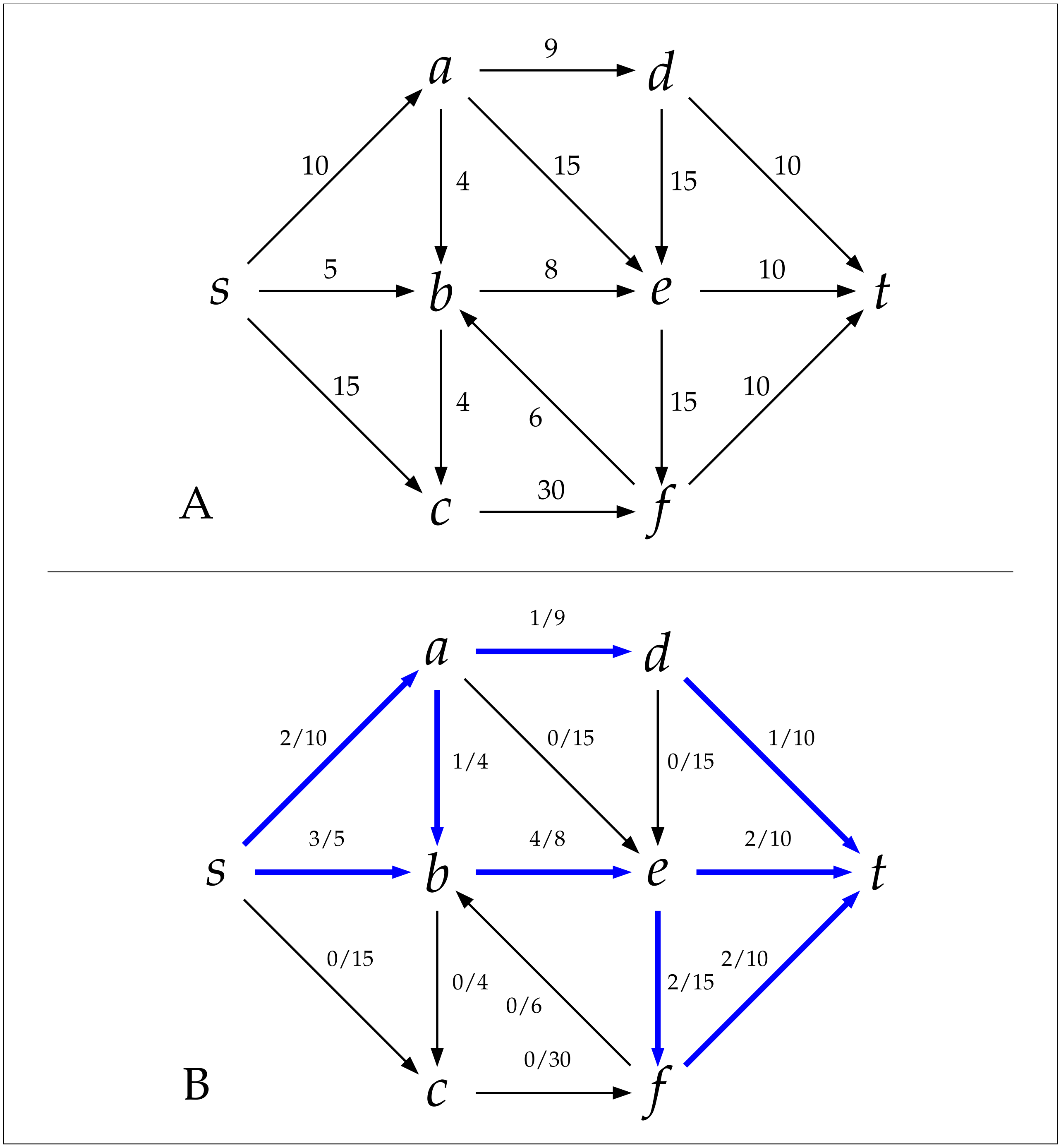
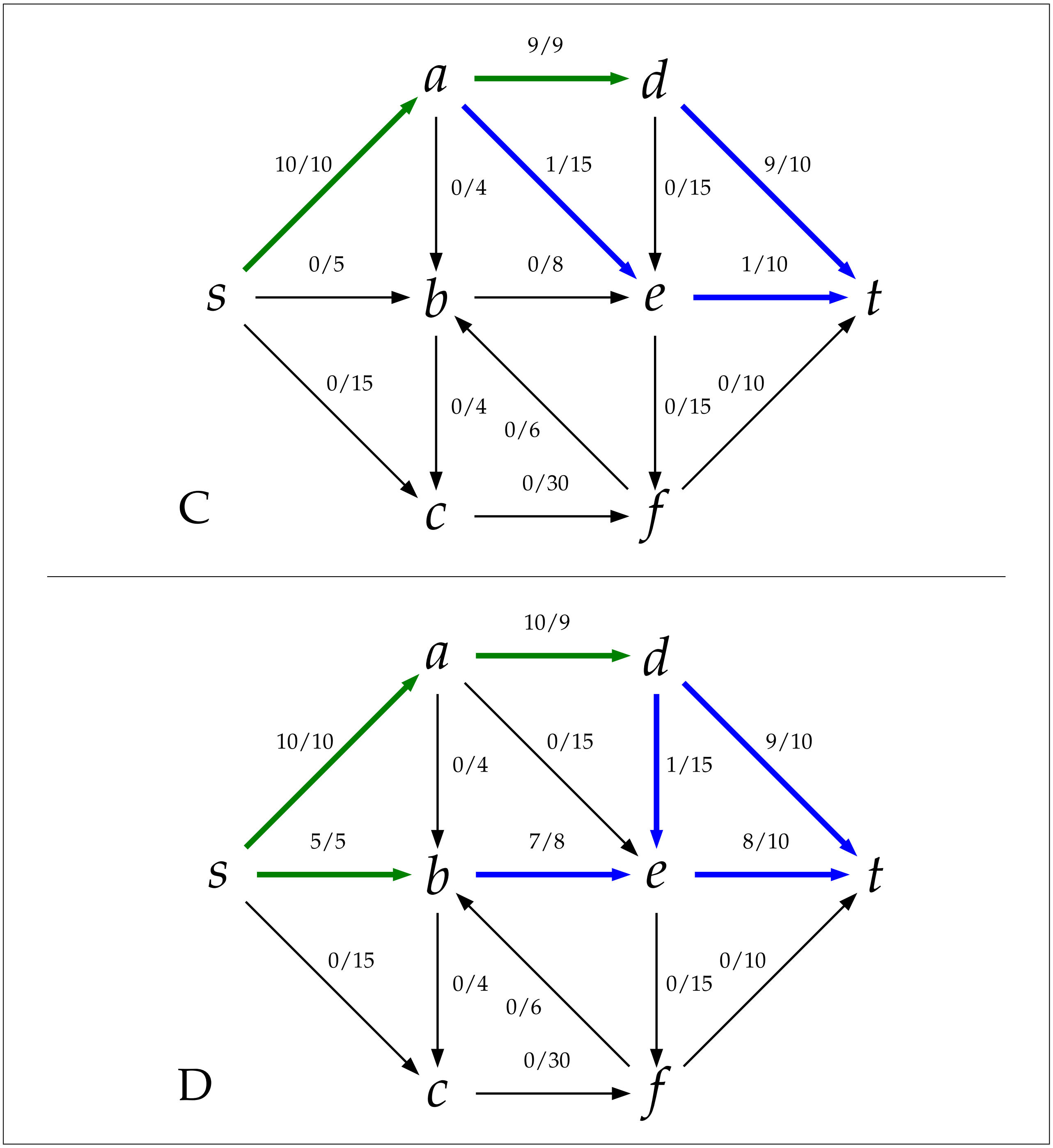
Algorithms: Introduction to Flow Networks

Model 1: Networks and flows



Model 1: (continued)



Consider graph A. Once again we have a directed graph with weighted edges. However, instead of thinking of the weights as some sort of length, we will now think of them as a *capacity*: the "maximum amount of stuff" that the edge can carry. For example, the capacity might be used to model things like:

- maximum gallons of water per minute that can flow through a pipe;
- maximum number of trucks per hour that can drive along a road; or
- maximum number of times a certain resource can be used before it is all used up.

1 Consider graph B. How is it related to graph A?

2 What do the blue edges in graph B all have in common?

3 What do you think the labels on the edges of graph B represent?

4 Now consider graph C. Why do you think some of the edges are green?

5 Graph D is invalid! In fact, there are two things wrong with it. What are they?

Definition 1. A *flow network* is a directed graph $G = (V, E)$ with

- a distinguished *source* vertex $s \in V$, with indegree 0;
- a distinguished *sink* or *target* vertex $t \in V$, with outdegree 0;
- a *capacity function* $c : E \to \mathbb{R}^+$ assigning a non-negative real number capacity $c(e)$ to each edge $e \in E$.

6 Is graph A a flow network? Why or why not?

Now let's define a *flow*. Both graphs B and C depict valid flows on A; graph D does not.

Definition 2. A *flow* on a flow network G is a function $f : E \to \mathbb{R}^+$ assigning a non-negative flow $f(e)$ to each edge, such that

1. ____________ $\leq f(e) \leq$ ____________ for every $e \in E$
2. At each vertex $v \in V$ other than s and t, ______________________

   ____.

Make sure graphs B and C are valid flows according to your definition, and that there are two different reasons why D is invalid according to your definition.

Definition 3. The *value* of a flow, $v(f)$, is the sum of the flow on all edges leaving s.

7 What is the value of the flow on graph B?

8 What is the value of the flow on graph C?

9 Make a conjecture about the relationship between the value of a flow and the amount of flow entering t.

10 For each amount, say whether you can construct a flow on graph *A* with the given value.

(a) 15

(b) 40

(c) 30

11 What is the value of the biggest flow you can construct on graph *A*?

12 (Bonus question) Brainstorm how you might create an algorithm to find the biggest possible flow for a given flow network.