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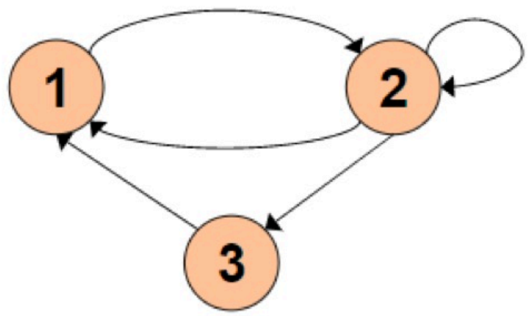
Homework 7

Exercise 1 Compute the steady-state probabilities of the following graph.



Solution.

We can get the matrix M as:

$$M = \begin{bmatrix} 0 & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix}$$

According to $\mathbf{r} = M \cdot \mathbf{r}$ and $|\mathbf{r}|_1 = 1$, we have

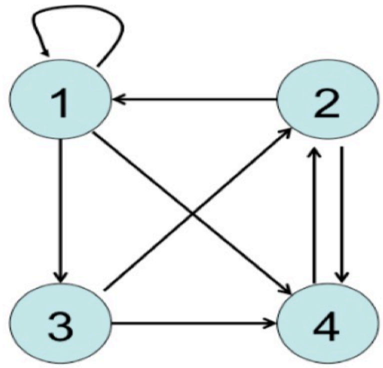
$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{3} & 1 \\ 1 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \cdot \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$\mathbf{r} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \\ \frac{1}{6} \end{bmatrix}$$

Also we can use equation $\mathbf{r}^{(k)} = M \cdot \mathbf{r}^{(k-1)}$ to get the result:

$$\mathbf{r}^{(0)} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}, \mathbf{r}^{(1)} = \begin{bmatrix} \frac{4}{9} \\ \frac{4}{9} \\ \frac{1}{9} \end{bmatrix}, \mathbf{r}^{(2)} = \begin{bmatrix} \frac{7}{27} \\ \frac{16}{27} \\ \frac{4}{27} \end{bmatrix}, \mathbf{r}^{(3)} = \begin{bmatrix} \frac{28}{81} \\ \frac{37}{81} \\ \frac{16}{81} \end{bmatrix}, \cdots, \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \\ \frac{1}{6} \end{bmatrix}$$

Exercise 2 Compute the PageRank of each node for the following graph:



1. Write down the column-stochastic matrix for the graph.

2. Write down the final column-stochastic matrix used for PageRank calculation. Use $\beta = 0.9$ for random teleportation.

3. Compute the PageRank value for each of the node in the graph (one iteration).

Solution.

The column-stochastic matrix for this graph is

$$M = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

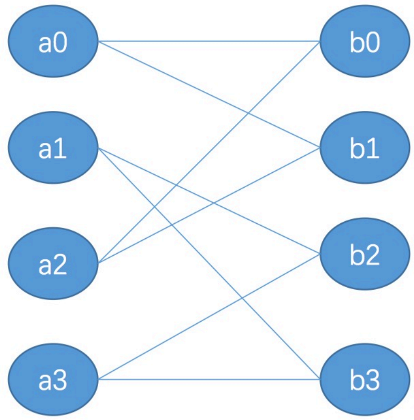
The final column-stochastic matrix for PageRank calculation is

$$A = \begin{bmatrix} \frac{39}{120} & \frac{19}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{1}{40} & \frac{19}{40} & \frac{37}{40} \\ \frac{39}{120} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{39}{120} & \frac{19}{40} & \frac{19}{40} & \frac{1}{40} \end{bmatrix}$$

After one iteration, we have:

$$\mathbf{r}^{(1)} = A \cdot \mathbf{r}^{(0)} = \begin{bmatrix} \frac{39}{120} & \frac{19}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{1}{40} & \frac{1}{40} & \frac{19}{40} & \frac{37}{40} \\ \frac{39}{120} & \frac{1}{40} & \frac{1}{40} & \frac{1}{40} \\ \frac{39}{120} & \frac{19}{40} & \frac{19}{40} & \frac{1}{40} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{51}{240} \\ \frac{29}{80} \\ \frac{1}{10} \\ \frac{39}{120} \end{bmatrix}$$

Exercise 3 A Graph has the following edges: (a0, b0), (a0, b1), (a1, b2), (a1, b3), (a2,b0), (a2,b1), (a3,b2) and (a3,b3). List all perfect matchings this graph have.



Solution.

1st: (a0,b0), (a2,b1), (a1,b2), (a3,b3)

2nd: (a0,b1), (a2,b0), (a1,b2), (a3,b3)

3rd: (a0,b0), (a2,b1), (a1,b3), (a3,b2)

4th: (a0,b1), (a2,b0), (a1,b3), (a3,b2)