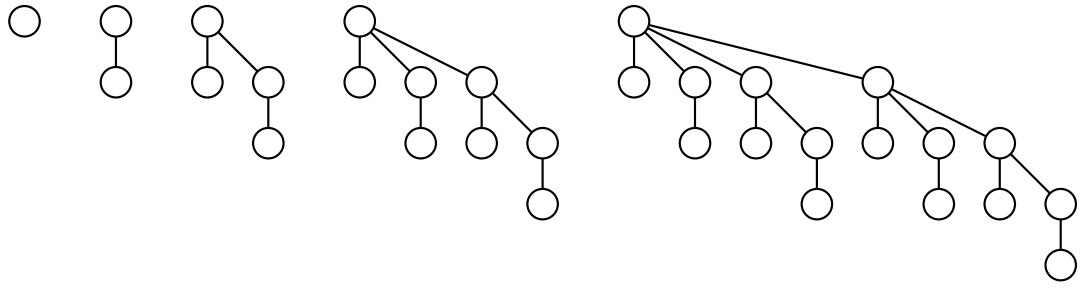
Algorithms: Binomial Heaps

Model 1: Binomial Trees

Learning objective: Students will describe binomial heaps and analyze the amortized running time for heap operations such as INSERT, DELETE-MIN, and MERGE.

Definition 1. A *binomial tree* of order n (for $n \geq 0$) consists of a root node with n subtrees. The leftmost subtree is of order 0, the next subtree is of order 1, and so forth, with the rightmost subtree being of order $n - 1$.

The image below depicts a series of *binomial trees* in increasing order. The leftmost tree is order 0, the next one is order 1, the third one is order 2, the fourth one is order 3, and the final tree is order 4.



1 What patterns do you notice? Write down at least three observations.

2 Send your reporter to some other groups to share your observations and collect as many additional observations as possible.

3 If it was not already among your observations from the previous question, explain how we can make a binomial tree of order n from two binomial trees of order $n - 1$.

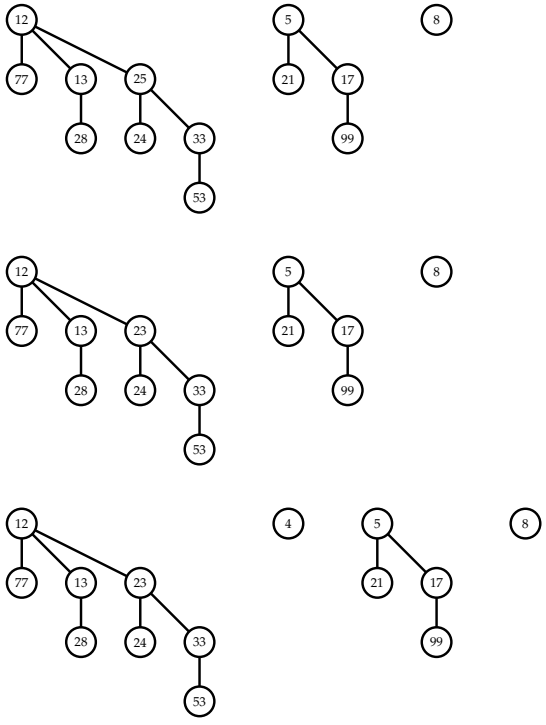
4 Assuming a binomial tree stores a root value and a list of children, how long does this operation (making two order-$(n-1)$ trees into an order-n tree) take?

5 How many total nodes does a binomial tree of order n have?
Why?

Model 2: Binomial Heaps

Definition 2. A *binomial heap* is a list of binomial trees such that:

- Each binomial tree satisfies the heap property, i.e., each node's value is less than or equal to the values of all its children.
- There is at most one binomial tree of any given order.



6 Two of the binomial heaps shown above are invalid, and one is valid. Which is which? Cross out the invalid ones.

7 How many total nodes does the valid binomial heap in Model 2 contain?

8 Draw a valid binomial heap with...

(a) 5 nodes

(b) 8 nodes

(c) 11 nodes

9 The table below displays total number of nodes (n) across the top row; each column records the number of trees of each order needed to make a binomial heap of a certain size. The number of trees of each order needed to make binomial heaps of sizes 0, 1, and 2 have already been filled in for you. Fill in the rest of the table.

n	0	1	2	3	4	5	6	7	8
Order 0 trees	0	1	0						
Order 1 trees	0	0	1						
Order 2 trees	0	0	0						
Order 3 trees	0	0	0						

10 What is the relationship between the number of elements in a binomial heap and the orders of the binomial trees that it contains?

11 If a binomial heap has a total of n elements, what is the maximum number of binomial trees it could contain?