

Fermat's principle states that a ray of light will follow the path that requires the shortest traveling time. For a two dimensional case, such a path is obtained by minimizing the integral

$$\int_a^b n(x, y) \sqrt{1 + y'^2} \, dx$$

where $n(x, y)$ is the index of refraction and $y' = dy/dx$.

- (a) For the special case that the integrand F does not depend explicitly on x , use the *Euler-Lagrange Equation* to prove that

$$F - y' \frac{\partial F}{\partial y'} = \text{constant}.$$

- (b) Find all possible $y(x)$ for the particular case $n = 1 + c|y|$ with $c > 0$.
(c) Take the limit $c \rightarrow 0$ of your answers from part (b), and discuss the results.