An infinite one-dimensional square-well potential defined as

$$V(x) = 0, \quad 0 \le x \le a,$$
$$V(x) = \infty, \quad 0 > x > a$$

has well-defined normalized energy eigenfunctions given by:

$\varphi_n(x) = \sqrt{\frac{2}{a}} Sin(nk_1 x),$ where $k_1 = \frac{\pi}{a}$. Answer the following questions:

a. For the 3rd excited state (n=3) what is the probability $P_n(x)$ that a particle is located between the interval at $x = \frac{a}{3}$ and $x + dx$, where $dx = \frac{a}{1000}$?

b. Now consider the momentum space representation of the wave functions for the one-dimensional square-well potential described above and let us call these functions $\phi_n(p)$, where p is the linear momentum of the particle in the nth excited state. Determine $\phi_n(p)$, and explain briefly the physical meaning of $\phi_n(p)$.

c. Determine the probability of finding a particle in the 3rd excited state with a momentum between $p = 2\hbar k_1$ and $p + dp$, where $dp = \frac{\hbar k_1}{1000}$.