Remarks on QM Selection Rules

1) The CPT table just compiled has use in deciding what sort of elementary EM interactions can occur in nature. Consider an interaction energy U = coupling of a charge or current (ρ or $\mathbb{J}$) to an EM field ($\mathbb{E}$ or $\mathbb{B}$). From the definition of energy (e.g. $U = \int \mathbb{F} \cdot d\mathbb{r}$), U must have $CPT = (+1, +1, +1)$.

Suppose $U \propto \mathbb{J} \cdot \mathbb{B}$ were a candidate. Its CPT signature is $(+1, -1, +1)$, and it is ruled out on the grounds that it is a pseudoscalar. Similarly, coupling of the system ∢ momentum $\mathbb{L}$ to $\mathbb{E}$, i.e. $U \propto q\,\mathbb{L} \cdot \mathbb{E}$ has $CPT = (+, -, -)$ and is ruled out because it is a T-odd pseudoscalar.

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2) For atoms, there are two basic couplings of charge/current to $\mathbb{E}/\mathbb{B}$. They are:

① **STARK EFFECT** : $U_s = e\,\mathbb{E} \cdot \mathbb{r} \leftarrow CPT = (+, +, +)$; is acceptable,

so// $\langle U_s \rangle = \int_\infty d^3x\, \psi_f^*(\mathbb{r}) [e\,\mathbb{E} \cdot \mathbb{r}]\, \psi_i(\mathbb{r}) \ldots$ applied $\mathbb{E}$ = cnst over atomic dimensions,

i.e.// $\langle U_s \rangle = e\,\mathbb{E} \cdot \int_\infty d^3x\, [\mathbb{r}\, \psi_f^*(\mathbb{r})\, \psi_i(\mathbb{r})]$ ∫ $\mathbb{r}$ is P-odd, so if $\langle U_s \rangle \neq 0$, must have $\psi_f^*(\mathbb{r})\psi_i(\mathbb{r})$ P-odd.

⇒ Selection Rule : $\mathbb{E}$ connects $\psi_i \to \psi_f$ only if the states have opposite parity.
I.e. $\psi_i \xrightarrow[\mathbb{E}]{} \psi_f$ involves ∢ momentum change: $\Delta J = \pm 1$.

② **ZEEMAN EFFECT** : $U_z = \mathbb{m} \cdot \mathbb{B} \leftarrow CPT = (+, +, +)$; is acceptable.

so// $\langle U_z \rangle = \int_\infty d^3x\, \psi_f^*(\mathbb{r}) [\mathbb{m} \cdot \mathbb{B}]\, \psi_i(\mathbb{r}) \ldots$ applied $\mathbb{B}$ = cnst over atomic dimensions,

i.e.// $\langle U_z \rangle = \mathbb{B} \cdot \int_\infty d^3x\, [\mathbb{m}\, \psi_f^*(\mathbb{r})\, \psi_i(\mathbb{r})]$ ∫ $\mathbb{m}$ is P-even; if $\langle U_z \rangle \neq 0$, must have $\psi_f^*(\mathbb{r})\psi_i(\mathbb{r})$ P-even.

⇒ Selection Rule : $\mathbb{B}$ connects $\psi_i \to \psi_f$ only if the states have same parity.
I.e. $\psi_i \xrightarrow[\mathbb{B}]{} \psi_f$ involves no ∢ momentum change: $\Delta J = 0$.

Selection rules are strict so long as $\mathbb{E}$ is polar, $\mathbb{B}$ is axial, and P is a "good" quantum #.