Wave-particle duality for photons.

ASIDE Photon dynamics.

As remarked above, the quantized units of EM radiation in the blackbody distribution are called "photons". NOTE: if the photon's energy is quantized, then so is its momentum, since--from special relativity...

$$\underline{\underline{E^2 = (pc)^2 + (mc^2)^2}} \quad \int \text{for } \underline{\text{any}} \text{ particle of total energy } E, \text{ linear momentum } p, \text{ rest mass } m. \tag{18}$$

For photons, $m \equiv 0$, so that the Newtonian momentum $mv/\sqrt{1-(v/c)^2}$ does not diverge as the propagation velocity $v \to c$. Then (18) relates p to E, as...

$$\boxed{E = pc, \text{ for photons}} \quad \int \text{for } m=0 \ \& \ v=c \text{ (only)}. \tag{19}$$

That energy $E \propto$ momentum p, here for <u>particles</u> of light, agrees with the classical finding for light <u>waves</u>, that the energy density $\propto$ momentum density. So (19) really isn't new. What is new is to invoke Planck's rule that $E = h\nu$, so that both E & p for the photon are fixed by the freq. ν:

$$\rightarrow E = pc = h\nu \Rightarrow p = h\nu/c, \text{ or } \boxed{p = h/\lambda}, \text{ since } \lambda\nu = c. \tag{20}$$

Photons at a given freq. ν will always have the <u>same</u> fixed energy $E = h\nu$, and fixed momentum $p = h/\lambda$ (known as the de Broglie relation). And we have generated a connection between the wave-like & particle-like photon properties:

$$\left[\begin{array}{l}\text{Wave-particle duality}\\ \text{for EM radiation (photons)}\end{array}\right\} \underbrace{(\overset{\text{frequency}}{\nu}, \overset{\text{wavelength}}{\lambda})}_{\substack{\text{waves}\\ \text{(e.g. in diffraction)}}} \longleftrightarrow \underbrace{(\overset{\text{energy}}{E} = h\nu, \overset{\text{momentum}}{p} = h/\lambda)}_{\substack{\text{particles}\\ \text{(e.g. BB radiation)}}} \tag{21}$$

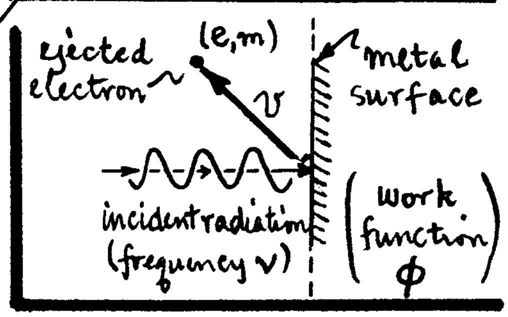
NOTE: later we will use the wave-vector $k = 2\pi/\lambda$ instead of λ, so that $p = \hbar k$, where $\hbar = h/2\pi$ is Planck's cnst divided by 2π. Then, in 3D, and with $\omega = 2\pi\nu$ the photon's circular freq., the duality relation is...

$$\boxed{(E, \mathbf{p}) = \hbar(\omega, \mathbf{k})}. \tag{22}$$

More direct evidence of photons.

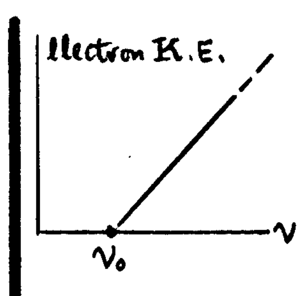
PhotoElectric & Compton Effects

The idea of photons was only implicit in Planck's work. Nobody had actually "seen" quantum packets of EM radiation interact with matter; one needed only imagine them to aid in the derivation of the correct BB radiation formula. Planck himself didn't believe there was any real physics in photons -- he spent a great deal of time trying to dis-invent them (unsuccessfully). Now, we seek more direct evidence of the existence of photons.



5) In the photoelectric effect, EM radiation incident on a metal surface supplies enough energy to eject electrons, as sketched at right. One has control over the freq. ν of the input radiation, and can measure the ejected electron velocity v; most often, the measurement data is presented as a plot of electron K.E. (kinetic energy) vs. rad$^{\underline{n}}$ freq. ν.

Classically, the electron K.E. should depend on the intensity of the input radiation, since the electrons should continuously absorb the radiation energy supplied, ultimately gaining enough energy to escape from the surface (over the attractive work fcn barrier ϕ). There will be a scattering of maxm K.E.'s, depending on how long the absorbing electron remains trapped in the metal. Also, this absorption process should not depend on frequency... ν should only determine the (relatively short) time-scale for the absorption.



$K = \text{cnst} \times (\nu - \nu_0)$

Observed: max. electron K.E. depends linearly (and exclusively) on the radiation freq. ν. And, below a certain critical freq. ν_0, no electrons at all are ejected, no matter how intense the radiation input. The above classical picture is spectacularly wrong!

Einstein's PhotoElectric Equation

<u>QM picture</u> Einstein invoked Planck's photon hypothesis to explain quite elegantly what was going on. An incident photon strikes an electron, giving up its energy $h\nu$ to that electron (with ~ zero time delay). For those electrons that escape, the energy $h\nu$ is distributed between the electron K.E. (viz. $K = \frac{1}{2}mv^2$) and the energy $e\phi$ necessary to surmount the work fcn barrier. So, by conservation of energy for the photon-electron <u>collision</u>...

$$\underline{\underline{K = h\nu - e\phi = \text{cnst}(\nu - \nu_0)}} \quad \left\{ \begin{array}{l} \text{cnst} = h, \text{ and:} \\ \nu_0 = e\phi/h \text{ (crit. freq.)}. \end{array} \right. \tag{23}$$

This Ansatz explains immediately why the max. electron K.E. depends only on the input radiation freq. ν. And, since the radiation intensity does not enter (23) at all, then -- for $\nu > \nu_0$ -- photoelectrons will be ejected even when the incident intensity $\rightarrow 0$. This last feature is in fact observed; it is incomprehensible from the classical standpoint (where, by continuous absorption, the electron would be ejected only after a very long time).

So, the photon idea wins big in the PhotoElectric Sweepstakes.

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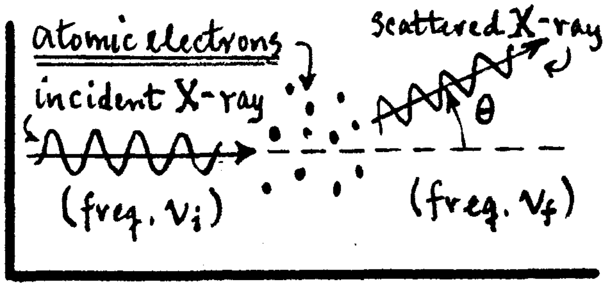
6) The most direct evidence for the existence of photons-as-particles is that from the <u>Compton effect</u>. Here radiation is incident on a scattering material, where the EM "waves" interact with atomic electrons. The incident radiation is usually high-energy X-rays (energies ~ 100 eV); on this energy scale, the electrons can be considered free, since their binding energies are much smaller (~ few eV). One has control over the incident radiation intensity and frequency ν_i, and one can measure the frequency ν_f of the scattered X-ray and the scattering ∡ θ.

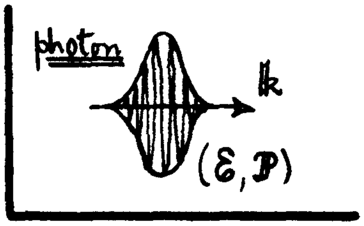


Compton effect viewed as a photon-electron collision.

Classically, the incident radiation at freq. ν_i sets the electrons oscillating at the *same* freq., whereupon they radiate (as electric dipoles) at freq. ν_i. At the same time, the incident wave "pushes" on the electron (via radiation pressure), so the electron recoils in the direction of the incident wave. Upon recoiling, the electron "sees" a lower (Doppler-shifted) incident frequency, and re-radiates that... so the electron is radiating a *band* of frequencies at $\nu \leq \nu_i$. The bandwidth must depend on the incident X-ray intensity and on the time that elapses for the electron to scatter the entire wave-train.

Observed: the X-rays scattered at a particular $\measuredangle\theta$ exhibit just *one* sharply defined freq. ν_f; there is *no* band of frequencies. $\nu_f \leq \nu_i$, but it does not depend on intensity; ν_f is a fcn of θ & ν_i only. This data suggests that the X-ray $\rightarrow$ electron scattering is an *instantaneous event*, rather than a gradual exchange of momentum between the wave-train and the electron. An instantaneous exchange is characteristic of a particle-particle collision rather than a wave-particle interaction.

QM picture. If you believe in photons, the above mystery is easily solved. Consider the photon as a *particle* (or localized wave) with a discrete energy $\mathcal{E}$ and momentum $\mathbb{P}$ specified by its wave vector $\mathbf{k}$...



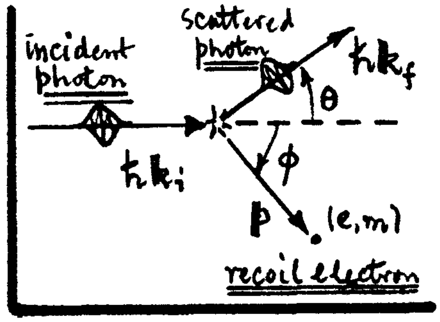
$$\left.\begin{array}{l}\text{photon}\\ \text{wave-vector}\end{array}\right\} \mathbf{k} = (2\pi/\lambda)\,\hat{n} \qquad \hat{n} = \text{unit vector in propagation direction},\ \lambda = \text{wavelength}$$

$$\left.\begin{array}{l}\text{so// energy: } \underline{\mathcal{E} = h\nu = \hbar c k}, \ \text{w/ } \hbar = h/2\pi \\ \text{momentum: } \underline{\mathbb{P} = (h/\lambda)\,\hat{n} = \hbar\mathbf{k}};\end{array}\right\} \text{and: } \underline{\underline{\mathbb{P} = (\mathcal{E}/c)\,\hat{n}}}. \qquad (24)$$

One can then impose conservation of energy & momentum on the photon-electron collision. The electron -- of mass m, energy E and momentum p -- has a more complicated momentum-energy relation than (24) [because $m \neq 0$]; it is...

$$\rightarrow E^2 = (pc)^2 + (mc^2)^2, \text{ or } \underline{p = \sqrt{(E/c)^2 + (mc)^2}}. \quad (25)$$

Still, E can be eliminated in favor of p, as in (24). Now if the photon-electron collision is described as in the diagram, and the electron is initially at rest (in effect -- its initial momentum is negligibly small w.r.t. that of the incident X-ray), then the conservation laws are...



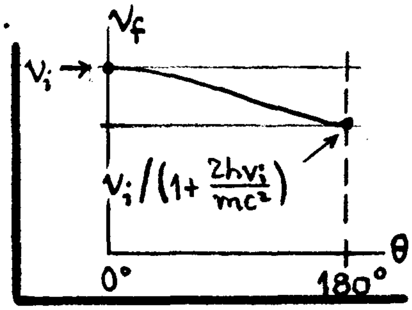
$$\left[\begin{array}{ll} \text{ENERGY}: & \hbar c k_i + mc^2 = \hbar c k_f + E; \\ \text{MOMENTUM}: & \hbar k_i = \hbar k_f \cos\theta + p\cos\phi \quad \text{(forward direction)}, \\ & 0 = \hbar k_f \sin\theta - p \sin\phi \quad \text{(transverse direction)}. \end{array}\right. \quad (26)$$

Eliminate E in favor of p, and use 2 of the 3 eqtns in (26) to eliminate the recoil electron variables p & ϕ. The remaining eqtn can be written as...

$$\rightarrow \frac{mc}{\hbar}\left(\frac{1}{k_f} - \frac{1}{k_i}\right) = 1 - \cos\theta$$

Substitute: $\nu = kc/2\pi$...

$$\text{so} \quad \boxed{\nu_f = \nu_i \Big/ \left[1 + \frac{h\nu_i}{mc^2}(1-\cos\theta)\right] \leq \nu_i} \quad (27)$$



This is Compton's formula -- it agrees with experiment, and it specifies the single, sharply-defined frequency for the X-ray scattered at ∡ θ. The important theoretical fact is that this result for ν_f is obtained only if the X-ray photons are considered as discrete particles, per Eqs. (26) above. So the notion of photon-as-particle is very vivid here.

ASIDE Experimental values for Planck's cnst h can be extracted from the photoelectric eqtn (23), and from Compton's formula (27). These values agree with the h from BB radiation analysis, Eq. (14), and further validate h as a new & universal constant of nature.