Scope of the Comprehensive Examination

After the first year in the physics graduate program, students are required to take a comprehensive examination. If you do not pass this exam the first time, you have a second and final attempt one year later.

The exam will consist of 10 questions spanning the subject areas of classical mechanics, quantum mechanics, electrodynamics, and thermal physics, from the list of topics below. Each question will test: 1) basic knowledge and understanding of physical concepts, and, 2) technical ability to calculate physical effects. You will have approximately one hour per question.

If you do not pass on your first attempt, you still might pass some of the four subject areas. In that case, you will not have to redo those subject areas on your second attempt. You will be informed after your first attempt what subject areas, if any, remain to be passed.

On each problem, you will be asked to do the following:

1. First describe the concepts and strategy you would use to solve the problem. You may also write equations to explain your approach, but do not solve them in your exposition.
2. Use the strategy you have outlined to solve the problem.

Each problem will be graded pass/fail. In order to pass a problem, you must demonstrate fundamental understanding of the physical system in question and technical competence.

Mathematical material will be provided for reference.

Topics

Mechanics

- Identification of symmetries and conserved quantities. Application of conservation laws.
- One-dimensional motion in a conservative system with an arbitrary potential. The concept of turning points.
- Equilibrium of mechanical systems.
- Lagrangian methods.
- The simple harmonic oscillator. Free and damped oscillations, inclusion of external forces, resonance. Calculation of the natural frequency for small oscillations about the minimum of an arbitrary potential.
- Normal modes of a system with coupled parts. Solution to initial-value problems.
- Rotational motion. The gyroscope.
- Oscillations and wave propagation of strings and membranes. Normal modes for particular boundary conditions. Solution to initial-value problems.

Quantum Mechanics

- Measurement in quantum mechanics: expectation values, measurement probability, Uncertainty Principle. The relevance of commutation relations to measurement.
- Multi-state systems. Eigenstates of operators. Solution to the initial-value problem for a superposition state.
- Bound-state problems, especially the 1-d infinite square well and the 1-d harmonic oscillator.
- First-order, time-independent, non-degenerate perturbation theory.
- Motion in one dimension. Quantum tunneling. Evanescent waves.
- Scattering in the Born approximation.
- Spin (half-integer and integer) and angular momentum. Addition rules. Coupling of spin to another spin and to orbital angular momentum.
- Energy of a spin in an external magnetic field. Spin precession.

Electrodynamics

- The electric field due to a static charge distribution. The magnetic field due to a static current distribution. Fields from electric and magnetic dipoles.
- Boundary value problems involving slabs and spheres.
- Fields in matter.
- Maxwell's equations.
- Electromagnetic wave propagation in vacua and in matter. Reflection and transmission for dielectric and conducting media.
- Motion of a charged particle under the Lorentz force. The Hall Effect.
- Dipoles (electric and magnetic) in external fields.
- Fundamental circuits; RC and LC circuits.

Thermal Physics

- The First and Second Laws of thermodynamics.
- Energy, heat, and work.
- Adiabatic, isothermal, and isochoric processes.
- Heat capacity and entropy.

- Thermodynamics of gases with translational and rotational degrees of freedom.
- Thermodynamics of first-order phase transitions. Heat of fusion and latent heat.
- The partition function. Microcanonical, canonical, and grand canonical ensembles.
- Maxwell and Boltzmann distributions.
- Thermodynamics of systems with discrete energy spectra. Calculation of energy, entropy, and heat capacity from the partition function.
- Bose and Fermi gases. Calculation of thermodynamic quantities from phase space integration. Electron degeneracy in metals. Bose-Einstein condensation.
- Blackbody radiation. Calculation of the number of photons in a photon gas, the heat capacity at constant volume, and the pressure. Energy flux from a blackbody.
- Elementary kinetic theory. Atomic/molecular composition of matter. Cross section, mean-free-path, collision time.
- Brownian motion. Thermal diffusion.

You should also be familiar with the classic experiments in physics such as the Stern-Gerlach experiment, the photo-electric effect, the Zeeman effect, and the Compton effect.

Your mathematical preparation should include separation of variables, Taylor series, complex variables, Fourier techniques to solve initial-value problems and Laplace's equation. Linearization of a non-linear ordinary differential equation and its solution. Stable and unstable solutions.

Examples

1. Consider light that is propagating along the z axis. If the beam is linearly polarized along the x axis, each photon is in quantum state $|x\rangle$. If the beam is linearly polarized along the y axis, each photon is in a state $|y\rangle$. Recall that the states of right- and left-circular polarization can be written:

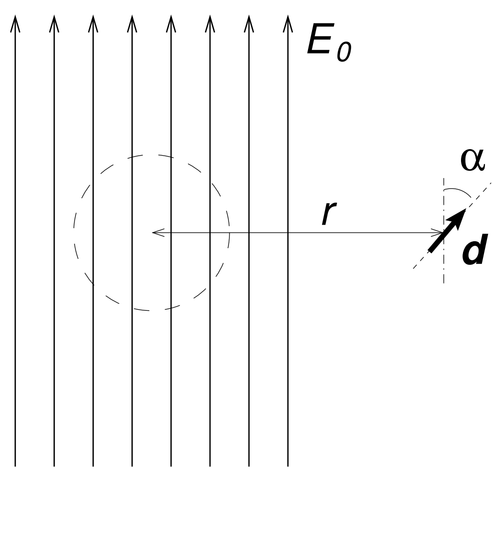
$$|R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right) \qquad |L\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right).$$

   Calcite is an example of a *birefringent* crystal. Calcite has an *optic axis*; light that is linearly polarized perpendicular to the optic axis is called the *ordinary* ray. Light polarized parallel to the optic axis is called the *extraordinary* ray. The index of refraction is different for the two rays - n_e for the extraordinary ray and $n_o > n_e$ for the ordinary ray.

   A slab of calcite lies in the $x - y$ plane, and has thickness L in the z direction. The optic axis is along x. Calculate the probability that a right-circularly polarized photon entering the calcite exits the slab as a left-circularly polarized photon.

2. An electric field $\mathbf{E}_0$ is uniform inside a certain region of space and is zero outside that region. Assume all the space is vacuum. A small electric dipole $\mathbf{d}$ initially oriented as shown, is placed into the region free from field.

   Find the energy of the dipole as a function of r and α, and its equilibrium orientation, when a sphere with dielectric constant ε, and radius a is placed fully into the field region as indicated.



3. An inverted pendulum with the bob of mass m and massless rod of length L is connected to a stationary pole by a spring k, as shown. The spring can follow the arch of circle L, as it stretches and compresses. The system can freely rotate around the z-axis. In equilibrium there is no motion and the angle at the base is θ_0.

   We set the system into motion by shifting the mass from the equlibrium by a *small* angle $\delta\theta$, and simultaneously providing a *small* kick into the page with angular velocity Ω. Find the coordinates of the mass at later times.