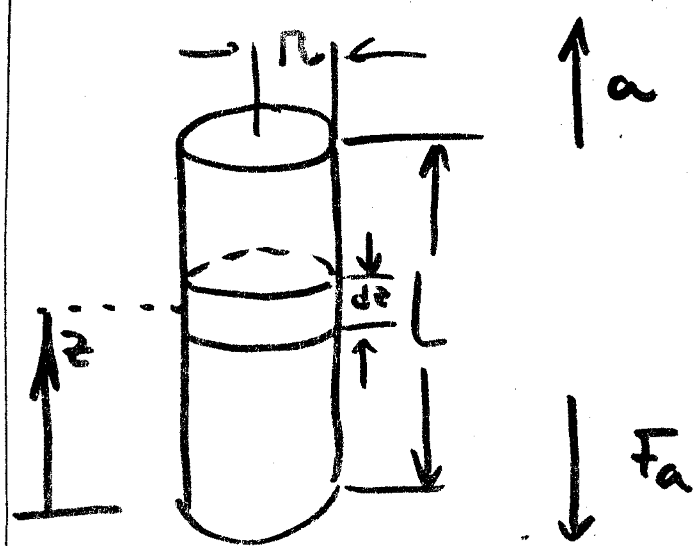
P2

ANSWER:



$a \Longleftrightarrow$ gravity force $F_a = \text{MASS} \cdot a$

MECHANICAL EQUILIBRIUM:

$$\pi r^2 \left[P(z - \tfrac{1}{2}dz) - P(z + \tfrac{1}{2}dz) \right] =$$

$$= \rho(z)\, \pi r^2 dz \cdot a$$

$$\Rightarrow \boxed{-\frac{dP(z)}{dz} = a\,\rho(z)}$$

THERMODYNAMIC EQUILIBRIUM:

$$P(z)\, \pi r^2 dz = n(z) RT$$

$$n(z) = \rho(z)\pi r^2 dz / N_A \cdot m$$

$$\therefore \boxed{P(z) = \frac{\rho(z)}{N_A \cdot m} RT}$$

ELIMINATING $\rho(z)$

7 $$\Longrightarrow P(z) = P(0)\, e^{-\frac{a N_A m}{RT} z} = P(0)\, e^{-\frac{a m}{k_0 T} z}$$

P_1 3

$$\pi R^2 \int_0^L \rho(z)\, dz = m N_A$$

$$\pi R^2 \int_0^L P(0) \frac{N_A m}{RT} e^{-\frac{am}{k_B T} z} dz = m N_A$$

$$\pi R^2 \frac{P(0)}{RT} \left(+\frac{k_B T}{am}\right)\left[1 - e^{-\frac{am}{k_B T} L}\right] = 1$$

3 $$\boxed{P(0) = N_A a m / \left[\pi R^2 \cdot \left(1 - e^{-\frac{am}{k_B T} L}\right)\right]}$$

4. a) An alpha particle of kinetic energy, T_α, makes a head-on collision with a nucleus of atomic number, Z, and mass number, A. Calculate the distance of closest approach, taking into account the recoil of the nucleus.

b) A proton with energy 0.2 MeV makes a head-on collision with an alpha particle at rest. What is the distance of closest approach (in Fermi)?

c) If an alpha particle makes a head-on collision with a proton at rest in the lab, what must the kinetic energy of the alpha particle be so that the distance of closest approach is identical to that in case (b), above?

Solution:

a) KE of $\alpha + (Z,A)$ in CM $= T_{LAB} - T$ of CM $= T_\alpha - \frac{1}{2}(M_\alpha + M_A) v_0^2$

v_0 of CM $= M_\alpha v_\alpha / (M_\alpha + M_A)$

$$T \text{ in CM} = T_\alpha - \frac{1}{2}(M_\alpha + M_A)(M_\alpha v_\alpha)^2/(M_\alpha + M_A)^2$$

$$= T_\alpha - \frac{1}{2} M_\alpha v_\alpha^2 \frac{M_\alpha}{M_\alpha + M_A} = T_\alpha - T_\alpha \frac{M_\alpha}{M_\alpha + M_A} = \frac{T_\alpha}{M_\alpha + M_A}(M_\alpha + M_A - M_\alpha)$$

$$= \frac{M_A}{M_\alpha + M_A} T_\alpha$$

If in LAB, D is Closest approach $T_\alpha = \frac{2Ze^2}{D}$, but taking into account recoil of nucleus, only $\frac{M_A}{M_\alpha + M_A} T_\alpha$ is available so

$2Ze^2/D = \frac{A}{A+4} T_\alpha$ (using mass numbers A & 4 as approximate masses in atomic mass units is adequate in most cases).

$$\boxed{\text{Then } D = \frac{2Ze^2}{T_\alpha} \frac{A+4}{A}}$$

b). Proton in lab $T_p = \frac{1 Ze^2}{D}$ but taking into account recoil

$T_{CM} = \frac{M_A}{M_A + M_p} T_p = \frac{4}{4+1} T_p = \frac{4}{5} T_p$ available for $p\,(M_p \simeq 1)$ on $\alpha\,(M_A \simeq 4)$

$D = \frac{Ze^2}{T_p}\frac{5}{4} = \frac{2Ze^2}{T_p} 5/4$, $e = 4.8 \times 10^{-10}$ esu, $T_p = 0.2$ MeV, $eV = 1.6 \times 10^{-12}$ erg

$$D = \frac{2(4.8 \times 10^{-10})}{0.2(1.6 \times 10^{-6})} \frac{5}{4} = 1.8 \times 10^{-12}\, cm = 18 \times 10^{-13}\, cm = \boxed{18\,F}$$

c) Incident α in LAB, $T_\alpha = 2e^2/D$ but $T_{CM} = \frac{1}{4+1} T_{LAB} = \frac{1}{5} T_\alpha$

$D = \frac{2e^2}{T_\alpha} 5$ must be same as $\frac{2e^2}{T_p}\frac{5}{4} = \frac{2e^2}{T_\alpha} 5 \rightarrow T_\alpha = 4 T_p = 4(0.2) = \boxed{0.8\,MeV}$