Activity 1 Properties of Triangles

A. What do we know about the sides of a triangle?

1. a. Can you make a triangle with sides of length 2 inches, 3 inches, and 6 inches?
   b. Use the pieces of length 2 inches and 3 inches to form two sides of a triangle. What happens to the length of the third side as you increase the angle between the first two sides?
   c. What is the longest that the third side could be? What is the smallest?
2. Two sides of a triangle are 6 centimeters and 8 centimeters long. What are the possible lengths of the third side?
3. Two sides of a triangle are p units and q units long. What are the possible lengths of the third side?

B. What do we know about the angles of a triangle?

1. Use the protractor to measure the three angles of the paper triangle in degrees. Now add up the three angles. What is their sum?
2. Tear off the three corners of the triangle. Place them side-by-side with their vertices (tips) at the same point. What do you find?
3. How are your answers to parts (1) and (2) related?

C. How are the sides and angles of a triangle related?

1. A standard way to label a triangle is to call the angles A, B, and C. The side opposite angle A is called a, the side opposite angle B is called b, and the side opposite angle C is called c. Sketch a triangle and label it with standard notation.
2. Using the ruler, carefully draw a triangle and label it with standard notation so that $a > b > c$. Now use the protractor to measure the angles and list them in order from largest to smallest. What do you observe?
3. Using the ruler, carefully draw another triangle and label it with standard notation so that $A > B > C$. Now use the ruler to measure the sides and list them in order from largest to smallest. What do you observe?

D. What do we know about right triangles?

1. The side opposite the $90°$ angle in a right triangle is called the **hypotenuse**. Why is the hypotenuse always the longest side of a right triangle?
2. The Pythagorean theorem states that:

   IF: a, b, and c are the sides of a right triangle, and c is the hypotenuse,
   THEN: $a^2 + b^2 = c^2$

   The "if" part of the theorem is called the **hypothesi**s, and the "then" part is called the **conclusion**. The **converse** of a theorem is the new statement you obtain when you interchange the hypothesis and the conclusion. Write the converse of the Pythagorean theorem.
3. The converse of the Pythagorean theorem is also true. Use the converse to decide whether each of the following triangles is a right triangle. Support your conclusions with calculations.

   a. $a = 9,\ b = 16,\ c = 25$ b. $a = 12,\ b = 16,\ c = 20$
   a. $a = \sqrt{8},\ b = \sqrt{5},\ c = \sqrt{13}$ b. $a = \frac{\sqrt{3}}{2},\ b = \frac{1}{2},\ c = 1$

Activity 2 Trigonometric Ratios

A. Using Ratios and Proportions

1. Two related quantities or variables are **proportional** if their ratio is always the same.
   a. On any given day, the cost of filling up your car's gas tank is proportional to the number of gallons of gas you buy. For each purchase below, compute the ratio

$$\frac{\text{total cost of gasoline}}{\text{number of gallons}}$$

Gallons of Gas Purchased	Total Cost	$\frac{\text{Dollars}}{\text{Gallon}}$
5	$14.45	
12	$34.68	
18	$52.02	

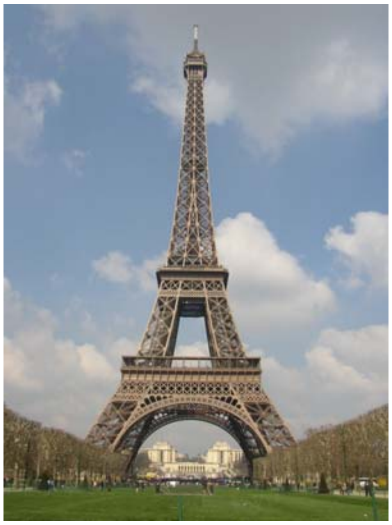
   b. Write an equation that you can solve to answer the question: How much does 21 gallons of gas cost? Use the ratio $\frac{\text{Dollars}}{\text{Gallon}}$ in your equation.
   c. Write an equation that you can solve to answer the question: How many gallons of gas can you buy for $46.24? Use the ratio $\frac{\text{Dollars}}{\text{Gallon}}$ in your equation.

2. A recipe for coffee cake calls for $\frac{3}{4}$ cup of sugar and $1\frac{3}{4}$ cup of flour.
   a. What is the ratio of sugar to flour? Write your answer as a common fraction, and then give a decimal approximation rounded to four places.

   For parts (b) and (c) below, write an equation that you can solve to answer the question. Use the ratio $\frac{\text{Amount of sugar}}{\text{Amount of flour}}$ in your equation.
   b. How much sugar should you use if you use 4 cups of flour? Compute your answer two ways: writing the ratio as a common fraction, and then writing the ratio as a decimal approximation. Are your answers the same?
   c. How much flour should you use if you use 4 cups of sugar? Compute your answer two ways: writing the ratio as a common fraction, and then writing the ratio as a decimal approximation. Are your answers the same?

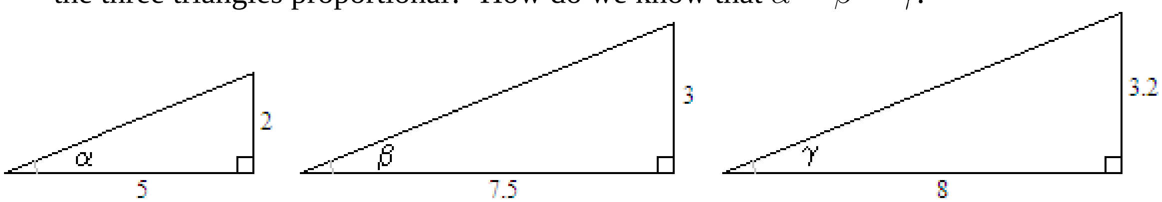
3. You are making a scale model of the Eiffel tower, which is 324 meters tall and 125 meters wide at its base.
   a. Compute the ratio of the width of the base to the height of the tower. Round your answer to four decimal places.

   Use your ratio to write equations and answer the questions below:
   b. If the base of your model is 8 inches wide, how tall should the model be?
   c. If you make a larger model that is 5 feet tall, how wide will the base be?

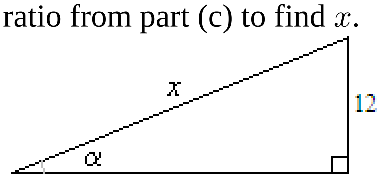


B. Similar Triangles

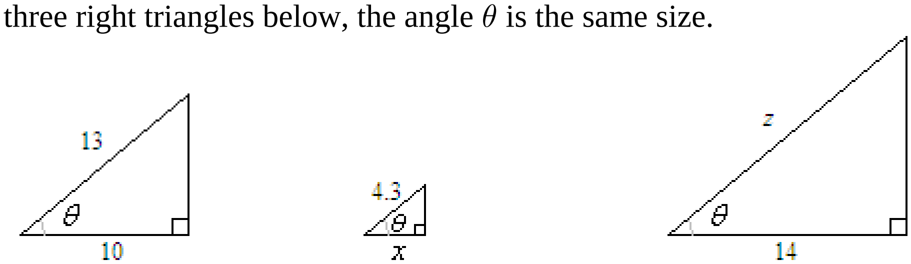
1. Recall that two triangles are **similar** if their corresponding sides are proportional. The corresponding angles of similar triangles are equal.
   a. What is the ratio of the two given sides in each triangle? Are the corresponding sides of the three triangles proportional? How do we know that $\alpha = \beta = \gamma$?



   b. Find the hypotenuse of each right triangle.
   c. Use the sides of the approporiate triangle to compute sin α, sin β, and sin γ. Round your answers to four decimal places. Does the sine of an angle depend on the lengths of its sides?
   d. How do you know that the triangle below is similar to the three triangles in part (a)? Write an equation using the ratio from part (c) to find x.

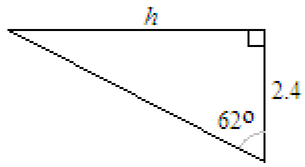


2. In the three right triangles below, the angle θ is the same size.

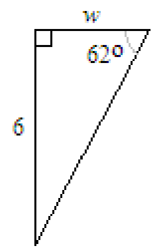


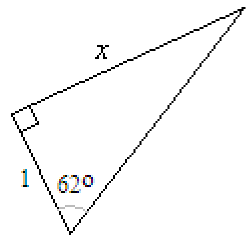
   a. Use the first triangle to calculate cos θ. Round your answer to four decimal places.
   b. In the second triangle, explain why $\dfrac{x}{4.3} = \dfrac{10}{13}$. Write an equation using your answer to part (a) and solve it to find x.
   c. Write and solve an equation to find z in the third triangle.

3. a. Use your calculator to find the value of $\dfrac{h}{2.4}$. What is the length of side h?
   (*Hint*: Which trig ratio should you use?)



b. What is the value of $\dfrac{6}{w}$ for the triangle below left? Write an equation and solve for w.





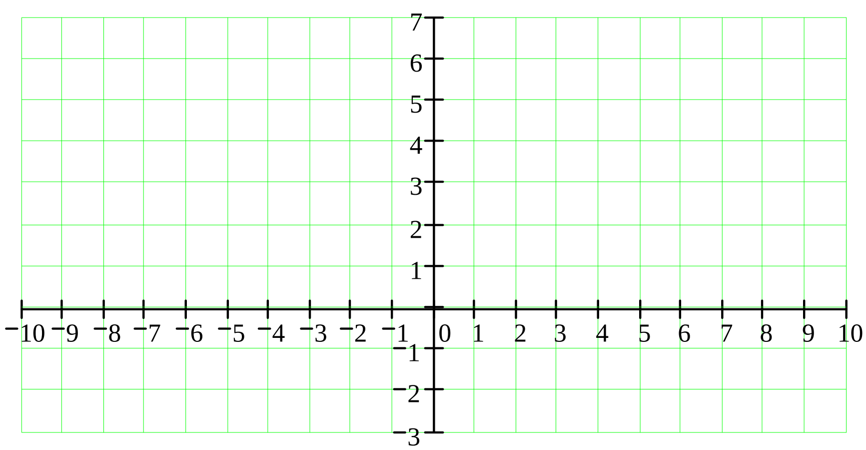
c. Write an equation and solve it to find x in the triangle above right.

Activity 3 Trigonometric Functions

Recall that we extend our definitions of the trigonometric ratios to all angles as follows. Place the angle θ in standard position and choose a point P with coordinates (x, y) on the terminal side. The distance from the origin to P is $r = \sqrt{x^2 + y^2}$. The trigonometric ratios of θ are defined as follows.

The Trigonometric Ratios

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

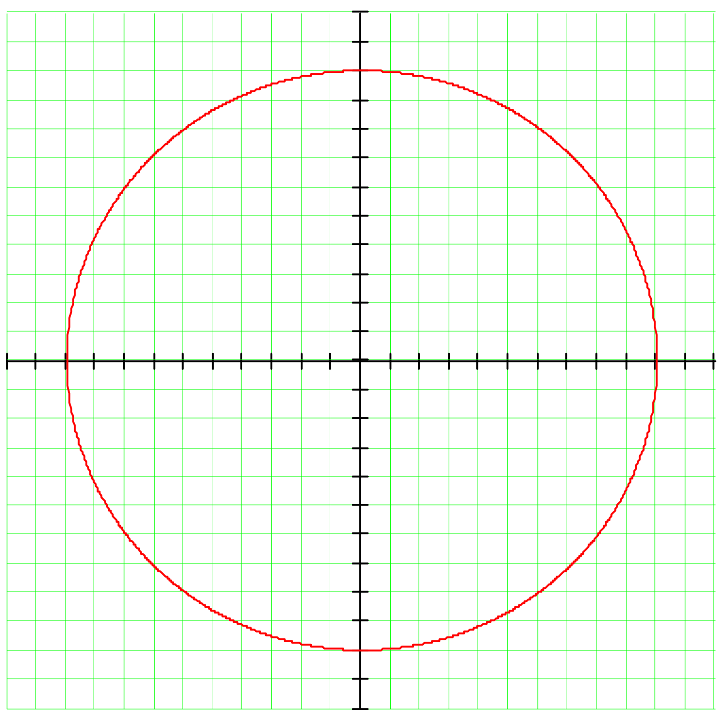


1. Draw an angle θ in standard position with the point $P(6, 4)$ on its terminal side.
2. Find r, the distance from the origin to P.
3. Calculate $\sin\theta$, $\cos\theta$, and $\tan\theta$. Give both exact answers and decimal approximations rounded to four places.
4. Use the inverse cosine key on your calculator to find θ. Use your calculator to verify the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ that you found in part (3).

5. Draw another angle ϕ in standard position with the point $Q(-6, 4)$ on its terminal side. Explain why ϕ is the supplement of θ. (*Hint*: Consider the right triangles formed by drawing vertical lines from P and Q.)
6. Can you use the right triangle definitions (using opposite, adjacent and hypotenuse) to compute the sine and cosine of ϕ? Why or why not?
7. Calculate sin ϕ, cos ϕ, and tan ϕ using the extended definitions listed above. How are the trig values of ϕ related to the trig values of θ?

8. Explain why θ and ϕ have the same sine but different cosines.
9. Use the inverse cosine key on your calculator to find ϕ. Use your calculator to verify the values of sin ϕ, cos ϕ, and tan ϕ that you found in part (6).
10. Compute $180° - \phi$. What answer should you expect to get?

Activity 4 Reference Angles



1. Use a protractor to draw an angle of $56°$ in standard position. Draw its reference triangle.
2. Use your calculator to find the sine and cosine of $56°$, rounded to two decimal places. Label the sides of the reference triangle with their lengths.
3. What are the coordinates of the point P where your angle intersects the unit circle?
4. Draw the reflection of your reference triangle across the y-axis, so that you have a congruent triangle in the second quadrant.
5. You now have the reference triangle for a second-quadrant angle in standard position. What is that angle?
6. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point Q where the angle intersects the unit circle.
7. Draw the reflection of your triangle from part (4) across the x-axis, so that you have a congruent triangle in the third quadrant.

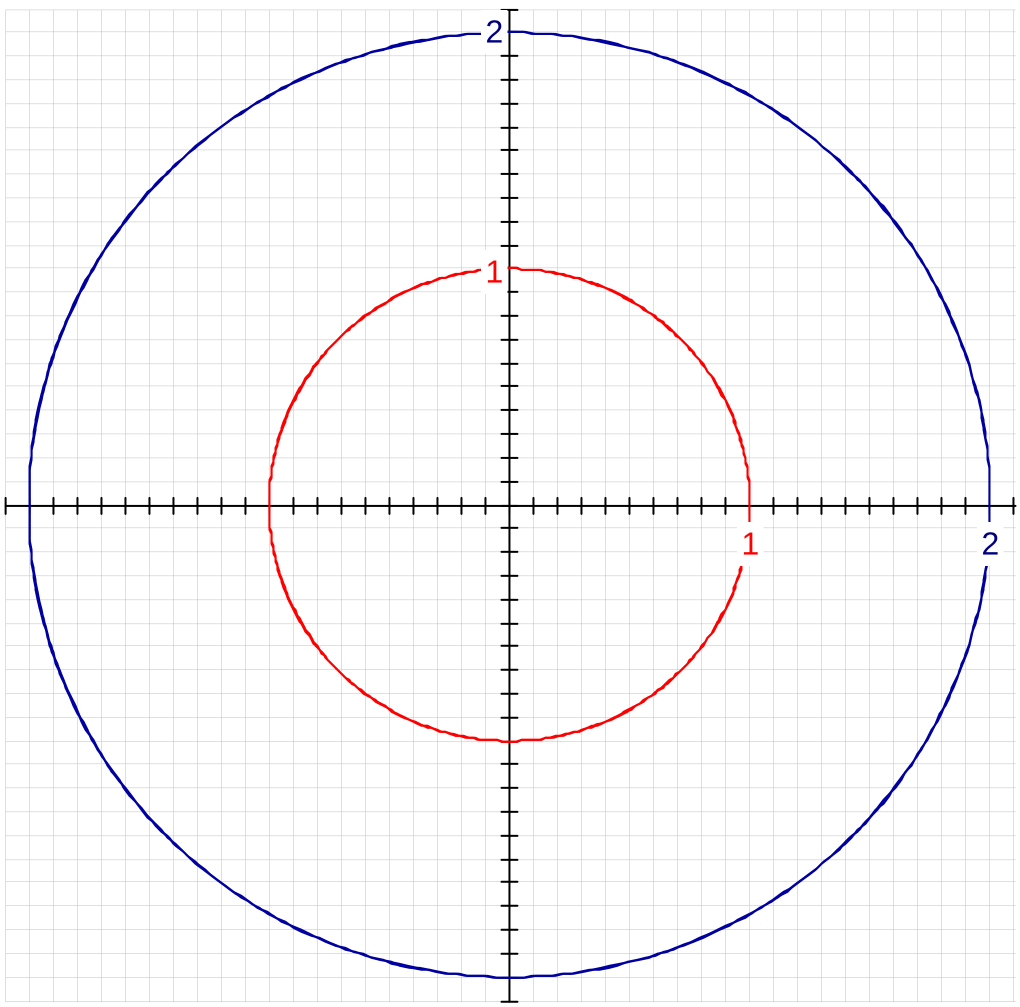
8. You now have the reference triangle for a third-quadrant angle in standard position. What is that angle?
9. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point R where the angle intersects the unit circle.
10. Draw the reflection of your triangle from part (7) across the y-axis, so that you have a congruent triangle in the fourth quadrant.
11. You now have the reference triangle for a fourth-quadrant angle in standard position. What is that angle?
12. Use your calculator to find the sine and cosine of your new angle. Label the coordinates of the point S where the angle intersects the unit circle.

Generalize: All four of your angles have the same reference angle, $56°$. For each quadrant, write a formula for the angle whose reference angle is θ.

Quadrant I: Quadrant II:

Quadrant III: Quadrant IV:

Activity 5 Unit Circles



1. Use a protractor to draw an angle $\theta = 36°$ in standard position.
   a. Estimate the coordinates of the point P where the terminal side of the angle intersects the circle of radius $r = 2$.
   a. Calculate approximate values for $\cos\theta$ and $\sin\theta$ using the coordinates of P.

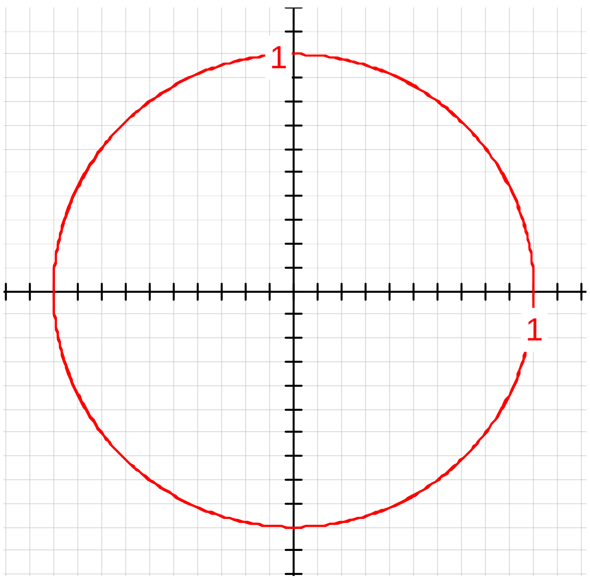
c. Estimate the coordinates of the point Q where the terminal side of the angle intersects the circle of radius $r = 1$.
d. Calculate approximate values for $\cos\theta$ and $\sin\theta$ using the coordinates of Q.

2. Use a protractor to draw an angle $\theta = 107°$ in standard position. Repeat parts (a)-(d) for this new angle.

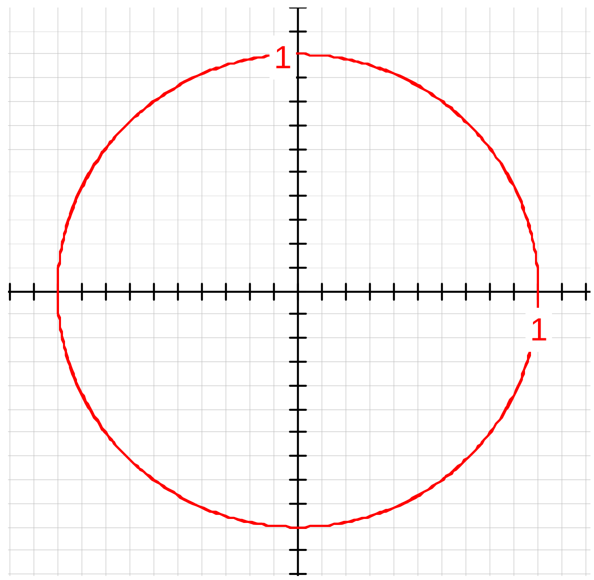
3. Use a protractor to draw an angle $\theta = 212°$ in standard position. Repeat parts (a)-(d) for this new angle.

4. Use a protractor to draw an angle $\theta = 325°$ in standard position. Repeat parts (a)-(d) for this new angle.

5. What do you notice about the coordinates of the point located on the unit circle by an angle and the values of the trig ratios of that angle?



Use this grid for #6 and #7



Use this grid for #8 and #9

6. Draw two different angles α and β in standard position whose sine is 0.6.
   a. Use a protractor to measure α and β.
   b. Find the reference angles for both α and β. Draw in the reference triangles.

7. Draw two different angles θ and ϕ in standard position whose sine is -0.8.
   a. Use a protractor to measure θ and ϕ.
   b. Find the reference angles for both θ and ϕ. Draw in the reference triangles.

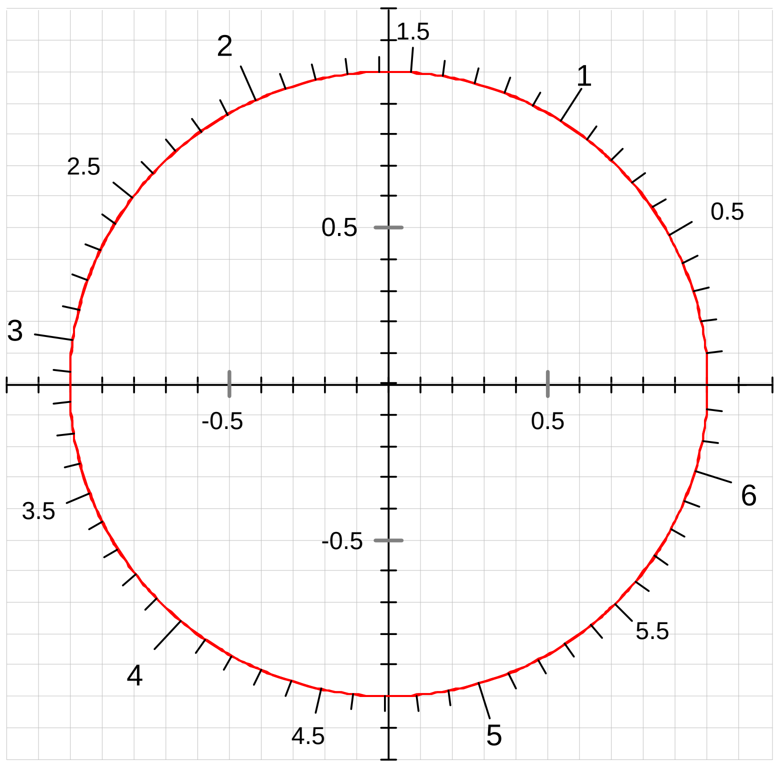
8. Draw two different angles α and β in standard position whose cosine is 0.3.
   a. Use a protractor to measure α and β.
   b. Find the reference angles for both α and β. Draw in the reference triangles.

9. Draw two different angles θ and ϕ in standard position whose cosine is -0.4.
   a. Use a protractor to measure θ and ϕ.
   b. Find the reference angles for both θ and ϕ. Draw in the reference triangles.

10. a. If you know one angle whose sine is a given positive number, how can you find the other angle?
    b. If you know one angle whose sine is a given negative number, how can you find the other angle?
    c. If you know one angle whose cosine is a given positive number, how can you find the other angle?
    d. If you know one angle whose cosine is a given negative number, how can you find the other angle?

11. Use your answers to the problems above to solve the equations for $0 \le \theta \le 360°$:
    a. $\sin\theta = 0.6$ b. $\sin\theta = -0.8$
    c. $\cos\theta = 0.3$ d. $\cos\theta = -0.4$

Activity 6 Radians



1. Use the unit circle to estimate the sine, cosine, and tangent of each arc of given length.
   a. 0.6 b. 2.3 c. 3.5 d. 5.3

2. Use the unit circle to estimate two solutions to each equation.
   a. $\cos t = 0.3$ b. $\sin t = -0.7$

3. Sketch the angle on the unit circle. Find the reference angle in radians, rounded to two decimal places, and sketch the reference triangle.
   a. 1.8 b. 5.2 c. 3.7

4. Give a decimal approximation to two places for each angle, then the degree measure of each.

Radians	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
Decimal Approx.													
Degrees													

Radians	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
Decimal Approx.													
Degrees													

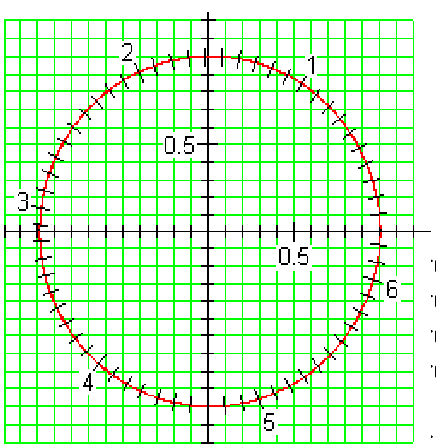
5. On the unit circle above, plot the endpoint of each arc in standard position.

   a. $\dfrac{\pi}{3}$ b. $\dfrac{7\pi}{6}$ c. $\dfrac{7\pi}{4}$

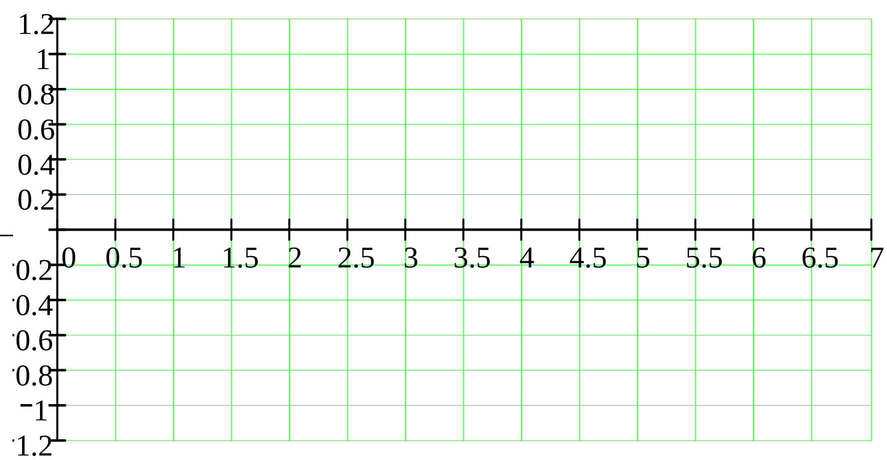
Activity 7 Graphs of Sine and Cosine

We are going to graph $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ from their definitions. The unit circle at the left of each grid is marked off in radians. (Each tick mark is 0.1 radian.) The x-axis of each grid is also marked in radians.

1. Choose a value of θ along the horizontal axis of the $f(\theta) = \sin\theta$ grid. This value of θ represents an angle in radians.
2. Now look at the unit circle and find the point P designated by that same angle in radians.
3. Measure the vertical (signed) distance that gives the y-coordinate of point P.
4. At the value of θ you chose in step 1, lightly draw a vertical line segment the same length as the y-coordinate of P. Put a dot at the top (or bottom) of the line segment.
5. Repeat for a bunch more values of θ. Connect the dots to see the graph of $f(\theta) = \sin\theta$.

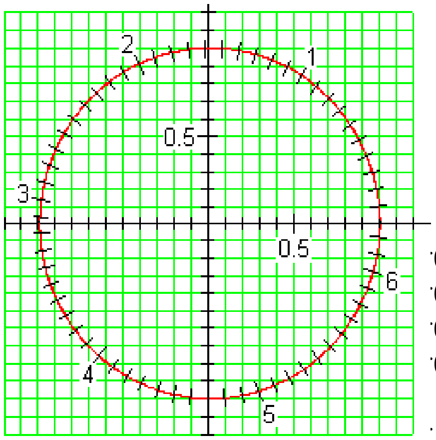


Unit circle

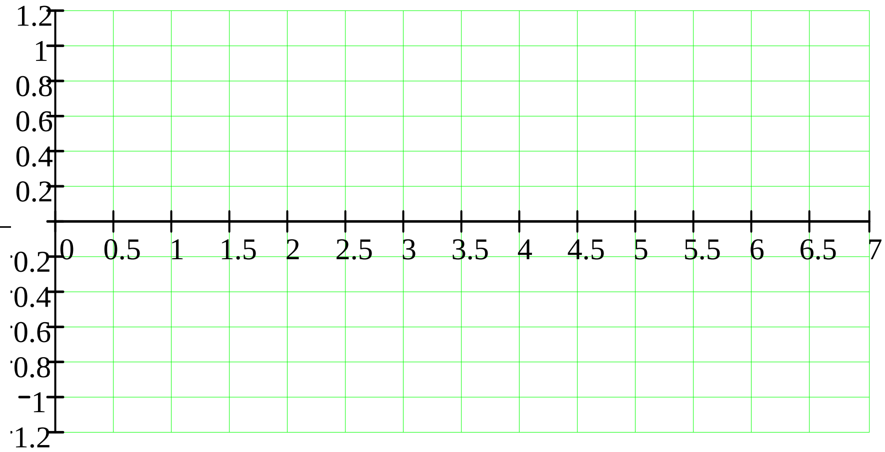


$f(\theta) = \sin\theta$

1. Choose a value of θ along the horizontal axis of the $g(\theta) = \cos\theta$ grid. This value of θ represents an angle in radians.
2. Now look at the unit circle and find the point P designated by that same angle in radians.
3. Measure the *horizontal* (signed) distance that gives the x-coordinate of point P.
4. At the value of θ you chose in step 1, lightly draw a *vertical* line segment the same length as the x-coordinate of P. Put a dot at the top (or bottom) of the line segment.
5. Repeat for a bunch more values of θ. Connect the dots to see the graph of $g(\theta) = \cos\theta$.

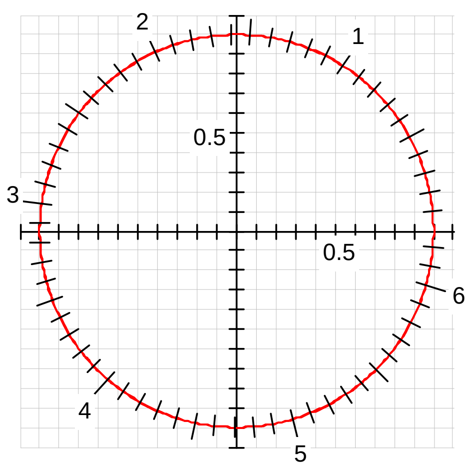


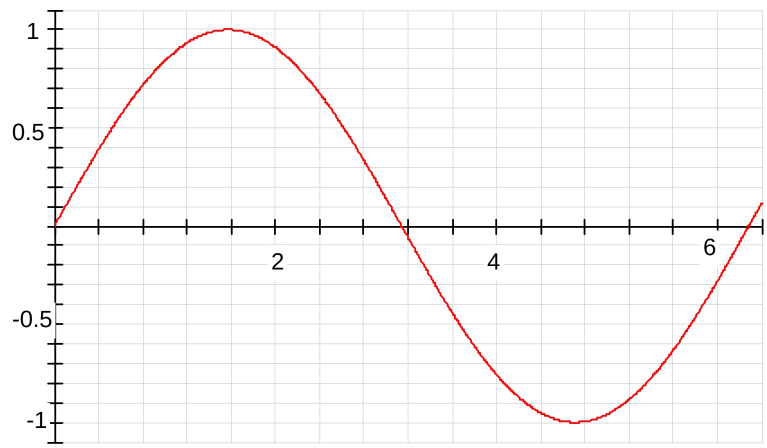
Unit circle



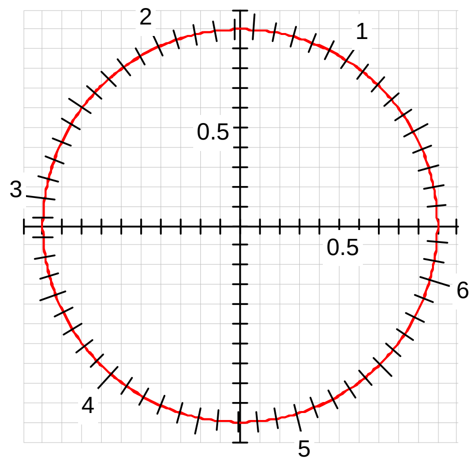
$g(\theta) = \cos\theta$

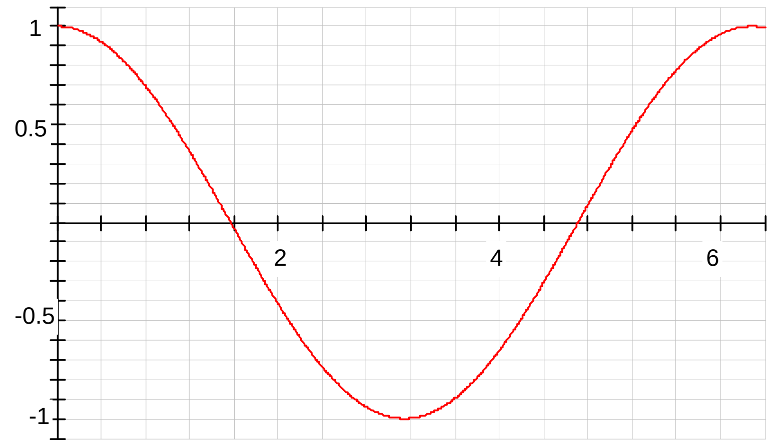
Activity 8 Solving Equations





1. a. Use the graph of $y = \sin x$ to estimate two solutions of the equation $\sin x = 0.65$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\sin x = 0.65$. Show your solutions on the circle.
   a. Use the graph of $y = \sin x$ to estimate two solutions of the equation $\sin x = -0.2$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\sin x = -0.2$. Show your solutions on the circle.





2. a. Use the graph of $y = \cos x$ to estimate two solutions of the equation $\cos x = 0.15$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\cos x = 0.15$. Show your solutions on the circle.
   a. Use the graph of $y = \cos x$ to estimate two solutions of the equation $\cos x = -0.4$. Show your solutions on the graph.
   b. Use the unit circle to estimate two solutions of the equation $\cos x = -0.4$. Show your solutions on the circle.

Activity 9 Identities

I Negative Angle Identities

1. a. Suppose that θ is a first-quadrant angle. In which quadrant would you find $-\theta$?
   b. Sketch an example for θ, $-\theta$, and the reference triangle for each.
   c. How is $\sin(-\theta)$ related to $\sin\theta$? What about $\cos(-\theta)$ and $\tan(-\theta)$?
2. Repeat part (1) for the case where θ is a second-quadrant angle.

II Sum of Angles Identities

1. Is it true that $\cos(\theta + \phi) = \cos\theta + \cos\phi$? Try it for $\theta = 60°$ and $\phi = 45°$.
2. a. Recall the distributive law, $a(b + c) = ab + ac$, where the parentheses denote **multiplication**. Is the same law true when the parentheses denote a **function**? In other words, is it true that $f(a + b) = f(a) + f(b)$?
   b. In the expression $\sin(\theta + \phi)$, do the parentheses denote multiplication or the application of a function? Does the distributive law apply to $\sin(\theta + \phi)$? Do you think that $\sin(\theta + \phi) = \sin\theta + \sin\phi$ is an identity?
3. a. Look at the Sum of Angles Identities in your textbook. Make some observations that will help you memorize these formulas.
   b. Do you think you would have to memorize these formulas if the equation $\sin(\theta + \phi) = \sin\theta + \sin\phi$ were an identity?

III Difference of Angles Identities, Tangent Identities

1. a. Yikes! More formulas. Compare the Difference of Angles Identities with the Sum of Angles Identities. If you have memorized the Sum formulas, how can you also memorize the Difference formulas?
   b. Comment on the sign patterns in the Sum and Difference Identities for Tangent.

2. a. Now let's use the formulas backwards: look at the expression below:

$$\frac{\tan 285° - \tan 75°}{1 + \tan 285° \tan 75°}$$

Does it remind you of the left side of one of the six new identities? Use that identity to simplify the expression.

b. Do the same thing for this expression: $\sin 4t \cos 0.7 - \cos 4t \sin 0.7$

.

IV Complementary and Supplementary Angles

1. Write out the complementary and supplementary angle identities in radians.
2. a. Choose a value of θ and substitute that value into the first identity.
   b. Evaluate both sides of the equation to verify the identity.
   c. Sketch both angles on a unit circle.
3. Repeat part (2) for each of the complementary and supplementary angle identities

V Double Angle Identities

1. a. Is it true that $\sin 2\theta = 2\sin\theta$? Choose a value for θ and try it. Sketch θ and 2θ on a unit circle and show the sine of each.
   b. Is it true that $\cos 2\theta = 2\cos\theta$? Repeat part (a) for cosine.
2. a. Substitute $\alpha = \theta$ and $\beta = \theta$ into the sum of angles formulas for sine, cosine, and tangent to derive the double angle formulas.
   b. Choose one of the three forms of the Double Angle Identity for Cosine. Use the Pythagorean Identity to convert from that form to each of the other two forms.

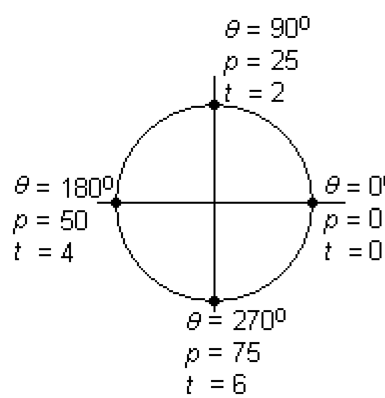
Radians Lesson

A. Arclength

Imagine that you are riding on a Ferris wheel of radius 100 feet, and each rotation takes 8 minutes. We can use angles in standard position to describe your location as you travel around the wheel.

The figure shows the locations indicated by $\theta = 0°,\ 90°,\ 180°,$ and $270°$. But degrees are not the only way to specify location on a circle. Here are some other ways to designate your location on the Ferris wheel.

Angle in standard position, θ	0°	90°	180°	270°	360°
Percent of one rotation, p	0	25	50	75	100
Time elapsed, t (min)	0	2	4	6	8

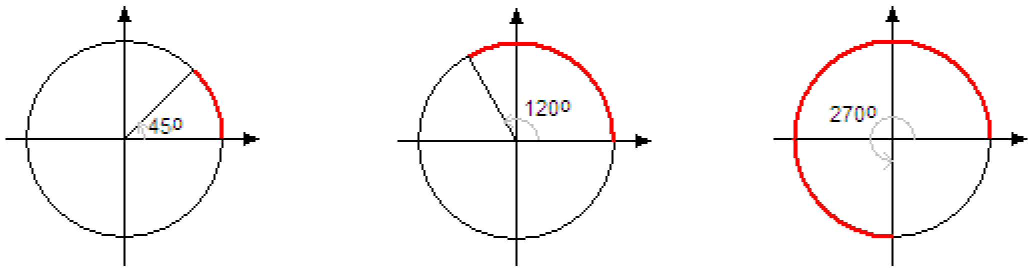


Another useful method uses distance traveled, or arclength, along the circle. How far have you traveled around the Ferris wheel at each of the locations shown? Recall that the circumference of a circle is proportional to its radius,

$$C = 2\pi r$$

If we walk around the entire circumference of a circle, the distance we travel is 2π times the length of the radius, or about 6.28 times the radius. If we walk only part of the way around the circle, then the distance we travel depends also on the angle we cover.

1. a. What fraction of a whole circle does each angle represent?
   b. What is the arclength spanned by each angle on a circle of radius 100 feet?



$$\boxed{\textbf{Arclength} = \textbf{(fraction of one revolution)} \cdot (2\pi r)}$$

B. Radian Measure

Notice that the degree measure of the spanning angle is not as important as the fraction of one revolution it covers. This suggests a new unit of angle measurement that is better suited to calculations involving arclength. We'll make one change in our formula for arclength, from

$$\textbf{Arclength} = \textbf{(fraction of one revolution)} \cdot (2\pi r)$$

to

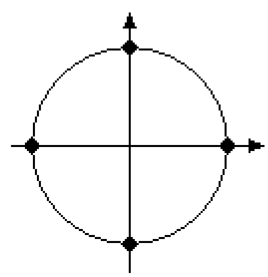
$$\textbf{Arclength} = (\textbf{fraction of one revolution} \times 2\pi) \cdot r.$$

We call the quantity in parentheses, (fraction of one revolution $\times\ 2\pi$), the **radian measure** of the angle.

> **Radians**
> The **radian measure** of an angle is given by
>
> $$\textbf{fraction of one revolution} \times 2\pi$$

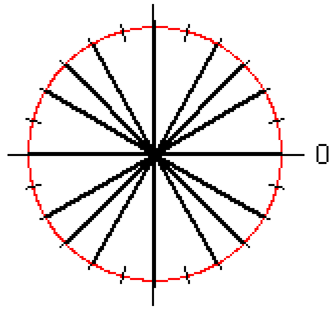
2. Find the radian measure of the quadrantal angles. For each angle, give both an exact value, as a multiple of π, and a decimal approximation.

Degrees	Radians: Exact Values	Radians: Decimal Approximations
0°		
90°		
180°		
270°		
360°		



3. a. In which quadrant would you find an angle of 2 radians? An angle of 5 radians?
   b. Draw a circle centered at the origin and sketch (in standard position) angles of approximately 3 radians, 4 radians, and 6 radians.

4. Give the radian measure of each angle.



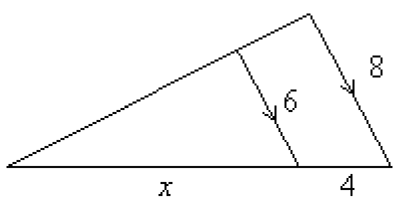
Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
Radians									

Degrees	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
Radians									

5. a. Use a proportion to find the degree measure of 1 radian.
   b. What ratio can you use to convert degrees to radians? What ratio can you use to convert radians to degrees?

Examples of Exercises That You Might Have Hoped Would Be Easy, But In Fact Are Not

1. Can two acute angles be supplementary? Why or why not?
2. For the triangles shown, which of the following equations is true? Explain why.



a. $\frac{4}{x} = \frac{6}{8}$ b. $\frac{x}{4} = \frac{6}{8}$

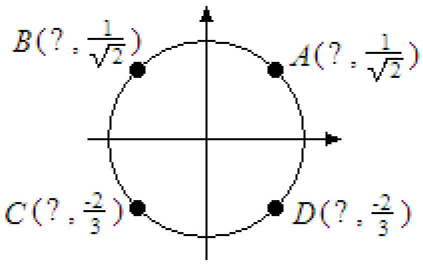
c. $\frac{x}{x+4} = \frac{6}{8}$ d. $\frac{x}{x+4} = \frac{6}{14}$

3. a. Find two points on the circle $x^2 + y^2 = 4$ with x-coordinate -1.
   b. Find two points on the circle $x^2 + y^2 = 1$ with y-coordinate $\frac{1}{2}$.

4. a. Write an expression for the distance between the points (x, y) and $(-3, 4)$.
   b. Write an equation that says "the distance between the points (x, y) and $(-3, 4)$ is 5 units."

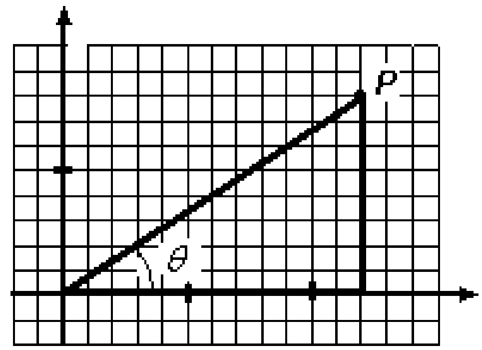
5. How long is the diagonal of a square of side 6 centimeters?
   a. Give an exact answer.
   b. Round your answer to hundredths.

6. a. Explain why the solutions of the equation $x^2 + y^2 = 100$ must have $-10 \le x \le 10$.
   b. What does part (a) tell you about the graph of the equation?

7. 

Find the missing coordinates of the points on the unit circle.

8. a. What is the slope of the line through the origin and point P?
   b. What is the tangent of the angle θ?
   c. On the same grid, sketch an angle whose tangent is $\frac{8}{5}$.

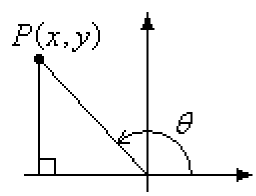


9. Which of the following numbers are equal to cos 45° ?

   a. $\frac{\sqrt{2}}{2}$ b. $\frac{1}{\sqrt{2}}$ c. $\frac{2}{\sqrt{2}}$ d. $\sqrt{2}$

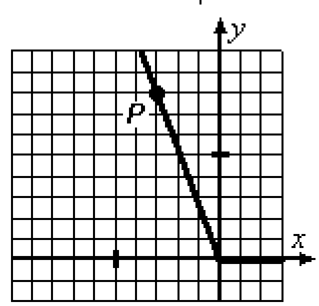
10. Is the acute angle larger or smaller than 60°?
    a. $\cos\theta = 0.75$ b. $\tan\phi = 1.5$ c. $\sin\psi = 0.72$

11. Explain why the length of the horizontal leg of the right triangle shown is $-x$.



12. Use your calculator to evaluate sin 118°, then evaluate $\sin^{-1}$(**ANS**). Explain the result.

13. a. Give the coordinates of point P on the terminal side of the angle.
    b. Find the distance from the origin to point P.
    c. Find $\cos\theta$, $\sin\theta$, and $\tan\theta$.



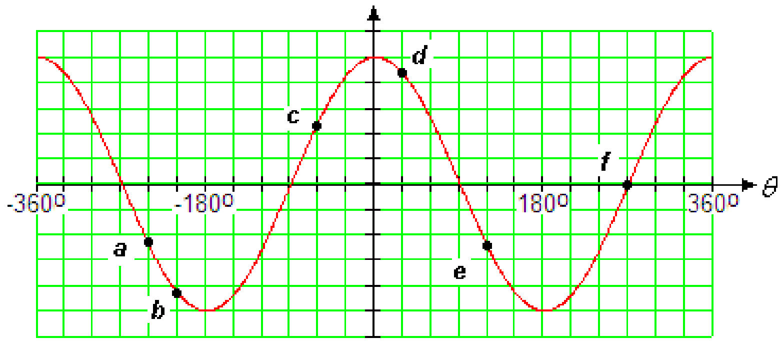
14. a. Draw four angles, one in each quadrant, whose reference angle is 60°.
    b. Find exact values for the sine, cosine, and tangent of each of the angles in part (a).
    c. Find all solutions for $0° \le \theta < 360°$ to the equation $\cos\theta = \frac{\sqrt{3}}{2}$.
    d. Find all solutions for $0° \le \theta < 360°$ to the equation $\cos\theta = \frac{-\sqrt{3}}{2}$.

15. Use the given values to find each trigonometric ratio. Do not use a calculator!

$$\cos 23° = 0.9205 \qquad \sin 46° = 0.7193$$

    a. cos 157° b. sin 226° c. sin 314° d. cos 203°

16. Give the coordinates of each point on the graph of $g(\theta) = \cos\theta$.

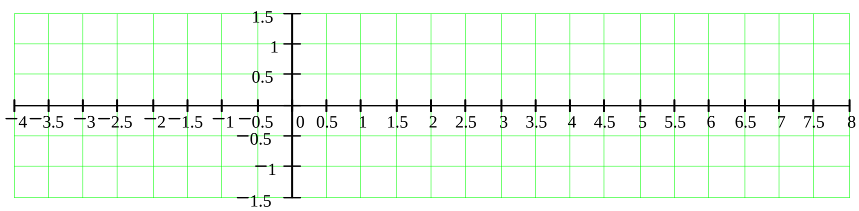


17. Decide whether the expressions are equivalent: $\sin(\theta - \phi)$, $\sin\theta - \sin\phi$

18. Simplify $\cos^2 3\alpha + \sin^2 3\alpha$

19. Scale the x-axis in multiples of $\frac{\pi}{12}$ from 0 to 2π. Reduce each fraction.

20. Sketch a graph of $y = \cos x$.



Presentation: Activities for Trigonometry

Intro: Welcome, and thanks for coming. If your experience is like ours, you have calculus students whose trigonometry skills are very weak. We decided to consider 2 questions:

1. Why is trig so hard for students?
2. What can we do about it?

Naturally, we don't have definitive answers for these questions yet. We're hoping that this talk can be informal, and that you will contribute your own experience and expertise to this investigation.

Slide 2: Why is trig so hard?
We've identified two broad areas to consider. (On slide)
Think about the introductory chapter in a typical trigonometry text:

Slide 3: "Before we get started ..."

There seem to be two types of trig courses, stand-alone courses, and those that are part of a precalculus course.
(What sort of course do you have at your school?)
But even the stand-alone courses tend to present a large number of function and angle ideas in a general setting before applying them to trigonometry.

No wonder students can't see the forest for the trees! **Slide 4**

Slide 5: What are the important ideas?
We decided to try to identify the big ideas in the subject, and to distinguish them from the details. Very generally, we have: (on slide)

Slide 6: What are some stumbling blocks?
We also tried to identify some of the main technical issues that hamper students in grasping the important ideas: (on slide)
(Do you have any additions to this list?)

Slide 7: Fundamental strategies
So next we came up with some strategies for structuring a course that would address these issues. (on slide)

Slide 8: Effective mathematics teaching
Last fall we were fortunate to hear a lecture at CSUN by James Hiebert, who headed the team that analyzed the TIMSS data. They videotaped eighth-grade classrooms in several nations to compare with US classrooms.

Slide 9: The teaching gap
Some of their findings are available in *The Teaching Gap*.

Slide 10: Guiding principles
What conclusions can we draw from their results? Prof. Hiebert identified two guiding principles:

- Teaching matters

Although many factors affect education, such as cultural differences, socio-economic forces, and so on, the way students are taught does make a difference.

- Effective teaching depends on a few key features
  This is the hopeful part: among the myriad of teaching styles and theories, and the controversy surrounding them, they were able to isolate some practices that actually made a measurable difference to students' learning. And here they are.

Slide 11: Features of effective teaching (on slide)

Slide 12: Procedures or connections (conceptual versus algorithmic)
With the exception of Japan, figures for the other nations are comparable.

Slide 13: Actual practice
Look at the last bar.

Slide 14: Example: interior angles of polygons

Slide 15: Resources
You can read more about the TIMSS findings at these websites.

Slide 16: What are the important ideas?
Back to trigonometry. Consider the important ideas again.

Slide 17: Trigonometric ratios
I'm a believer in starting with concrete ideas and building towards the abstract, rather than the other way round. Historically , trigonometry began with triangles, and that's where we begin our course. We are still learning at just how basic a level we must start in order for students to find the ideas concrete.

Slides 18-20: British museum, Louvre, survey of India
On the first day, I like to show some of the slides we ran before our talk, to show students why triangles are important: they are the basic building block for actual real-life construction, and for the slightly more abstract application of surveying.

You might enjoy reading about the Great Trigonometrical Survey of India: www.thegreatarc.net

Activity 1: Properties of Triangles

This is an introductory activity that pulls together facts that students probably already know about triangles, although applying the triangle inequality is hard for them, and we get to review a little about solving inequalities. I also use it to review the terms hypothesis, conclusion, and converse.

Activity 2: Trigonometric ratios

As soon as we define the trigonometric ratios, we run into our first stumbling block: students are not comfortable with ratio and proportion.

For example, many students don't recognize 2.89 or 0.4286 as ratios, and hence don't see asn equation such as

$$\frac{46.24}{x} = 2.89 \text{ or } \frac{x}{4} = 0.4286$$

as using proportional reasoning. (Plus, they can't solve the first one.) Problem 3 of part A of this activity addresses the fact that we can equate ratios of quantities in different units, such as inches and meters.

Because of these very real conceptual issues to overcome, we find it more effective to define only three trig ratios for acute angles only at first.

Slide 17: Trigonometric functions of angles

Which brings us to our next stumbling block: moving beyond right triangles and acute angles. We use oblique triangles and the laws of sines and cosines to motivate the transition as a progression of knowledge about triangles and their properties.

(**Slides 17a, b, c:** Include triangle slides here?)

Activity 3: Trigonometric functions

Once again, we focus on the new idea by limiting the scope of the generalization: we first study second-quadrant angles only. Again, underlying difficulties are revealed.

- Many students could not do part 2, find r. Although they knew the distance formula, it didn't occur to them to use it here.
- Most students could not do part 5: they couldn't see that θ and ϕ are supplementary. Why? Perhaps because the angles are not physically adjacent. So saying that supplementary angles add up to 180 degrees does not indicate understanding of the idea.
- Getting them to do parts 6 and 7 was quite hard, even with the definitions in plain view. They want to use the old opp, adj and hyp.

Activities 4 and 5: Reference angles and unit circles

After talking about obtuse angles, we then treat the laws of sines and cosines. In the next chapter we consider angles as rotations and introduce the trig functions in all four quadrants.

I find that students are more ready to accept this notion after working with obtuse angles for a bit.

Now we are tackling the first really difficult set of new concepts, the use of reference angles and the unit circle. The identification of an angle with a pointon the unit circle is especially slippery for students, and of course, crucial to understanding radians a little later.

As you can see, these Activities have students explicitly demonstrate some of the properties of reference angles and the unit circle for specific angles. Besides helping students grasp these ideas, we find that using them with angles in degrees sets the stage for radians.

Question: Does anyone know the history behind the unit circle?

Slides 18, 19, and 20: Very quickly, here are the goals we set for ourselves in teaching the other big ideas. (On slides.)

Radians Lesson: There are four more Activities for you to look at, and on p. 12 is a synopsis of an introduction to radians that we like. The punchline of the lesson is at the bottom of p. 12, where we move one parenthesis to get the definition of raian measure.

We've also included some exercise we've found useful in ferreting out students' misunderstandings and helping them to think about the ideas.

Slide 21: Thank you!

Activities for Trigonometry

Bruce and Katherine Yoshiwara

yoshiwbw@piercecollege.edu
yoshiwka@piercecollege.edu

CMC^3-South Annual Conference
March 1, 2008