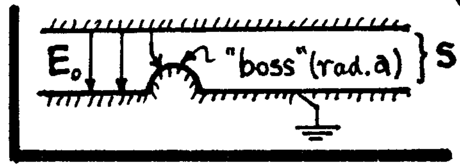
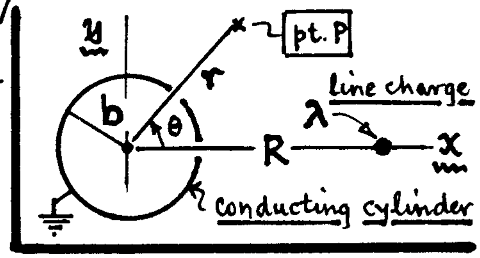
ϕ519 Problems



① [15pts, Jkⁿ #(2.6)]. One plate of a large ∥-plate capacitor (separation s) has a small hemispherical "boss" of radius $a \ll s$; this plate is grounded. The other plate is at potential V so that far from the boss, the interplate field is $E_0 = V/s =$ cnst. **(A)** Using spherical cds, find the potential between the plates. Then, calculate the surface charge density on the boss, and at an arbitrary pt. on the grounded plane. **(B)** Show that the total charge on the boss has size $\frac{3}{4}E_0 a^2$. **(C)** If -- instead of the upper plate charged to V -- a pt. charge q were placed at distance $d > a$ above the boss, show that the charge induced on the boss is: $\tilde{q} = -q\left[1-(1-\epsilon^2)/\sqrt{1+\epsilon^2}\right]$, w/ $\epsilon = \frac{a}{d}$. For $\epsilon \ll 1$, show: $\tilde{q} \simeq -\frac{3}{2}\epsilon^2 q$.



② [15pts, ~Jkⁿ (2.7)]. A long line at uniform charge per unit length λ lies ∥ to a long (grounded) conducting cylinder of radius b and at distance $R > b$ from the axis.

(A) Find the size and location of the image charge λ' induced inside the cylinder.

(B) Find the potential ϕ(pt. P) in the polar cds (r, θ), w/ $r > b$. As $r \to \infty$, $\phi(r) \to$ what?

(C) Find the surface charge density $\sigma(b, \theta)$ on the cylinder. Plot σ vs. θ for $R = 2b$.

(D) Calculate the force acting between the line and the cylinder.

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③ Consider the ODE: $\mathcal{D}(u) = 0$, w/ the operator: $\mathcal{D} = p_2(x)\frac{d^2}{dx^2} + p_1(x)\frac{d}{dx} + p_0(x)$, and the interval $x \in [a, b]$. $\mathcal{D}$ is not self-adjoint unless $p_1 = dp_2/dx$; this rarely occurs spontaneously. But show that a function $\mu(x)$ can be constructed from the $p_i(x)$ such that $\tilde{\mathcal{D}} = \mu(x)\mathcal{D}$ is self-adjoint. Find $\mu(x)$ explicitly. What conditions on the $p_i(x)$ are needed for $\mu(x)$ to exist? What use is this overall procedure?

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④ Jackson [Sec. (2.11)] solves the 2D wedge problem using polar cds (ρ, ϕ). B. Chang notes that for wedge ∡ $\beta = k\frac{\pi}{2}$ $(k = 1, 2, 3, \ldots)$, the problem should be doable in rectangular cds (x, y). Show how (or whether) this suggestion works.