Week 4 lecture 2 challenges

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Challenges II

Question 1

We can use the following function to generate a $n \times n$ symmetric positive matrix, where $n =$ `n`.

```
rsympdmat <- function(n) {
# function to generate symmetric positive definite matrix
# of dimension n x n
# n is an integer
A <- matrix(rnorm(n * n), n)
crossprod(A, diag(abs(rnorm(n))) %*% A)
}
```

The following forms a 5×5 matrix, which we'll call `A`, and then calculates its Cholesky decomposition in the upper triangular form $\mathbf{A} = \mathbf{U}^\top \mathbf{U}$.

```
A <- rsympdmat(5)
U <- chol(A)
```

The following then confirms that $\mathbf{A} = \mathbf{U}^\top \mathbf{U}$.

```
all.equal(A, crossprod(U))
```

```
## [1] TRUE
```

We can use `chol2inv()` to invert $\mathbf{A}$ via its Cholesky decomposition

```
iA <- chol2inv(U)
```

which can be checked with `solve()` as follows.

```
all.equal(iA, solve(A))
```

```
## [1] TRUE
```

The following calculates the logarithm of its determinant, given that $\log \det \mathbf{A} = 2 \sum_{i=1}^5 \log U_{ii}$,

```
ldet1 <- 2 * sum(log(diag(U)))
```

which is the same as using `determinant()`

```
ldet2 <- as.vector(determinant(A)$modulus)
all.equal(ldet1, ldet2)
```

```
## [1] TRUE
```