# A Non-homogeneous Time Mixed Integer LP Formulation for Traffic Signal Control

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## ABSTRACT

We build on the body of work in mixed integer linear programming (MILP) approaches that attempt to jointly optimize traffic signal control over an *entire traffic network* (rather than focus on arterial routes) and specifically on improving the scalability of these methods for large urban traffic networks. Our primary insight in this work stems from the fact that MILP-based approaches to traffic control used in a receding horizon control manner (that replan at fixed time intervals) need to compute high fidelity control policies only for the early stages of the signal plan; therefore, coarser time steps can be employed to "see" over a long horizon to preemptively adapt to distant platoons and other predicted long-term changes in traffic flows. To this end, we contribute the queue transmission model (QTM) which blends elements of cell-based and link-based modeling approaches to enable a non-homogeneous MILP formulation of traffic signal control. We then experiment with this novel QTM-based MILP control in a range of networks demonstrating the improved scalability possible with non-homogeneous time steps in comparison to the best homogeneous time step. Our experiments also provide near-optimal traffic control policies for larger horizons and larger networks than shown in previous implementations of MILP-based traffic signal control.


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[1]

[1] Make sure to follow instructions and author guide: `http://onlinepubs.trb.org/onlinepubs/AM/InfoForAuthors.pdf` `http://onlinepubs.trb.org/onlinepubs/am/2015/WritingForTheTRRecord.pdf`

Also note this example related paper from Steve Smith (formatted to TRB specs): `https://www.ri.cmu.edu/pub_files/2014/1/TRB14UTC.pdf`

**INTRODUCTION**

As cities rapidly grow in population while urban traffic infrastructure often adapts at a slower pace, it is critical to maximize capacity and throughput of existing road infrastructure through optimized traffic signal control. Unfortunately, many large cities still use some degree of *fixed-time* control (e.g., Toronto (*1*)) even if they also use *actuated* or *adaptive* control methods such as SCATS (*2*) or SCOOT (*3*). However, there is further opportunity to improve traffic signal control even beyond adaptive methods through the use of *optimized* controllers as evidenced in a variety of approaches ranging from mixed integer (linear) programming (*4, 5, 6, 7, 8, 9*) to heuristic search (*10, 11*) to scheduling (*12*) to reinforcement learning (*1*). While such optimized controllers hold the promise of maximizing existing infrastructure capacity by finding more complex (and potentially closer to optimal) jointly coordinated intersection policies than arterially-focused master-slave approaches such as SCATS and SCOOT, such optimized methods are computationally demanding and either (a) do not guarantee jointly optimal solutions over a large intersection network (often because they only consider coordination of neighboring intersections or arterial routes) or (b) fail to scale to large intersection networks simply for computational reasons (which is the case for many mixed integer programming approaches).

In this work, we build on the body of work in mixed integer linear programming (MILP) approaches that attempt to jointly optimize traffic signal control over an *entire traffic network* (rather than focus on arterial routes) and specifically on improving the scalability of these methods for large urban traffic networks. In our investigation of existing approaches in this vein, namely exemplar methods in the spirit of (*6, 8, 9*) that use a (modified) cell transmission model (CTM) (*13, 14*) for their underlying prediction of traffic flows, we remark that a major drawback is the CTM-imposed requirement to choose a predetermined homogeneous (and often necessarily small) time step for reasonable modeling fidelity. This need to model large number of CTM cells with a small time step leads to MILPs that are exceedingly large and intractable to solve.

Our primary insight in this work stems from the fact that MILP-based approaches to traffic control used in a receding horizon control manner (that replan at fixed time intervals) need to compute high fidelity control policies only for the early stages of the signal plan; therefore, coarser time steps can be employed to "see" over a long horizon to preemptively adapt to distant platoons and other predicted long-term changes in traffic flows. This need for non-homogeneous control in turn spawns the need for an additional innovation: we require a traffic flow model that permits non-homogeneous time steps and properly models the travel time delay between lights. To this end, we might consider CTM extensions such as the variable cell length CTM (*15*), stochastic CTM extensions (*16, 17*), extensions for better modeling freeway-urban interactions (*18*) including CTM hybrids with link-based models (*19*), assymmetric CTMs for better handling flow imbalances in merging roads (*20*), the situational CTM for better modeling of boundary conditions (*21*), and the lagged CTM for improved modeling of the flow density relation (*22*). However, despite the widespread varieties of the CTM and the usage of the CTM (*23*) for a range of applications, there seems to be no extension that permits non-homogeneous time steps as required in our novel MILP-based control approach.

For this reason, as a major contribution of this work to enable our non-homogeneous time MILP-based model of joint intersection control, we contribute the queue transmission model (QTM) which blends elements of cell-based and link-based modeling approaches with the following key benefits:

- unlike previous joint intersection control work (*6, 8, 9*), it is inherently intended for *non-homogeneous* time steps that can be used for control over large horizons,
- any length of roadway with no merges or diverges can be modeled as a single queue leading to compact models of large traffic networks thus maintaining relatively compact MILPs for large traffic networks (i.e., larges numbers of cells are not required between intersections), and
- it accurately models fixed travel time delays critical to green wave coordination as in (*4, 5, 7*) through the use of a non-first order Markovian update[2] model and combines this with the more global intersection signal optimization approach of (*6, 8, 9*).

In the remainder of this paper, we first formalize our novel QTM model of traffic flow with non-homogeneous time steps and show how to encode it as a linear program for simulating traffic. We proceed to allow the traffic signals to become discrete variables subject to a delay minimizing optimization objective and standard cycle and phase time constraints leading to our final MILP formulation of traffic signal control. We then experiment with this novel QTM-based MILP control in a range of networks demonstrating the improved scalability possible with non-homogeneous time steps in comparison to the best homogeneous time step. These experiments also provide near-optimal traffic control policies for larger horizons and larger networks than shown in previous implementations of MILP-based traffic signal control. [3] [4] [5]

## THE QUEUE TRANSMISSION MODEL

A Queue Transmission Model (QTM) is the tuple $(\mathcal{Q}, \mathcal{L}, \vec{\Delta t}, \mathbf{I})$, where $\mathcal{Q}$ and $\mathcal{L}$ are, respectively, the set of queues and lights; $\vec{\Delta t}$ is a vector of size $\mathrm{N}$ representing the discretization of the simulation horizon $[0, \mathrm{T}]$ and the duration in seconds of the $n$-th time interval is denoted as $\Delta t_n$; and $\mathbf{I}$ is a matrix $|\mathcal{Q}| \times \mathrm{T}$ in which $I_{i,n}$ represents the flow of cars requesting to enter queue $i$ from the outside of the network at time $n$.

A **traffic light** $\ell \in \mathcal{L}$ is defined as the tuple $(\Psi_\ell^{\min}, \Psi_\ell^{\max}, \mathcal{P}_\ell, \vec{\Phi}_\ell^{\min}, \vec{\Phi}_\ell^{\max})$, where:

- $\mathcal{P}_\ell$ is the set of phases of $\ell$;
- $\Psi_\ell^{\min}$ ($\Psi_\ell^{\max}$) is the minimum (maximum) allowed cycle time for $\ell$; and
- $\vec{\Phi}_\ell^{\min}$ ($\vec{\Phi}_\ell^{\max}$) is a vector of size $|\mathcal{P}_\ell|$ and $\Phi_{\ell,k}^{\min}$ ($\Phi_{\ell,k}^{\max}$) is the minimum (maximum) allowed time for phase $k \in \mathcal{P}_\ell$.

[2] This is not explicitly mentioned later on

[3] Paper should follow the basic progression outlined in last paragraph above. Need to be careful to maintain the thread of the story throughout the paper and the summarize it in the conclusion with the major take-home results — longer horizons and larger networks for MILP-based control!

[4] We could really use some pictures in the Intro to refer to here and subsequently – both a traffic network divided into queues, and the concept of the piecewise linear evolution of traffic flow with **non-homogeneous** (dilated) time steps, something like I had provided in my early writeup. I think these help visually explain much of the context for the paper and its approach and are critical for reviewer understanding on a time budget for reading this They may only read the first 2-3 pages and then skim!

[5] A picture is worth a 1000 words but we only pay 250, hence a 4X ROI on pictures!

A **queue** $i \in \mathcal{Q}$ represents a segment of road that vehicles traverse at free flow speed; once traversed, the vehicles are vertically stacked in a stop line queue. Formally, a queue $i$ is defined by the tuple $(\mathrm{Q}_i, \mathrm{T}_i^{\text{prop}}, \mathrm{F}_i^{\text{out}}, \vec{F}_i, \vec{Pr}_i, \mathcal{Q}_i^{\mathcal{P}})$ where:

- $\mathrm{Q}_i$ is the maximum capacity of $i$;
- $\mathrm{T}_i^{\text{prop}}$ is the time required to traverse $i$ and reach the stop line;
- $\mathrm{F}_i^{\text{out}}$ represents the maximum traffic flow from $i$ to the outside of the modeled network;
- $\vec{F}_i$ and $\vec{Pr}_i$ are vectors of size $|\mathcal{Q}|$ and their $j$-th entry (i.e., $\mathrm{F}_{i,j}$ and $\mathrm{Pr}_{i,j}$) represent the maximum flow from queue $i$ to $j$ and the turn probability from $i$ to $j$ ($\sum_{j \in \mathcal{Q}} \mathrm{Pr}_{i,j} = 1$), respectively; and
- $\mathcal{Q}_i^{\mathcal{P}}$ denotes the set of traffic light phases controlling the outflow of queue $i$.

Differently than CTM (8, 13), QTM does not assume that $\Delta t_n = \mathrm{T}_i^{\text{prop}}$ for all $n \in \{1, \dots, \mathrm{N}\}$, that is, the QTM can represent non-homogeneous time intervals. The only requirement over $\Delta t_n$ is that no traffic light maximum phase time is smaller than any $\Delta t_n$ since phase changes occur only between time intervals; formally, $\Delta t_n \leq \min_{\ell \in \mathcal{L}, k \in \mathcal{P}_\ell} \Phi_{\ell,k}^{\max}$ for all $n \in \{1, \dots, \mathrm{N}\}$. [6]

## Traffic Flow Simulation with QTM

In this section, we present how to simulate traffic flow in a network using QTM and non-homogeneous time intervals $\Delta t$. We assume for the remainder of this section that a *valid* control plan for all traffic lights is fixed and given as parameter; formally, for all $\ell \in \mathcal{L}$, $k \in \mathcal{P}_\ell$, and interval $n \in \{1, \dots, N\}$, the binary variable $p_{\ell,k,n}$ is known a priori and indicates if phase $k$ of light $\ell$ is active (i.e., $p_{\ell,k,n} = 1$) or not on interval $n$.

We represent the problem of finding the flow between queues as a Linear Program (LP) over the following variables defined for all interval $n \in \{1, \dots, \mathrm{N}\}$ and queues $i$ and $j$:

- $q_{i,n} \in [0, \mathrm{Q}_i]$: traffic volume of queue $i$ during interval $n$;
- $f_{i,n}^{\text{in}} \in [0, I_{i,n}]$: inflow to the network via queue $i$ during interval $n$;
- $f_{i,n}^{\text{out}} \in [0, \mathrm{F}_i^{\text{out}}]$: outflow from the network via queue $i$ during interval $n$; and
- $f_{i,j,n} \in [0, \mathrm{F}_{i,j}]$: flow from queue $i$ into queue $j$ during interval $n$.

The maximum traffic flow from queue $i$ to queue $j$ is enforced by constraints (C1) and (C2). (C1) ensures that only the fraction $\mathrm{Pr}_{i,j}$ of the total internal outflow of $i$ goes to $j$, and (C2) forces the flow from $i$ to $j$ to be zero if all phases controlling $i$ are inactive (i.e., $p_{\ell,k,n} = 0$ for all $k \in \mathcal{Q}_i^{\mathcal{P}}$). If more than one phase $p_{\ell,k,n}$ is active, then (C2) is subsumed by the domain upper bound of $f_{i,j,n}$.

$$f_{i,j,n} \leq \mathrm{Pr}_{i,j} \sum_{k=1}^{|\mathcal{Q}|} f_{i,k,n} \tag{C1}$$

$$f_{i,j,n} \leq \mathrm{F}_{i,j} \sum_{p_{\ell,k,n} \in \mathcal{Q}_i^{\mathcal{P}}} p_{\ell,k,n} \tag{C2}$$

[6]**To Iain**: Maybe bring forward a small network and any other figure that would help illustrate the model and comment about it.

To simplify the presentation of remainder of the LP, we define the helper variables $q^{\text{in}}_{i,n}$ (C3), $q^{\text{out}}_{i,n}$ (C4), and $t_n$ (C5) to represent the volume of traffic to enter and leave queue $i$ during interval $n$, and the time elapsed since the beginning of the simulation until the end of interval $\Delta t_n$. [7]

$$q^{\text{in}}_{i,n} = \Delta t_n (f^{\text{in}}_{i,n} + \sum_{j=1}^{|\mathcal{Q}|} f_{j,i,n}) \tag{C3}$$

$$q^{\text{out}}_{i,n} = \Delta t_n (f^{\text{out}}_{i,n} + \sum_{j=1}^{|\mathcal{Q}|} f_{i,j,n}) \tag{C4}$$

$$t_n = \sum_{x=1}^{n} \Delta t_x \tag{C5}$$

In order to account for the misalignment of the different $\Delta t$ and $\mathrm{T}^{\text{prop}}_i$, we need to find the volume of traffic that was able to arrive at queue $i$, traverse it (i.e., wait $\mathrm{T}^{\text{prop}}_i$ seconds), and reach the stop line before $\Delta t_n$ is over. This volume of traffic is obtained by integrating over the current interval with the rate at which traffic was arriving during the interval $m$ containing the time $t_n - \mathrm{T}^{\text{prop}}_i$, where $m$ is the interval such that $t_m \leq t_n - \mathrm{T}^{\text{prop}}_i < t_{m+1}$. The input rate of queue $i$ during interval $m$ is found by dividing $q^{\text{in}}_{i,m}$ with $\Delta t_m$, and then the total traffic arriving during the interval $n$ is found by multiplying this rate by $\Delta t_n$. The flow conservation principle for non-homogeneous time steps is presented in (C6). If $\Delta t$ is homogeneous for all $n$ then (C6) reduces to $q_{i,n} = q_{i,n-1} - q^{\text{out}}_{i,n-1} + q^{\text{in}}_{i,m}$. To insure that the total volume of traffic travelling down the queue and waiting at the stop line does not exceed the capacity if the queue, we apply (C7).

$$q_{i,n} = q_{i,n-1} - q^{\text{out}}_{i,n-1} + \frac{\Delta t_n}{\Delta t_m} q^{\text{in}}_{i,m} \tag{C6}$$

$$\frac{\Delta t_n}{\Delta t_m} q^{\text{in}}_{i,m} + \sum_{k=m+1}^{n} q^{\text{in}}_{i,k} \leq \mathrm{Q}_i - q_{i,n-1} \tag{C7}$$

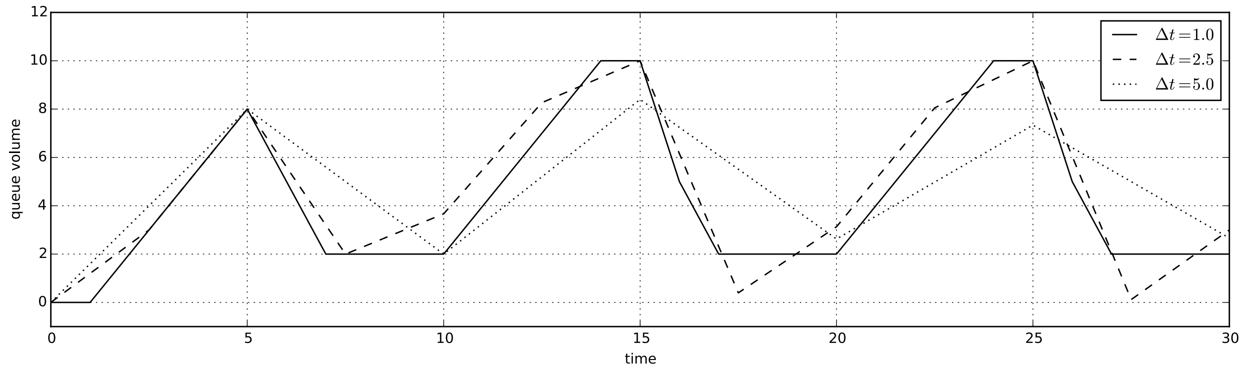
As with MILP formulations of CTM (e.g. Lin and Wang (*8*)), QTM is also susceptible to *withholding traffic*, i.e., the optimizer might prevent cars from moving from $i$ to $j$ even though the associated traffic phase is active and $j$ is not full. We address this issue through our objective function (O1) by maximizing the total outflow $q^{\text{out}}_{i,n}$ (i.e., both internal and external outflow) of $i$ plus the inflow $f^{\text{in}}_{i,n}$ from the outside of the network to $i$. This quantity is weighted by the remaining time until the end of the simulation horizon T to force the optimizer to allow as much traffic volume as possible into the network and move traffic to the outside the network as soon as possible. (O1) is analogous to minimizing delay in CTM models, e.g., (O1) is equivalent to the objective function (O3) in Lin and Wang (*8*) for their parameters $\alpha = \beta = 1$. [8] [9]

$$\max \sum_{n=1}^{\mathrm{N}} \sum_{i=1}^{|\mathcal{Q}|} (\mathrm{T} - t_n + 1)(q^{\text{out}}_{i,n} + f^{\text{in}}_{i,n}) \tag{O1}$$

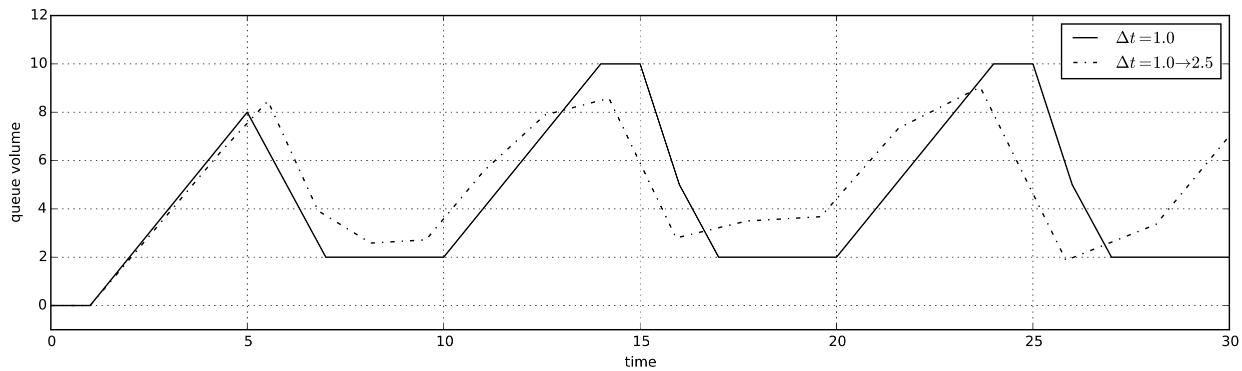
[7] **To Iain**: Triple check my explanation.

[8] **To Iain**: Add a paragraph linking the plots with the objective function.

[9] Show diagrams with traffic predictions converging as time increment gets smaller. Validates that large time-steps are rough approximations while model behavior converges for small time steps.




(a)




(b)

**FIGURE 1 An example showing the evolution of traffic volume in a queue over time. (a) Convergce with increasing refinement of $\Delta t$ from $5.0$ down to $1.0$. (b) Dilation of $\Delta t$ from $1.0$ to $2.5$ compared to a fixed $\Delta t$ of $1.0$.**

The objective function (O1) and constraints (C1–C7) form the LP representing the dynamic, piecewise linear model of flow in a QTM network over time when a control plan $p_{\ell,k,n}$ is given as an input parameter.

Figures 1(a), 1(a) and 2 show the results of applying the LP formulation to a simple model with a fixed signal plan, using both homogeneous $\vec{\Delta t}$ and non-homogeneous $\vec{\Delta t}$.

## TRAFFIC CONTROL WITH QTM AS AN MILP

In this section, we remove the assumption that a valid control plan for all traffic lights is given and extend the LP (O1, C1–C7) to an Mixed-Integer LP (MILP) that also computes the optimal control plan. Formally, for all $\ell \in \mathcal{L}$, $k \in \mathcal{P}_\ell$, and interval $n \in \{1, \dots, N\}$, the phase activation parameter $p_{\ell,k,n} \in \{0, 1\}$ becomes a free variable to be optimized. In order to obtain a valid control plan, we enforce that one phase of traffic light $\ell$ is always active at any interval $n$ (C8) and that phase changes happen sequentially (C9), i.e., if phase $k$ was active during interval $n - 1$ and has become inactive in interval $n$, then phase $k + 1$ must be active in interval $n$. (C9) assumes that $k + 1$ equals 1 if $k = |\mathcal{P}_\ell|$. [10]

[10]**To Iain**: I removed the constraint $p_{\ell,k,n} + p_{\ell,k+1,n} \leq 1$ because it is subsumed by $p_{\ell,k,n} \in \{0, 1\}$ and (C8)

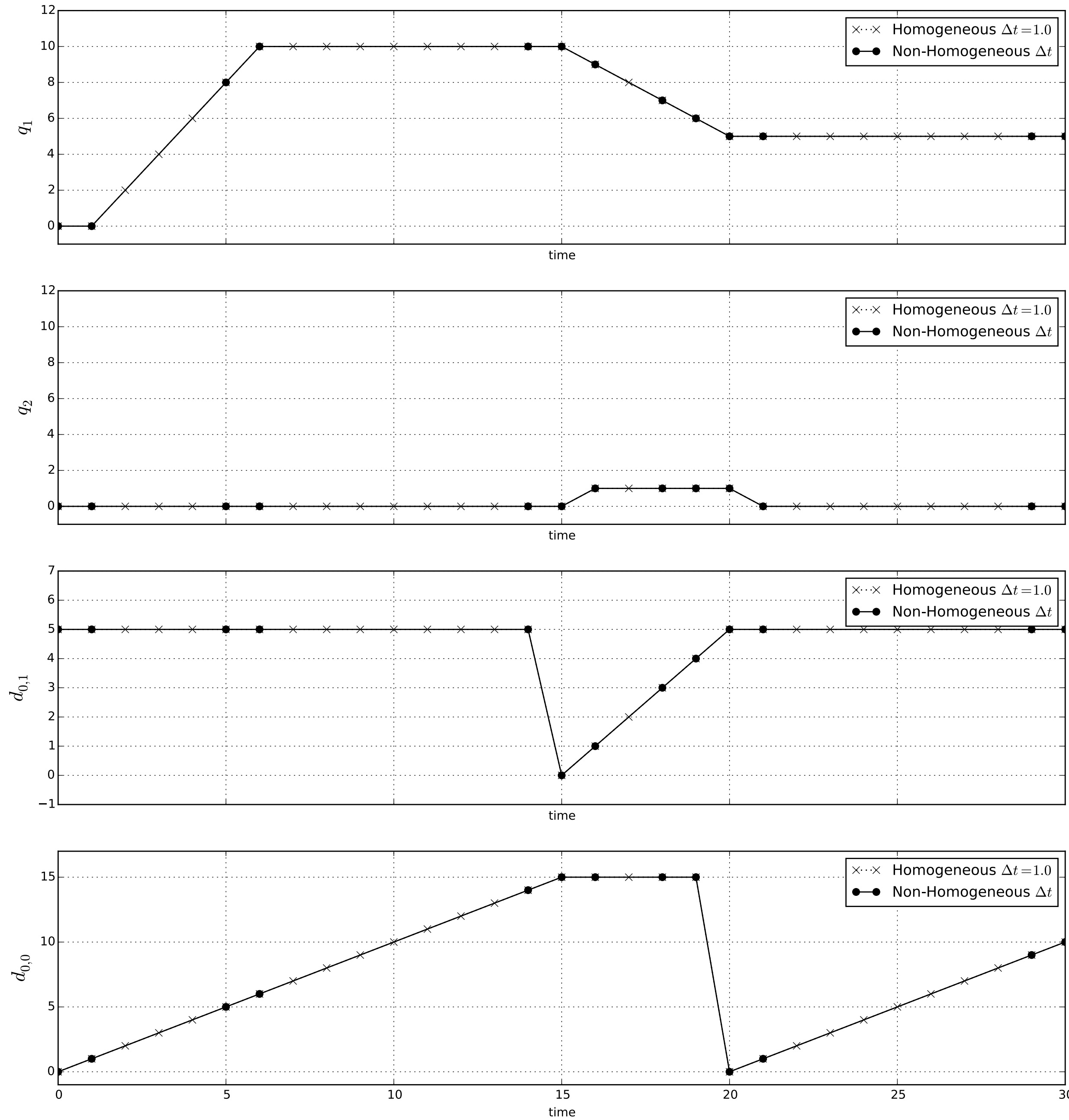



**FIGURE 2 An example showing the convergence between a homogeneous solution with $\Delta t = 1.0$ and a non-homogeneous solution over 30 seconds for the same network. By using non-homogeneous time steps the same solution is found with only 14 sample points compared to 30 for homogeneous solution.**

$$\sum_{k=1}^{|\mathcal{P}_\ell|} p_{\ell,k,n} = 1 \tag{C8}$$

$$p_{\ell,k,n-1} \leq p_{\ell,k,n} + p_{\ell,k+1,n} \tag{C9}$$

Next, we enforce the minimum and maximum phase durations (i.e., $\Phi^{\min}_{\ell,k}$ and $\Phi^{\max}_{\ell,k}$) for each phase $k \in \mathcal{P}_\ell$ of traffic light $\ell$. To encode these constraints, we use the helper variable $d_{\ell,k,n} \in [0, \Phi^{\max}_{\ell,k}]$ defined by constraints (C10–C14) that: (i) holds the elapsed time since the start of phase $k$ when $p_{\ell,k,n}$ is active (C10,C11) (see Figure 3(a)); (ii) is constant and holds the duration of the last phase until the next activation when $p_{\ell,k,n}$ is inactive (C12,C13) (see Figure 3(b)); and (iii) is restarted when phase $k$ changes from inactive to active (C14) (see Figure 3(c)). Notice that (C10–C14) employs the *big-M* method to turn the cases that should not be active into subsumed constraints based on the value of $p_{\ell,k,\cdot}$. We use $\Phi^{\max}_{\ell,k}$ as our large constant since $d_{\ell,k,n} \leq \Phi^{\max}_{\ell,k}$ and $\Delta t_n \leq \Phi^{\max}_{\ell,k}$ by assumption (Section 2.1). Similarly, constraint (C15) ensures the minimum phase time of $k$ and is not enforced while $k$ is still active.

$$d_{\ell,k,n} \leq d_{\ell,k,n-1} + \Delta t_{n-1} p_{\ell,k,n-1} + \Phi^{\max}_{\ell,k}(1 - p_{\ell,k,n-1}) \tag{C10}$$
$$d_{\ell,k,n} \geq d_{\ell,k,n-1} + \Delta t_{n-1} p_{\ell,k,n-1} - \Phi^{\max}_{\ell,k}(1 - p_{\ell,k,n-1}) \tag{C11}$$
$$d_{\ell,k,n} \leq d_{\ell,k,n-1} + \Phi^{\max}_{\ell,k} p_{\ell,k,n-1} \tag{C12}$$
$$d_{\ell,k,n} \geq d_{\ell,k,n-1} - \Phi^{\max}_{\ell,k} p_{\ell,k,n} \tag{C13}$$
$$d_{\ell,k,n} \leq \Phi^{\max}_{\ell,k}(1 - p_{\ell,k,n} + p_{\ell,k,n-1}) \tag{C14}$$
$$d_{\ell,k,n} \geq \Phi^{\min}_{\ell,k}(1 - p_{\ell,k,n}) \tag{C15}$$

Finally, we constrain the sum of all the phase durations for light $\ell$ to be within the cycle time limits $\Psi^{\min}_\ell$ (C16) and $\Psi^{\max}_\ell$ (C17) (see Figure 3(d)). In both (C16) and (C17), we use the duration of phase 1 of $\ell$ from the previous interval $n-1$ instead of the current interval $n$ because (C14) forces $d_{\ell,1,n}$ to be 0 at the beginning of each cycle; however, from the previous end of phase 1 until $n-1$, $d_{\ell,1,n-1}$ holds the correct elapse time of phase 1. Additionally, (C16) is enforced right after the end of the each cycle, i.e., when its first phase is changed from inactive to active. [11]

$$d_{\ell,1,n-1} + \sum_{k=2}^{|\mathcal{P}_\ell|} d_{\ell,k,n} \geq \Psi^{\min}_\ell (p_{k,1,n} - p_{k,1,n-1}) \tag{C16}$$
$$d_{\ell,1,n-1} + \sum_{k=2}^{|\mathcal{P}_\ell|} d_{\ell,k,n} \leq \Psi^{\max}_\ell \tag{C17}$$

The MILP that encodes the problem of finding the optimal traffic control plan in a QTM network is defined by (O1, C1–C17).

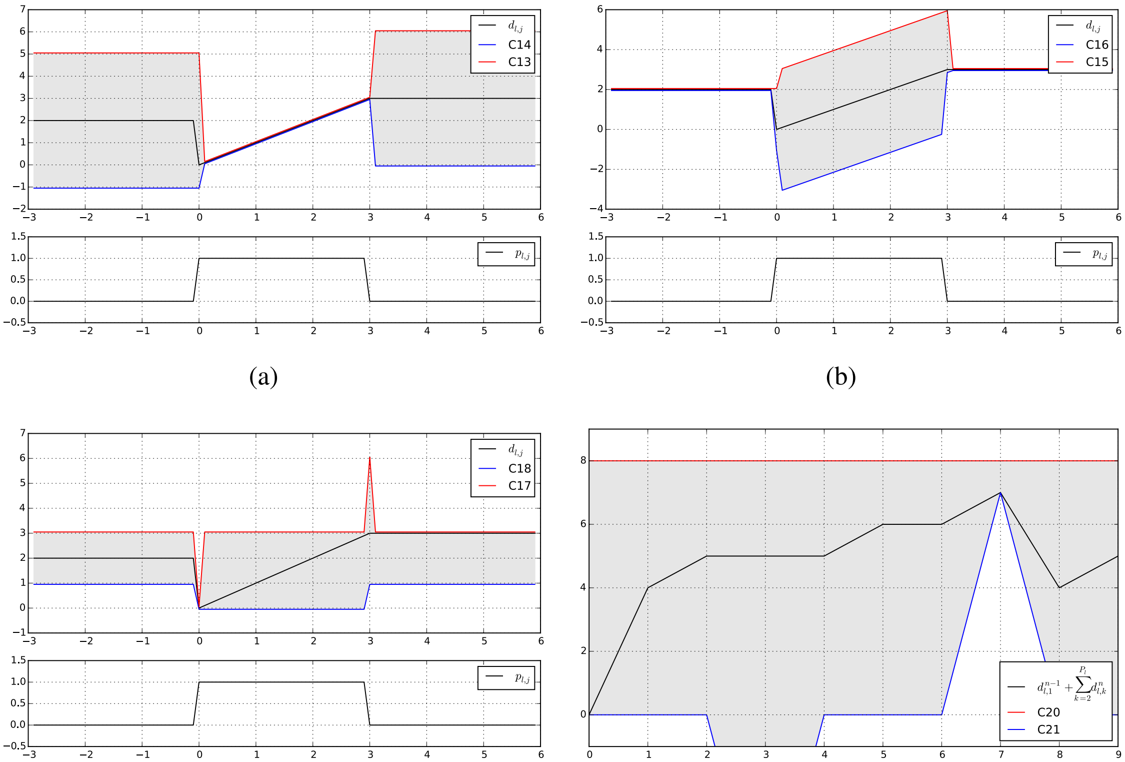
# EMPIRICAL EVALUATION

[12]

In this section we compare the performance of non-homogeneous solutions with homogeneous solutions. Our hypothesis is that an aggregate plan formed from a set of shorter planning steps, will converge on the optimum plan as the amount of look ahead at each step is increased. We then exploit the non-homogeneity of the QTM to vary $\vec{\Delta t}$ such that we sample with reducing resolution as we approach the planning horizon. Compared with a homogenous $\vec{\Delta t}$ of the same

[11] **To Iain**: Relate the phase and cycle constraints with the plots

[12] Start this section with the experimental rationale... what hypotheses are we trying to evaluate in this section?




(a) (b) (c) (d)

**FIGURE 3 An example showing the phase and cycle time constraint envelopes. In (a), (b) and (c), $\Phi^{\min}_{\ell,k} = 1$ and $\Phi^{\max}_{\ell,k} = 3$, the duration of the previous activation was 2 and the duration of the current activation is 3. In (d), the total cycle time is 7 with $\Psi^{\min}_{\ell} = 7$, $\Psi^{\max}_{\ell} = 8$**

length, the non-homogenous solution will have a greater look ahead, but with decreasing resolution. Reducing the resolution with look ahead seems reasonable as the accuracy of the plan will also reduce over time when compared with the actual state of the network that later eventuates. [13]

## Networks

To demonstrate the scalability of the QTM, we evalute three networks of increasing complexity in the comparison. The first consists of an avenue crossed by three side streets at controlled intersections, as shown in Figure 5(a). The second introduces a second parallel avenue to form a gridded network with a total of three controlled intersections, as shown in Figure 5(b). And the third is a more complex grid network with 9 controlled intersections between six avenues, with a seventh avenue running through at a diagonal as shown in Figure 5(c).

The traversal time of each queue, $\mathrm{T}^{\mathrm{prop}}$ in all three networks is set at 9s between intersections (a distance of about 100m with a free flow speed of 50km/h). The maximum capacity of each queue not leading in from the boundary of the network is set at 60 cars. For queues leading in from the boundary, the capacity is increased sufficiently to buffer any spill back from the stop

[13] Add a planning horizon to the planning figure

line and prevent interruption of the input demand profile. Flows are defined from the head of each queue into the tail of the next. There is no turning traffic ($\mathrm{Pr}_{i,j} = 1$), and in all cases the maximum flow rate between queues, $\mathrm{F}_{i,j}$, is set at 5 cars per second. Each intersection in networks 1 and 2 has two phases - North-South (NS) and East-West (EW). In network 3, lights 2, 4 and 6 have an additional Northeast-Southwest phase to control the diagonal avenue. For networks 1 and 2 for all phases $\Phi^{\min}_{,}$ is 1s and $\Phi^{\max}_{,}$ is 3s, and for all intersections $\Psi^{\min}$ is 2s and $\Psi^{\max}$ is 6s. In network 3 for all phases $\Phi^{\min}_{,}$ is 1s and $\Phi^{\max}_{,}$ is 6s, and for all intersections $\Psi^{\min}$ is 2s and $\Psi^{\max}$ is 12s except for lights 2, 4 and 6 which have a $\Psi^{\min}$ of 3s and $\Psi^{\max}$ of 18s.

**Experimental Methodology**

For each network a background level of inflow is first established and left to run for 55s to allow the solver to settle on a stable policy. Then a spike in demand is introduced to some of the input queues for 15s to trigger a policy change, with the expectation that plans generated with longer look ahead will produce a more coordinated global policy change. The demand is then returned to the background level for another 15s before being reduced to zero, and finally sufficient time is given to allow the network to clear of traffic. By clearing the network we can easily measure the total travel time for all the traffic as the area between the cumulative arrival and departure curves measured at the boundaries of the network. The details of the demand profile per queue are given in Table 1.

For both the homogeneous and non-homogeneous test points, we use the MILP QTM formulation described in the previous section to generating an optimum signal plan for a longer horizon than needed (the look ahead). We then keep the first part of this plan where the accuracy is highest and discard the rest. We call the long horizon plan a major frame, and the first part of the major frame that we retain, a minor frame. While the minor frame plan is being executed on the network, we generate another major frame starting from the end of the current minor frame and repeat (see Figure 4). For each test point we use minor frames of 10s but increase major frame sizes across the test points from 20s upwards. We generated plans for all three networks with both a homogeneous $\Delta t$ of 0.25s and a non-homogeneous $\Delta t$ ranging from fixed 0.25s increments during the minor frame and then increasing linearly until reaching 1s at the end of the major frame. Once we have generated a set of minor frames, we aggregated them into a single plan and simulate the flow through the network using the QTM LP formulation with a fixed $\Delta t$ of 0.25s. As a ground truth reference for each network, we also solve a single optimal signal plan spanning all the minor frames with a fixed $\Delta t$ of 0.25s, and then simulate the flow with the QTM LP formulation. [14] To solve each major frame and the reference, we use Gurobi™ with 12 threads on an AMD Opteron™ 4334 Processor with 12 cores running at 3.1GHz. We limit MIP gap accuracy to 0.1% and, while we can solve non- homogeneous major frames up to the convergence point in real time, we extend the solve time limit to 3000s for all test points for a fair comparison with the homogeneous test points. At each test point we take the average of five runs of the solver. (The reference signal plans take considerably longer to solve and have no solve time limit set).

**Results**

We compare the performance of non-homogeneous and homogeneous solutions in two ways: comparing the decrease in total travel time with increasing major frame time (greater look ahead), and

[14] Do we need to justify why we use the QTM as the simulator over say a micro simulator?

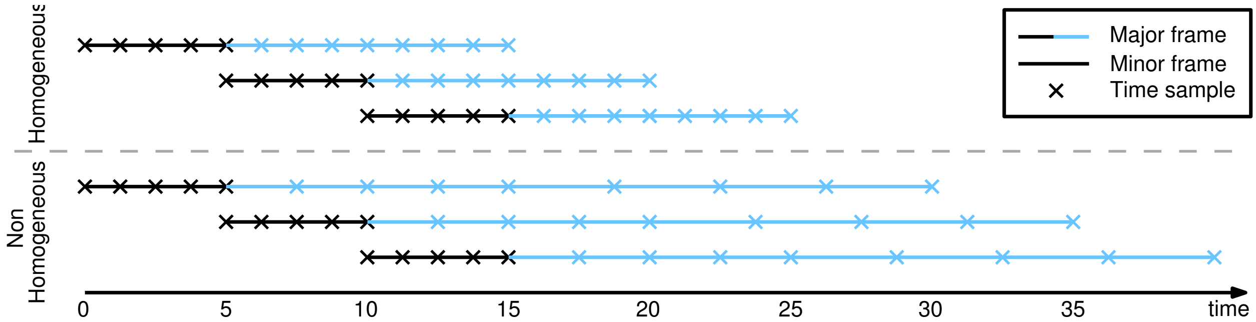



**FIGURE 4 Multi-step planning**

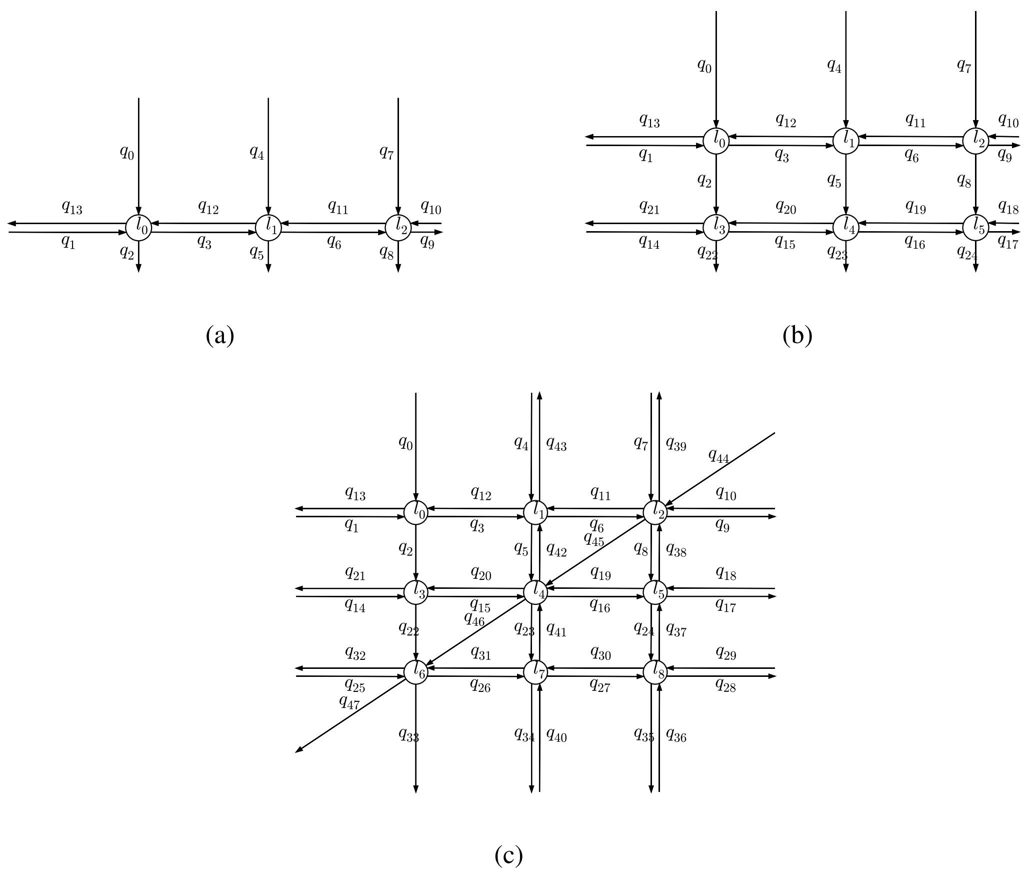



**FIGURE 5 Networks used to evaluate the QTM performance.**

analysing the distribution of delay in each queue of the network. Figures 6(a), 6(c) and 6(e) show a comparison between the number of time samples used in the major frame vs the % improvement in total travel time, where 0% is the reference solution total travel time. It can be seen that for all three networks, using a non homogeneous $\Delta t$ converges towards the optimum total travel time more quickly than the homogeneous $\Delta t$.

Figures 6(b), 6(d) and 6(f) show a comparison of distribution of delay across the network. This gives us an indication of the quality of the solution in terms of the number of vehicles that

**TABLE 1 Network Demand Profiles (vehicles per second)**

| | Inflow Queues | 0 - 55 s | 55 - 70 s | 70 - 85 s | > 85 s |
|---|---|---|---|---|---|
| Network 1 | $q_0$ | 1 | 1 | 1 | 0 |
| | $q_4, q_7$ | 4 | 4 | 4 | 0 |
| | $q_1, q_{10}$ | 2 | 4 | 2 | 0 |
| Network 2 | $q_0$ | 1 | 1 | 1 | 0 |
| | $q_4, q_7, q_{14}$ | 4 | 4 | 4 | 0 |
| | $q_1, q_{10}, q_{18}$ | 2 | 4 | 2 | 0 |
| Network 3 | $q_0$ | 1 | 1 | 1 | 0 |
| | $q_4, q_7, q_{14}, q_{25},$ | 4 | 4 | 4 | 0 |
| | $q_1, q_{10}, q_{18}, q_{29}, q_{36}, q_{40}, q_{44}$ | 2 | 4 | 2 | 0 |

experience significant delay and if the solution may be starving some parts of the network. The box plots show three comparisons: at the point where the non-homogeneous $\Delta t$ first converges on the optimum solution, where the homogeneous $\Delta t$ first converges on the optimum solution, and the optimum solution itself. With all three networks the quality of the solution is better or the same using a non-homogeneous $\Delta t$ compared to a homogeneous $\Delta t$ with the same number of sample points. Finally, Figure 7 shows the cumulative arrival and departure curves and the how delay evolves over time for $q_1$ of network 2. Figure 7(a) shows the comparison at the point where the non-homogeneous $\Delta t$ first converges and shows that with the longer major frame time of the non-homogeneous $\Delta t$, the solver is able to find a co-ordinated signal policy along the avenue to dissipate the extra traffic that arrives at the 55s point, while the homogeneous $\Delta t$ with its shorter major frame fails to find a coordinated policy along the avenue and experiences more delay. Once the homogeneous $\Delta t$ has converged in Figure 7(b), both solutions are close to the optimum solution which is shown in Figure 7(c).

## CONCLUSION

In this paper, we showed how to formulate a novel queue transmission model (QTM) model of traffic flow with non-homogeneous time steps as a linear program. We then proceeded to allow the traffic signals to become discrete variables subject to a delay minimizing optimization objective and standard traffic signal constraints leading to a final MILP formulation of traffic signal control. We experimented with this novel QTM-based MILP control in a range of networks and demonstrated that by exploiting the non-homogeneous time steps supported by the QTM, we are able to scale the model up to larger networks whilst maintaining the same quality of a homogeneous solution using more binary variables. Altogether, this work represents a major step forward in the scalability of MILP-based jointly optimized traffic signal control via the use of a non-homogeneous traffic models and thus helps pave the way for fully optimized joint urban traffic signal controllers as an improved successor technology to existing signal control methods.

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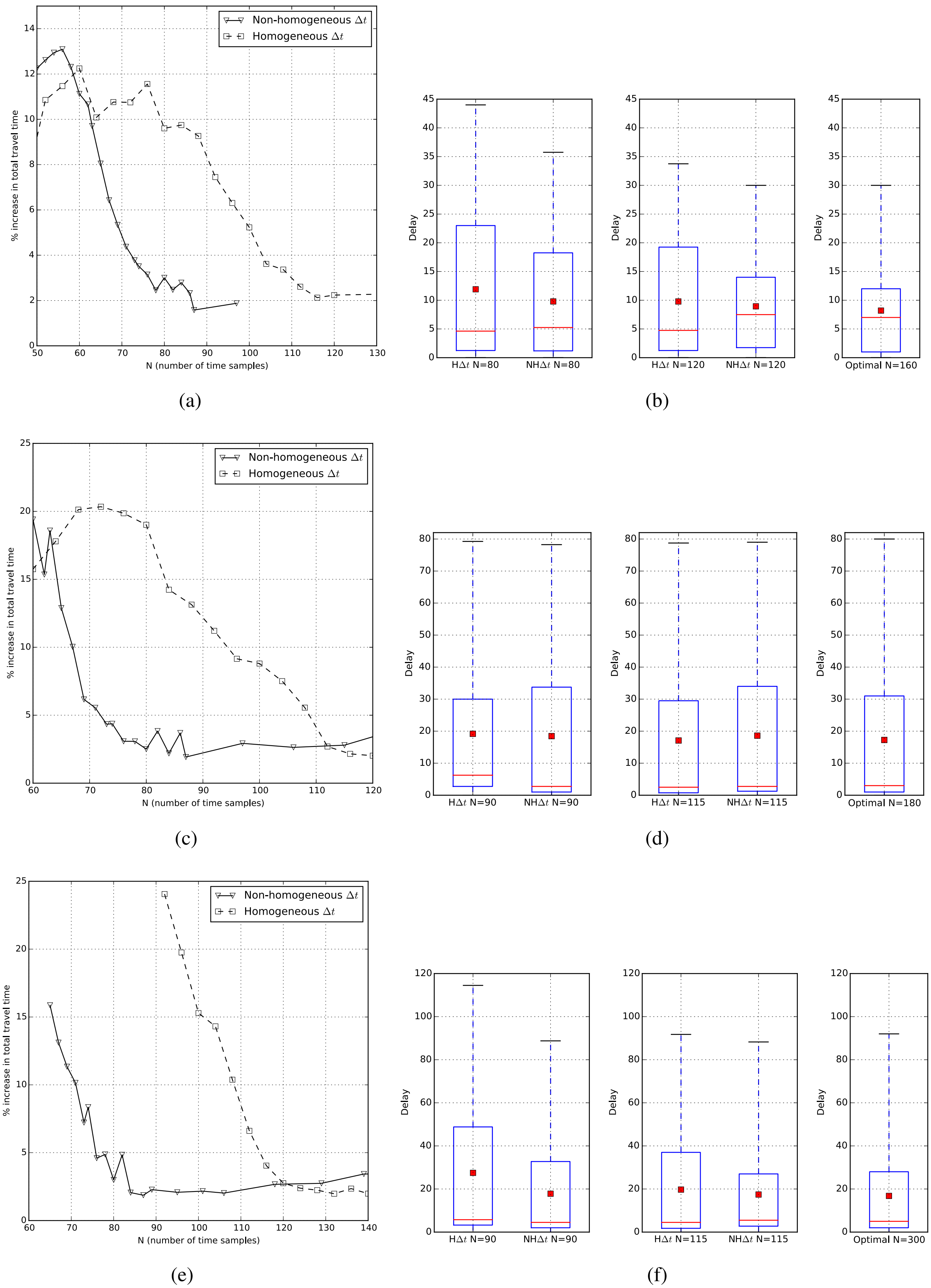
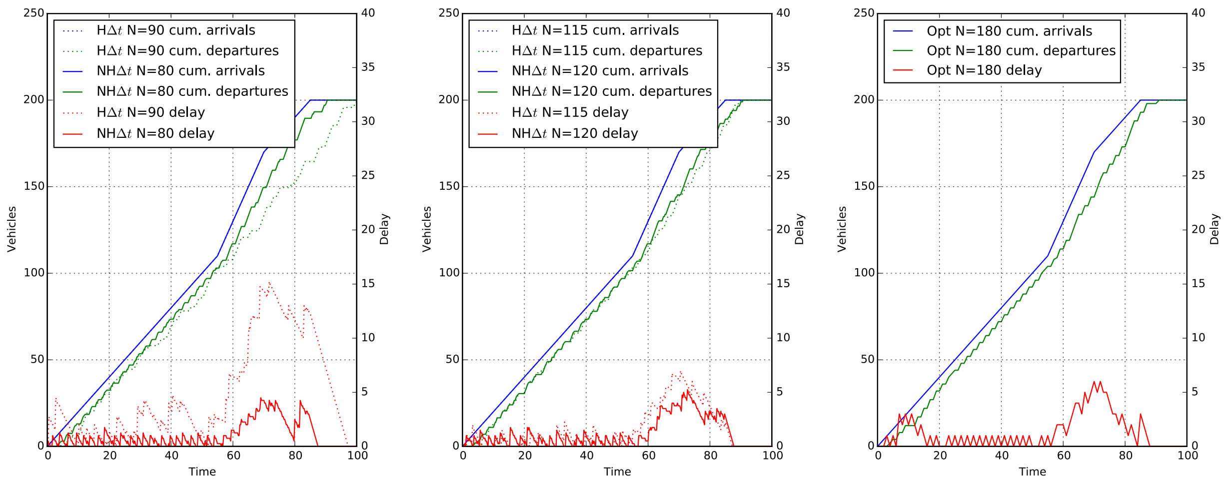



**FIGURE 6 Results for the three networks showing the comparitive % increase in total travel time for the network between using a homogeneous $\Delta t$ and a non-homogeneous $\Delta t$, and the distribution of delay time at the convergence point of non-homogeneous $\Delta t$, the convergence point of homogeneous $\Delta t$ and for the fully solved optimal solution. (a) and (b) 3 light avenue, (c) and (d) 6 light grid, and (e) and (f) 9 light grid,**




(a) (b) (c)

**FIGURE 7 Cumulative arrival and departure curves and delay for queue 1 in the 6 light grid. (a) at the convergence point of the non-homogeneous $\Delta t$ it is near to the optimum solution while homogeneous $\Delta t$ lags behind (b) at the convergence point of homogeneous $\Delta t$ both are near optimum, and (c) the fully solved optimal solution**