14. Find the quantum-mechanical eigenfunction $\psi_n(k)$ and energy bands $E_n(k)$ of a one-dimensional empty lattice [$V(x)=0$] with lattice constant a; n and k are the band index and wave-vector. Illustrate your results with a sketch of the energy bands. Hint: Use Bloch's theorem to represent $\psi_n(k)$ in terms of its periodic part $u_n(k)$.

$$\psi_n(k) = e^{ikx} u_k^{(n)}(x)$$

$$\nabla_x^2 \psi_n(k) = \frac{2m(V-E)}{\hbar^2} \psi_n(k)$$

$$e^{ikx}(\nabla_x^2 + 2ik\nabla_x - k^2) u_k^{(n)}$$

$$(\nabla_x^2 + 2ik\nabla_x) u_k^{(n)} = \left(k^2 - \frac{2mE}{\hbar^2}\right) u_k^{(n)}$$

with BC : $u_k^{(n)}(0) = u_k^{(n)}(a)$

Try plane wave soln: $u_k^{(n)} \propto e^{i\sigma x}$

BC. $e^{i\sigma a} = 1$, $\sigma a = 2\pi n$, $n = 0, \pm 1, \pm 2 \cdots$

put into D.E.

$$\sigma_n^2 - 2\sigma_n k + k^2 = \frac{2m E(k, n)}{\hbar^2}$$

$$E(k,n) = \frac{\hbar^2}{2m}(\sigma_n + k)^2 = \frac{\hbar^2}{2m}\left(k + \frac{2\pi n}{a}\right)^2$$

$$u_k^{(n)} = \frac{1}{\sqrt{a}} e^{\frac{2\pi n}{a} x}$$

$n = 0, \pm 1, \pm 2$

$-\frac{\pi}{a} < k \leq \pi/a$

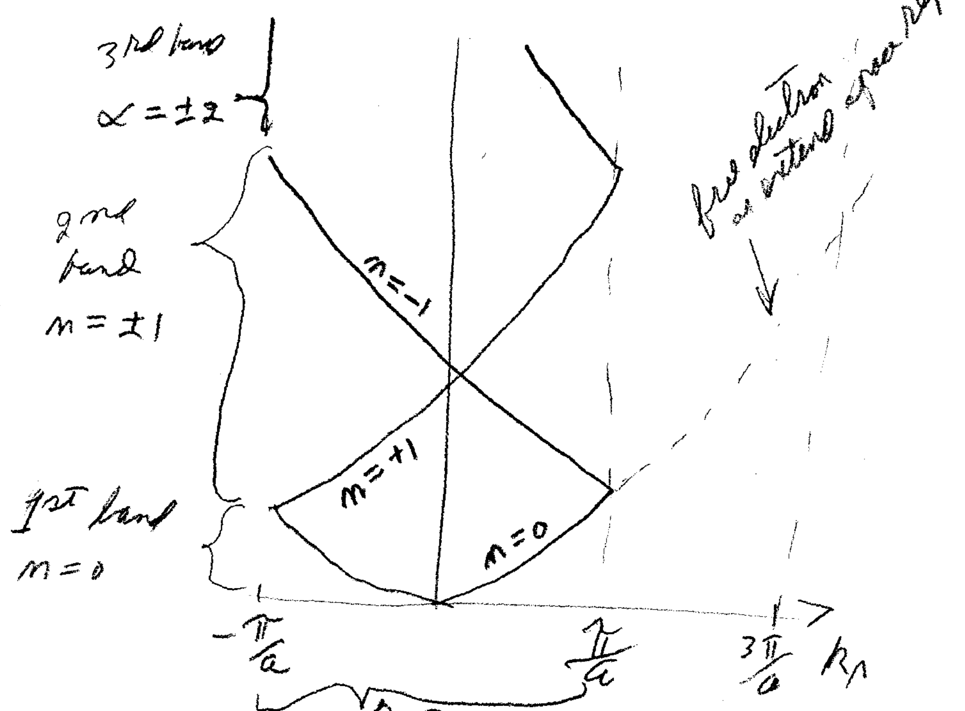
$$\rightarrow \psi_n(k) = \frac{1}{\sqrt{Na}} e^{i(k + \frac{2\pi n}{a})x}$$

$n=1$, $k + \frac{2\pi n}{a} = \frac{\pi}{a}$ for $k = -\frac{\pi}{a}$

$= +\frac{3\pi}{a}$ " $k = +\frac{\pi}{a}$

$n=-1$ $k + \frac{2\pi n}{a} = -\frac{\pi}{a}$ for $k = +\frac{\pi}{a}$

$= -\frac{3\pi}{a}$ " $k = -\frac{\pi}{a}$



15. The mercury atom has the following energy levels expressed in terms of energy units $1/\lambda$.

$6s^2$	^1S_o	0
$6s6p$	^3P_o	37,645 cm^{-1}
$6s6p$	$^3P_1$	39,412 cm^{-1}
$6s6p$	$^3P_2$	44,043 cm^{-1}
$6s7s$	$^3S_1$	62,350 cm^{-1}

a) Explain the meaning of the spectroscopic notation above.

b) What transitions will occur between these energy levels in a gas discharge? Explain in moderate detail.

c) Briefly outline an experimental method for verifying the total angular momenta J assigned to the levels above.