

## ***A Fresh Look at Teaching Trigonometry***

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## **Why is trig so hard?**

- Trigonometry inherently involves a great deal of technical detail, which can be allowed to obscure the main ideas.
- Trigonometry is often taught with the analytical rigor appropriate to a precalculus course, before students have acquired the necessary facility with functions.

## **“Before we get started...”**

An introductory chapter often includes some or all of

- coterminal and reference angles,
- minutes and seconds, radians,
- arc length, angular velocity,
- domain and range,
- symmetry, transformations,
- composition, inverse functions

No wonder the  
students can't  
see the forest for  
the trees!



## **What Are the Important Ideas?**

- Triangles and trigonometric ratios
- Trigonometric functions of angles
- Radians
- Graphs of trigonometric functions
- Solving trigonometric equations
- Using trigonometric identities

## **What Are Some Stumbling Blocks for Students?**

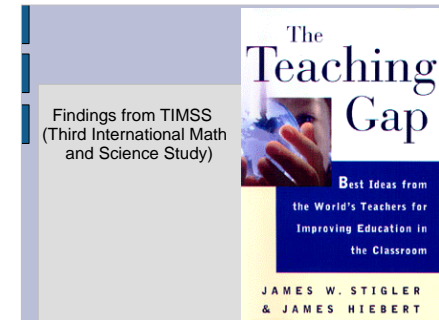
- ratio and proportion
- irrational numbers: exact values vs approximations
- fractional multiples of  $\pi$
- function notation
- connection between graphs and equations

## Fundamental Strategies

- Begin with concrete ideas before introducing abstractions.
- Keep the calculations simple.
- Introduce ideas by level of complexity, not by topic.
- Return to each skill several times in different contexts.

## Effective Mathematics Teaching: What does the Research Say?

James Hiebert  
University of Delaware  
October 2, 2007  
presentation at  
California State University Northridge



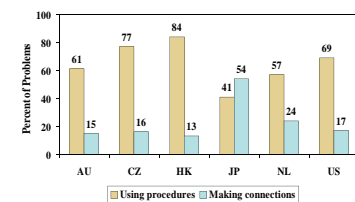
## Guiding Principles

- Teaching matters.
- Effective teaching depends on a few key features.

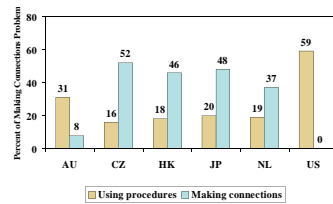
## Features of Effective Teaching

- Make conceptual relationships clear.  
Attend explicitly to connections between ideas.
- Allow students to wrestle with ideas.  
Avoid reducing conceptual problems to procedural problems.

*Percentage of Problems Presented as Each Type*



Percentage of Making Connections Problems Worked On in Each Way



Example: Transforming a Problem From Making Connections  $\Rightarrow$  Using Procedures

- Problem: Find a pattern for the sum of the interior angles of polygons
- Worked on as Making Connections
  - ▢ Measure the sum of the angles for 3-, 4-, and 5-sided polygons; predict for 6-sided polygons; for n-sided polygons; develop a general formula
- Worked on as Using Procedures
  - ▢ Present formula [ $\text{Sum} = 180(n - 2)$ ] and ask students to practice

## Resources

- Complete TIMSS Video report plus video clips <http://nces.ed.gov/timss/Video.asp>
- Public release videos <http://lessonlab.com/bkstore/>
- Slides available at <http://www.udel.edu/soe/mathed/>

## Important Ideas (again)

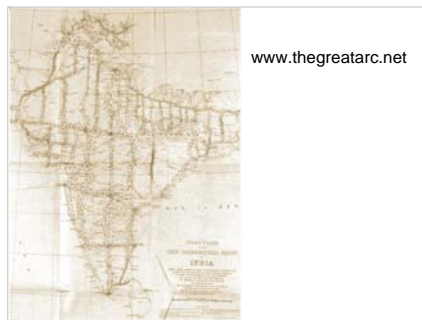
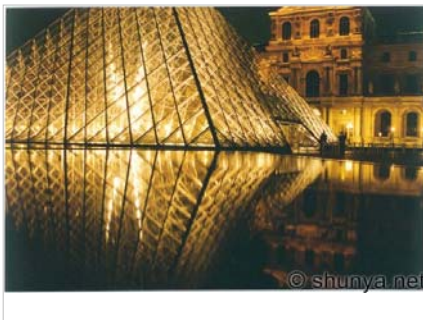
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- Radians
- Graphs of trigonometric functions
- Solving trigonometric equations
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## Trigonometric ratios

To start the course:

- Begin with properties of triangles.
- Define sine, cosine, and tangent as ratios of sides of right triangles.
- Consider only three trig ratios and only one quadrant.



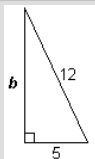


www.thegreatarc.net

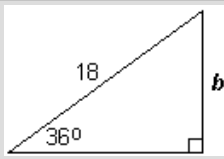
## Trigonometric Functions of Angles

- Begin with obtuse angles and laws of sines and cosines.
- Introduce standard position and reference angles.
- Graph trigonometric functions of degrees.

## Right Triangle Trigonometry

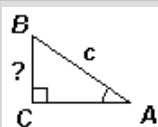


We can find side  $b$  with the Pythagorean theorem.

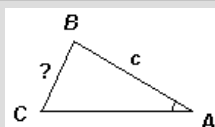


Can we find side  $b$ ?

## Law of Sines

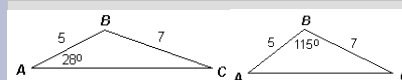


We can find  $a$  if we know  $A$  and  $c$  (and  $C = 90^\circ$ ).



Can we find  $a$  if we know  $c$  and  $A$  and  $C$ ?

## Law of Cosines



Two sides and an angle opposite one of them. We can use the Law of Sines.

Two sides and the included angle. We cannot use the Law of Sines.

## Radians

- Motivate radian measure by arclength.
- Discuss radians in both decimal form and as multiples of  $\pi$ .
- Connect graphs of sine and cosine with the unit circle.

## Identities

- Begin with the Pythagorean and tangent identities only.
- Increase the algebraic complexity gradually.
- Emphasize using identities rather than proving them.

## Graphing

- Graph trig functions in degrees before considering radians.
- Emphasize finding an equation for a given graph.
- Use graphs for solving equations and verifying identities.

## Thank you!

[www.piercecollege.edu/faculty/yoshiwb/Talks/](http://www.piercecollege.edu/faculty/yoshiwb/Talks/)

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