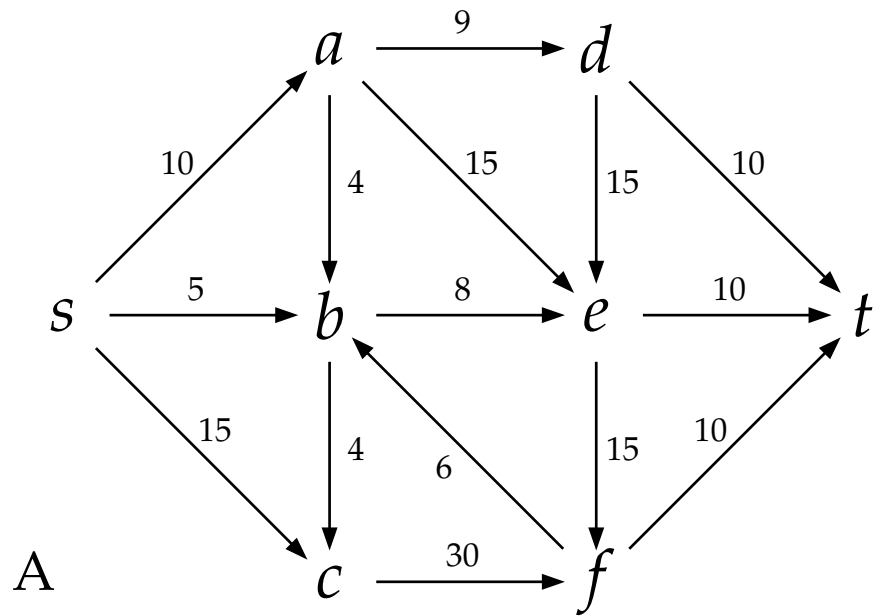
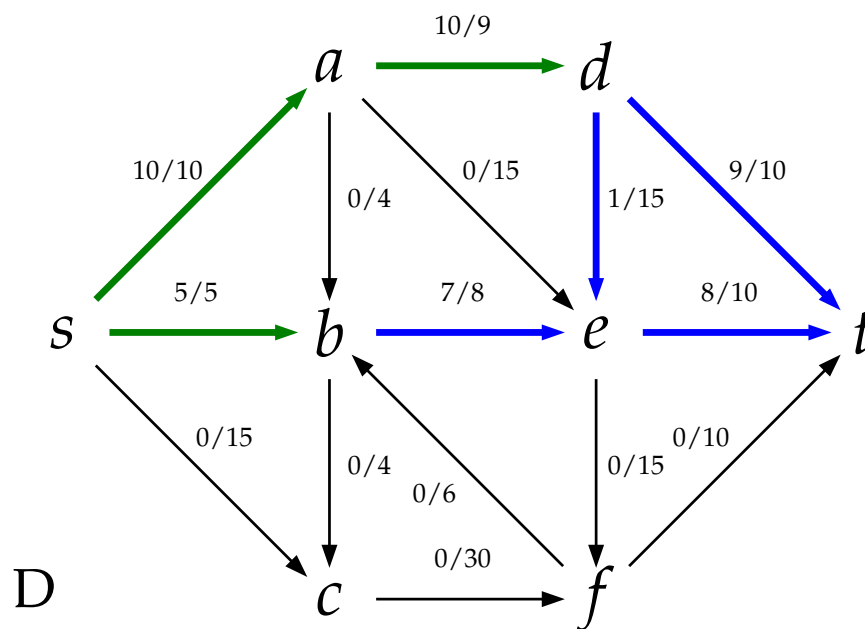
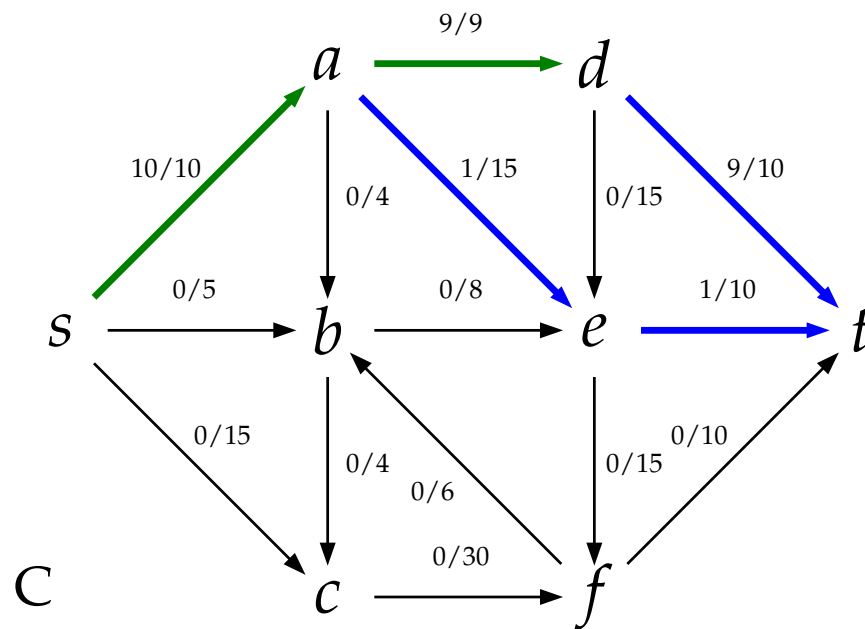


## Algorithms: Introduction to Flow Networks

### Model 1: Networks and flows



Model 1: (continued)



Consider graph  $A$ . Once again we have a directed graph with weighted edges. However, instead of thinking of the weights as some sort of length, we will now think of them as a *capacity*: the “maximum amount of stuff” that the edge can carry. For example, the capacity might be used to model things like:

- maximum gallons of water per minute that can flow through a pipe;
- maximum number of trucks per hour that can drive along a road;  
or
- maximum number of times a certain resource can be used before it is all used up.

- 1 Consider graph  $B$ . How is it related to graph  $A$ ?
  
- 2 What do the blue edges in graph  $B$  all have in common?
  
- 3 What do you think the labels on the edges of graph  $B$  represent?
  
- 4 Now consider graph  $C$ . Why do you think some of the edges are green?
  
- 5 Graph  $D$  is invalid! In fact, there are two things wrong with it. What are they?



**Definition 1.** A *flow network* is a directed graph  $G = (V, E)$  with

- a distinguished *source* vertex  $s \in V$ , with indegree 0;
- a distinguished *sink* or *target* vertex  $t \in V$ , with outdegree 0;
- a *capacity function*  $c : E \rightarrow \mathbb{R}^+$  assigning a non-negative real number capacity  $c(e)$  to each edge  $e \in E$ .

6 Is graph  $A$  a flow network? Why or why not?

Now let's define a *flow*. Both graphs  $B$  and  $C$  depict valid flows on  $A$ ; graph  $D$  does not.

**Definition 2.** A *flow* on a flow network  $G$  is a function  $f : E \rightarrow \mathbb{R}^+$  assigning a non-negative flow  $f(e)$  to each edge, such that

1. \_\_\_\_\_  $\leq f(e) \leq$  \_\_\_\_\_ for every  $e \in E$
2. At each vertex  $v \in V$  other than  $s$  and  $t$ , \_\_\_\_\_  
\_\_\_\_\_.

**Definition 3.** The *value* of a flow,  $v(f)$ , is the sum of the flow on all edges leaving  $s$ .

7 What is the value of the flow on graph  $B$ ?

8 What is the value of the flow on graph  $C$ ?

9 Make a conjecture about the relationship between the value of a flow and the amount of flow entering  $t$ .

10 For each amount, say whether you can construct a flow on graph  $A$  with the given value.

(a) 15



(b) 40

(c) 30

- 11 What is the value of the biggest flow you can construct on graph  $A$ ?
- 12 (Bonus question) Brainstorm how you might create an algorithm to find the biggest possible flow for a given flow network.

