Scope of the Comprehensive Examination

After the first year in the physics graduate program, students are required to take a comprehensive examination. If you do not pass this exam the first time, you have a second and final attempt one year later.

The exam will consist of 10 questions spanning the subject areas of classical mechanics, quantum mechanics, electrodynamics, and thermal physics, from the list of topics below. Each question will test: 1) basic knowledge and understanding of physical concepts, and, 2) technical ability to calculate physical effects. You will have approximately one hour per question.

If you do not pass on your first attempt, you still might pass some of the four subject areas. In that case, you will not have to redo those subject areas on your second attempt. You will be informed after your first attempt what subject areas, if any, remain to be passed.

On each problem, you will be asked to do the following:

1. First describe the concepts and strategy you would use to solve the problem. You may also write equations to explain your approach, but do not solve them in your exposition. In your explanation, imagine that you are outlining the solution of the problem to a person trained in physics that has not thought about the problem. That person should then be able to solve the problem with the concepts and strategy you have provided. See the attached examples.
2. Use the strategy you have outlined to solve the problem.

Each problem will be graded pass/fail. In order to pass a problem, you must demonstrate fundamental understanding of the physical system in question and technical competence.

Mathematical material will be provided for reference.

Topics

Mechanics

- Identification of symmetries and conserved quantities. Application of conservation laws.
- One-dimensional motion in a conservative system with an arbitrary potential. The concept of turning points.
- Equilibrium of mechanical systems.
- Lagrangian methods.
- The simple harmonic oscillator. Free and damped oscillations, inclusion of external forces, resonance. Calculation of the natural frequency for small oscillations about the minimum of an arbitrary potential.

- Normal modes of a system with coupled parts. Solution to initial-value problems.
- Rotational motion. The gyroscope.
- Oscillations and wave propagation of strings and membranes. Normal modes for particular boundary conditions. Solution to initial-value problems.

Quantum Mechanics

- Measurement in quantum mechanics: expectation values, measurement probability, Uncertainty Principle. The relevance of commutation relations to measurement.
- Multi-state systems. Eigenstates of operators. Solution to the initial-value problem for a superposition state.
- Bound-state problems, especially the 1-d infinite square well and the 1-d harmonic oscillator.
- First-order, time-independent, non-degenerate perturbation theory.
- Motion in one dimension. Quantum tunneling. Evanescent waves.
- Scattering in the Born approximation.
- Spin (half-integer and integer) and angular momentum. Addition rules. Coupling of spin to another spin and to orbital angular momentum.
- Energy of a spin in an external magnetic field. Spin precession.

Electrodynamics

- The electric field due to a static charge distribution. The magnetic field due to a static current distribution. Fields from electric and magnetic dipoles.
- Boundary value problems involving slabs and spheres.
- Fields in matter.
- Maxwell's equations.
- Electromagnetic wave propagation in vacua and in matter. Reflection and transmission for dielectric and conducting media.
- Motion of a charged particle under the Lorentz force. The Hall Effect.
- Dipoles (electric and magnetic) in external fields.
- Fundamental circuits; RC and LC circuits.

Thermal Physics

- The First and Second Laws of thermodynamics.

- Energy, heat, and work.
- Adiabatic, isothermal, and isochoric processes.
- Heat capacity and entropy.
- Thermodynamics of gases with translational and rotational degrees of freedom.
- Thermodynamics of first-order phase transitions. Heat of fusion and latent heat.
- The partition function. Microcanonical, canonical, and grand canonical ensembles.
- Maxwell and Boltzmann distributions.
- Thermodynamics of systems with discrete energy spectra. Calculation of energy, entropy, and heat capacity from the partition function.
- Bose and Fermi gases. Calculation of thermodynamic quantities from phase space integration. Electron degeneracy in metals. Bose-Einstein condensation.
- Blackbody radiation. Calculation of the number of photons in a photon gas, the heat capacity at constant volume, and the pressure. Energy flux from a blackbody.
- Elementary kinetic theory. Atomic/molecular composition of matter. Cross section, mean-free-path, collision time.
- Brownian motion. Thermal diffusion.

You should also be familiar with the classic experiments in physics such as the Stern-Gerlach experiment, the photo-electric effect, the Zeeman effect, and the Compton effect.

Your mathematical preparation should include separation of variables, Taylor series, complex variables, Fourier techniques to solve initial-value problems and Laplace's equation. Linearization of a non-linear ordinary differential equation and its solution. Stable and unstable solutions.

Examples

1. Consider light that is propagating along the z axis. If the beam is linearly polarized along the x axis, each photon is in quantum state $|x\rangle$. If the beam is linearly polarized along the y axis, each photon is in a state $|y\rangle$. Recall that the states of right- and left-circular polarization can be written:

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right) \qquad |L\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right).$$

   Calcite is an example of a *birefringent* crystal. Calcite has an *optic axis*; light that is linearly polarized perpendicular to the optic axis is called the *ordinary* ray. Light polarized parallel to the optic axis is called the *extraordinary* ray. The index of refraction is different for the two rays - n_e for the extraordinary ray and $n_o > n_e$ for the ordinary ray.

   A slab of calcite lies in the $x - y$ plane, and has thickness L in the z direction. The optic axis is along x. Calculate the probability that a right-circularly polarized photon entering the calcite exits the slab as a left-circularly polarized photon.

   Solution outline:

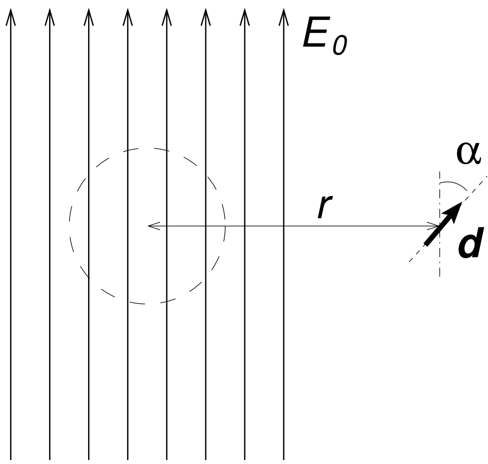
   Suppose the beam exits the slab in state $|\Psi\rangle$. The probability that any photon in the exiting beam will be found in state $|L\rangle$ is

$$P = |\langle L|\Psi\rangle|^2.$$

   To obtain $|\Psi\rangle$, note that the entering beam is a superposition of $|x\rangle$ and $|y\rangle$ as given above. Upon propagation through the slab, the $|x\rangle$ contribution acquires a phase e^{ik_eL}, where k_e is the wavenumber of the extraordinary component. The $|y\rangle$ contribution acquires a phase e^{ik_oL}, where k_o is the wavenumber of the ordinary component. As a result, the exiting beam will no longer be in state $|R\rangle$, but will have a component of $|L\rangle$. The wavenumbers are related by $\omega = k_ec/n_e = k_oc/n_o$, where ω is the frequency of the light.

2. An electric field $\mathbf{E}_0$ is uniform inside a certain region of space and is zero outside that region. Assume all the space is vacuum. A small electric dipole $\mathbf{d}$ initially oriented as shown, is placed into the region free from field.

   Find the energy of the dipole as a function of r and α, and its equilibrium orientation, when a sphere with dielectric constant ε, and radius a is placed fully into the field region as indicated.



Solution outline:

Without the dielectric sphere, there is no force on the charges in the dipole because it is in a region where the field is zero; the energy is independent of r and orientation α.

When the dielectric sphere is introduced, the field $\mathbf{E}_0$ polarizes the sphere, inducing a dipole field $\mathbf{E}_i$ around the sphere. This field extends to infinity. The energy of the dipole now depends on r and α. The induced field has the form

$$\mathbf{E}_i(\mathbf{r}) = \frac{3(\mathbf{P}\cdot\hat{r})\hat{r} - \mathbf{P}}{r^3}$$

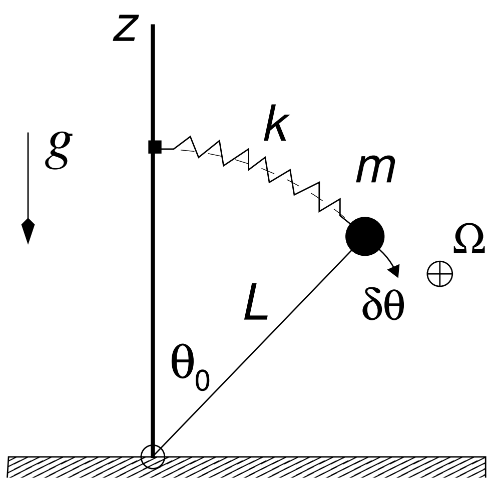
where $\hat{r} = \mathbf{r}/r$ is a unit vector along the radius and $\mathbf{P}$ is the induced dipole moment in the sphere. Note that $\mathbf{P}\cdot\hat{r} = 0$ by symmetry. The energy of the small dipole $\mathbf{d}$ in this field is

$$W = -\mathbf{E}_i(\mathbf{r})\cdot\mathbf{d} = \frac{\mathbf{P}\cdot\mathbf{d}}{r^3} = \frac{Pd}{r^3}\cos\alpha$$

The technical procedure is to obtain the induced dipole moment $\mathbf{P}$; it will be proportional to $\mathbf{E}_0$. I will obtain $\mathbf{P}$ by solving Laplace's equation for the electrostatic potential everywhere subject to boundary conditions on the surface of the sphere. The general solutions inside and outside the sphere are sums over Legendre polynomials. The boundary conditions are continuity in the tangential $\mathbf{E}$ and normal $\mathbf{D}$ at the surface of the sphere. Outside the sphere, the gradient of the potential gives the applied $\mathbf{E}_0 + \mathbf{E}_i$, from which I will obtain the induced dipole moment.

3. An inverted pendulum with the bob of mass m and massless rod of length L is connected to a stationary pole by a spring k, as shown. The spring can follow the arch of circle L, as it stretches and compresses. The system can freely rotate around the z-axis. In equilibrium there is no motion and the angle at the base is θ_0.

   We set the system into motion by shifting the mass from the equlibrium by a *small* angle $\delta\theta$, and simultaneously providing a *small* kick into the page with angular velocity Ω. Find the coordinates of the mass at later times.



Solution outline:

If only θ is perturbed, θ will oscillate about θ_o. The frequency of that oscillation depends on the strengths of gravity and the spring. When the system is set rotating about the z axis, the motion will be more complicated. But since there is no torque about the z axis, the angular momentum along that axis will be conserved. As θ increases, the angular frequency about z will decrease, and vice versa. Oscillations will be accompanied by changes in the angular velocity about the z axis.

The system has two degrees of freedom: $\delta\theta$ and ϕ (the angle about the z axis). I will solve for these variables using Lagrangian methods. Since $\delta\theta$ and $\Omega = \dot{\phi}(t=0)$ are both small, I will proceed to leading order in these quantities.

The kinetic energy will have contributions involving $\delta\dot{\theta}$ and $\dot{\phi}$. The potential energy will have a contribution from the spring, proportional to $\delta\theta$, and a contribution from gravity, proportional the height of the mass above the ground. The angle ϕ will not appear explicitly in the Lagrangian, so ϕ is an ignorable coordinate corresponding to the conserved z-component of the angular momentum.