A Didactic Approach to Wave Mechanics.

Properties and Structure of Schrodinger's Wave Mechanics

The QM analysis we can do by this juncture is to look for solutions $\psi = \psi(\mathbf{r},t)$ that obey Schrödinger's Eqtn: $\underline{i\hbar\,\partial\psi/\partial t = \mathcal{H}\psi}$, w/ $\mathcal{H} = \mathbf{p}^2/2m + V(\mathbf{r},t)$, and $\mathbf{p} = -i\hbar\nabla$. The wave fcn ψ specifies the evolution of the QM system in an external potential V... finding ψ is as close as we can come to specifying the system's classical trajectory $\mathbf{r} = \mathbf{r}(t)$, but ψ can only give us "most probable" values for the system's dynamical variables $q(\mathbf{r},t)$, via the so-called expectation values: $\underline{\langle q\rangle = \int_\infty \psi^*(\mathbf{r},t)\{q(\mathbf{r},t)\}\psi(\mathbf{r},t)\,d^3r}$. The exercise of finding the wave fcn ψ for a particular potential V, and then calculating $\langle q\rangle$-values for relevant quantities q as they evolve in time, is called "wave mechanics."

Before we plunge into detailed wave-mechanical solutions, we shall look at some general aspects of the theory--namely, what kind of solutions ψ can we expect, what restrictions might there be on the ψ's, what is the detailed time-evolution of the $\langle q\rangle$'s? Also, in developing the QM formalism a bit further, we will be able to state the uncertainty relations (the driving engine of QM) in a much more precise fashion--we do this for "fun". In this section, we treat the following topics...

$$\left.\begin{array}{l}
\bullet\ \text{General Properties of the QM System's Hamiltonian } \mathcal{H}.\\
\bullet\ \text{Functional Conditions on Acceptable QM Wave Fcns } \psi.\\
\bullet\ \text{The QM Version of an Equation-of-Motion for } q = q(\mathbf{r},\mathbf{p};t).\\
\bullet\ \text{Heisenberg's Version of the Uncertainty Relations: } \Delta p\Delta x \sim \hbar,\ \Delta E\Delta t \sim \hbar.
\end{array}\right\} \quad (1)$$

When we are finished with this list, we will: (A) feel better about QM in general, (B) have some new tricks up our sleeve, (C) be ready to do some wave mechanics.