CCK Special Topics Problem: Surface Tension
2007-Jul-16

The surface tension, σ, is a constant force per unit length that tends to minimize the free surface area (*i.e.*, the area not in contact with matter) of a fluid. Consequently, it is useful to define the surface energy

$$U = \sigma A,$$

where A is the free surface area of the fluid. A long, thin column of water initially has cylindrical shape $\rho = R$, where ρ is the radial coordinate. We now perturb the shape of the column so that

$$\rho = R\left[1 - \alpha + \epsilon \sin(kz)\right],$$

where $\epsilon \ll 1$. In the following, work to lowest nontrivial order in the perturbation amplitude, ϵ.

A. In order for the above description to be valid, the total volume of water must be independent of ϵ. Use this requirement to calculate the correction term α.

B. Calculate the total surface energy as a function of ϵ and k.

C. For what values of k does the perturbation increase (or decrease) the surface energy?

D. Use your results to explain, qualitatively, why a stream of water from a squirt bottle will break up into drops. Predict the minimum size (volume) of a drop.