Introduction Properties of Triangles

A. What do we know about the sides of a triangle?

1. a. Can you make a triangle with sides of length 2 inches, 3 inches, and 6 inches?
   b. Use the pieces of length 2 inches and 3 inches to form two sides of a triangle. What happens to the length of the third side as you increase the angle between the first two sides?
   c. What is the longest that the third side could be? What is the smallest?
2. Two sides of a triangle are 6 centimeters and 8 centimeters long. What are the possible lengths of the third side?
3. Two sides of a triangle are p units and q units long. What are the possible lengths of the third side?

B. What do we know about the angles of a triangle?

1. Use the protractor to measure the three angles of the paper triangle in degrees. Now add up the three angles. What is their sum?
2. Tear off the three corners of the triangle. Place them side-by-side with their vertices (tips) at the same point. What do you find?
3. How are your answers to parts (1) and (2) related?

C. How are the sides and angles of a triangle related?

1. A standard way to label a triangle is to call the angles A, B, and C. The side opposite angle A is called a, the side opposite angle B is called b, and the side opposite angle C is called c. Sketch a triangle and label it with standard notation.
2. Using the ruler, carefully draw a triangle and label it with standard notation so that $a > b > c$. Now use the protractor to measure the angles and list them in order from largest to smallest. What do you observe?
3. Using the ruler, carefully draw another triangle and label it with standard notation so that $A > B > C$. Now use the ruler to measure the sides and list them in order from largest to smallest. What do you observe?

D. What do we know about right triangles?

1. The side opposite the $90°$ angle in a right triangle is called the **hypotenuse**. Why is the hypotenuse always the longest side of a right triangle?
2. The Pythagorean theorem states that:

   IF: a, b, and c are the sides of a right triangle, and c is the hypotenuse,
   THEN: $a^2 + b^2 = c^2$

   The "if" part of the theorem is called the **hypothesi**s, and the "then" part is called the **conclusion**. The **converse** of a theorem is the new statement you obtain when you interchange the hypothesis and the conclusion. Write the converse of the Pythagorean theorem.
3. The converse of the Pythagorean theorem is also true. Use the converse to decide whether each of the following triangles is a right triangle. Support your conclusions with calculations.

   a. $a = 9,\ b = 16,\ c = 25$ b. $a = 12,\ b = 16,\ c = 20$
   a. $a = \sqrt{8},\ b = \sqrt{5},\ c = \sqrt{13}$ b. $a = \frac{\sqrt{3}}{2},\ b = \frac{1}{2},\ c = 1$

Activity I Trigonometric Ratios

A. Using Ratios and Proportions

1. Two related quantities or variables are **proportional** if their ratio is always the same.
   a. On any given day, the cost of filling up your car's gas tank is proportional to the number of gallons of gas you buy. For each purchase below, compute the ratio

$$\frac{\text{total cost of gasoline}}{\text{number of gallons}}$$

Gallons of Gas Purchased	Total Cost	$\frac{\text{Dollars}}{\text{Gallon}}$
5	\$14.45	
12	\$34.68	
18	\$52.02	

   b. Write an equation that you can solve to answer the question: How much does 21 gallons of gas cost? Use the ratio $\frac{\text{Dollars}}{\text{Gallon}}$ in your equation.
   c. Write an equation that you can solve to answer the question: How many gallons of gas can you buy for \$46.24? Use the ratio $\frac{\text{Dollars}}{\text{Gallon}}$ in your equation.

2. A recipe for coffee cake calls for $\frac{3}{4}$ cup of sugar and $1\frac{3}{4}$ cup of flour.
   a. What is the ratio of sugar to flour? Write your answer as a common fraction, and then give a decimal approximation rounded to four places.

   For parts (b) and (c) below, write an equation that you can solve to answer the question. Use the ratio $\frac{\text{Amount of sugar}}{\text{Amount of flour}}$ in your equation.

   b. How much sugar should you use if you use 4 cups of flour? Compute your answer two ways: writing the ratio as a common fraction, and then writing the ratio as a decimal approximation. Are your answers the same?
   c. How much flour should you use if you use 4 cups of sugar? Compute your answer two ways: writing the ratio as a common fraction, and then writing the ratio as a decimal approximation. Are your answers the same?

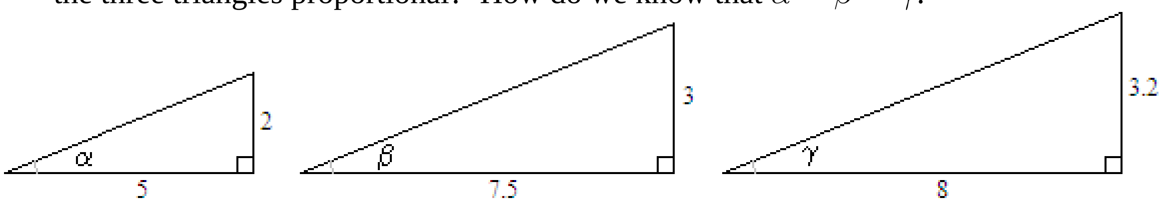
3. You are making a scale model of the Eiffel tower, which is 324 meters tall and 125 meters wide at its base.
   a. Compute the ratio of the width of the base to the height of the tower. Round your answer to four decimal places.

   Use your ratio to write equations and answer the questions below:

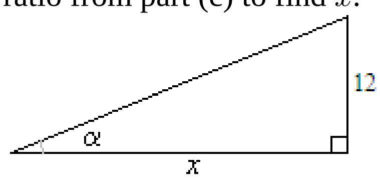
   b. If the base of your model is 8 inches wide, how tall should the model be?
   c. If you make a larger model that is 5 feet tall, how wide will the base be?

B. Similar Triangles

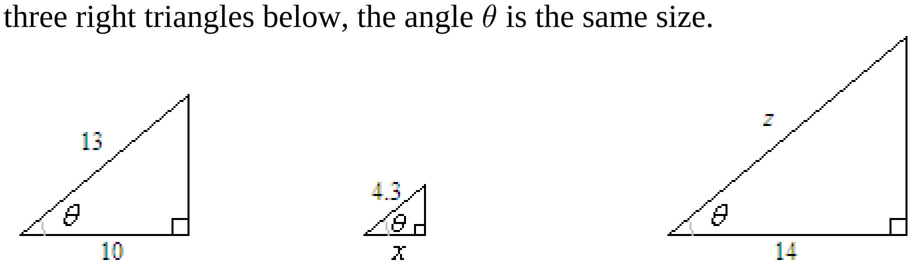
1. Recall that two triangles are **similar** if their corresponding sides are proportional. The corresponding angles of similar triangles are equal.
   a. What is the ratio of the two given sides in each triangle? Are the corresponding sides of the three triangles proportional? How do we know that $\alpha = \beta = \gamma$?



   b. Find the hypotenuse of each right triangle.
   c. Use the sides of the appropriate triangle to compute sin α, sin β, and sin γ. Round your answers to four decimal places. Does the sine of an angle depend on the lengths of its sides?
   d. How do you know that the triangle below is similar to the three triangles in part (a)? Write an equation using the ratio from part (c) to find x.

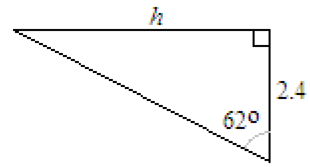


2. In the three right triangles below, the angle θ is the same size.

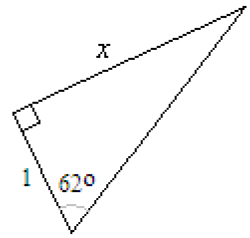


   a. Use the first triangle to calculate cos θ. Round your answer to four decimal places.
   b. In the second triangle, explain why $\dfrac{x}{4.3} = \dfrac{10}{13}$. Write an equation using your answer to part (a) and solve it to find x.
   c. Write and solve an equation to find z in the third triangle.

3. a. Use your calculator to find the value of $\dfrac{h}{2.4}$.
(*Hint*: Which trig ratio should you use?)
What is the length of side h?



b. What is the value of $\dfrac{6}{w}$ for the triangle below left? Write an equation and solve for w.



c. Write an equation and solve it to find x in the triangle above right.

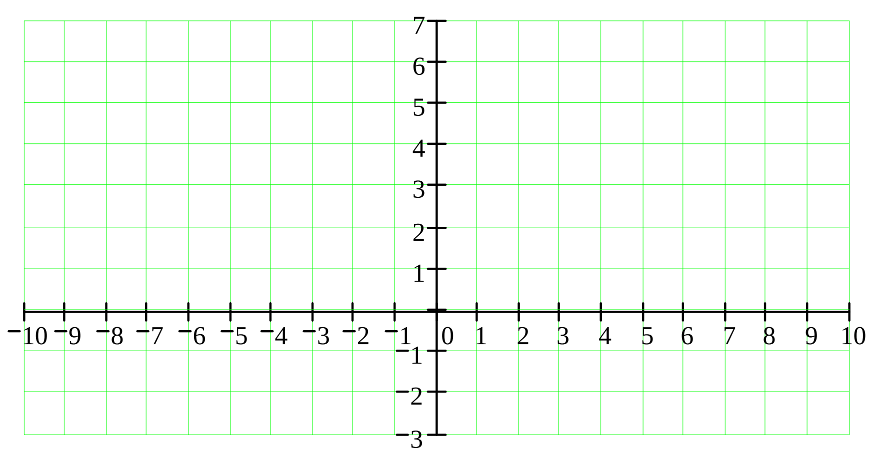
Activity II Trigonometric Functions

Recall that we extend our definitions of the trigonometric ratios to all angles as follows. Place the angle θ in standard position and choose a point P with coordinates (x, y) on the terminal side. The distance from the origin to P is $r = \sqrt{x^2 + y^2}$. The trigonometric ratios of θ are defined as follows.

The Trigonometric Ratios

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

A. Second-Quadrant Angles

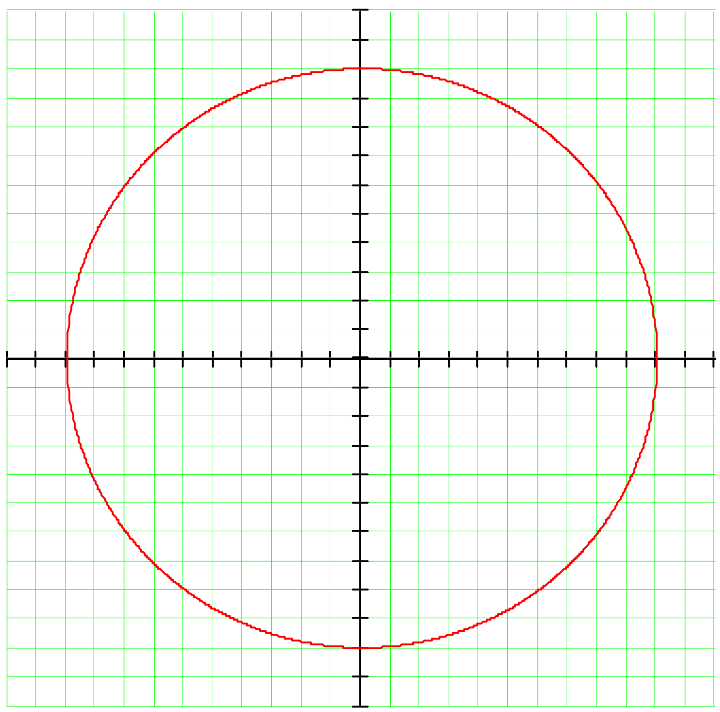


1. Draw an angle θ in standard position with the point $P(6, 4)$ on its terminal side.
2. Find r, the distance from the origin to P.
3. Calculate $\sin\theta$, $\cos\theta$, and $\tan\theta$. Give both exact answers and decimal approximations rounded to four places.
4. Use the inverse cosine key on your calculator to find θ. Use your calculator to verify the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ that you found in part (3).

5. Draw another angle ϕ in standard position with the point $Q(-6, 4)$ on its terminal side. Explain why ϕ is the supplement of θ. (*Hint*: Consider the right triangles formed by drawing vertical lines from P and Q.)
6. Can you use the right triangle definitions (using opposite, adjacent and hypotenuse) to compute the sine and cosine of ϕ? Why or why not?
7. Calculate sin ϕ, cos ϕ, and tan ϕ using the extended definitions listed above. How are the trig values of ϕ related to the trig values of θ?

8. Explain why θ and ϕ have the same sine but different cosines.
9. Use the inverse cosine key on your calculator to find ϕ. Use your calculator to verify the values of sin ϕ, cos ϕ, and tan ϕ that you found in part (6).
10. Compute $180° - \phi$. What answer should you expect to get?

B. Reference Angles



1. Use a protractor to draw an angle of $56°$ in standard position. Draw its reference triangle.
2. Use your calculator to find the sine and cosine of $56°$, rounded to two decimal places. Label the sides of the reference triangle with their lengths.
3. What are the coordinates of the point P where your angle intersects the unit circle?
4. Draw the reflection of your reference triangle across the y-axis, so that you have a congruent triangle in the second quadrant.
5. You now have the reference triangle for a second-quadrant angle in standard position. What is that angle?
6. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point Q where the angle intersects the unit circle.
7. Draw the reflection of your triangle from part (4) across the x-axis, so that you have a congruent triangle in the third quadrant.
8. You now have the reference triangle for a third-quadrant angle in standard position. What is that angle?

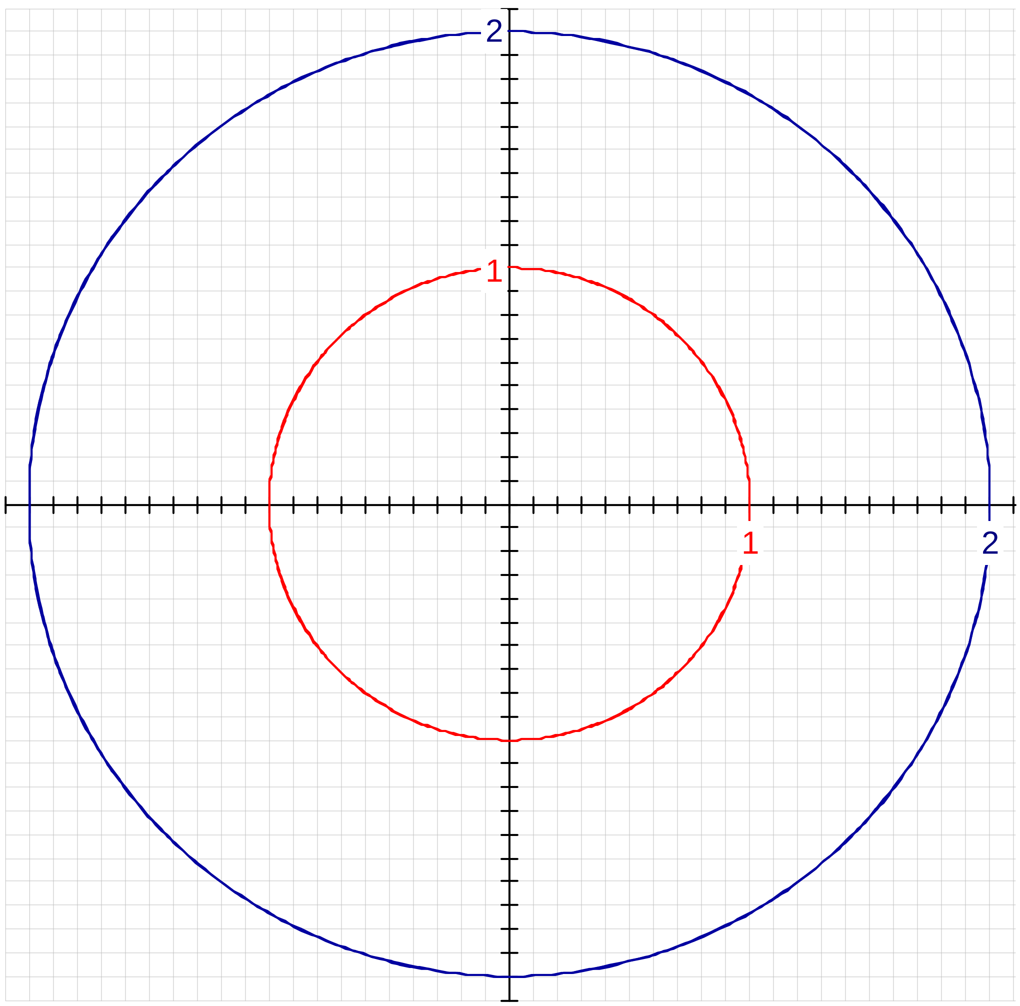
9. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point R where the angle intersects the unit circle.
10. Draw the reflection of your triangle from part (7) across the y-axis, so that you have a congruent triangle in the fourth quadrant.
11. You now have the reference triangle for a fourth-quadrant angle in standard position. What is that angle?
12. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point S where the angle intersects the unit circle.

Generalize: All four of your angles have the same reference angle, $56°$. For each quadrant, write a formula for the angle whose reference angle is θ.

Quadrant I: Quadrant II:

Quadrant III: Quadrant IV:

C. Unit Circles



1. Use a protractor to draw an angle $\theta = 36°$ in standard position.
   a. Estimate the coordinates of the point P where the terminal side of the angle intersects the circle of radius $r = 2$.
   a. Calculate approximate values for $\cos\theta$ and $\sin\theta$ using the coordinates of P.
   c. Estimate the coordinates of the point Q where the terminal side of the angle intersects the circle of radius $r = 1$.

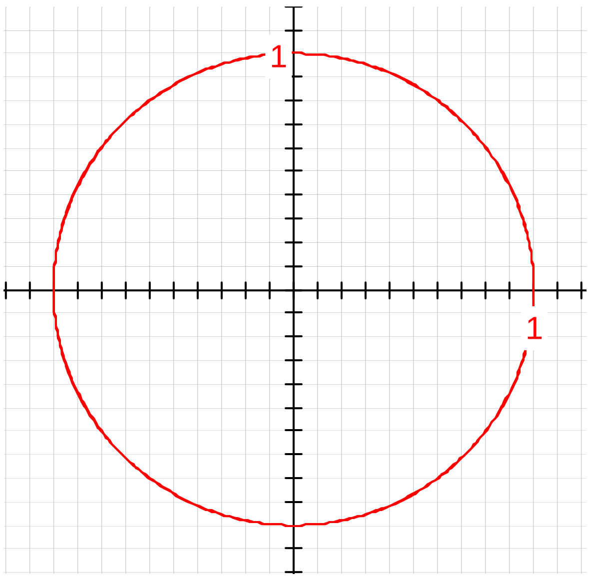
d. Calculate approximate values for $\cos\theta$ and $\sin\theta$ using the coordinates of Q.

2. Use a protractor to draw an angle $\theta = 107°$ in standard position. Repeat parts (a)-(d) for this new angle.

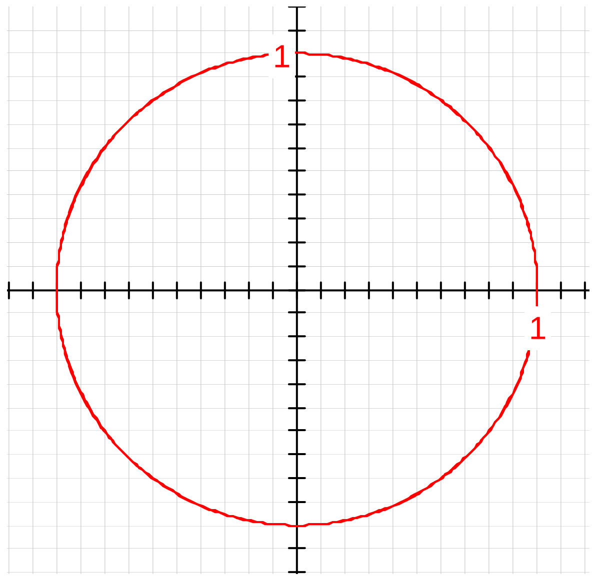
3. Use a protractor to draw an angle $\theta = 212°$ in standard position. Repeat parts (a)-(d) for this new angle.

4. Use a protractor to draw an angle $\theta = 325°$ in standard position. Repeat parts (a)-(d) for this new angle.

5. What do you notice about the coordinates of the point located on the unit circle by an angle and the values of the trig ratios of that angle?



Use this grid for #6 and #7



Use this grid for #8 and #9

6. Draw two different angles α and β in standard position whose sine is 0.6.
   a. Use a protractor to measure α and β.
   b. Find the reference angles for both α and β. Draw in the reference triangles.

7. Draw two different angles θ and ϕ in standard position whose sine is -0.8.
   a. Use a protractor to measure θ and ϕ.
   b. Find the reference angles for both θ and ϕ. Draw in the reference triangles.

8. Draw two different angles α and β in standard position whose cosine is 0.3.
   a. Use a protractor to measure α and β.
   b. Find the reference angles for both α and β. Draw in the reference triangles.

9. Draw two different angles θ and ϕ in standard position whose cosine is -0.4.
   a. Use a protractor to measure θ and ϕ.
   b. Find the reference angles for both θ and ϕ. Draw in the reference triangles.

10. a. If you know one angle whose sine is a given positive number, how can you find the other angle?

b. If you know one angle whose sine is a given negative number, how can you find the other angle?
c. If you know one angle whose cosine is a given positive number, how can you find the other angle?
d. If you know one angle whose cosine is a given negative number, how can you find the other angle?

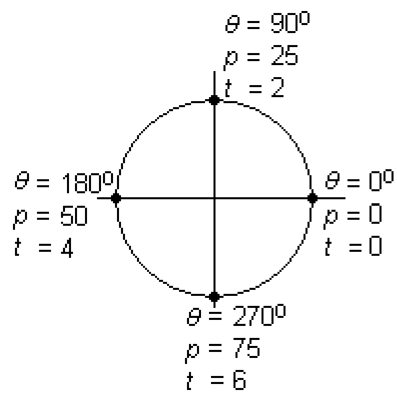
Radians Lesson

A. Arclength

Imagine that you are riding on a Ferris wheel of radius 100 feet, and each rotation takes 8 minutes. We can use angles in standard position to describe your location as you travel around the wheel.

The figure shows the locations indicated by $\theta = 0°, 90°, 180°$, and $270°$. But degrees are not the only way to specify location on a circle. Here are some other ways to designate your location on the Ferris wheel.

Angle in standard position, θ	0°	90°	180°	270°	360°
Percent of one rotation, p	0	25	50	75	100
Time elapsed, t (min)	0	2	4	6	8

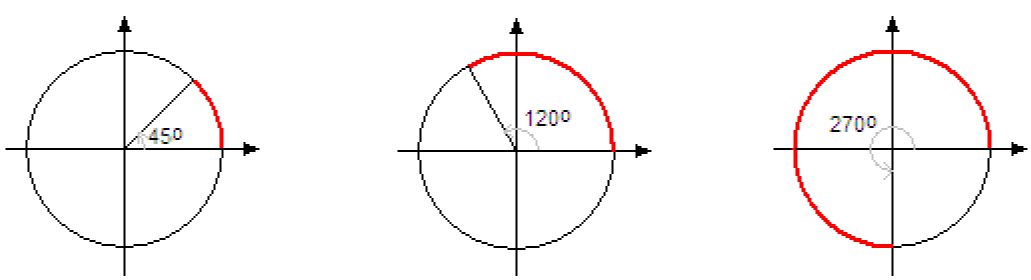


Another useful method uses distance traveled, or arclength, along the circle. How far have you traveled around the Ferris wheel at each of the locations shown? Recall that the circumference of a circle is proportional to its radius,

$$C = 2\pi r$$

If we walk around the entire circumference of a circle, the distance we travel is 2π times the length of the radius, or about 6.28 times the radius. If we walk only part of the way around the circle, then the distance we travel depends also on the angle we cover.

1. a. What fraction of a whole circle does each angle represent?
   b. What is the arclength spanned by each angle on a circle of radius 100 feet?



$$\textbf{Arclength} = \textbf{(fraction of one revolution)} \cdot (2\pi r)$$

B. Radian Measure

Notice that the degree measure of the spanning angle is not as important as the fraction of one revolution it covers. This suggests a new unit of angle measurement that is better suited to calculations involving arclength. We'll make one change in our formula for arclength, from

$$\textbf{Arclength} = \textbf{(fraction of one revolution)} \cdot (2\pi r)$$

to

$$\textbf{Arclength} = \textbf{(fraction of one revolution} \times 2\pi) \cdot r.$$

We call the quantity in parentheses, (fraction of one revolution $\times\, 2\pi$), the **radian measure** of the angle.

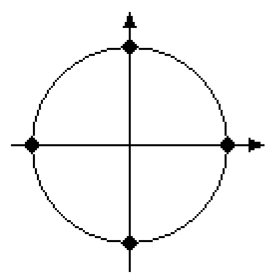
Radians

The **radian measure** of an angle is given by

$$\textbf{fraction of one revolution} \times 2\pi$$

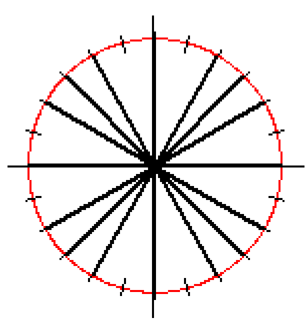
2. Find the radian measure of the quadrantal angles. For each angle, give both an exact value, as a multiple of π, and a decimal approximation.

Degrees	Radians: Exact Values	Radians: Decimal Approximations
$0°$		
$90°$		
$180°$		
$270°$		
$360°$		



3. a. In which quadrant would you find an angle of 2 radians? An angle of 5 radians?
   b. Draw a circle centered at the origin and sketch (in standard position) angles of approximately 3 radians, 4 radians, and 6 radians.

4. Give the radian measure of each angle.



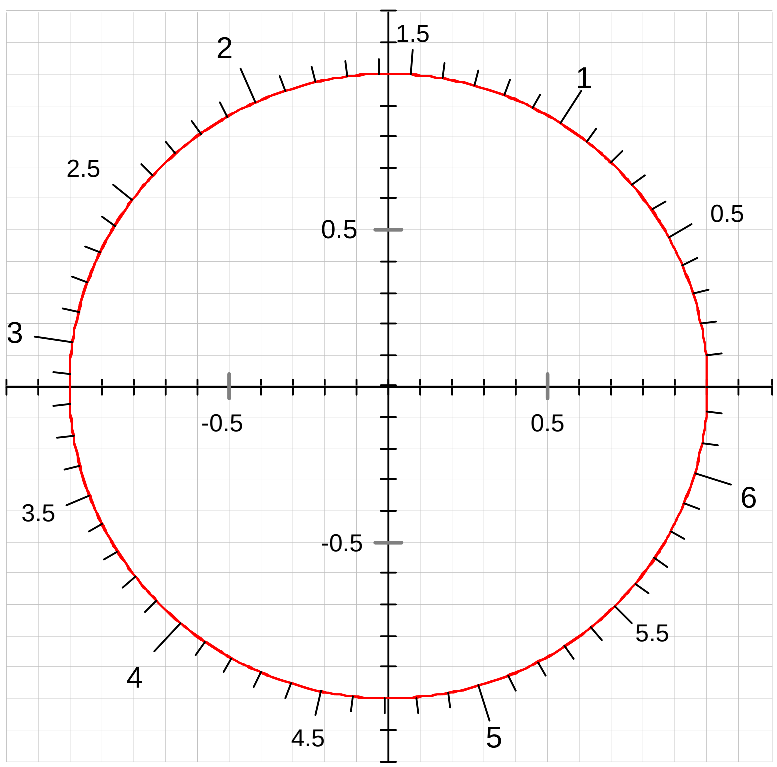
Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians									

Degrees	180°	210°	225°	240°	270°	300°	315°	330°	360°
Radians									

5. a. Use a proportion to find the degree measure of 1 radian.
   b. What ratio can you use to convert degrees to radians? What ratio can you use to convert radians to degrees?

Activity III Radians

A. The Unit Circle



1. Use the unit circle to estimate the sine, cosine, and tangent of each arc of given length.
   a. 0.6 b. 2.3 c. 3.5 d. 5.3

2. Use the unit circle to estimate two solutions to each equation.
   a. $\cos t = 0.3$ b. $\sin t = -0.7$

3. Sketch the angle on the unit circle. Find the reference angle in radians, rounded to two decimal places, and sketch the reference triangle.
   a. 1.8 b. 5.2 c. 3.7

4. Give the degree measure of each angle.

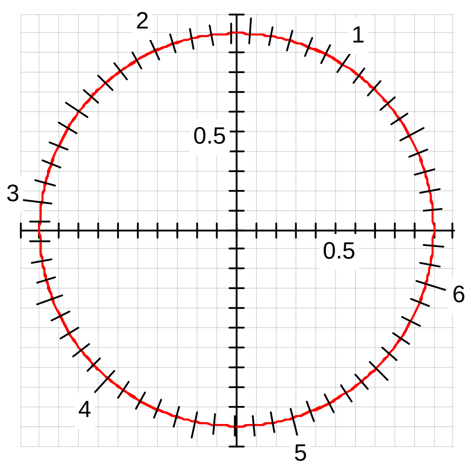
Radians	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
Degrees													

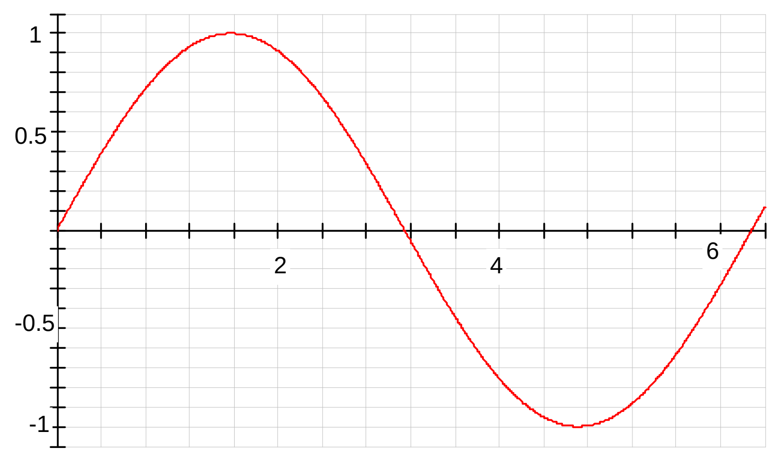
Radians	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
Degrees													

5. On the unit circle above, plot the endpoint of each arc in standard position.

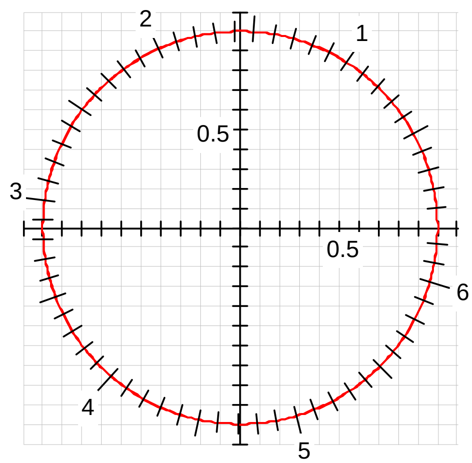
a. $\dfrac{\pi}{3}$ b. $\dfrac{7\pi}{6}$ c. $\dfrac{7\pi}{4}$

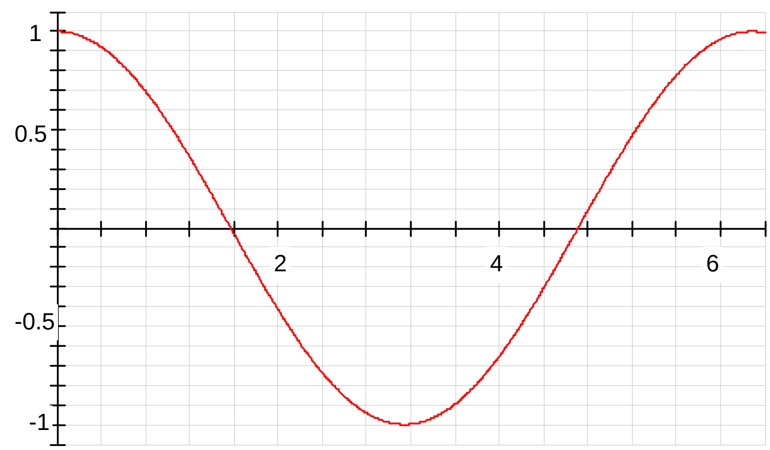
B. The Circular Functions





6. a. Use the graph of $y = \sin x$ to estimate two solutions of the equation $\sin x = 0.65$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\sin x = 0.65$. Show your solutions on the circle.
   a. Use the graph of $y = \sin x$ to estimate two solutions of the equation $\sin x = -0.2$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\sin x = -0.2$. Show your solutions on the circle.

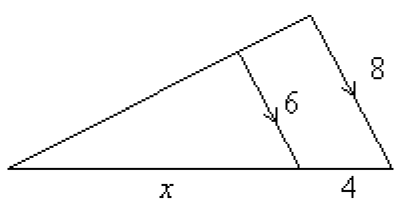




7. a. Use the graph of $y = \cos x$ to estimate two solutions of the equation $\cos x = 0.15$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\cos x = 0.15$. Show your solutions on the circle.
   a. Use the graph of $y = \cos x$ to estimate two solutions of the equation $\cos x = -0.4$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\cos x = -0.4$. Show your solutions on the circle.

Examples of Exercises That You Might Have Hoped Would Be Easy, But In Fact Are Not

1. Can two acute angles be supplementary? Why or why not?
2. For the triangles shown, which of the following equations is true? Explain why.



a. $\frac{4}{x} = \frac{6}{8}$ b. $\frac{x}{4} = \frac{6}{8}$
c. $\frac{x}{x+4} = \frac{6}{8}$ d. $\frac{x}{x+4} = \frac{6}{14}$

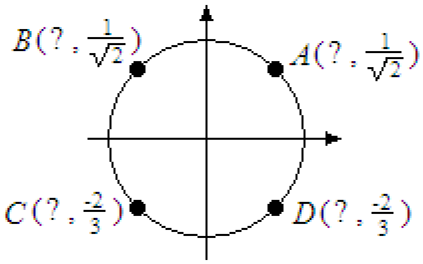
3. a. Find two points on the circle $x^2 + y^2 = 4$ with x-coordinate -1.
   b. Find two points on the circle $x^2 + y^2 = 1$ with y-coordinate $\frac{1}{2}$.

4. a. Write an expression for the distance between the points $(x,\ y)$ and $(-3,\ 4)$.
   b. Write an equation that says "the distance between the points $(x,\ y)$ and $(-3,\ 4)$ is 5 units."

5. How long is the diagonal of a square of side 6 centimeters?
   a. Give an exact answer.
   b. Round your answer to hundredths.

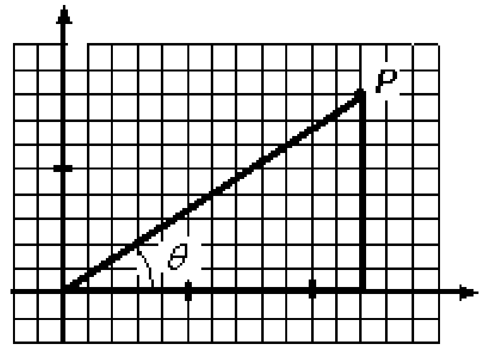
6. a. Explain why the solutions of the equation $x^2 + y^2 = 100$ must have $-10 \le x \le 10$.

b. What does part (a) tell you about the graph of the equation?

7. 

Find the missing coordinates of the points on the unit circle.

8. a. What is the slope of the line through the origin and point P?
   b. What is the tangent of the angle θ?
   c. On the same grid, sketch an angle whose tangent is $\frac{8}{5}$.

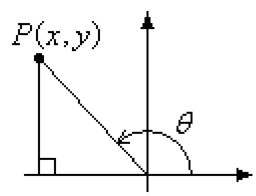


9. Which of the following numbers are equal to cos 45° ?

   a. $\frac{\sqrt{2}}{2}$ b. $\frac{1}{\sqrt{2}}$ c. $\frac{2}{\sqrt{2}}$ d. $\sqrt{2}$

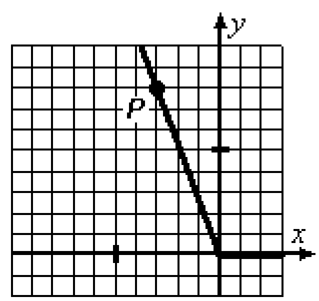
10. Is the acute angle larger or smaller than 60°?
   a. $\cos\theta = 0.75$ b. $\tan\phi = 1.5$ c. $\sin\psi = 0.72$

11. Explain why the length of the horizontal leg of the right triangle shown is $-x$.



12. Use your calculator to evaluate sin 118°, then evaluate $\sin^{-1}$(**ANS**). Explain the result.

13. a. Give the coordinates of point P on the terminal side of the angle.
   b. Find the distance from the origin to point P.
   c. Find $\cos\theta$, $\sin\theta$, and $\tan\theta$.



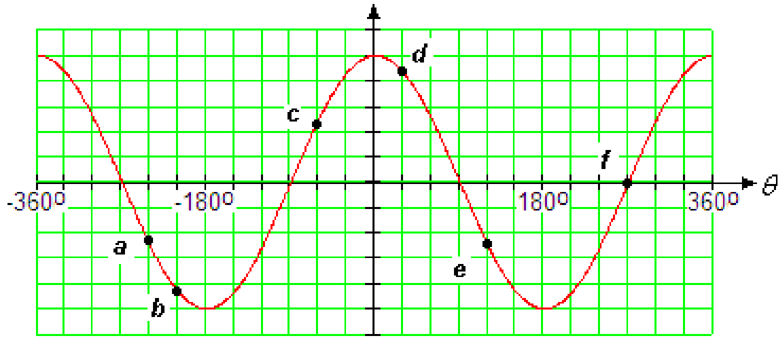
14. a. Draw four angles, one in each quadrant, whose reference angle is 60°.
   b. Find exact values for the sine, cosine, and tangent of each of the angles in part (a).
   c. Find all solutions for $0° \le \theta < 360°$ to the equation $\cos\theta = \frac{\sqrt{3}}{2}$.
   d. Find all solutions for $0° \le \theta < 360°$ to the equation $\cos\theta = \frac{-\sqrt{3}}{2}$.

15. Use the given values to find each trigonometric ratio. Do not use a calculator!

$$\cos 23° = 0.9205 \qquad \sin 46° = 0.7193$$

   a. cos 157° b. sin 226° c. sin 314° d. cos 203°

16. Give the coordinates of each point on the graph of $g(\theta) = \cos\theta$.

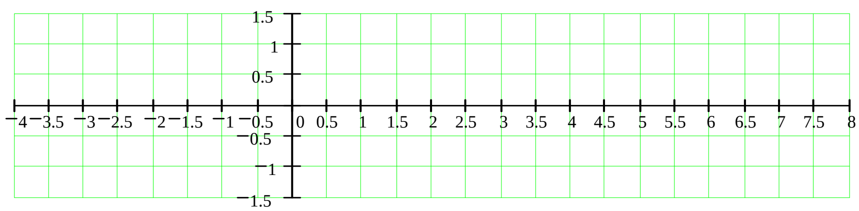


17. Decide whether the expressions are equivalent: $\sin(\theta - \phi)$, $\sin\theta - \sin\phi$

18. Simplify $\cos^2 3\alpha + \sin^2 3\alpha$

19. Scale the x-axis in multiples of $\frac{\pi}{12}$ from 0 to 2π. Reduce each fraction.

20. Sketch a graph of $y = \cos x$.

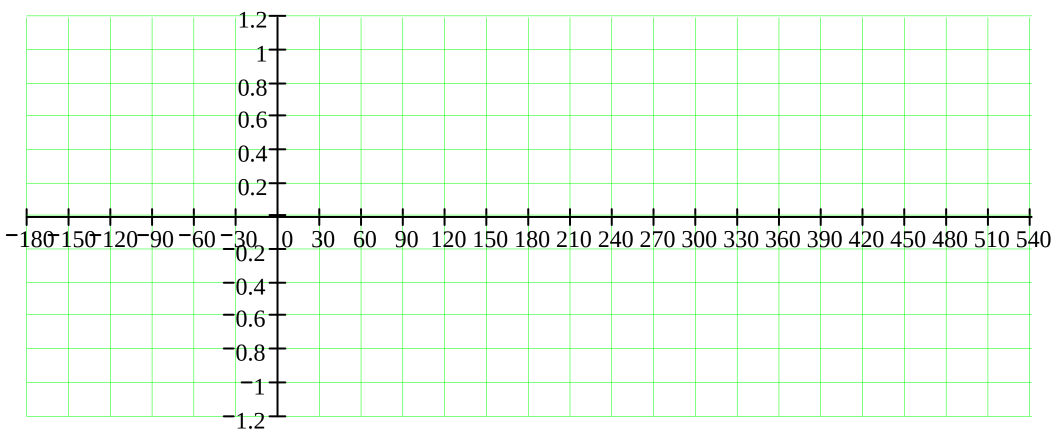


Graphing

Lesson 1 Graphs of Sine and Cosine in Degrees

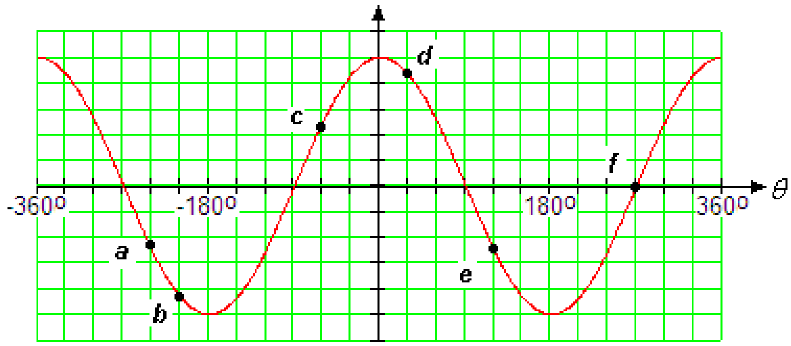
Exercise Complete the table below with values rounded to two decimal places. Use the table and your knowledge of reference angles to graph the cosine function, $f(\theta) = \cos\theta$, from $\theta = -180°$ to $\theta = 540°$.

θ	$0°$	$10°$	$20°$	$30°$	$40°$	$50°$	$60°$	$70°$	$80°$	$90°$
$\cos\theta$										

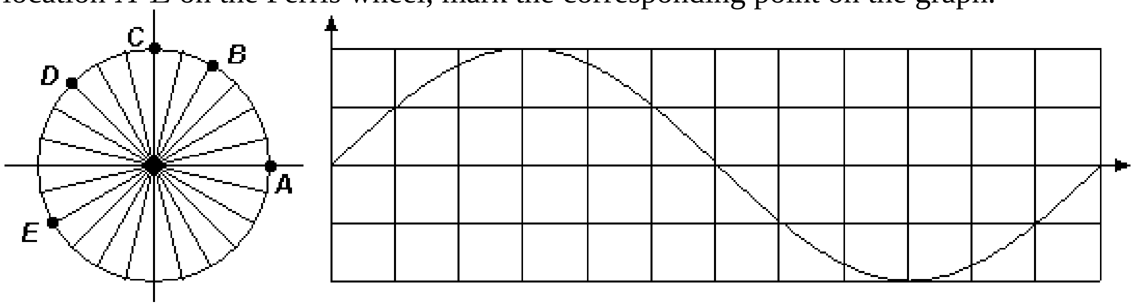


b. How does the graph of cosine differ from the graph of sine?

1. a. Prepare a graph with the horizontal axis scaled from 0° to 360° in multiples of 45°.
   b. Sketch a graph of $f(\theta) = \sin\theta$.
2. Give the coordinates of each point on the graph of $f(\theta) = \sin\theta$ or $g(\theta) = \cos\theta$.



3. The graph shows your height as a function of angle as you ride the Ferris wheel. For each location A-E on the Ferris wheel, mark the corresponding point on the graph.



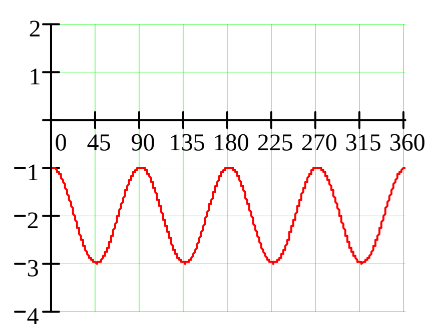
Lesson 2 Period , Midline, and Amplitude

Exercise Sketch a graph for each of the following functions. Describe how each is different from the graph of $y = \cos\theta$.

a. $f(\theta) = 2 + \cos\theta$ ***b.*** $g(\theta) = \cos 2\theta$ ***c.*** $h(\theta) = 2\cos\theta$

1. Graph the function in the **Trig** window (**ZOOM** **7**), but change Ymin to -10 and Ymax to 10. State the amplitude, period, and midline.
   a. $y = 3 + 4\cos\theta$ b. $y = 5\sin 2\theta$ c. $f(\theta) = -4 + 3\sin 3\theta$

2. a. State the amplitude, period, and midline of the graph shown.
   b. Write an equation for the graph using sine or cosine.



3. a. Write an equation for a sine function with amplitude 6.
   b. Write an equation for a cosine function with midline 2.
   c. Write an equation for a sine function with period 90°.

4. The table describes a sine or cosine function. Find an equation for the function.

θ	0°	45°	90°	135°	180°	225°	270°	315°	360°
$f(\theta)$	7	5.56	2	-1.54	-3	-1.54	2	5.54	7

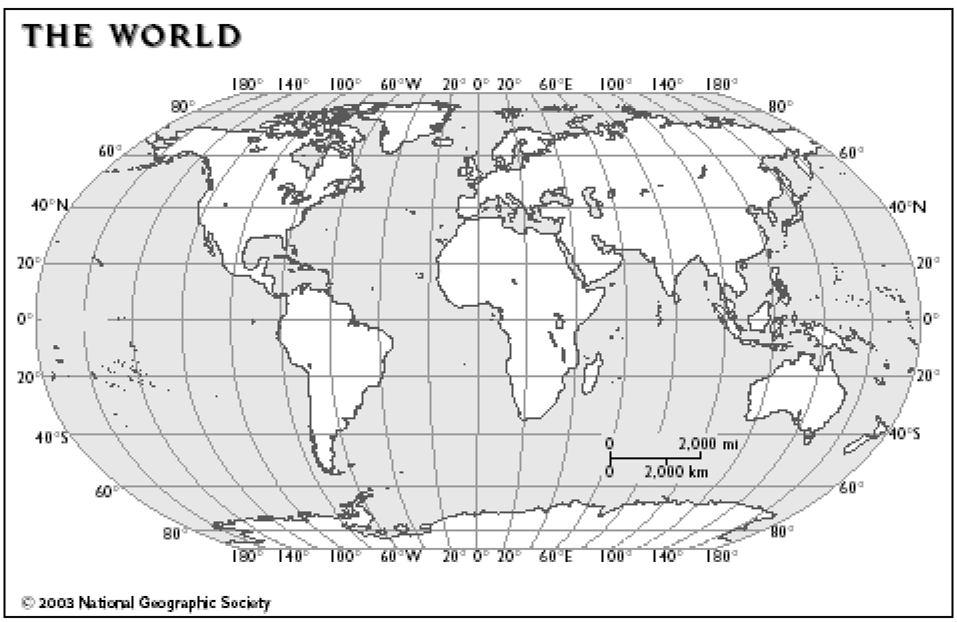
5. Write the equation of a sine or cosine function with maximum points at $(0°, 5)$ and $(360°, 5)$, and a minimum point at $(180°, 1)$.

Lesson 3   Applications: Periodic Functions

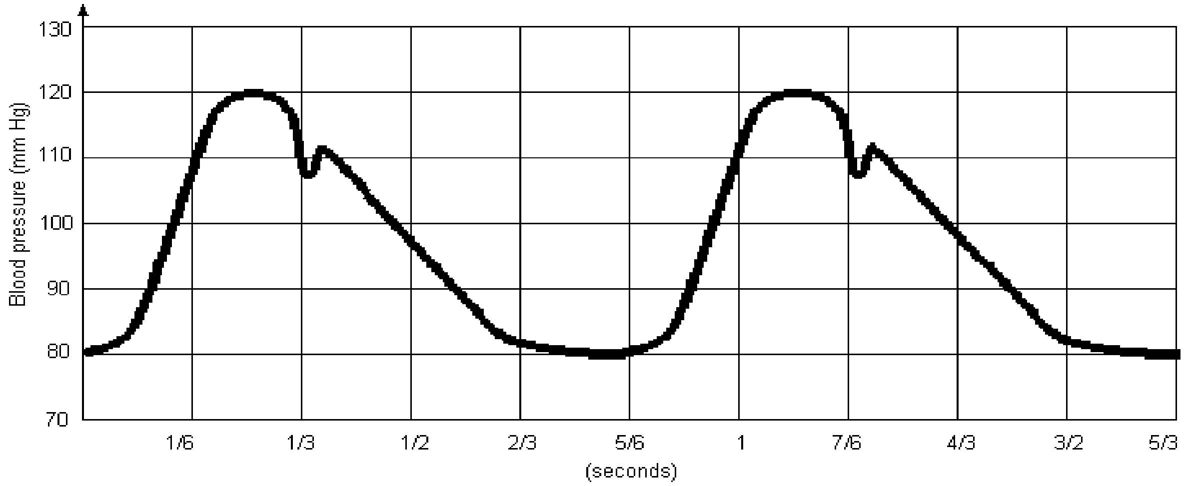
1. Francine adds water to her fish pond once a week to keep the depth at 30 centimeters. During the week the water evaporates at a constant rate of 0.5 centimeters per day.
   a. Sketch a graph of $D(t)$, the depth of the water as a function of time.
   b. What is the period of $D(t)$?

2. The population of mosquitoes at Marsh Lake is a sinusoidal function of time. The population peaks around June 1 at about 6000 mosquitoes per square kilometer, and is smallest on December 1, at 1000 mosquitoes per square kilometer.
   a. Sketch a graph of $M(t)$, the number of mosquitoes as a function of the month, where $t = 0$ on January 1.
   b. Give the period, midline and amplitude of your graph.

3. The path of a satellite orbiting above the earth makes a sinusoidal graph on a map of the earth, with its midline at the equator. On the map below, sketch a graph for a satellite that orbits the earth every 90 minutes, and strays no farther than 4000 km from the equator. (One degree of latitude is equal to 111 kilometers.) The satellite passes over $0°$ latitude and $0°$ longitude at time $t = 0$. Label a scale on the equator to serve as a time axis for your graph.



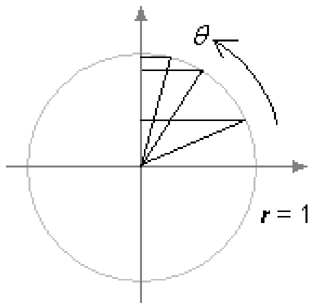
4. The graph shows arterial blood pressure, measured in millimeters of mercury (mmHg), as a function of time.

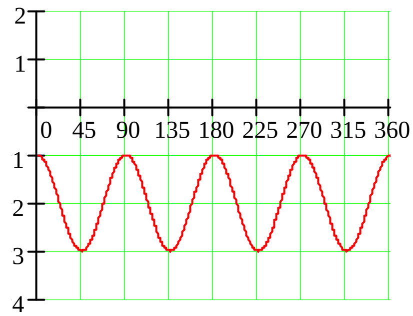


a. What are the maximum (systolic) and minimum (diastolic) pressures? The *pulse pressure* is the difference of systolic and diastolic pressures. What is the pulse pressure?
b. The *mean arterial pressure* is the diastolic pressure plus one-third of the pulse pressure. Calculate the mean arterial pressure, and draw a horizontal line on the graph at that pressure.
c. The blood pressure graph repeats its cycle with each heartbeat. What is the heart rate, in beats per minute, of the person whose blood pressure is shown in the graph?

Lesson 4 Graphs of Circular Functions

21. Use the figure at left below to help you fill in the blanks.
    a. As θ increases from 0° to 90°, $f(\theta) = \cos\theta$ _________ from _____ to _____ .
    b. As θ increases from 90° to 180°, $f(\theta) = \cos\theta$ _________ from _____ to _____ .
    c. As θ increases from 180° to 270°, $f(\theta) = \cos\theta$ _________ from _____ to _____ .
    d. As θ increases from 270° to 360°, $f(\theta) = \cos\theta$ _________ from _____ to _____ .



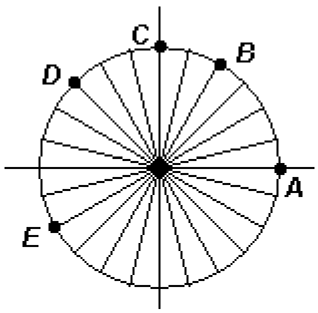


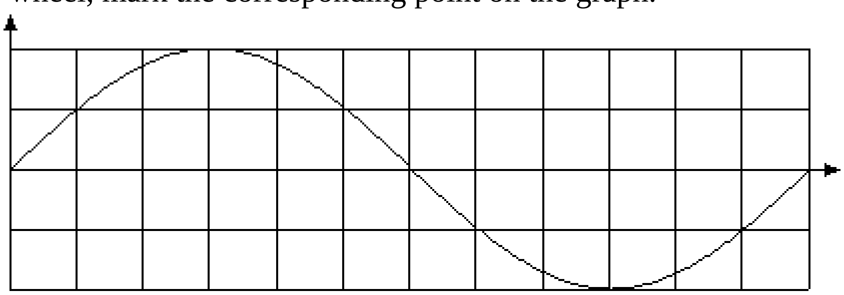
22. a. State the amplitude, period, and midline of the graph shown above at right.
    b. Write an equation for the graph using sine or cosine.

23. The table gives data from a sinusoidal function. Find an equation for the function.

θ	0°	45°	90°	135°	180°	225°	270°	315°	360°
$f(\theta)$	1	3.12	4	3.12	1	-1.12	-2	-1.12	1

24. The graph shows your height as a function of angle as you ride a Ferris wheel. For each location A-E on the Ferris wheel, mark the corresponding point on the graph.



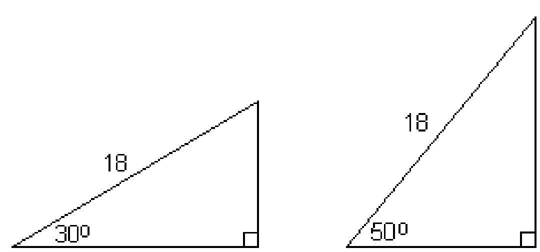


25. Let $\tilde{\theta} = f(\theta)$ be the function that gives the reference angle of θ. For example, $f(110°) = 70°$ because the reference angle for $110°$ is $70°$.
    a. Fill in the table of values.

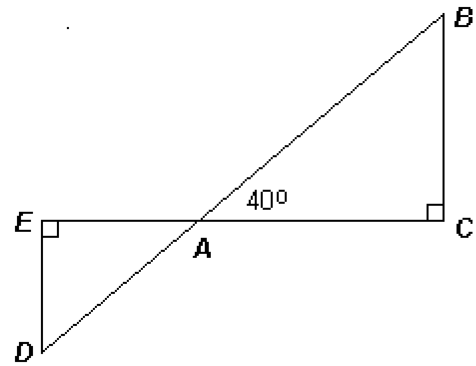
θ	30	60	90	120	150	180	210	240	270	300	330	360
$f(\theta)$												

    b. Choose appropriate scales for the axes and graph the function for $-360 \le \theta \le 360$.

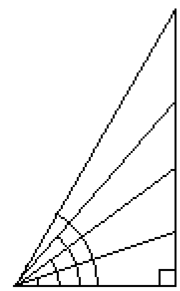
8. Consider the figures at right. Do you expect the sine of 50° to be larger or smaller than the sine of 30°? Do you expect the sine of 50° to be larger or smaller than 1?



9. Here are two right triangles with a 40° angle.
   a. Measure the sides AB and AC with a ruler. Use the lengths to estimate cos $40°$.
   b. Measure the sides AD and AE with a ruler. Use the lengths to estimate cos $40°$.
   c. Use your calculator to look up cos 40°. Compare your answers. How close were your estimates?



10. a. Use the figure to explain what happens to tan θ as θ increases, and why.
    b. Use the figure to explain what happens to cos θ as θ increases, and why.

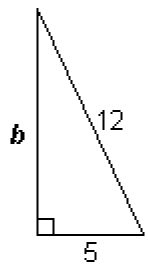


11. a. Use your calculator to complete the table, rounding your answers to four decimal places.

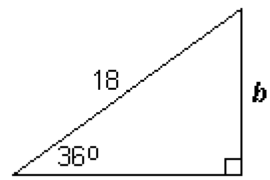
θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin θ										

    b. What do you notice about the values of sin θ as θ increases from 0° to 90°? If you plot the values of sin θ against the values of θ, will the graph be a straight line? Why or why not?

Right Triangle Trigonometry

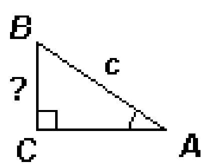


We can find side b with
the Pythagorean theorem.

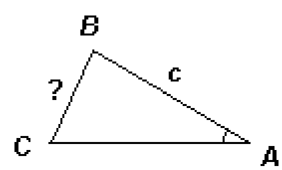


Can we find side b?

Law of Sines

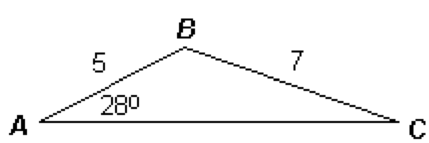


We can find a if we know A and c
(and $C = 90°$).

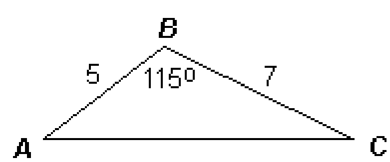


Can we find a if we know A
and c and C?

Law of Cosines



Two sides and an angle opposite one
of them. We can use the Law of Sines.



Two sides and the included angle.
We cannot use the Law of Sines.

Activities for Trigonometry

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