Comp 2013

1) The coherent state $|\alpha\rangle$ of a 1-D quantum harmonic oscillator is often defined as the eigenstate of the destruction (also known as annihilation or lowering) operator $\hat{a}$:

$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, where the eigenvalue α can be a complex number.

a) Expand the coherent state $|\alpha\rangle$ in terms of a superposition of the number states $|n\rangle$

$$|\alpha\rangle = \sum_{n=0} c_n |n\rangle$$

and use the destruction operator $\hat{a}$ applied to this superposition of number states to find the coefficients c_n for the coherent state $|\alpha\rangle$ in terms of the complex number α and the coefficient c_o for the ground number state $|n=0\rangle$. Hint: $a|n\rangle = \sqrt{n}|n-1\rangle$.

b) Using your result of part a) and the normalization condition for the coherent state $\langle\alpha|\alpha\rangle = 1$, find the needed value of the ground state coefficient c_o in terms of the eigenvalue α .

c) Then use your results for part a) and b) to write the coherent state superposition of number states just in terms of the complex number α .

d) Find the time dependence of the coherent state $|\alpha(t)\rangle$. As you find this time dependence, let ω be the classical oscillation frequency of the harmonic oscillator.

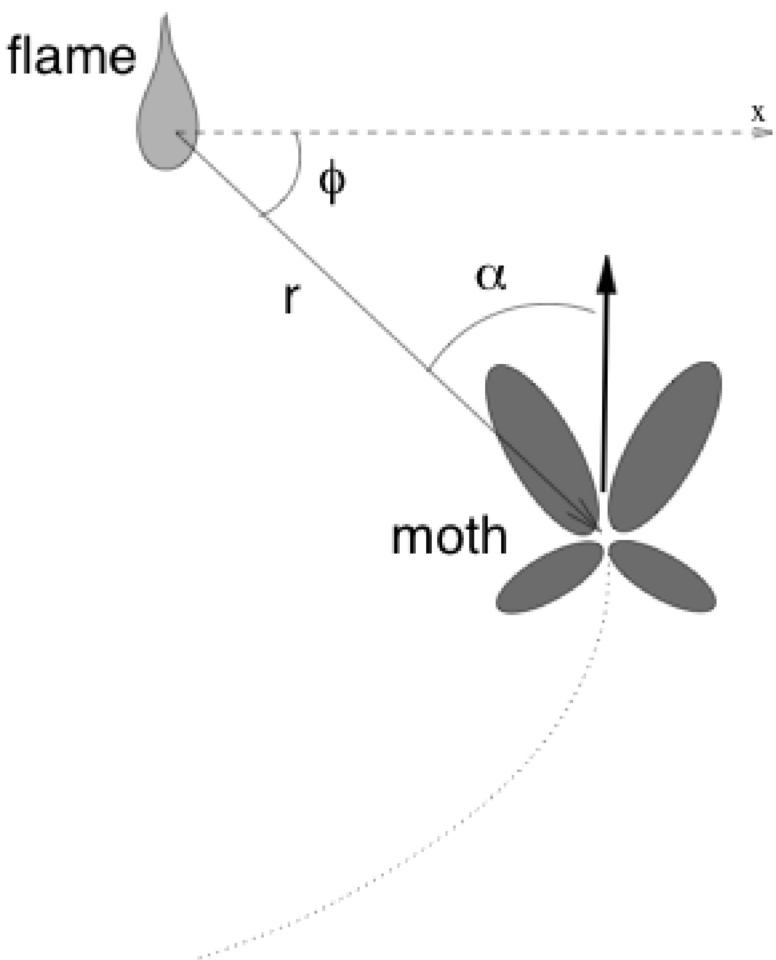
2) A parallel-plate capacitor is formed from two circular plates of radius b and spacing s. It is being discharged by a current I. Assuming $s \ll b$ and a vacuum exists between the plates. Find the magnetic field B between the plates at a radius r from the center. State the direction of B.

3) A moth flies at constant speed, always in a direction making an angle α from the line toward a candle flame located at the origin ($\alpha < \pi/2$). Sadly, the moth flies in the same horizontal plane as the flame.

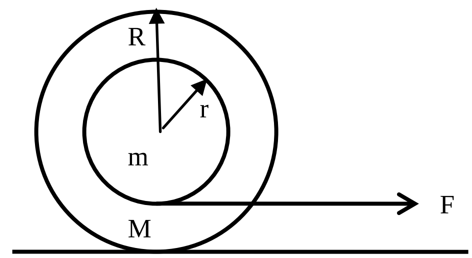
a) Write down the path $r(\phi)$ describing by the moth's flight beginning at a point (r_0,ϕ_0)

b) Write down the **acceleration vector** of the moth as a function of r and ϕ.

c) Write down the radius of curvature of the moth's path as a function of r and ϕ.



4) A spool is made from two solid discs, each of radius R and each of mass M, each attached to an axle of radius r and mass m. Thus the total mass of the spool is $2M+m$. A very lightweight thread is wound around the axle. The spool is then pulled along the floor without slipping by a horizontal force F applied to the thread coming off the bottom of the axle of the spool as shown in the figure. The surface has a coefficient of static friction of μ.



a) Find the maximum acceleration of the spool.
b) Does the spool wind or unwind the thread?
c) Find the acceleration a in the limit when $\mu=1$ and $R \gg r$ and $M \gg m$.

5)

a) Determine the average number of particles occupying single state of energy ε in a Fermi gas at thermodynamical equilibrium whose chemical potential is μ at temperature T .

b) Determine the average number of particles occupying state of energy ε in a Boson gas at thermodynamical equilibrium whose chemical potential is $\mu < \varepsilon$ at temperature T .

(*Hint:* The simplest approach is to start with the probability $P(n)$ of finding n particles occupying state of energy ε. Alternatively you can use the grand potential, given by $\Phi(T,V,\mu) = -kT \ln Z$ where Z is the grand canonical partition function and $d\Phi = E\, dT - p\, dV - N\, d\mu$).

6) A Helmholtz coil is a device to produce a fairly uniform **B**-field in a given volume. The coil consists of two identical loops of radius a with N turns carrying current I. The loops share an axis and are separated by distance b. Let the axis be the z-axis, and place the loops at $z = \pm \frac{b}{2}$.

a) It is possible to choose b so that the axial field near the center of the coil (i.e. $z \rightarrow 0$) goes as:

$$B(z) = B_z(0)\,(\,1 - k(z/a)^4 + \text{higher order terms}).$$

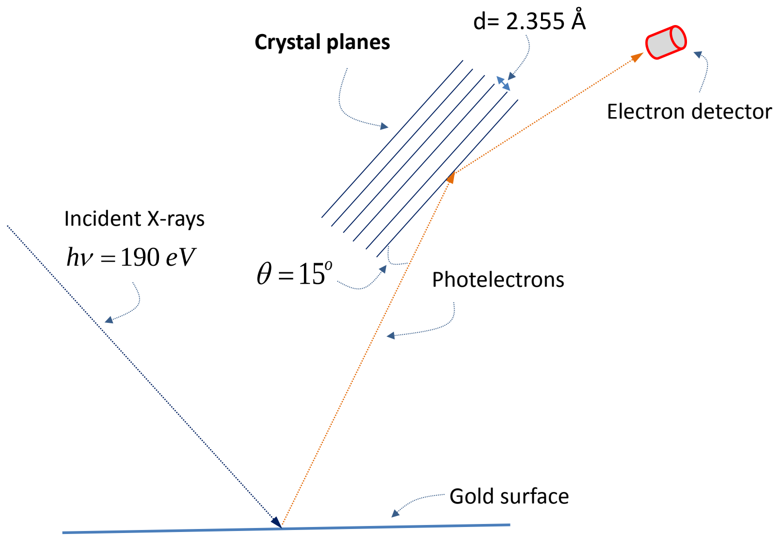
Find the condition on b which makes this so. (Express b with the given parameters in the equation.)

(b) Find the radial field $B\rho$ close to the axis and Bz near the coil center.

7) A spider sits on a horizontal mirror. The spider’s body is located 1 mm above the glass top surface. The mirror is made of a slab of 10 mm thick glass (n_{glass}=1.5) with a silver reflective coating on the back side of the glass. Find the distance d from the spider to it’s image.

8) The heat capacity of water is 4180 J/kg•K. A kilogram of water at 273 K is placed in thermal contact with a heat bath at 373 K, and equilibrium is reached. Determine the change in entropy of the water, heat bath, and the total system.

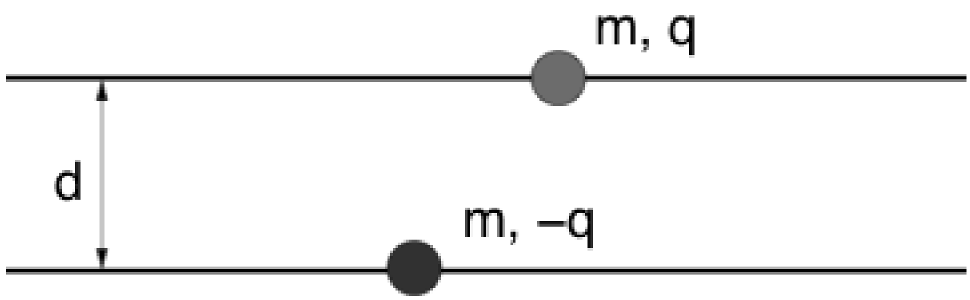
9) A beam of $h\nu = 190$ eV soft X rays impinges on a gold surface, resulting in the ejection of photoelectrons of various kinetic energies. These photoelectrons are directed onto a flat crystal monochromator with a lattice spacing of $d = 2.355$ Å aligned as shown in the figure below. A maximum reflection of electrons is observed at a minimum $\theta = 15^o$, as is also shown in the figure below. Based on this observation, determine the binding energy (analogous to the work function), E_o, of these electrons in eV. Results within an order of magnitude accuracy will be acceptable provided that you set up the solution correctly.



10) Two balls have identical mass m, and opposite electrical charges $\pm q$. They slide frictionlessly along parallel rods separated by distance d.

a) What is the frequency of small oscillation about equilibrium?

b) From its stationary equilibrium the positive ball is given a very small initial velocity v_0, while the negative ball is untouched. Write down the position of the negative ball for all later times.



11) A particle is in a normalized state $\psi(x)$ given by

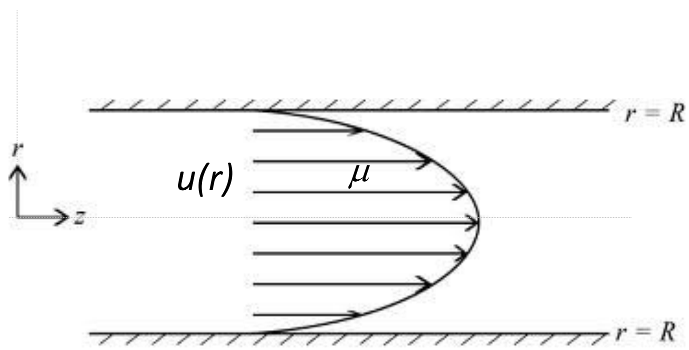
$$\psi(x) = \frac{1}{\sqrt{2a}} e^{-|x|/(2a)}$$

a) Find the expectation value of the position operator $\langle x \rangle$.
b) Find the expectation value of the position operator squared $\langle x^2 \rangle$.
c) Find the expectation value of the momentum operator $\langle p \rangle$.
d) Find the expectation value of the momentum operator $\langle p^2 \rangle$.
e) Use your results from parts a), b), c), and d) to check this state to see how close it is to the minimum given by the Heisenberg uncertainty principle.

12) Consider a uniform cylindrical pipe of inner radius R and an incompressible fluid with a viscosity constant of μ flowing through it from left to right as shown in the figure below. Do not panic: you only need freshman physics and a bit of math to solve this problem. At the steady state under fully developed laminar flow conditions the velocity $u = u(r)$ of each streamline remains constant along the z–axis but varies with radius r as shown in the figure, reaching a maximum at the center and zero at the edge. This means moving layers exert shear force on each other. While the fast-moving layer applies a shear force to drag the slow stream forward, the slow-moving layer applies an equal and opposite force on the fast-moving layer to slow it down. Recall that the shear stress, τ , is defined as the shear force per unit area acting on the surface (in this case) of the fluid directed along the z-axis. The value of $\tau(r)$ is given by Newton's law of viscosity: $\tau(r) = -\mu \dfrac{du}{dr}$. A steady flow is maintained by an external pressure, $P(z)$ (with dP/dz =constant), acting on the surface of the incompressible fluid element. Answer the following questions:

a) Draw a free-body diagram of a ring-shaped differential fluid element of radius $r \le R$, thickness dr and length dz oriented coaxially with the horizontal pipe. You only need to show the forces along the z-direction. Note that the pressure acts on the surfaces perpendicular to the z-axis while the shear stress acts on the surfaces parallel to the z-axis. Using this free-body diagram show that $\dfrac{dP}{dz} - \dfrac{\mu}{r}\dfrac{d}{dr}\left(r\dfrac{du}{dr}\right) = 0$; from this determine velocity $u(r)$ in terms of r, $\dfrac{dP}{dz}$, μ and R.

b) Using the fact that at the steady state the mass flow rate, Q, is constant and is given by $Q = u_o A$ where u_o is the average speed of the fluid and $A = \pi R^2$ is the area of the cross section, show that the mass flow rate Q is related to the pressure gradient via $Q = -\dfrac{\pi R^4}{8\mu}\left(\dfrac{dP}{dz}\right)$.

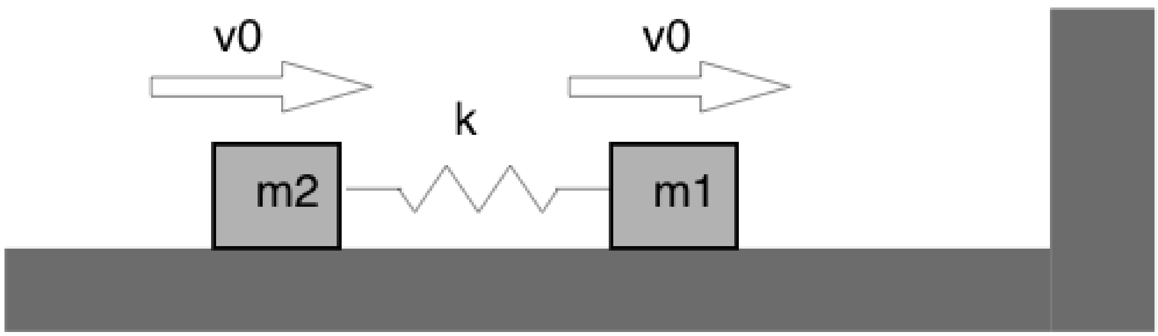


13) Two blocks, masses m_1 and m_2 (with $m_1 > m_2$), connected by a spring, with spring constant k, slide without friction along a one-dimensional rail. Initially both masses travel together at the same constant speed v_0. The system then strikes a solid wall and bounces off. Assume a single **perfectly elastic** collision between the wall and mass m_1.

a) What is the system's **center-of-mass velocity** following the collision?

b) What is the maximum extension of the spring following collision?

c) For what values of m1, m2, k, and v0 will mass m1 reverse directions periodically as the system travels away from the wall?



14) f(z) = u(x,y) + iv(x,y) is an analytic function of z = x + iy, with real functions u(x,y) and v(x,y). Find the missing quantities below, and find f(z) as a function of z and an arbitrary constant C.

(a) u(x,y) = e^x cos y, and then what is v(x,y) and f(z)?

(b) v(x,y) = y $(3x^2 - y^2 - 1)$, and then what is u(x,y) and f(z)?

(Hint: An analytic function satisfies the Cauchy-Riemann equations, which are: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.)

15) The diagram below is a schematic of a mass spectrometer. It consists of a magnetic field B = 0.500 tesla pointing out of the page in a perpendicular fashion (indicated by dots). The charged parallel plates are perpendicular to the page; this region of the mass spectrometer is the **velocity selector**. The top and bottom plates are charged to opposite polarity producing an electric field perpendicular to the plates *and* the magnetic field.

(a) A singly-ionized particle must travel along the path indicated by the dashed line so that it can exit through the small opening on the right side of the velocity selector. If the speed of the ion is $8.00\text{x}10^4$ m/s, determine the magnitude and direction of the electric field. Explain how you determined the direction of the electric field.

(b) After exiting the velocity selector, the electric field is negligible. The ion strikes one detector at a position 5.34 cm from the velocity selector opening. Indicate, with a dashed line, the path that the ion takes from the opening to the detector. Determine the ion mass.

(c) If the molar mass of nitrogen is 14.0 g/mole, oxygen is 16.0 g/mole, and fluorine is 19.0 g/mole, which of these were the ion that struck the detector.