Fundamental Concepts

$S_k \left|S_k; \pm\right\rangle = \frac{\hbar}{2} \left|S_k; \pm\right\rangle$ Eigenkets of operator S_k (S. 1.2.6)

$|\alpha\rangle = \sum_{a'} c_{a'} \left|a'\right\rangle$ Exapansion of an arbitrary ket $|\alpha\rangle$ (S. 1.2.8)

$\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$ Complex conjugate of bra-ket (S. 1.2.12)

$\langle\alpha|\alpha\rangle \geq 0$ Postulate of positive definite metric (S. 1.2.13)

$X = X^\dagger$ Condition of a hermitian operator (S. 1.2.25)

$(XY)^\dagger = X^\dagger Y^\dagger$ Hermitian conjugate distributivity (S. 1.2.29)

$A\left|a'\right\rangle = a'\left|a'\right\rangle$ Eigenkets of the hermitian operator A (S. 1.3.1)

$\left\langle a''\right| A = a''^* \left\langle a''\right|$ Hermitian operator A on a bra (S. 1.3.2)

$\left\langle a''\middle|a'\right\rangle = \delta_{a'',a'}$ Condition of an orthonormal set (S. 1.3.6)

$c_{a'} = \left\langle a'\middle|\alpha\right\rangle$ Technique for finding the expansion coefficient (S. 1.3.8)

$\sum_{a'} = \left|a'\right\rangle\left\langle a'\right| = 1$ Identity operator (S. 1.3.11)

$\sum_{a'} \left|c_{a'}\right|^2 = 1$ Normalization condition of expansion coefficients (S. 1.3.13)

$\Lambda_{a'} \equiv \left|a'\right\rangle\left\langle a'\right|$ Definition of the projection operator (S. 1.3.15)

$P_{a'} = \left|\left\langle a'\middle|\alpha\right\rangle\right|^2$ Probability for a' (S. 1.4.4)

$\langle A\rangle \equiv \langle\alpha|A|\alpha\rangle$ Definition of expectation value (S 1.4.5)

$S_z = \frac{\hbar}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$ Spin-z operator in z-basis (S. 1.3.36)

$S_x = \frac{\hbar}{2}\left(|+\rangle\langle-| + |-\rangle\langle+|\right)$ Spin-x operator in z-basis (S. 1.4.18a)

$S_y = \frac{\hbar}{2}\left(-i|+\rangle\langle-| + i|-\rangle\langle+|\right)$ Spin-y operator in z-basis (S. 1.4.18b)

$|S_x; \pm\rangle = \frac{1}{\sqrt{2}}|+\rangle \pm \frac{1}{\sqrt{2}}|-\rangle$ Spin-x eigenkets in z-basis (S. 1.4.17a)

$|S_y; \pm\rangle = \frac{1}{\sqrt{2}}|+\rangle \pm \frac{i}{\sqrt{2}}|-\rangle$ Spin-y eigenkets in z-basis (S. 1.4.17b)

$|\mathbf{S}\cdot\hat{\mathbf{n}}; +\rangle = \cos\left(\frac{\theta}{2}\right)|+\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|-\rangle$
Arbitrary axis spin-up eigenkets in z-basis (S. Pr. 1.11)

$\left[S_i, S_j\right] = i\epsilon_{ijk}\hbar S_k$ Spin operator commutation relations (S. 1.4.20)

$\left\{S_i, S_j\right\} = \frac{1}{2}\hbar^2\delta_{ij}$ Spin operator anticommutation relations (S. 1.4.21)

$[A, B] \equiv AB - BA$ Definition of the commutator (S. 1.4.22a)

$\{A, B\} \equiv AB + BA$ Definition of the anticommutator (S. 1.4.22b)

$[A, B] = 0$ Condition for two observables to be compatible (S. 1.4.26)

$A\left|a', b'\right\rangle = a'\left|a', b'\right\rangle$
$B\left|a', b'\right\rangle = b'\left|a', b'\right\rangle$ Property of a simultaneous eigenket of A and B

$\left\langle(\Delta A)^2\right\rangle = \left\langle A^2\right\rangle - \langle A\rangle^2$ Dispersion of A (S. 1.4.51)

$\left\langle(\Delta A)^2\right\rangle\left\langle(\Delta B)^2\right\rangle \geq \frac{1}{4}\left|\langle[A, B]\rangle\right|^2$ Uncertainty relation (S. 1.4.53)

$\int d\xi' \left|\xi'\right\rangle\left\langle\xi'\right| = 1$ Identity operator for continuous variables (S. 1.6.2b)

$\mathcal{J}(d\mathbf{x}')\left|\mathbf{x}'\right\rangle = \left|\mathbf{x}' + d\mathbf{x}'\right\rangle$ Infinitesimal translation operator on ket (S. 1.6.12)

$\mathcal{J}(d\mathbf{x}') = 1 - i\mathbf{p}\cdot d\mathbf{x}'/\hbar$ Definition of infinitesimal translation operator (S. 1.6.32)

$\mathcal{J}(\Delta x'\hat{\mathbf{x}})\left|\mathbf{x}'\right\rangle = \left|\mathbf{x}' + \Delta x'\hat{\mathbf{x}}\right\rangle$ Finite translation operator on ket (S. 1.6.35)

$\mathcal{J}(\Delta x'\hat{\mathbf{x}}) = \exp\left(-\frac{ip_x\Delta x'}{\hbar}\right)$ Definition of finite translation operator (S. 1.6.36)

$\left\langle(\Delta x)^2\right\rangle\left\langle(\Delta p_x)^2\right\rangle \geq \hbar^2/4$ WH's position-momentum uncertainty relation (S. 1.6.34)

$[x_i, x_j] = 0, \quad [p_i, p_j] = 0, \quad [x_i, p_j] = i\hbar\delta_{ij}$ Canonical commutation relations (S. 1.6.46)

$[A, BC] = [A, B]C + B[A, C]$ Commutator distributivity (S. 1.6.50e)

$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$ Jacobi Identity (S. 1.6.50f)

$x\left|x'\right\rangle = x'\left|x'\right\rangle$ Definition of the position ket (S. 1.7.1)

$\left\langle x''\middle|x'\right\rangle = \delta(x - x')$ Position ket normalization condition (S. 1.7.2)

$\left\langle x'\middle|\alpha\right\rangle = \psi_\alpha(x')$ Definition of the position-space wave function (S. 1.7.5)

$p = -i\hbar\frac{\partial}{\partial x'}$ Momentum operator in terms of (S. 1.7.18)

$\psi_\alpha(\mathbf{x}') = \left[\frac{1}{(2\pi\hbar)^{3/2}}\right]\int d^3x' \exp\left(\frac{i\mathbf{p}'\cdot\mathbf{x}'}{\hbar}\right)\phi_\alpha(\mathbf{p}')$
Fourier transform from momentum to position space (S. 1.7.51a)

$\phi_\alpha(\mathbf{p}') = \left[\frac{1}{(2\pi\hbar)^{3/2}}\right]\int d^3x' \exp\left(\frac{-\mathbf{p}'\cdot\mathbf{x}'}{\hbar}\right)\psi_\alpha(\mathbf{x}')$
Fourier transform from position to momentum space (S. 1.7.51b)

Quantum Dynamics

$\mathcal{U}(t, t_0) = \exp\left[\frac{-iH(t - t_0)}{\hbar}\right]$ Time-evolution operator (S. 2.1.28)

$[A, H] = 0 \Rightarrow H\left|a'\right\rangle = E_{a'}\left|a'\right\rangle$ Definition of energy eigenkets (S. 2.1.34)

$|\alpha, t_0 = 0; t\rangle = \exp\left(\frac{-iHt}{\hbar}\right)|\alpha, t_0 = 0\rangle = \sum_{a'}\left|a'\right\rangle\left\langle a'\middle|\alpha\right\rangle\exp\left(\frac{-iE_{a'}t}{\hbar}\right)$
Time-evolution of expansion coefficients (S. 2.1.38)

$H = -\left(\frac{eB}{m_ec}\right)S_z$ Hamiltonian of a spin-1/2 particle in uniform magnetic field (S. 2.1.50)

$\omega \equiv \frac{|e|B}{m_ec}$ Definition of the transition frequency (S. 2.1.52)

$\mathcal{U}_s(t, 0) = \exp\left(\frac{-i\omega S_zt}{\hbar}\right)$ Time-evolution of spin states (S. 2.1.54)

$[x_i, F(\mathbf{p})] = i\hbar\frac{\partial F}{\partial p_i}$ Commutator of position and function of momentum (S. 2.2.23a)

$[p_i, G(\mathbf{x})] = -i\hbar\frac{\partial G}{\partial x_i}$ Commutator of momentum and function of position (S. 2.2.23b)

$H = \frac{\mathbf{p}^2}{2m} + V(x)$ General expression for the Hamiltonian (S. 2.2.31)

$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\langle\mathbf{x}\rangle = \frac{\mathrm{d}\langle\mathbf{p}\rangle}{\mathrm{d}t} = -\langle\boldsymbol{\nabla}V(\mathbf{x})\rangle$ Ehrenfest's theorem (S. 2.2.36)

$H = \frac{p^2}{2m} + \frac{m\omega^2x^2}{2}$ Hamiltonian of the simple harmonic oscillator (S. 2.3.1)

$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ Energy eigenvalues of the SHO (S. 2.3.9)

$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right)$ Definition of the lowering operator (S. 2.3.2)

$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$ Definition of the raising operator (S. 2.3.2)

$a|n\rangle = \sqrt{n}|n - 1\rangle$ Behaviour of lowering operator (S. 2.3.16)

$a^\dagger|n\rangle = \sqrt{n + 1}|n + 1\rangle$ Behaviour of the raising operator (S. 2.3.17)

$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$ Position operator in terms of raising/lowering operators (S. 2.3.24)

$p = i\sqrt{\frac{m\hbar\omega}{2}}(-a + a^\dagger)$ Momentum operator in terms of raising/lowering ops. (S. 2.3.24)

$x_0 \equiv \sqrt{\frac{\hbar}{m\omega}}$ Characteristic length scale for SHO (S. 2.3.29)

$\left\langle x'\middle|0\right\rangle = \left(\frac{1}{\pi^{1/4}\sqrt{x_0}}\right)\exp\left[-\frac{1}{2}\left(\frac{x'}{x_0}\right)^2\right]$ Ground state wavefunction for SHO (S. 2.3.30)

$\left\langle x'\middle|1\right\rangle = \left\langle x'\right|a^\dagger|0\rangle$ Technique for evaluating higher wavefunctions of the SHO (S. 2.3.31)

$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x}', t) = -\left(\frac{\hbar^2}{2m}\right)\boldsymbol{\nabla}'^2\psi(\mathbf{x}') + V(\mathbf{x}')\psi(\mathbf{x}', t)$
Schrödinger's time-dependent wave equation (S. 2.4.8)

$-\left(\frac{\hbar^2}{2m}\right)\boldsymbol{\nabla}'^2u_E(\mathbf{x}') + V(\mathbf{x}')u_E(\mathbf{x}') = Eu_E(\mathbf{x}')$
Schrődinger's time-independent wave equation (S. 2.4.11)

$\left\{\frac{1}{[V(x) - E]^{1/4}}\right\}\exp\left[-\frac{1}{\hbar}\int_x^{x_1}dx'\sqrt{2m[V(x') - E]}\right]$
$\rightarrow\left\{\frac{2}{[E - V(x)]^{1/4}}\right\}\cos\left[\frac{1}{\hbar}\int_{x_1}^{x}dx'\sqrt{2m[E - V(x')]} - \frac{\pi}{4}\right]$ WKB I $\rightarrow$ II (S. 2.5.48)

$\left\{\frac{1}{[V(x) - E]^{1/4}}\right\}\exp\left[-\frac{1}{\hbar}\int_{x_2}^{x}dx'\sqrt{2m[V(x') - E]}\right]$
$\rightarrow\left\{\frac{2}{[E - V(x)]^{1/4}}\right\}\cos\left[\frac{1}{\hbar}\int_{x}^{x_2}dx'\sqrt{2m[E - V(x')]} - \frac{\pi}{4}\right]$ WKB III $\rightarrow$ II (S. 2.5.48)

$\int_{x_1}^{x_2}dx\sqrt{2m[E - V(x)]} = \left(n + \frac{1}{2}\right)\pi\hbar$ WKB quantization condition (S. 2.5.50)

$E = V(x_1), \quad E = V(x_2)$ Classical turning points (S. 2.5.53)

Theory of Angular Momentum

$[J_i, J_j] = i\hbar\epsilon_{ijk}J_z$ Fundamental commutation relations of angular momentum (S. 3.1.20)

$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \quad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \quad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ Pauli spin matrices (S. 3.2.32)

$\{\sigma_i, \sigma_j\} = 2\delta_{ij}$ Anticommutation relation for Pauli matrices (S. 3.2.34)

$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$ Commutation relation for Pauli matrices (S. 3.2.35)

$\mathcal{D}(\hat{\mathbf{n}}, \phi) = \exp\left(\frac{-i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}}{2}\right) = \mathbf{1}\cos\left(\frac{\phi}{2}\right) - i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\sin\left(\frac{\phi}{2}\right)$ Spin-1/2 rotation (S. 3.2.44)

$\mathcal{D}(\alpha, \beta, \gamma) = \begin{pmatrix}e^{-i(\alpha+\gamma)/2}\cos(\beta/2) & -e^{-i(\alpha-\gamma)/2}\sin(\beta/2)\\e^{i(\alpha-\gamma)/2}\sin(\beta/2) & e^{i(\alpha+\gamma)/2}\cos(\beta/2)\end{pmatrix}$ Euler angle $\mathcal{D}$ (S. 3.3.21)

$U(a, b) = \begin{pmatrix}a & b\\-b^* & a^*\end{pmatrix}$ General unitary unimodular matrix (S. 3.3.7)

$|a|^2 + |b|^2 = 1$ Unimodular condition (S. 3.3.8)

$U(a, b)U^\dagger(a, b)$ Unitary condition (S. 3.3.9)

$\sum_i w_i = 1$ Normalization condition for fractional populations (S. 3.4.5)

$\rho \equiv \sum_i w_i\left|\alpha^{(i)}\right\rangle\left\langle\alpha^{(i)}\right|$ Definition of the density operator (S. 3.4.8)

$[A] = \mathrm{tr}(\rho A)$ Ensemble average (S. 3.4.10)

$\mathrm{tr}(\rho^2) = 1$ Property of a pure ensemble (3.4.15)

$\rho(t) = \mathcal{U}(t, t_0)\rho(t_0)U^\dagger(t, t_0)$ Time-evolution of an ensemble (S. Pr. 3.11)

$\mathbf{J}^2 \equiv J_x^2 + J_y^2 + J_z^2$ Definition of the total angular momentum operator (TAM) (S. 3.5.1)

$[\mathbf{J}^2, J_k] = 0, \ (k = 1, 2, 3)$ TAM commutivity (S. 3.5.2)

$\mathbf{J}^2 |j, m\rangle = j(j + 1)\hbar^2 |j.m\rangle$ TAM operator eigenvalue (S. 3.5.34a)

$J_z |j, m\rangle = m\hbar |j, m\rangle$ TAM z-component operator eigenvalue (S. 3.5.34b)

$J_+ |j, m\rangle = \sqrt{(j - m)(j + m + 1)}\hbar |j, m + 1\rangle$ TAM raising operator eigenvalue (S. 3.5.39)

$J_- |j, m\rangle = \sqrt{(j + m)(j - m + 1)}\hbar |j, m - 1\rangle$ TAM lowering operator eigenval. (S. 3.5.40)

$\mathcal{D}^{(j)}_{m'm} = \left\langle j, m' \right| \exp\left(\frac{-i\mathbf{J} \cdot \hat{\mathbf{n}}\phi}{\hbar}\right) |j, m\rangle$ Wigner functions (S. 3.5.42)

$\mathcal{D}_{m'm}(R^{-1}) = \mathcal{D}^*_{mm'}(R)$ Unitary property of the rotation operator (S. 3.5.47)

$\mathcal{D}^{(j)}_{m'm}(\alpha, \beta, \gamma) = e^{-i(m'\alpha+m\gamma)} d^{(j)}_{m'm}(\beta)$ Redefinition of the rotation op. (S. 3.5.50)

$d^{(j)}_{m'm}(\beta) \equiv \left\langle j, m' \right| \exp\left(\frac{-iJ_y\beta}{\hbar}\right) |j, m\rangle$ Rotation operator j-dependence (S. 3.5.51)

$d^{(1/2)} = \begin{pmatrix} \cos\left(\frac{\beta}{2}\right) & -\sin\left(\frac{\beta}{2}\right) \\ \sin\left(\frac{\beta}{2}\right) & \cos\left(\frac{\beta}{2}\right) \end{pmatrix}$ Spin-1/2 case (S. 3.5.52)

$d^{(1)}(\beta) = \begin{pmatrix} \frac{1}{2}(1 + \cos\beta) & -\frac{1}{\sqrt{2}}\sin\beta & \frac{1}{2}(1 - \cos\beta) \\ \frac{1}{\sqrt{2}}\sin\beta & \cos\beta & -\frac{1}{\sqrt{2}}\sin\beta \\ \frac{1}{2}(1 - \cos\beta) & \frac{1}{\sqrt{2}} & \frac{1}{2}(1 + \cos\beta) \end{pmatrix}$ Spin-1 case (S. 3.5.57)

$\mathbf{L} = \mathbf{x} \times \mathbf{p}$ Definition of orbital angular momentum (OAM) (S. 3.6.1)

$[L_i, L_j] = i\epsilon_{ijk}\hbar L_k$ OAM commutation relations (S. 3.6.2)

$\mathbf{L}^2 = \mathbf{x}^2\mathbf{p}^2 - (\mathbf{x} \cdot \mathbf{p})^2 + i\hbar\mathbf{x} \cdot \mathbf{p}$ OAM operator identity (S. 3.6.16)

$\left\langle \mathbf{x}' \middle| n, l, m \right\rangle = R_{nl}(r)Y_l^m(\theta, \phi)$ Energy eigenfunctions, seperable solution (S. 3.6.22)

$\langle \hat{\mathbf{n}}|l, m\rangle = Y_l^m(\theta, \phi) = Y_l^m(\hat{\mathbf{n}})$ Angular dependence of solution (S. 3.6.23)

$L_z |l, m\rangle = m\hbar |l, m\rangle$ Eigenvalue of the OAM z-component operator (S. 3.6.24)

$\mathbf{L}^2 |l, m\rangle = l(l + 1)\hbar^2 |l, m\rangle$ Eigenvalue of the OAM operator (S. 3.6.27)