A Note on Magnetic Monopoles [Ref. Jackson, Secs. (6.12) & (6.13)]

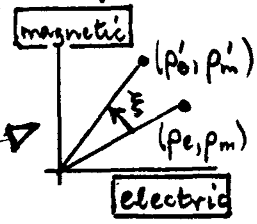
1) In ϕ 519 Prob^m (41), we did the arithmetic of how Maxwell's Eqtns...

$$\left[\!\left[\ \nabla\cdot\begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix} = 4\pi\begin{pmatrix}\rho_e\\ \rho_m\end{pmatrix},\quad \nabla\times\begin{pmatrix}\mathbf{H}\\ -\mathbf{E}\end{pmatrix} = \frac{1}{c}\frac{\partial}{\partial t}\begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix} + \frac{4\pi}{c}\begin{pmatrix}\mathbf{J}_e\\ \mathbf{J}_m\end{pmatrix},\ \right]\!\right] \tag{1}$$

here augmented for the existence of MM's (magnetic monopoles) -- with scalar charge density ρ_m & vector current density $\mathbf{J}_m$ -- behave under the "duality transform..."

$$\rightarrow \left\{ \begin{pmatrix}\mathbf{D}'\\ \mathbf{B}'\end{pmatrix}, \begin{pmatrix}\mathbf{E}'\\ \mathbf{H}'\end{pmatrix}; \begin{pmatrix}\rho_e'\\ \rho_m'\end{pmatrix}, \begin{pmatrix}\mathbf{J}_e'\\ \mathbf{J}_m'\end{pmatrix} \right\} = \mathcal{R}(\xi) \left\{ \begin{pmatrix}\mathbf{D}\\ \mathbf{B}\end{pmatrix}, \begin{pmatrix}\mathbf{E}\\ \mathbf{H}\end{pmatrix}; \begin{pmatrix}\rho_e\\ \rho_m\end{pmatrix}, \begin{pmatrix}\mathbf{J}_e\\ \mathbf{J}_m\end{pmatrix} \right\},$$

w/ $\mathcal{R}(\xi) = \begin{pmatrix}\cos\xi & -\sin\xi\\ \sin\xi & \cos\xi\end{pmatrix}$, a "rotation" in "EM space". (2)



Conclusions were that...

1. energy density : $u \rightarrow u' \equiv u$,
Poynting vector : $\mathbf{S} \rightarrow \mathbf{S}' \equiv \mathbf{S}$,
Stress tensor : $T_{ik} \rightarrow T_{ik}' \equiv T_{ik}$

are all form-invariant under a duality transform. As well: (Max. Eqs.) $\rightarrow$ (Max. Eqs.)' don't change form. The physics is exactly the same for any value of the "mixing angle" ξ.

2. Since EM theory is insensitive to values of ξ, the choice of names for what we call electric & magnetic (i.e. ρ_e & ρ_m, $\mathbf{E}$ & $\mathbf{B}$, etc.) is arbitrary. The convention is:

$\rightarrow$ for electrons : $(\rho_m/\rho_e) \equiv 0$, and $\xi \equiv 0$ $\leftarrow$ by CONVENTION. (3)

But other particles (e.g. protons, μ-mesons, etc) could have $(\rho_m/\rho_e) \neq 0$ and $\xi \neq 0$.

2) It becomes an experimental question to measure $\left(\frac{q_m}{q_e}, \xi\right)$ for every particle. The best information is on the proton, for which it is known...

$\rightarrow \big|\, |q_e(\text{proton})| - |q_e(\text{electron})| \,\big| < 10^{-20} e$, $|q_m(\text{proton})| < 2\times 10^{-24} e$ — Jackson p. 252 (4)

On this basis, it seems a good approx^n to fix $(\rho_m/\rho_e, \xi) = (0,0)$ for all EM particles. This is the evidence for claiming that MM's don't exist, and for setting ρ_m & $\mathbf{J}_m \equiv 0$.

) A theoretical argument against MM's goes as follows. The magnetic source eqtn would be:

$\rightarrow \nabla\cdot\mathbf{B} = 4\pi\rho_m$ — if CPT = (−, +, −) for $\mathbf{B}$, then CPT = (−, −, −) for ρ_m, so ρ_m is a T-odd pseudoscalar (and $\mathbf{J}_m = \rho_m \mathbf{v}$ is a T-even axialvector). (5)

Now, under a duality transform, the electric charge density (e.g.) goes as

$\rightarrow \rho_e \rightarrow \rho_e' = \rho_e \cos\xi - \rho_m \sin\xi$ ‖ This implies that EM theory is PT invariant, but

PT = (+,+) (−,−) ‖ violations of P and T could occur separately. (6)

EM theory is certainly PT invariant, but violations of P-invariance and T-invariance separately have never been observed for particles coupled by EM fields alone. Such violations would be unwelcome (except for getting your power meter to run backwards).

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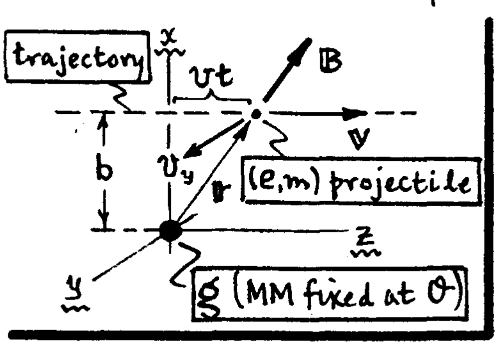
4) Nevertheless, the idea of the existence of MM's still has some theoretical appeal, on endorsement by Dirac. He showed (1931) that if a MM existed, then its charge is

MM charge : $\boxed{g = (n/2\alpha)\, e}$ $\int$ e = electron charge; $n = 0, \pm 1, \pm 2, \ldots$; $\alpha = e^2/\hbar c \simeq 1/137$, finestructure cnst. (7)

So, g would be quantized as is e. Conversely, the existence of a $g \neq 0$ "explains" the quantization of e. That is why people still search for Dirac monopoles.

Dirac argued from QM to get to above relation (Jkn pp. 257-60). We will just repeat a semi-classical argument by Goldhaber (1965... also in Jackson, p. 254). One considers the scattering of a particle (e,m) by a MM of charge g fixed at the origin. So (e,m) is moving in a magnetic field $\mathbf{B} = (g/r^2)\hat{r}$, and...



Force on e : $\mathbf{F} = \frac{e}{c}\mathbf{v}\times\mathbf{B} = -\left(\frac{eg}{mc}/r^3\right)\mathbf{L}$, along y-axis,

w/ $\mathbf{L} = \mathbf{r}\times m\mathbf{v}$ = ∡ momentum of m about origin. (8)

During the scattering, the only average nonzero force acting on (e,m) is $F_y = (ev/c)\,B_x$. If the overall particle deflection is "small", then its x cd $\simeq b$ always, so that:

$\rightarrow F_y = (ev/c)\,B_x \simeq (ev/c)\dfrac{gb}{[b^2+(vt)^2]^{3/2}}$. (9) Action of F_y constitutes an impulse Δp_y o m, and subsequent motion along y; calculate : $\Delta p_y = \int_{-\infty}^{\infty} F_y\, dt = 2eg/bc$. (10)

Then $v_y \neq 0$ after scattering, and we've developed an ∡ momentum about the z-axis of size : $\boxed{\Delta L_z = b\,\Delta p_y = 2eg/c}$ (11). Quantize, $\Delta L_z = n\hbar$, to get Dirac's Eq. (7).