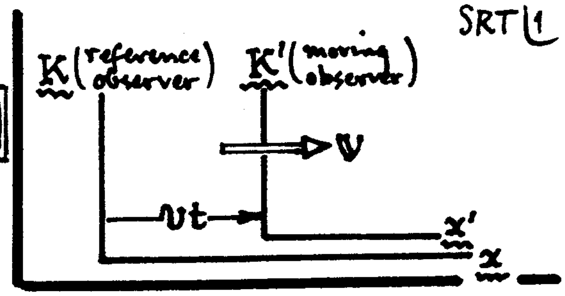
Trouble with the Galilean Transformation.

[Ref. Jk^{n} Ch. 11]

S$^{\text{pecial}}$R$^{\text{elativity}}$T$^{\text{heory}}$: Postulates and Lorentz Transformation.



1) By 1905, A. Einstein had pondered the following <u>dilemma</u> for observers in uniform relative motion at velocity $\mathbf{v}$ = cnst...

<u>A.</u> Newtonian mechanics <u>was</u> invariant under a Galilean Transf^{n}: $x' = x - vt$, $t' = t$.
(i.e. if $\mathbf{F} = d\mathbf{p}/dt$ in K, then -- under Galilean Transf^{n} -- $\mathbf{F}' = d\mathbf{p}'/dt'$ in K′).

<u>B.</u> Maxwell's E&M theory was <u>not</u> invariant under the same Galilean Transf^{n}, e.g.

if// $\left\{\left(\frac{\partial}{\partial x}\right)^2 - \frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2\right\}\psi = 0 \leftarrow$ <u>wave eqtn for a light pulse in frame K</u>;

and// $K \rightarrow K'$ under a Galilean Transf^{n}: $x \rightarrow x' = x - vt$, $t \rightarrow t' = t$;

then/ $\left\{\left[1 - \frac{v^2}{c^2}\right]\left(\frac{\partial}{\partial x'}\right)^2 + \frac{2v}{c^2}\left(\frac{\partial^2}{\partial x'\partial t'}\right) - \frac{1}{c^2}\left(\frac{\partial}{\partial t'}\right)^2\right\}\psi' = 0 \leftarrow$ <u>wave eqtn for "same" light pulse in frame K′.</u> (1)

Evidently, light appears to behave differently in the relatively moving frames.

If you adopt the point-of-view that physics should operate the same way everywhere in the universe (same physics here as in Andromeda galaxy), the dilemma was that mechanics and E&M could not <u>both</u> be right, since they transform from K to K′ in different ways under a GT (Galilean Transformation). There were <u>3 alternatives</u>:

<u>1.</u> E&M was wrong. A correct version would be invariant under a GT, just as mechanics.

<u>2.</u> GT & mechanics were OK, but E&M did not show a proper GT because it was valid in only one <u>preferred</u> frame, viz. $v = 0$ (i.e. "ether frame", w/ lightspeed $\equiv c$).

<u>3.</u> Mechanics and the GT were wrong. The correct $K \rightarrow K'$ transf^{n} would preserve form-invariance for <u>both</u> E&M <u>and</u> a corrected version of mechanics.

→ Einstein thought alternative #3 was the most sensible, since ~ nobody believed #1 (that E&M could be wrong), and no expt. had verified #2 (that "ether" existed).

2) To implement alternative #3 above, Einstein adopted two postulates:

① The laws of physics are of the same form in all inertial frames.¶ (2)

② The speed of light, c, is a universal cnst, independent of motion between source & observer.

From these two (reasonable) postulates comes all of Special Relativity Theory.

REMARKS

1. Postulate ① is "just" a claim that the laws of physics are valid everywhere in the universe (!), so attempts to verify the postulates have focussed on checking ② -- i.e. trying to find a situation where lightspeed c depends on source-observer motion.

2. Checks on ② started with the Michelson-Morley expt (1887), an attempt to detect $\Delta c/c$ due to earth's motion through the "luminiferous aether", and such checks continue to the present day -- with sophisticated expts involving Mössbauer Effect [Jkⁿ pp. 508-512] and observations on high-energy (~ 6 GeV) gamma rays. On a lab scale, one finds: $|\Delta c/c| \sim (0 \pm 1)\times 10^{-4}$, due to source-observer motion.

3. Lightspeed would not be a universal cnst, and postulate ② would be wrong, if c were frequency-dependent (relatively moving observers see different frequencies because of Doppler shifts; they would then claim to have measured different c's). The most sensitive limits on $\Delta c/c$ come from data on possible frequency-dependence of c.

MEASUREMENT of PHOTON MASS: m_γ

photon: energy: $\mathcal{E} = \hbar\omega$; mom^m: $p = \hbar k$; mass: $m_\gamma \neq 0$

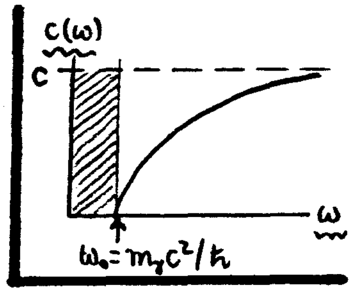
$$\mathcal{E}^2 = (pc)^2 + (m_\gamma c^2)^2 \Rightarrow \omega = c\sqrt{k^2+\mu^2}, \quad \mu = \frac{m_\gamma c}{\hbar}. \tag{3}$$

The energy transport velocity for this photon is the group velocity:

$$\rightarrow v_g = \partial\omega/\partial k = ck/\sqrt{k^2+\mu^2} = c(\omega) \int \text{detected lightspeed, i.e.} \; \boxed{c(\omega) = c\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^{1/2}} \int \omega_0 = \mu c. \tag{4}$$

¶ "Inertial frames" are coordinate systems in uniform (relative) translational motion, w/ Newton I holds.

SRT Introdn Effect of photon mass $m_\gamma \neq 0$ on lightspeed c.



For a massive photon, the lightspeed $c(\omega)$ in Eq. (4) shows a <u>low frequency cutoff</u>; no photons can move at frequencies <u>below</u>:

$$\rightarrow \omega_0 = \frac{m_\gamma c^2}{\hbar} = \frac{m_\gamma}{m_e}\left(\frac{c}{\hbar/m_e c}\right) = \frac{m_\gamma}{m_e} \times 2\pi \times 1.24 \times 10^{20}\ \mathrm{Hz} \qquad (5)$$

Here $m_e = 0.511\ \mathrm{MeV}/c^2$ is the electron mass. In terms of wavelength, a massive photon cannot propagate at wavelengths <u>above</u> the limiting value:

$$\rightarrow \lambda_0 = 2\pi c/\omega_0 = 0.0243\,(m_e/m_\gamma),\ \text{Å};\quad \lambda > \lambda_0 \text{ is forbidden.} \qquad (6)$$

If we detect some long wavelength λ_{det}, then: $\frac{m_\gamma}{m_e} < \frac{0.0243\,\text{Å}}{\lambda_{det}}$. We can certainly broadcast & detect radio-frequencies with $\lambda_{det} \sim 100\,\mathrm{m}$; this gives $m_\gamma/m_e < 2.4 \times 10^{-14}$. One can also detect very low frequency EM resonances in the earth's ionosphere (which acts like a waveguide)... here $\lambda_{det} \sim$ earth radius $\sim 6.37 \times 10^8$ cm, corresponding to $m_\gamma/m_e < 3.8 \times 10^{-19}$. This last limit implies a cutoff @ $\omega_0 = 2\pi c/\lambda_0 \lesssim 2\pi \times 47\ \mathrm{Hz}$.

What m_γ/m_e has to do with $\Delta c/c$ is seen by expanding Eq. (4) for $\omega \gg \omega_0$:

$$\frac{\Delta c}{c} = \frac{c - c(\omega)}{c} \simeq \frac{1}{2}\left(\frac{\omega_0}{\omega}\right)^2;\quad \text{assume } \omega_0 \lesssim 2\pi \times 47\ \mathrm{Hz}; \qquad (7)$$

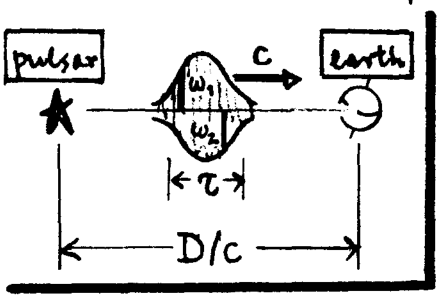
So//

$\omega/2\pi$, Hz	10^3	10^6 (radio)	10^9 (μwave)	10^{12} (far IR)	10^{15} (visible)
$\Delta c/c$, max.	1.1×10^{-3}	1.1×10^{-9}	1.1×10^{-15}	1.1×10^{-21}	1.1×10^{-27}

The best measured ratio is $m_\gamma/m_e < 4.3 \times 10^{-21}$ (from measurements on dipole character of earth's magnetic field -- see Jackson, p. 6); this drives the photon low-freq. cutoff down to $\omega_0 \lesssim 2\pi \times 0.53\ \mathrm{Hz}$, and -- as a consequence -- we have $\Delta c/c < 14$ ppm even at freqs $\omega = 2\pi \times 100\ \mathrm{Hz}$.

→ There is <u>no</u> evidence of a detectable value of $\Delta c/c$ resulting from expts which effectively measure the photon mass m_γ. The notion of $m_\gamma \neq 0$ is the "easiest" way of producing a frequency-dependent lightspeed $c(\omega)$.

MEASUREMENT of PULSAR SIGNAL DURATION



A simple limit on $\Delta c/c$ is possible from the fact that pulsar signals arrive as coherent pulses with finite durations τ. The pulse must contain frequencies spread over a small range $\Delta\omega \sim 1/\tau$ (Fourier Theorem), and -- if c depends on ω -- there would be a small velocity spread Δc. If the pulsar lies at distance D from earth, then transit time is D/c, and

$$\left(\begin{matrix}\text{dispersion}\\ \text{in transit}\end{matrix}\right) \leqslant \left(\begin{matrix}\text{observed}\\ \text{pulsewidth}\end{matrix}\right) \Rightarrow \Delta c \cdot \frac{D}{c} \lesssim c\tau, \text{ or } \boxed{\left|\frac{\Delta c}{c}\right| \lesssim \frac{\tau}{L}} \qquad (8)$$

$L = D/c$ is the distance in light years. Data from pulsar in Crab Nebula; $L = 6000$ l.y., $\tau < 3$ msec $\Rightarrow \underline{|\Delta c/c| < 1.6 \times 10^{-14}}$, @ freqs $\nu \simeq 10$ GHz.

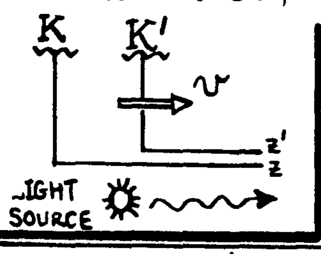
> There seems no serious doubt that in fact <u>c = universal constant</u>, independent of the EM signal frequency, and of any relative motion between source & observer.

4. C has of course been <u>measured</u> in many expts†. Recent numerical values are:

$$c/(10^8\,\text{m/sec}) = 2.997\ 9250\ (\pm 0.3\,\text{ppm}) \leftarrow 1969 \text{ adjustment (NBS)} \qquad (9)$$
$$= 2.997\ 92458\ (\text{EXACT}) \leftarrow 1986 \quad \text{(NBS)}$$

The "exact" value of c is now used to define the standard of length.

5. The consequences of c = universal cnst are immediate for two relatively moving observers K & K'. They will have to adjust their length & time scales so that...



$$c = \begin{cases}\Delta z/\Delta t, \text{ meas. by } K;\\ \Delta z'/\Delta t', \text{ meas. by } K.\end{cases} \text{ both must measure } c = \frac{\Delta z}{\Delta t} = \frac{\Delta z'}{\Delta t'}. \qquad (10)$$

This does not work for GT: $\Delta z' = \Delta z - v\Delta t \Rightarrow c' = c - v$.

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† Beginning with measurement by Olaf Roemer (1675) on eclipses of Jupiter's moons. Roemer found $c = 2.3 \times 10^8$ m/sec, within ~25% of current value.