Consider a two-state quantum system subject to a time-independent Hamiltonian with energy eigenvalues $\hbar\omega_a$ and $\hbar\omega_b$ corresponding to eigenstates $|a\rangle$ and $|b\rangle$. In this system we have a mixed state, such that at $t = 0$ the system is in the state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$ with 25% probability, in the state $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|a\rangle - |b\rangle)$ with 25% probability, and in the state $|\psi_3\rangle = \frac{1}{\sqrt{2}}(|a\rangle + i\,|b\rangle)$ with 50% probability.

(a) Is this a bound-state system or not? Explain your reasoning.

(b) The *density operator* for a mixed ensemble of states is defined by $\rho \equiv \sum_k w_k \,|\psi_k\rangle\,\langle\psi_k|$, where w_k is the relative population of the state $|\psi_k\rangle$. Determine the *density matrix* at $t = 0$ in the basis of energy eigenstates.

(c) Determine the density matrix at time $t > 0$ for the given mixed state in both the Schrödinger and the Heisenberg representation.