Physics 505 Hour Exam 15 Nov. 1971

① By considering the Fourier pair

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \varphi(k,t) e^{+ikx} dk, \quad \varphi(k,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x,t) e^{-ikx} dx,$$

Show directly that if $\psi$ satisfies the Schrodinger equation in coordinate space, namely

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \psi(x,t),$$

then $\varphi$ satisfies the counterpart momentum space equation

$$i\hbar \frac{\partial}{\partial t} \varphi(k,t) = \left[ \frac{\hbar^2 k^2}{2m} + V\left(i \frac{\partial}{\partial k}\right) \right] \varphi(k,t).$$

Clearly state the assumptions which must be made concerning the behavior of $\psi$ and the potential function V.

② For a one-dimensional system described by the Hamiltonian: $H = (p^2/2m) + V(x)$, obtain an expression for the time rate of change of kinetic energy, $d\langle p^2/2m \rangle/dt$. Give your answer in terms of the force F acting on the particle. What relation does your result have to the classical work-energy theorem?

③ The total energy of a one-dimensional harmonic oscillator (mass m, natural frequency $\omega$) can be written as; $E = (p^2/2m) + \frac{1}{2} m\omega^2 x^2$, where p is the momentum and x is the position. Use the Uncertainty Relations to estimate the minimum energy of the oscillator.

④ A particle is in a state described by the wave function

$$\psi(x) = A(a^2 - x^2), \text{ for } -a \leq x \leq +a; \quad \psi(x) \equiv 0, \text{ for } |x| > a.$$

Here A is a normalization constant. What is the probability that a measurement of the particle's position will yield a value between $-\frac{1}{2}a$ and $+\frac{1}{2}a$?