Trigonometric Identities

$\sin^2 u + \cos^2 u = 1$	Sine pythagorean identity
$1 + \tan^2 u = \sec^2 u$	Tangent pythagorean identity
$1 + \cot^2 u = \csc^2 u$	Cotangent pythagorean identity
$\sin\left(\frac{\pi}{2} - u\right) = \cos u$	Sine co-function identity
$\cos\left(\frac{\pi}{2} - u\right) = \sin u$	Cosine co-function idenity
$\tan\left(\frac{\pi}{2} - u\right) = \cot u$	Tangent co-function identity
$\sin(-u) = -\sin u$	Sine even-odd identity
$\cos(-u) = \cos u$	Cosine even-odd identity
$\tan(-u) = -\tan u$	Tangent even-odd identity
$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$	Sine sum-difference formula
$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$	Cosine sum-difference formula
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$	Tangent sum-difference formula
$\sin(2u) = 2 \sin u \cos u$	Sine double angle formula
$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$	Cosine double angle formula
$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$	Tangent double angle formula
$\sin^2 u = \frac{1 - \cos(2u)}{2}$	Sine half-angle formula
$\cos^2 u = \frac{1 + \cos(2u)}{2}$	Cosine half-angle formula
$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$	Tangent half-angle formula
$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$	Sine sum-to-product formula
$\sin u - \sin v = 2\sin\left(\frac{u-v}{2}\right)\cos\left(\frac{u+v}{2}\right)$	Sine difference-to-product formula
$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$	Cosine sum-to-product formula
$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$	Cosine difference-to-product formula
$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$	Sine product-to-sum formula
$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right]$	Cosine product-to-sum formula
$\sin u \cos v = \frac{1}{2}\left[\sin(u-v) + \sin(u+v)\right]$	Hybrid product-to-sum formula

Vector Formulas

$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$	Dot product of cross product and vector
$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$	BAC-CAB rule
$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$	Dot product of cross products
$\boldsymbol{\nabla} \times \boldsymbol{\nabla}\psi = 0$	Curl of a gradient
$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \mathbf{a}) = 0$	Divergence of a curl
$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{a}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$	Curl of a curl
$\boldsymbol{\nabla} \cdot (\psi\mathbf{a}) = \mathbf{a} \cdot \boldsymbol{\nabla}\psi + \psi \boldsymbol{\nabla} \cdot \mathbf{a}$	Divergence chain rule
$\boldsymbol{\nabla} \times (\psi\mathbf{a}) = \boldsymbol{\nabla}\psi \times \mathbf{a} + \psi \boldsymbol{\nabla} \times \mathbf{a}$	Curl chain rule
$\boldsymbol{\nabla}(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \boldsymbol{\nabla})\mathbf{b} + (\mathbf{b} \cdot \boldsymbol{\nabla})\mathbf{a} + \mathbf{a} \times (\boldsymbol{\nabla} \times \mathbf{b}) + \mathbf{b} \times (\boldsymbol{\nabla} \times \mathbf{a})$	Gradient of dot product
$\boldsymbol{\nabla} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\boldsymbol{\nabla} \times \mathbf{a}) - \mathbf{a} \cdot (\boldsymbol{\nabla} \times \mathbf{b})$	Divergence of cross product
$\boldsymbol{\nabla} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\boldsymbol{\nabla} \cdot \mathbf{b}) - \mathbf{b}(\boldsymbol{\nabla} \cdot \mathbf{a}) + (\mathbf{b} \cdot \boldsymbol{\nabla})\mathbf{a} - (\mathbf{a} \cdot \boldsymbol{\nabla})\mathbf{b}$	Curl of cross product

Potentially Useful Mathematical Identities

$$\delta(f(x)) = \sum_i \frac{1}{\left|\frac{\mathrm{d}f}{\mathrm{d}x}(x_i)\right|}\delta(x - x_i)$$ Jackson Dirac delta function Rule 5

$$(a+x)^n \approx a^n + na^{n-1}x + \ldots, \quad f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \ldots$$ Taylor Expansions

$$J_m(k\rho) \propto (k\rho)^m,\ Y_m(k\rho) \propto (k\rho)^{-m},\ I_m(k\rho) \propto (k\rho)^m,\ K_m(k\rho) \propto (k\rho)^{-m},\ \text{As } \rho \to 0$$

$$\begin{pmatrix}\hat{\boldsymbol{\rho}}\\ \hat{\boldsymbol{\phi}}\\ \hat{\mathbf{z}}\end{pmatrix} = \begin{pmatrix}\cos\phi & \sin\phi & 0\\ -\sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\\ \hat{\mathbf{z}}\end{pmatrix}, \quad \begin{pmatrix}\hat{\mathbf{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\phi}}\end{pmatrix} = \begin{pmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\ -\sin\phi & \cos\phi & 0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\ \hat{\mathbf{y}}\\ \hat{\mathbf{z}}\end{pmatrix}$$

$$\begin{pmatrix}\hat{\mathbf{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\phi}}\end{pmatrix} = \begin{pmatrix}\sin\theta & 0 & \cos\theta\\ \cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\hat{\boldsymbol{\rho}}\\ \hat{\boldsymbol{\phi}}\\ \hat{\mathbf{z}}\end{pmatrix}, \quad \begin{pmatrix}\hat{\boldsymbol{\rho}}\\ \hat{\boldsymbol{\phi}}\\ \hat{\mathbf{z}}\end{pmatrix} = \begin{pmatrix}\rho/\sqrt{\rho^2+z^2} & z/\sqrt{\rho^2+z^2} & 0\\ 0 & 0 & 1\\ z/\sqrt{\rho^2+z^2} & -\rho/\sqrt{\rho^2+z^2} & 0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\phi}}\end{pmatrix}$$

$$R_x(\theta) = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{pmatrix}$$ Rotation matrix about x-axis

$$R_y(\theta) = \begin{pmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{pmatrix}$$ Rotation matrix about y-axis

$$R_z(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}$$ Rotation matrix about z-axis

Todo:

- Properties of special functions : spherical harmonics, Bessel functions, Legendre polynomials, etc.
- How to use Green's functions