$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ Maxwell's Eqns. (6.6)

General Equations of Electrostatics

$\oint_S \mathbf{E} \cdot \mathbf{n}\, da = \frac{1}{\epsilon_0} \int_V \rho(\mathbf{x}) d^3x$ Gauss' Law (1.11)

$\mathbf{E} = -\nabla \Phi, \quad \Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$ Scalar potential (1.16, 1.17)

$\nabla^2 \Phi = -\rho/\epsilon_0$ Electrostatic Poisson Eequation (1.28)

$V_i = \sum_{j=1}^{n} p_{ij} q_j, \qquad Q_i = \sum_{j=1}^{n} C_{ij} V_j$ Capacitance matrices (1.61)

$q' = -\frac{a}{r} q, \quad r' = \frac{a^2}{r}$ Magnitude and position of image charge on sphere (2.4)

$q = \int \rho(\mathbf{x}')\, d^3x'$ Electric Monopole (4.4)

$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}')\, d^3x'$ Electric dipole (4.8)

$Q_{ij} = \int (3x_i'x_j' - r'\delta_{ij})\rho(\mathbf{x}') d^3x' = 3M_{ij} - \mathrm{Tr}(\mathbf{M}\delta_{ij})$ Electric Quadrupole (4.9)

$M_{ij} = \int x_i' x_j' \rho(x')\, d^3x$ Dana definition of $\mathbf{M}$ matrix

$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{x}}{r^3} + \frac{1}{2}\sum_{i,j} Q_{ij}\frac{x_i x_j}{r^5} + \ldots\right]$ Electric multipole Expansion (4.10)

$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p}\cdot\mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0|\mathbf{x} - \mathbf{x}_0|^3}$ $\mathbf{E}$-field due to dipole $\mathbf{p}$ (4.13)

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}, \quad \mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E}$ Torque, force on electric dipole (G. 4.4, 4.5)

$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ Electric displacement(4.34)

$\mathbf{D} = \epsilon \mathbf{E}$ Electric displacement (linear materials) (4.37)

$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} = (\epsilon - \epsilon_0)\mathbf{E}$ Induced polarization (linear materials) (4.36)

$\epsilon = \epsilon_0(1 + \chi_e)$ Electric permittivity (linear materials) (4.38)

$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}, \quad \rho_b = -\nabla \cdot \mathbf{P}$ Electric bound charge density (G. 4.11)

$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1)\cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \Rightarrow \Phi_2 = \Phi_1 = V \end{cases}$ Electric JC's (evaluate at boundary) (4.40)

$W = \int \rho(\mathbf{x})\Phi(\mathbf{x})\, d^3x = \frac{1}{2}\int \mathbf{E}\cdot\mathbf{D}\, d^3x$ Energy to bring charges from ∞ (4.83,89)

$W = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} V_i V_j$ Potential energy of capacitor system (1.62)

$W = q\Phi(0) - \mathbf{p}\cdot\mathbf{E}(0) - \frac{1}{6}\sum_i\sum_j Q_{ij}\frac{\partial E_j}{\partial x_i}(0) + \ldots$ Work multipole expansion (4.24)

Specific Cases in Electrostatics

$\begin{cases} \Phi_{\text{in}} = -\left(\frac{3}{\epsilon/\epsilon_0 + 2}\right) E_0 r \cos\theta \\ \Phi_{\text{out}} = -E_0 r\cos\theta + \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0 \frac{a^3}{r^2}\cos\theta \end{cases}$ Dielectric sphere in $\mathbf{E} = E_0\hat{\mathbf{z}}$ (4.54)

$\Phi = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta$ Electric potential of conducting sphere in $\mathbf{E} = E_0\hat{\mathbf{z}}$ (2.14)

$\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$ $\mathbf{E}$ of parallel-plate capacitor ($\hat{\mathbf{n}}$ points from pos. to neg.) (G. Ex. 2.6)

$\mathbf{E} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}}\right)$ Electic dipole at origin pointing in $\hat{\mathbf{z}}$ (4.12)

General Equations of Magnetostatics

$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$ Ampère's law (5.25)

$\mathbf{F} = I\int (d\mathbf{l} \times B)$ Magnetic force along a current element (G. 5.16)

$\nabla \cdot \mathbf{J} = 0$ Condition of magnetostatics (5.3)

$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_M$ Poisson equation in terms of magnetic vector potential (5.101)

$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{l}' \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}$ Biot-Savart law (G. 5.34)

$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x}), \quad \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$ Magnetic vector potential (5.27,5.32)

$\mathbf{m} = \frac{1}{2}\int \mathbf{x}' \times \mathbf{J}(\mathbf{x}')\, d^3x'$ Magnetic dipole (5.54)

$\mathbf{m} = IA\hat{\mathbf{n}}$ Magnetic moment of plane loop (5.57)

$\mathbf{m} = \int \mathbf{M}\, d^3x$ Total magnetic moment (J. pg. 197)

$\Phi_M(\mathbf{x}) = \frac{\mathbf{m}\cdot\mathbf{x}}{4\pi r^3}, \quad \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$ Magnetic potentials of a dipole (5.55)

$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\left[\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{m}) - \mathbf{m}}{|\mathbf{x}|^3}\right]$ Magnetic field of dipole ($\mathbf{n}$ parallel to $\mathbf{x}$) (5.56)

$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}, \quad \mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$ Torque and force on magnetic dipole (5.1, 5.69)

$\mathbf{H} = -\nabla\Phi_M$ Magnetic scalar potential (valid if $\mathbf{J}_f = 0$) (5.93)

$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$ Definition of $\mathbf{H}$ (5.81)

$\mathbf{B} = \mu\mathbf{H}$ Property of linear permeable materials (5.84)

$\begin{cases} (\mathbf{B}_2 - \mathbf{B}_1)\cdot\mathbf{n} = 0 \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f, \ \mathbf{K}_f = 0 \Rightarrow \Phi_1 = \Phi_2 \end{cases}$ Magnetic JC's (eval. at boundary) (5.86)

$\mathbf{J}_M = \nabla \times \mathbf{M}, \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{n}$ Bound current density (G. 6.13,14)

$\rho_M = -\nabla\cdot\mathbf{M}, \quad \sigma_M = \mathbf{n}\cdot\mathbf{M}$ Effective magnetic charge density (5.96,99)

$F = \int_S \mathbf{B}\cdot\mathbf{n}\, da = \oint \mathbf{A}\cdot d\mathbf{l}$ Magnetic flux (5.133)

$\mathscr{E} = -\frac{dF}{dt} = \oint \mathbf{E}\cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{a}$ EMF due to Faraday's Law (5.135)

$M_{ij} = \frac{1}{I_j} F_{ij}$ Mutual inductance (5.156)

$\mathcal{L} = \frac{1}{I^2}\int_{\mathcal{C}} \mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})\, d^3x = \frac{1}{I^2}\int \frac{\mathbf{B}\cdot\mathbf{B}}{\mu}\, d^3x$ Formulae for inductance (5.154, 5.157)

$W = \frac{1}{2}\int \mathbf{J}\cdot\mathbf{A}\, d^3x = \frac{1}{2}\int \mathbf{H}\cdot\mathbf{B}\, d^3x$ Energy in magnetic field (5.149,5.148)

$W = \frac{1}{2}\sum_{i=1}^{N}\mathcal{L}_i I_i^2 + \sum_{i=1}^{N}\sum_{j>i}^{N} M_{ij} I_i I_j$ Potential energy in inductor system(5.152)

Specific Cases of Magnetostatics

$\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2} \Rightarrow \mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$ Field of dipole (G 5.87,88)

$\mathbf{B}(z) = \frac{\mu_0 I}{2}\left[\frac{a^2}{(a^2 + z^2)^{3/2}}\right]\hat{\mathbf{z}}$ On-axis magnetic field of current loop (G. 5.41)

$\mathbf{B} = \frac{\mu_0 NI}{L}\hat{\mathbf{z}}, \quad \mathbf{B} = \frac{\mu_0 NI}{2\pi\rho}\hat{\boldsymbol{\phi}}$ Magnetic field inside (solenoid, toroidal coil) (G. 5.59,60)

$\Phi_M(r,\theta) = \frac{1}{3}M_0 a^2 \frac{r_<}{r_>^2}\cos\theta$ Sphere with uniform $\mathbf{M} = M_0\hat{\mathbf{z}}$ $[(r_<, r_>), (r, a)]$ (5.104)

$\mathbf{M} = \frac{3}{\mu_0}\left(\frac{\mu - \mu_0}{\mu + 2\mu_0}\right)\mathbf{B}_0$ Permeable sphere in uniform magnetic field $\mathbf{B}_0$ (5.115)

$\mathcal{L} = \frac{\mu_0 \pi a^2 N^2}{L}$ Inductance of a solenoid with N turns per unit length L (L. HW9 Pr. 2)

Electrodynamics

$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t}, \quad \nabla\cdot\mathbf{A}' + \frac{1}{c^2}\frac{\partial\Phi'}{\partial t} = 0$ Lorenz gauge (6.9, 6.17)

$\nabla\cdot\mathbf{J} + \frac{\partial\rho}{\partial t} = 0, \quad \frac{\partial u}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{J}\cdot\mathbf{E}$ Continuity Equations (6.3, 6.108)

$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\rho/\epsilon_0, \quad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J}$ EM wave equations (6.15,6.16)

$\Phi(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0}\int \frac{[\rho(\mathbf{x}', t')]_{\text{ret}}}{|\mathbf{x} - \mathbf{x}'|}\, d^3x$

$\mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\int \frac{[\mathbf{J}(\mathbf{x}', t')]_{\text{ret}}}{|\mathbf{x} - \mathbf{x}'|}\, d^3x'$ Retarded potentials where $t' = t - |\mathbf{x} - \mathbf{x}'|/c$ (6.48)

$r \gg c\tau$ (radiation zone), $\quad r \ll c\tau$ (Static zone) (G. 11.10, 11.13)

$u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})$ Total electromagnetic energy density (6.106)

$\mathbf{S} = \mathbf{E} \times \mathbf{H}$ Poynting vector definition (6.109)

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ Lorentz force law (6.113)

$\mathbf{g} = \frac{1}{c^2}(\mathbf{E} \times \mathbf{H})$ Electromagnetic momentum density (6.118)

$\mathbf{J} = \sigma\mathbf{E}, \quad V = IR; \qquad P = IV = I^2R$ Ohm's Law, Joule Heating Law (G. 7.3, 7.4; 7.7)

$R = \rho\ell/A, \quad \sigma = 1/\rho$ Pouillet's Law (ℓ is length, A is cross-sectional area) (Wikipedia)

Telegrapher's Equations

$\left.\begin{aligned} \frac{\partial I}{\partial t} &= -\frac{1}{\mathcal{L}}\frac{\partial V}{\partial z} \\ \frac{\partial V}{\partial t} &= -\frac{1}{\mathcal{C}}\frac{\partial I}{\partial z} \end{aligned}\right\} \Rightarrow \frac{\partial^2 V}{\partial t^2} = \frac{1}{\mathcal{LC}}\frac{\partial^2 V}{\partial z^2} = c^2\frac{\partial^2 V}{\partial z^2}$ Telegrapher's equations (L. 21, 22, 23)

$\mathcal{C} = \frac{2\pi\epsilon}{\ln(b/a)}, \quad \mathcal{L} = \frac{\mu}{2\pi}\ln(b/a)$ Capacitance and inductance of coaxial cable (L. 17, 19)

$V(z,t) = f(t - z/c) + g(t + z/c), \ I(z,t) = \frac{1}{Z}[f(t - z/c) - g(t + z/c)]$ (L. 25, 30)

$Z = c\mathcal{L} = \sqrt{\mathcal{L}/\mathcal{C}}, \quad k = \pm\omega/c$ Impedance definiton, wave vector value (L. 31)

$V(\ell, t) = RI(\ell, t) \Rightarrow g(t + \ell/c) = \frac{R - Z}{R + Z}f(t - \ell/c)$ Resistive BC (L. 37,39)

$V(z,t) = \Re[\tilde{A}(z)e^{-i\omega t}] \Rightarrow \frac{d^2\tilde{A}}{dz^2} = -(w/c)^2\tilde{A} \Rightarrow \tilde{A}(z) = \tilde{A}_0 e^{ikz}$ (L. 48, 49, 50)

$V(z,t) = |\tilde{A}_0|\cos[\omega(t \mp z/c) - \delta]$ Sinusoidal solutions to telegrapher's equation (L. 53)

$V(z,t) = \frac{V_0}{\sin(\omega\ell/c)}\cos(\omega t)\sin[\omega(\ell - z)/c]$ Short circuit, $V(0,t) = V_0\cos(\omega t)$ (L. 62)

	Wiggly	Decaying
x, y, z	$e^{\pm ik_n x}, \ A\cos(k_n x) + B\sin(k_n x)$	$e^{\pm k_n x}, \ A\cosh(k_n x) + B\sinh(k_n x)$
ρ, ϕ, z	$e^{im\phi}, \ AJ_m(k_n\rho) + BY_m(k_n\rho)$	$AI_m(k_n\rho) + BK_m(k_n\rho)$
ρ, ϕ	$\sin(m\phi + \alpha_m)$	$A_0 + B_0\ln\rho + \sum A_m\rho^m + B_m\rho^{-m}$
r, θ	$P_\ell(\cos\theta)$	$A\left(\frac{r}{a}\right)^\ell + B\left(\frac{r}{a}\right)^{-(\ell+1)}$
r, θ, ϕ	$Y_{\ell m}(\theta, \phi)$	$A\left(\frac{r}{a}\right)^\ell + B\left(\frac{r}{a}\right)^{-(\ell+1)}$