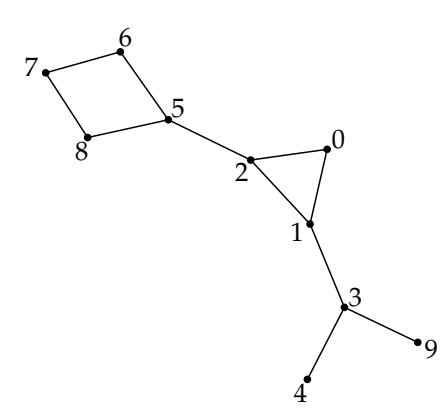
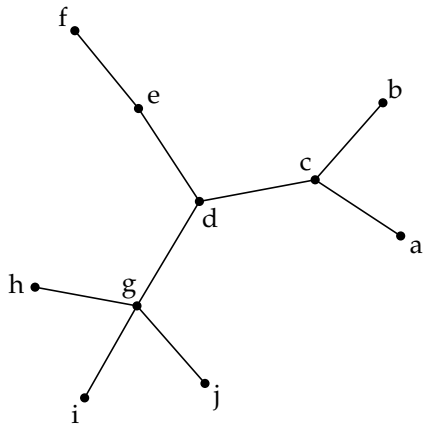
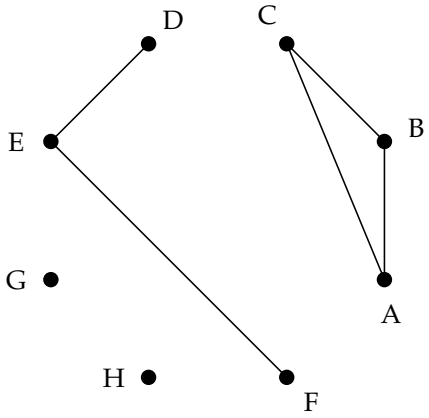
Algorithms: Introduction to Graphs

Model 1: Graphs and graph terms







$G = (V, E)$

$V = \{\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta\}$

$E = \{\{\alpha, \delta\}, \{\theta, \eta\}, \{\beta, \alpha\}, \{\zeta, \delta\}, \{\epsilon, \eta\}, \{\gamma, \alpha\}\}$

Key: α = alpha, β = beta, γ = gamma, δ = delta, ϵ = epsilon, ζ = zeta, η = eta, θ = theta

Definition 1. A (simple) *graph* $G = (V, E)$ is a set of *vertices* V (singular *vertex*) together with a set E of *edges*, where each edge consists of a set of two vertices.

Above are shown four example graphs. In clockwise order from the upper-left, we will refer to them as the numeric graph, the lowercase graph, the Greek graph, and the uppercase graph. Here are some facts about the graphs:

- C and B are *adjacent* (aka *neighbors*). So are 6 and 5. C and F are not adjacent. f and d are not adjacent. H is not adjacent to itself.
- The *degree* of vertex C is 2. The degree of vertex H is 0. The degree of vertex g is 4.
- Vertices D, F, 4, 9, a, b, f, h, i, j are *leaves*. No other vertex is a leaf.

Learning objective: Students will understand and apply graph terms *edge*, *vertex*, *adjacent/neighbor*, *degree*, *leaf*, *path*, *connected*, *connected component*, *cycle*, *cyclic*, *acyclic*, and *tree*.

Don't worry if you don't remember all these graph terms! In fact, you should probably try to forget what you think you might remember and just focus on the information in the model. Part of the point of this exercise is to help you either recall these terms, or learn them for the first time.

1 How many vertices does the numeric graph have? How many edges?

Important note: the singular of "vertices" is "vertex". "Vertice" is not a word. Every time you say it, a puppy dies.

Figure 1: A sad puppy who does not want to die. Photo by Karen Arnold, CC0 Public Domain.

2 How many vertices and edges does the Greek graph have?

3 Make a drawing of the Greek graph.

4 If you compared your group's drawing of the Greek graph to another group's drawing, do you think they would be identical, or different?

5 What does your answer to the previous question tell you about the relationship between graphs and *drawings of* graphs?

6 Give another example (besides the given ones) of two vertices that are adjacent.

7 Give another example of two vertices that are not adjacent.

8 What does it mean for two vertices to be *adjacent*?

9 Are α and β adjacent? What about α and ζ?

10 What is the *degree* of a vertex? Use the term *adjacent* in your answer.

11 What is the degree of α?

12 What is the definition of a *leaf*? Use the term *degree* in your answer.

13 Which vertices are leaves in the Greek graph?

14 In the space below, draw a graph with at least three leaves, exactly one vertex with degree five, and at least one pair of vertices that are not adjacent.

Model 2: More graph terms

Here are a few more facts about the graphs from Model 1:

- Vertices 7–8–5–2–0 are a *path*. 2–1–0–2–1 is also a path. *H* by itself is a path. *H*–*F*–*B* is not a path. 8–2–1 is also not a path.
- Vertices *f* and *g* are *connected*. So are 7 and 8. *C* and *F* are not connected. *H* is connected to itself.
- The graph with numbers is a *connected graph*. So is the graph with lowercase letters. The graph with uppercase letters is not a connected graph (it is *disconnected*).
- The numeric and lowercase graphs have one *connected component* each. The uppercase graph has four connected components.
- 8–7–6–5–8 is a *cycle*. So is *C*–*A*–*B*–*C*. *h*–*g*–*i*–*h* is not a cycle. *h*–*g*–*h* is not a cycle either. Nor is *H*–*H*. Nor is 2–5–8–7–6–5.
- The numeric graph and uppercase graph are *cyclic* graphs. The lowercase graph is *acyclic*.
- The lowercase graph is a *tree*. None of the other graphs are trees (not even the Greek one).

15 What do you think is the definition of a *path*?

16 Give an example of a path in the Greek graph.

17 Can two vertices be connected but not adjacent? If so, give an example.

18 Can two vertices be adjacent but not connected? If so, give an example.

19 What do you think it means for two vertices to be connected? Be sure to use the term *path* in your answer.

20 Is the Greek graph connected?

21 What do you think is the definition of a *connected graph*?

22 How many vertices can be in a connected component?

23 How many *connected components* does the Greek graph have?

24 Is the set of vertices $\{E, F\}$ a connected component? Why or why not?

25 Write a definition for *connected component*.

26 Write an "if and only if" statement using the terms *connected graph* and *connected component*.

27 What is a *cycle*? Use the term *path* in your answer.

28 Does your definition for *cycle* correctly explain why *h–g–h* is not a cycle? If not, revise it so it does.

29 Does the Greek graph have a cycle?

30 Is the Greek graph cyclic or acyclic?

31 What do you think is the definition of a tree? You should use two of the other graph terms in your definition.

Warning—a tree graph is not quite the same thing as a tree data structure!