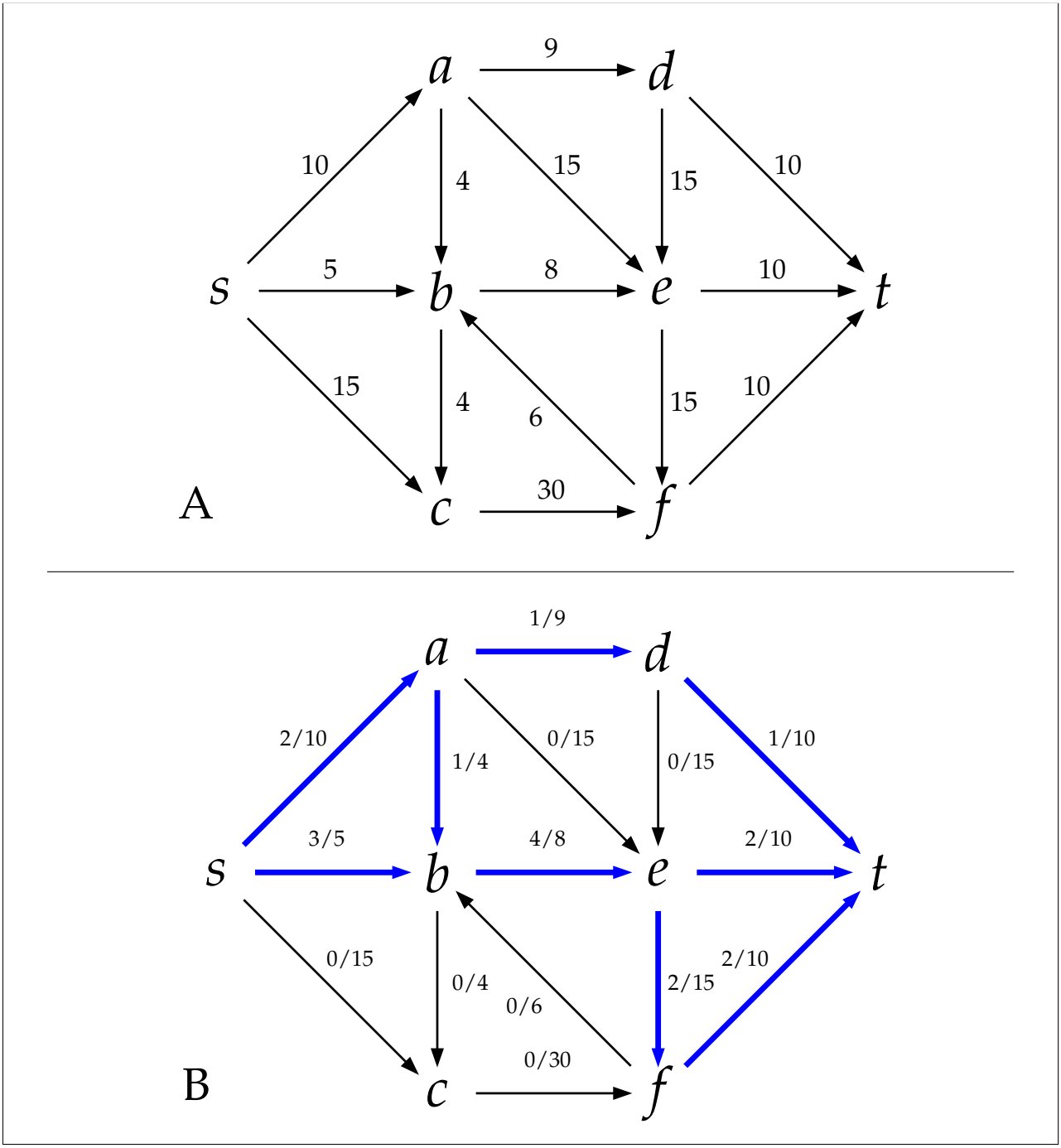
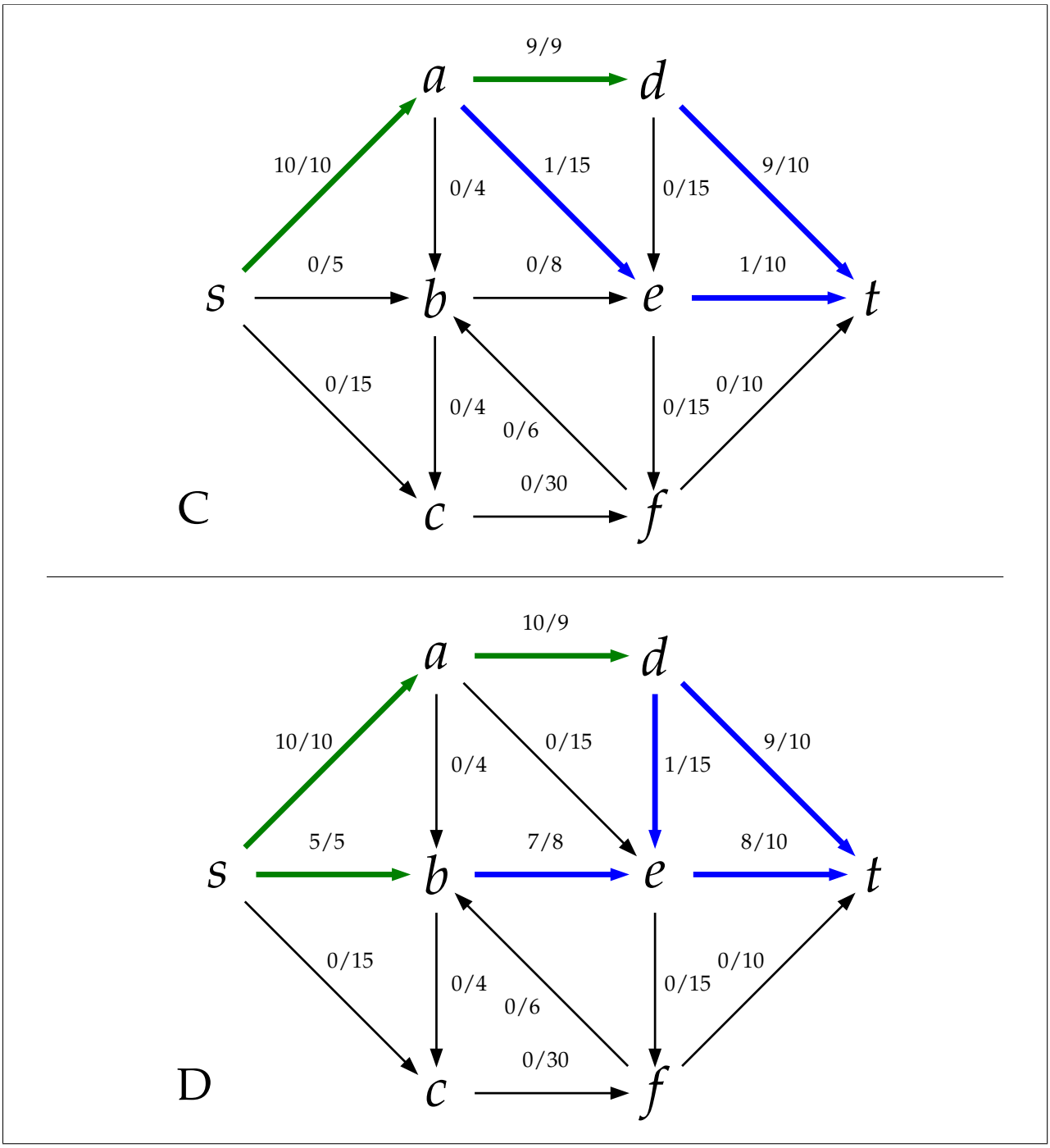

Model 1: Networks and flows



Model 1: (continued)



Consider graph A. Once again we have a directed graph with weighted edges. However, instead of thinking of the weights as some sort of length, we will now think of them as a *capacity*: the "maximum amount of stuff" that the edge can carry. For example, the capacity might be used to model things like:

- maximum gallons of water per minute that can flow through a pipe;
- maximum number of trucks per hour that can drive along a road; or
- maximum number of times a certain resource can be used before it is all used up.

1 Consider graph B. How is it related to graph A?

2 What do the blue edges in graph B all have in common?

3 What do you think the labels on the edges of graph B represent?

4 Now consider graph C. Why do you think some of the edges are green?

5 Graph D is invalid! In fact, there are two things wrong with it. What are they?

Definition 1. A *flow network* is a directed graph $G = (V, E)$ with

- a distinguished *source* vertex $s \in V$, with indegree 0;
- a distinguished *sink* or *target* vertex $t \in V$, with outdegree 0;
- a *capacity function* $c : E \to \mathbb{R}^+$ assigning a non-negative real number capacity $c(e)$ to each edge $e \in E$.

6 Is graph A a flow network? Why or why not?

Now let's define a *flow*. Both graphs B and C depict valid flows on A; graph D does not.

Definition 2. A *flow* on a flow network G is a function $f : E \to \mathbb{R}^+$ assigning a non-negative flow $f(e)$ to each edge, such that

1. _____ $\leq f(e) \leq$ _____ for every $e \in E$

2. At each vertex $v \in V$ other than s and t, _____

   _____.

Make sure graphs B and C are valid flows according to your definition, and that there are two different reasons why D is invalid according to your definition.

Definition 3. The *value* of a flow, $v(f)$, is the sum of the flow on all edges leaving s.

7 What is the value of the flow on graph B?

8 What is the value of the flow on graph C?

9 Make a conjecture about the relationship between the value of a flow and the amount of flow entering t.

10 For each amount, say whether you can construct a flow on graph A with the given value.

(a) 15

(b) 40

(c) 30

11 What is the value of the biggest flow you can construct on graph A?

12 (Bonus question) Brainstorm how you might create an algorithm to find the biggest possible flow for a given flow network.