φ566 : Aut. 94

φ566 ('94)

	Date	#	Lecture	Assignment
①	Wed. 24 Aug	1	Introdⁿ. ODEs : Topics ①-③, pp. 1-4.	① Prob. # 1 (20 pts) ✗ (copied)
	Fri. 26 "	2	ODEs : topics ④-⑨, pp. 4-9.	–
②	Mon. 29 "	3	ODEs : topics ⑩-⑪, pp. 9-14.	② Probs. # 2-5 (50 pts). ✗
	Wed. 31 "	4	ODEs : topics ⑫-⑭, pp. 14-20.	– (ODEs)
	Fri. 2 Sept.	5	ODEs ; topic ⑭ Exs : SHO & Bessel's Eq., pp. 21-24.	–
③	Mon. 5 "	–	HOLIDAY (Labor Day)	③ Probs # 6-8 (45 pts). ✗
	Wed. 7 "	6	Finish ODEs : pp. 25-27 | Start WKB, pp. 1-2.	– (ODEs)
	Fri. 9 "	7	WKB, pp. 2-7 (through LRC cct example).	–
④	Mon. 12 "(37)	8	WKB, pp. 7-10 (Through Neumann series). [4.63]	④ Probs # 9-11 (45 pts) ✗
	Wed. 14 "	9	WKB, pp. 11-15 (solns to Airy Eqtn).	– (ODEs)
	Fri. 16 "	10	WKB, pp. 15-19 (connection formulas).	–
⑤	Mon. 19 "	11	∞ Series I : convergence; prelim. & compⁿ tests, pp 1-5.	⑤ Probs. # 12-15 (45 pts). ✗
	Wed. 21 "	12	∞ Series II : thru evaluation of series, pp. 5-9.	– (WKB)
	Fri. 23 "	13	∞ Series III : thru properties of power series, pp 9-13.	–
⑥	Mon. 26 "(64)	14	∞ Series IV : Conv. in the mean, transfⁿ, etc pp. 14-18 [4.57]	⑥ Probs. 16-19 (50 pts). ✗
	Wed 28 "	15	SL Theory I : thru orthonormality, pp. 1-4.	– (∞ series)
	Fri. 30 "	16	SL Theory II : thru $\Lambda(u_\lambda) = \lambda$, pp. 5-8.	–
⑦	Mon. 3 Oct.	17	SL Theory III : review (p. 17); $\Lambda(u_\lambda) > \lambda$, pp. 9-10.	⑦ Probs. 20-23 (40 pts). ✗
	Wed. 5 "	18	SL Theory IV : $\lambda_n \to \infty$. Start $D_n \to 0$. pp. 11-14.	– (SL problems)
	Fri. 7 "	19	MidTerm Preview. Finish SL Thy. Begin CV, pp. 1-1.	–
⑧	Mon. 10 "	–	HOLIDAY (Columbus Day).	no assignment
	Wed. 12 "(85)	20	Compl. Var. I : pp. 1-4 (thru Integral Thm). [4.25]	–
	Fri. 14 "	21	Compl. Var. II : pp. 5-8 (at Laurent Thm).	–
⑨	Mon. 17 "	22	MIDTERM : 7-9 PM. Probs. ①-⑤ (200 pts).	⑧ Probs. 24-27 (50 pts). ✗
	Wed. 19 "	23	Review. Laurent Thm & remarks. Residues. pp. 8-10.	– (Complex Variables)
	Fri. 21 "	24	Compl. Var. IV : Residues & Contour Integration, pp. 10-13.	–
⑩	Mon. 24 "	25	Compl. Var. V : Contour Integration & Real Integrals, 13-17.	⑨ Probs. 28-30 (40 pts) ✗
	Wed. 26 "	26	Compl. Var. VI : Branch Cuts → thru B_{2n} #5, pp. 18-21.	– (contour integrals; Leibniz)
	Fri. 28 "(106)	27	Compl. Var. VII : thru Gauss Sumⁿ ; pp. 22-25. [4.08]	–
⑪	Mon. 31 "	28	SpF I : Legendre Eq. & Generating Fcns. pp. SpF 1-4.	⑩ Probs. 31-33 (50 pts). ✗
	Wed. 2 Nov.	29	CANCELLED (CAR opⁿ). Made up on 11/21/94.	– (Gram-Schmidt; G-fcns).
	Fri. 4 "	30	SpF II : Generating Fcns & Legendre Eq. pp SpF 4-7.	–
⑫	Mon. 7 "	31	SpF III : Properties of the P_n & Q_n, pp. SpF 7-11.	⑪ Probs. 34-37 (50 pts) ✗
	Wed. 9 "	32	SpF IV : Assoc. Leg. Fcns $P_n^m(x)$. Sph. Harmonics, pp. 12-16.	– (all Legendre fcns)
	Fri. 11 "	33	SpF V : Sph. Harmᶜs, pp. 16-18. ASIDE on Γ-fcns, p. Γ1.	–
⑬	Mon. 14 "	34	Finish Γ-fcns, pp. Γ2-Γ3. Start Bessel fcns, pp. SpF 19-19.	⑫ Probs. 38-41 (40 pts). ✗
	Wed. 16 "	35	SpF VII : Bessel fcns (thru $g \to J_n$ (series)) : pp. 19-22.	– (P_n's, Γ-fcn, and J_ν's)
	Fri. 18 "	36	Sp.F VIII : Bessel → Hankel & mod. Bessel fcns : pp. 22-25.	–
⑭	Mon. 21 "(142)	37	SpF IX & X : Bessel, p. 26 ; HG Eq. pp 27-33. [3.94]	no assignment
	Wed. 23 "	–	HOLIDAY (Thanksgiving)	–
	Fri. 25 "	–	HOLIDAY	–
⑮	Mon. 28 "	38	SpF XI : Review HG Eq. Start CHG Eq. : pp. 33-36.	⑬ Probs. 42-45 (40 pts). ✗
	Wed. 30 "	39	SpF XII : Singularities & Riemann's Eqtn : pp. 36-39.	– (Bessel, HG & CHG fcns).
	Fri. 2 Dec.	40	SpF XIII : CHG fcn particulars : pp. 40-45.	–
⑯	Mon. 5	41	Evaluation of Integrals (partial intgn, etc) : pp. 1-3.	no assignment
	Wed. 7	42	Evaluation of Integrals (thru Fresnel) : pp. 4-8.	–
	Fri. 9 (165)	43	Exam Preview. Integrals : pp. 9-10. EVALUATION. [3.93]	Final on Ths. 12/15/94.

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	#	PTS	SUBJECT	REMARKS
① (20)	①	20	Analyse eqtns for a two-level (QM) system.	#①, ϕ566('87).
② (50)	②	20	Solve 4 ODEs from M & W, p. 38.	#②, ϕ566('83).
	③	10	Solve: $y''+ay'+by=f(x)$, a & b = cnst, when roots $\mu_1=\mu_2$.	#③, ϕ566('83).
	④	10	Examine linear dependence/independence of $\{f_n(x)\}$ via Wronskian.	#④, ϕ566('83).
	⑤	10	Convert $y''+p(x)y'+q(x)y=0$ to Riccati's Eqtn.	- new -
③ (45)	⑥	10	Solve 2 ODEs from M & W, p. 38.	#⑥, ϕ566('83).
	⑦	20	Find series solns to hypergeometric eqtn: $x(1-x)y''+\ldots=0$.	#⑦, ϕ566('83).
	⑧	15	Find a particular sol^n to: $\ddot{P}+2\gamma\dot{P}+\omega_0^2P=\omega_p^2E(x,t)$.	#⑧, ϕ566('83).
④ (45)	⑨	15	Do Green's fcn soln. form for: $p(x)y''+q(x)y'+r(x)y=0$.	#⑨, ϕ566('83).
	⑩	20	Solve: $xy''+2y'+(\epsilon-x)y=0$, w/ $y(0)=1$, $y(\infty)=0$. What ϵ's?	#⑩, ϕ566('83).
	⑪	10	Find energies of a QM SHO via Bohr-Sommerfeld formula.	#⑪, ϕ566('83).
⑤ (45)	⑫	10	Find asymptotic form for $J_\nu(x)$ as $x\to\infty$, by WKB method.	#39, ϕ506('93).
	⑬	10	Iterate $u_{n+1}(s)$ Neumann series [from p. 10 of WKB notes].	#40, ϕ506('93).
	⑭	10	Review pp. 6-10 of WKB notes: employ cct model $\Omega=\omega_0 e^{-\alpha t}$.	#17, ϕ566('87).
	⑮	15	Solution to: $y''+\alpha\xi^n y=0$. Asymptotic form for $y(\xi)$ as $\xi\to\infty$.	#23, ϕ507('92).
⑥ (50)	⑯	15	Prove two convergence tests: (A) $\sum_n \lvert a_n\rvert$ conv. $\Rightarrow \sum_n a_n$ conv., (B) Root Test.	#14+15, ϕ566('83).
	⑰	15	Test 8 series for convergence & identify tests used.	#16, ϕ566('83).
	⑱	10	Sum 3 series in closed form and find numerical values.	#17, ϕ566('83).
	⑲	10	Sum $f(x)=\sum_n [(-)^{n+1}n^2/(2n-1)!]x^{2n-1}$ in closed form.	#18, ϕ566('83).
⑦ (40)	⑳	10	Stack books in an harmonic fashion.	#24, ϕ566('87).
	㉑	10	$\mathcal{O}=f(d^2/dx^2)+g(d/dx)+h$ is self-adjoint $\Rightarrow$ f, g, h related?	#4, ϕ566('87) MidTm.
(295 btl)	㉒	10	Write hypergeometric ODE in Sturm-Liouville form.	#22, ϕ566('87).
	㉓	10	Estimate lowest eigenvalue for $u''+k^2u=0$ via fcnal $\Lambda(u)$.	#24, ϕ566('87).
⑧ (50)	㉔	10	For $f(z)=u+iv$ analytic, fill in the blanks.	#⑧, ϕ567('88).
	㉕	10	Prove Liouville's Thm: $f(z)$ bounded for all $z\Rightarrow f(z)=$ cnst.	#⑨, ϕ567('88).
	㉖	15	Prove Fundamental Thm of Algebra: $P_n(z)=0$ has n roots.	#⑩, ϕ567('88).
	㉗	15	Evaluate 3 integrals by means of contour integration.	#⑪, ϕ567('88).
MID-TERM (200 pts)	①	45	Eq. of motion for a falling raindrop: evaporation + viscous drag.	#⑥, ϕ566('87).
	②	40	Convergence & summation of: $S(x)=1+3x^2+5x^4+7x^6+\ldots$	#②, ϕ566('82) MidTm.
	③	35	Wronskian for: $f(x)y''+g(x)y'+h(x)y=0$ ($W\sim\exp[-\int(g/f)dx]$).	#①, ϕ566('83) MidTm
	④	40	Orthogonality of SL eigenfcn derivatives: $\int u_m' u_n' p\,dx=\lambda_m\delta_{mn}$.	#23, ϕ566('87).
	⑤	45	Reflection of a radio-wave by the ionosphere via WKB.	#12, ϕ520('94).
⑨ (40)	㉘	20	Evaluation of Fresnel Integrals by contour integration of $\oint e^{-z^2}dz$.	#22, ϕ566('83).
	㉙	10	Evaluation of $I(\mu)=\int_{-\infty}^{+\infty}[x\sin x/(x^2-\mu^2)]dx$ by contour integration.	#25, ϕ566('83).
	㉚	10	Proof of Leibniz Thm: $(d/dx)^n[A(x)B(x)]=\ldots$ by math. induction.	#38, ϕ567('83).
⑩ (50)	㉛	15	Gram-Schmidt orthogonalization procedure.	#48, ϕ507('94).
	㉜	20	Apply G-S orthogonalization to $\{x^n\}$ on $\lvert x\rvert\le 1$ $\Rightarrow$ get $P_n(x)$.	#②, ϕ567('83) MidTm.
	㉝	15	Generating fcn for f_n's obeying: (A) $(n+1)f_{n+1}=xf_n-f_{n-2}$, etc.	#29, ϕ566('87).

(over)

ϕ566 ('94) Problems (cont'd)

	#	PTS	SUBJECT	REMARKS
(11)	(34)	10	Potential generated by a linear electric quadrupole.	#(28), ϕ566('87).
(50)	(35)	15	Legendre → Parseval Thm. σ_{tot} for partial wave analysis.	#(30), ϕ566('87).
	(36)	15	Legendre polynomial series from Laplace's Integral.	-new-
	(37)	10	m-reflection rule for associated Legendre polynomials $P_n^m(x)$.	#(31), ϕ566('87).
(12)	(38)	10	on Rodrigues' formula for $P_n(x)$: $(d/dx)^n (x^2)^{n-k}$ over $[\frac{n}{2}]+1 \le k \le n$.	#(37) ϕ567('83).
(40)	(39)	10	Evaluation of Gaussian integrals: $\int_0^\infty x^\nu e^{-ax^2} dx$.	new [Arfken #(10.1.11)]
	(40)	10	For Bessel's Eq., show J_ν & Y_ν are linearly independent.	#(43) ϕ567('83).
	(41)	10	Prove: $J_n(x) = (-)^n x^n (\frac{1}{x} d/dx)^n J_0(x)$, by mathematical induction.	#(33) ϕ566('87).
(13)	(42)	10	For Bessel fns, analyse: $(\nu^2 - \mu^2) \int_a^b \frac{dx}{x} Z_\mu(x) Z_\nu(x)$.	#(6) ϕ566('87) Final.
(40)	(43)	10	Show $F(a,b;c;x)$ & $x^{1-c} F(1+a-c, 1+b-c; 2-c; x)$ are dept. when $c \to N$.	#(1) ϕ567('87).
[565pts total]	(44)	10	From a linear transf^n on $F(a,b;c;z)$, find $M(a,c,x \to \infty)$.	#(4) ϕ567('87).
	(45)	10	For the CHG Eq., find Wronskian for $M(a,c,x)$ & $x^{1-c} M(1+a-c, 2-c, x)$.	#(6) ϕ567('87).
	(1)	50	Solve: (A) $xy' = \alpha y + x^\beta$, α & β = consts, and (B) $y' = xy + 1$.	ϕ566('75) & ϕ566('83)
FINAL	(2)	40	WKB soln to: $y'' + x^2 y = 0$, near $x=10$, w/ $y=0$, $y'=1$ @ $x=10$.	#(3), ϕ566('87) Final.
EXAM	(3)	50	Sum series to closed forms: (A) $S_p = \sum_{n=1}^{\infty} (1/n p^n)$, (B) $S_k(\theta) = \sum_{n=0}^{\infty} k^n \cos n\theta$.	ϕ566('83) Mid Term.
	(4)	40	Rudimentary analysis of Laguerre's ODE, using S-L Theory.	Arfken (3rd ed.) #9.1.6.
(350pts)	(5)	50	Evaluate by contour integration: $I(a,b,c) = \int_0^{2\pi} d\phi / (a + b\cos\phi + c\sin\phi)$.	#(2)(D), ϕ566('82) Final
	(6)	35	Legendre polynomials: Show $\int_{-1}^{+1} P_n(x)\, dx = 0$, for $n \ge 1$.	-new (as of Dec '94)-
	(7)	35	Show $\operatorname{erf} x \propto M(\frac{1}{2}, \frac{3}{2}, -x^2)$. Find asymptotic erf x as $x \to \infty$.	#(7), ϕ567('87),
	(8)	50	Evaluate two integrals, by series integration & parametric diff^n.	#(5) B & C, ϕ566('83).