INTRODUCTION TO GEOMETER'S SKETCHPAD

Participants are introduced to the basic applications of Geometer's Sketchpad.

Lesson Goals
❑ Learn ways to integrate Geometer's Sketchpad software into a mathematics classroom ❑ Use Geometer's Sketchpad to create fractals ❑ Use Geometer's Sketchpad to model basic geometry constructions

Word Bank	
❑ Geometer's Sketchpad	❑ Duplication
❑ Geometry Constructions	❑ Perpendicular Bisector
❑ Straightedge	❑ Angle
❑ Compass	❑ Fractal
❑ Segment	

ISTE National Educational Technology Standards
❑ I-A: Demonstrate introductory knowledge, skills, and understanding of concepts related to technology ❑ II-A: Design developmentally appropriate learning opportunities that apply technology-enhanced instructional strategies to support the diverse needs of learners ❑ II-C: Identify and locate technology resources and evaluate them for accuracy and suitability ❑ II-E: Plan strategies to manage student learning in a technology-enhanced environment ❑ III-A: Facilitate technology-enhanced experiences that address content standards and student technology standards ❑ III-B: Use technology to support learner-centered strategies that address the diverse needs of students ❑ III-C: Apply technology to develop students' higher order skills and creativity
Mathematics- In Brief
• Create straightedge and compass constructions • Create and understand fractals

Straightedge and Compass Constructions

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FRACTAL CREATION AND INVESTIGATION
A PROJECT

When you find the next term in a given sequence by applying a rule to the term before it, you are using **recursive rules**. Some picture patterns are generated by recursive rules. You find the next picture in the sequence by looking at the picture before it and comparing that to the picture before that, and so on. Geometer Sketchpad (a computer application) allows you to create scripts that can have recursive steps. These recursive steps tell the script to replay itself on designated parts of the script. You can use scripts with recursive steps to create different stages of geometric figures called **fractals**. Some fractals have strange properties, such as an infinite perimeter surrounding a finite area. Fractals exhibit **self-similarity**, meaning that if you zoom in on one part of the figure, it looks like the whole figure. A true fractal is a theoretical figure---the recursive rule is applied infinitely many times. Making an actual fractal would require infinite time and a computer with infinite memory. In this project, you will use a Sketchpad script to create the first few stages of a fractal called **Sierpinski's Triangle** also known as the **Sierpinski Gasket.**

Part I: Construction

Step 1 Use the segment toolbox to construct a triangle. Use the letter "A" toolbox to label one point A, the second point B and the last point C.

Step 2 Using the arrow toolbox, go over to the sketch area and click in empty space so that nothing is highlighted. Then click on segment AB, BC and AC. Go to the Construct menu at the top of the screen and click on Midpoint. This should create a midpoint on each of the segments.

Step 3 Using the letter "A" toolbox, click on the midpoint of AB. Double click on it and change the label to F. Click on the midpoint of BC. Double click on it and change the label to D. Click on the midpoint of AC. Double click on it and change the label to E.

Step 4 Using the arrow, click into empty space again so that nothing is highlighted. Then click on points A, B and C. Go to the Transform menu at the top of the screen and click on Iterate. A box should appear.

Step 5 Click on point F, then point B and then point D so that the box reads:

Pre-Image	to	First Image
A	$\Rightarrow$	F
B	$\Rightarrow$	B
C	$\Rightarrow$	D

Step 6 Underneath all this is a place to click on the word "Structure." Click on it. Go down to Add New Map and click on it.

Step 7 Now click on points E, D and then C. Your table should now read:

Pre-Image	to	Map #2	Map #1
A	$\Rightarrow$	E	F
B	$\Rightarrow$	D	B
C	$\Rightarrow$	C	D

Step 8 Click on “Structure” again. Go down to Add New Map and click on it again. Click on points A, F, and E. Now your table should read:

Pre-Image	to	Map #3	Map #2	Map #1
A	⇒	A	E	F
B	⇒	F	D	B
C	⇒	E	C	D

Step 9 Next to the “Structure” button there is a button that says “Display.” Click on it. Go down to Decrease Iterations and click on it twice to decrease the iterations from 3 to 1.

Step 10 Click on the Iterate button at the bottom of the box.

Step 11 Using the arrow tool, click in space so that nothing is highlighted. Then click on points D, E and F. Go to the Construct menu at the top of the page and choose Triangle Interior.

Step 12 Using the letter “A” toolbox, go to the sketch area and drag a box underneath the picture you just created. Type in the box “Stage 1.” You just created Stage 1 of Sierpinski’s Triangle also known as Sierpinski’s Gasket. It is a big triangle (ΔABC) with a center triangle (ΔDEF) cut out of it.

The next steps will help you to create Stages 2, 3 and 4.

Step 13 Using the arrow tool, drag a big square around the Stage 1 picture you just created, go to the Edit menu at the top of the page and choose Copy. Then go to the Edit menu again and choose paste. Click on the image you just created and drag it to an empty area near your Stage 1 picture. If necessary, change the name of the points to A, B, and C.

Step 14 Repeat steps 4 through 8.

Step 15 Click on the Display button. Go down to Decrease Iterations and click on it twice to decrease the iterations from 3 to 1. Click on the Iterate button at the bottom of the box.

Step 16 Using the letter “A” toolbox, go to the sketch area and drag a box underneath the picture you just created. Type in the box “Stage 2”

Step 17 Repeat step 13 and 14.

Step 18 Click on the Display button. Go down to Decrease Iterations and click on it once to decrease the iterations from 3 to 2. Click on the Iterate button at the bottom of the box.

Step 19 Using the letter “A” toolbox, go to the sketch area and drag a box underneath the picture you just created. Type in the box “Stage 3”

Step 20 Repeat steps 13 & 14

Step 21 This time you don’t have to click on the Display button. Just click on Iterate in the bottom corner.

Step 22 Using the letter “A” toolbox, go to the sketch area and drag a box underneath the picture you just created. Type in the box “Stage 4”

You have now created Stages 1 – 4 of Sierpinski’s Gasket. Let’s go back and create Stage 0.

Step 23 Repeat Step 13

Step 24 Using the arrow tool, click into empty space so that nothing is highlighted. Then click on points D, E and F. Go to the Edit menu and choose Cut. Points D, E and F along with that center triangle should disappear. All that should remain is the original ΔABC.

Step 25 Using the letter "A" toolbox, go to the sketch area and drag a box underneath te picture you just created. Type in the box "Stage 0"

To Print Your Work…

Finally, go to the File menu at the top of the page. Go down and choose Print Preview. If your pictures are on two pages, then go to the top of the screen and click on "Fit to one page". Then click Print at the top of the page. You should get a print out of Stages 0 – 4 of Sierpinski's Gasket. ☺

Part II: Investigation

1. Use the table provided to record the number of (shaded) triangles at each gasket stage.

2. Consider your data for the number of shaded triangles and answer the questions below.

   a) Predict how many (shaded) triangles will make up a Stage 5 gasket. Explain your reasoning.
   b) Determine how many (shaded) triangles will make up a Stage 60 gasket. Explain your reasoning.

3. Consider a Stage 0 gasket having an area of 1 unit. What would be the area of a Stage 1 gasket? A Stage 2 gasket? Record your answers up to a Stage 4 gasket in the table provided and then answer the questions below.

   a) Predict the shaded area of a Stage 5 gasket. Explain your reasoning.
   b) Determine the shaded area of a Stage 60 gasket. Explain your reasoning.
   c) What would be the shaded area of a stage infinity gasket? Explain.

Part III: Further Research

Go on the Internet and/or to the library and further research fractals. Write a one page summary of what you discover and include a picture example of another fractal besides Sierpinski's Triangle. Be sure to include a bibliography!

What to include when handing in your project:
1. Answer to questions from Part II
2. Data tables and gasket printouts
3. Fractal research from Part III

This project is due: _____

Data Tables

Stage	0	1	2	3	4
Number of triangles (shaded)					

Stage	0	1	2	3	4
Area (shaded)					

SKETCHPAD CONSTRUCTIONS

PART I: DUPLICATING A LINE SEGMENT

1. Click the mouse on the line segment toolbox (located at the left side of screen).
2. Move cursor over to the sketch area and click and drag the mouse. A line segment should appear.
3. Click the mouse on the letter "A" toolbox that enables you to label items. Then click on one of the points on the line segment. A letter label should appear. Double click (if necessary) to change the letter to A.
4. Repeat for the second endpoint. Label this endpoint B. You should now have line segment AB.
5. Click the mouse on the point toolbox and then click the mouse on the sketch area. A point should appear.
6. Click the mouse on the letter "A" toolbox, then click on this point you just created and label it point C.
7. Click on the arrow toolbox. Then, in the sketch area, click in empty space. This will make it so that no points or line segments are highlighted/selected. Now click on segment AB and point C. Both should now be highlighted/selected.
8. Go to the Construct menu and choose Circle by Center and Radius. A circle should appear with point C as the center with the radius the same length as AB.
9. Using the arrow, click in space so that nothing is highlighted. Now click the mouse on the circle. Go to the Construct menu and choose Point on Circle. A point on the circle will appear. (You can drag this point to anywhere on this circle.)
10. Click the mouse on the letter "A" toolbox and label this point D.
11. Click on the arrow toolbox and then in the sketch area click in space so that nothing is highlighted. Then click on point C and D. Go to the Construct menu and choose Segment. Line segment CD will now appear.
12. Using the arrow key, click in space so that nothing is highlighted. Then click on points A and B. Go to the Measure menu at the top of the screen and choose length or distance. The length or distance of line segment AB should appear on the screen.
13. Repeat for line segment CD. Line segments AB and CD should have the same measurement.

You just duplicated a line segment <u>as if</u> you were using a compass (just like we did in class with our compasses). Using other features of Sketchpad, there is a much easier way to duplicate a line. Can you find it?

PART II: CONSTRUCTING PERPENDICULAR BISECTORS

1. Construct line segment AB.

2. Using the arrow key, click in space so that nothing is highlighted/selected. Then click on points A and B in that order. Go to the Construct menu and choose Circle by Center and Point. A circle with center A and radius AB should appear.

3. Using the arrow, click in space so that nothing is selected/highlighted. Then click on points B and A in that order. Go to the Construct menu and choose Circle by Center and Point. A circle with center B and radius AB should appear.

4. Using the arrow, click in space so that nothing is selected. Next, click on the first circle and then on the second circle. Go to the Construct menu and choose Intersection. The two points of intersection for the two circles should appear.

5. Use the label (letter "A") toolbox button and label these two points of intersection C and D.

6. Using the arrow, click in space so that nothing is selected. Then click on points C and D. Go to the Construct menu and choose Segment. Line segment CD should appear.

7. Click in space. Then click on segment AB and segment CD. Go to the Construct menu and choose Intersection. The point of intersection of the two line segments should appear.

8. Use the letter toolbox button and label this point E.

9. Using the arrow, click in space. Then click anywhere on CD, go to the Construct menu and choose Point on Segment. A point should appear Use the letter toolbox button to label this point F.

10. Using the arrow, click on the first circle. Go the Display menu and select Hide Circle. Repeat this step for the second circle.

11. Click in space so that nothing is selected. Then click on points A, E, and F. Go to the measure menu and select Angle. The measure of this angle should appear.

12. Repeat step 11 for points B, E, and F. The measurements of these angles should confirm that CD is perpendicular to AB.

13. Select points A and E. Go to the measure menu and select length or distance. The length or distance of line segment AE should appear.

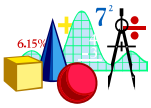
14. Repeat step 13 for points E and B. The length of AE and EB should be the same. This should confirm that CD bisects AB.

15. We have confirmed that CD is a perpendicular bisector of AB.

You have just constructed a perpendicular line segment <u>as if</u> you were using a compass (just like we did in class with our compasses). Using other features of Sketchpad, there is a much easier way to make a perpendicular bisector. Can you find it?

Don't forget to call the teacher over when you finish with duplicating a line segment (finding <u>two</u> ways) and constructing a perpendicular bisector (finding <u>two</u> ways). The teacher will check your constructions, mark off that you completed them and then give you the next sheet.

Sketchpad Constructions



Part I: CREATING AN ANGLE

1. Click on the line segment toolbox.

2. Move cursor over to the sketch area and click and drag the mouse. A line segment should appear. (We are going to use line segments instead of rays for the moment).

3. Still using the segment toolbox, go to one endpoint of the line segment and click and drag the mouse again. A second segment should appear. This creates an angle.

4. Click the mouse on the arrow toolbox and then click the mouse on one of the segments so it is highlighted. Go up to the Construct menu and choose Point on Segment. A point on the segment should appear.

5. Click the mouse in empty space so that nothing is highlighted. Then repeat the previous step, creating a random point on the second segment. You can move these points anywhere along their segment by clicking on the point and moving the arrow along the segment.

6. Click on the letter "A" toolbox that enables you to label items. Then click on one of the points that is on one of the segments. A letter label should appear. Double click to change the label to point A.

7. Repeat for the vertex angle and label it point B.

8. Repeat for the point on the second segment and label it point C. You have now created <ABC.

Part II: DUPLICATING AN ANGLE

1. Using the same sketch of <ABC, click on the segment toolbox and then click and drag in the sketch area. A new segment should appear. Use the label toolbox and label the new line segment PQ.

2. Using the arrow toolbox, first click in space so that nothing is highlighted/selected. Then go to the original <ABC and click on point B (the vertex angle) and point A **IN THAT ORDER.**

3. Go to the Construct menu and choose Circle by Center and Point. A circle should appear with vertex B as the center and it should pass through point A.

4. Click in space again so that nothing is selected/highlighted. Then click on points A and B in that order. Go to the Construct menu and choose segment. This will create a segment between points A and B.

5. Using the arrow key, select both point P and segment AB. Go to the Construct menu and choose Circle by Center and Radius. A circle should appear with center point P and should appear the same size as the circle created on the original angle.

6. Select both the new circle and line segment PQ. Go to the Construct menu and choose Intersection to create a point where the circle and line segment intersect.

7. Use the label toolbox (the letter "A" toolbox) and label this point of intersection point R.

8. Using the arrow, first click in space so that nothing is selected. Then go to <ABC and select both the circle and segment BC. Go to the Construct menu and choose Intersection. Label this point D. Point D should be on line segment BC.

9. Using the arrow toolbox, click in space so that nothing is selected. Then click on point D and point A (in that order). Go to the Construct menu and choose Circle by Center and Point. A circle with center point D should appear and it should pass through point A.

10. Click in space. Then select points D and A. Go to the Construct menu and choose segment. Line segment DA should appear.

11. Using the arrow, click in space so that nothing is selected. Then select point R and the line segment DA that you just created. Go to the Construct menu and choose Circle by Center and Radius. A circle should appear with center R and it should appear to be the same size as circle DA on the original angle.

12. Click in space so that nothing is selected. Then click on the two circle that have been created on line segment PQ. Go to the Construct menu and choose Intersection.

13. Label one of the points of intersection point S.

14. Select points P and S. Go to the Construct menu and choose segment. Line segment PS should appear.

15. Using the arrow, click in space so that nothing is selected. Then click on points A,B, and C in that order. Go to the Measure menu and choose angle. The degree measurement of <ABC should appear on your screen.

16 Repeat for <SPQ. Angles <SPQ and <ABC should have the same measurement.

You just duplicated an angle <u>as if</u> you were using a compass. Using other features of Sketchpad, there is a much easier way to duplicate an angle. Can you find it?

Part III: <u>INVESTIGATING WITH A PARTNER OR ON YOUR OWN</u>

If you are sharing a computer with another student, then you may continue to work with them. If not, you may work alone or join another student if you'd like.

1. Using Sketchpad, investigate how to construct a perpendicular through a given point NOT on the given line. You must find two different methods. At least one of the methods must be as if you were using a compass. Print out your final sketches and write up a step-by-step description of each method. You may want to type out your steps before you print out your sketch…or you may want to write them separate from your sketch. It is up to you.

2. Use Sketchpad to investigate how to construct a perpendicular through a given point ON the given line. Again, you must find two different methods, and at least one of the methods must be as if you were using a compass. Print out your final sketches and write up a step by step description of each method. You may want to type out your steps before you print your sketch…or you may want to write them separate from your sketch. It is up to you.

My Geometer's Sketchpad Notes and Pictures