Remarks on Parity-Violation in Atomic Physics

1) The selection rules we have just derived, viz

① **STARK**: $\langle U_s \rangle = e\mathbb{E}\cdot\int d^3x\,[\mathbb{r}\,\psi_f^*\,\psi_i]$ ∫ couples, or drives transitions $\psi_i \to \psi_f$, only when ψ_f is opposite parity to ψ_i (i.e. $\Delta J = \pm 1$); (1)

② **ZEEMAN**: $\langle U_z \rangle = \mathbb{B}\cdot\int d^3x\,[\mathbb{m}\,\psi_f^*\,\psi_i]$ ∫ couples, or drives transitions $\psi_i \to \psi_f$, only when ψ_f is same parity as ψ_i (i.e. $\Delta J = 0$); (2)

depend on the assumptions: (1) $\mathbb{E}$ & $\mathbb{B}$ have definite parities (−) & (+) resp. [so then $\mathbb{r}$ & $\mathbb{m}$ have definite parities (−) & (+) resp.], (2) the quantum states ψ_i & ψ_f have definite parities [usu. $(-)^l$, for state of orbital ∡ momentum l].

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2) Bound states ψ_i & ψ_f in atoms are generated principally by electromagnetic couplings (mainly Coulomb) between the proton (nucleus) and its electron(s). Then if parity P is a "good" (conserved) quantum # for EM couplings, P will also be good for the atomic states ψ_i & ψ_f, and above rules are absolute.

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3) BUT, suppose the P-conserving EM coupling between proton & electron has a small admixture of a P-nonconserving interaction... this is the case for the modern "electroweak" theory (Weinberg, Glashow, Salam; 1979) which unifies EM & weak interactions into one (combined) field. Then parity P is almost, but not quite, a good quantum # for atomic states ψ, and ψ becomes a parity-mixed state. To lowest order in the parity-mixing, we write

$$\left[\ \psi \to \tilde{\psi} = \psi + \kappa\varphi \quad \begin{cases} \psi \text{ has nominal state parity, } \varphi \text{ is opp. parity to } \psi; \\ \kappa = \text{parity-mixing parameter, w/ } |\kappa| \ll 1. \end{cases} \right. \qquad (3)$$

Here $\kappa \sim$ (weak coupling strength)/(EM coupling strength) is very small; for a single proton – single electron interaction: $|\kappa| \sim 10^{-10}$. But $\kappa \neq 0$ violates parity for the state since: $P\tilde{\psi} = \pm(\tilde{\psi} - 2\kappa\varphi)$, when ψ has ($\pm$) parity.

4) Parity-violation in atoms can be searched for as follows. The wavefunction combinations which occur in the Stark & Zeeman matrix elements above are

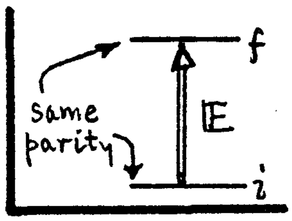
$$\psi_f^*\psi_i \longrightarrow \tilde{\psi}_f^*\tilde{\psi}_i \simeq \psi_f^*\psi_i + \kappa_i\left[\psi_f^*\varphi_i\right] + \kappa_f^*\left[\varphi_f^*\psi_i\right],\ \text{to } \mathcal{O}(\kappa). \tag{4}$$

these terms have parity opposite to $\psi_f^*\psi_i$

The Stark matrix element (w.r.t. parity-mixed $\tilde{\psi}$'s) picks up new terms...

$$\rightarrow \langle U_S\rangle = e\mathbb{E}\cdot\int d^3x\left[\mathbf{r}\psi_f^*\psi_i\right] + \kappa_i\, e\mathbb{E}\cdot\int d^3x\left[\mathbf{r}\psi_f^*\varphi_i\right] + \kappa_f^*\, e\mathbb{E}\cdot\int d^3x\left[\mathbf{r}\varphi_f^*\psi_i\right], \tag{5}$$

and likewise the Zeeman matrix element $\langle U_Z\rangle$ acquires terms in κ. Now, consider driving the transition $\psi_i \rightarrow \psi_f$ by an electric field $\mathbb{E}$, when the states i & f have the same "parity". For parity-pure states, such a transition is forbidden by the selection rule in Eq. (1); i.e. the matrix element $e\mathbb{E}\cdot\int d^3x\left[\mathbf{r}\psi_f^*\psi_i\right] \equiv 0$. But for the parity-mixed states, the terms in κ in Eq. (5) are non-zero, so we have



$$\rightarrow \langle U_S\rangle = e\mathbb{E}\cdot\left\{\kappa_i\int d^3x\left[\mathbf{r}\psi_f^*\varphi_i\right] + \kappa_f^*\int d^3x\left[\mathbf{r}\varphi_f^*\psi_i\right]\right\}, \tag{6}$$

as a transition amplitude for an otherwise forbidden transition $i \rightarrow f$ w/ $\Delta J = 0$. So, if we see a violation of the selection rule $\Delta J = \pm 1$ for $\mathbb{E}$-field driven transitions (i.e. we detect a $\Delta J = 0$ transition driven by $\mathbb{E}$), we can blame it on electroweak parity-nonconserving (PNC) effects. Likewise, a violation of $\Delta J = 0$ for $\mathbb{B}$-field driven transitions implies PNC for an atom.

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5) Atomic ϕ expts have in fact shown the existence of forbidden transitions and PNC effects in atoms. They are ferociously difficult, because the measurable violation rates go as $|\langle U\rangle|^2 \propto |\kappa|^2 \lll 1$. See R.T. Robiscoe & W.L. Williams, Nucl. Instr. Methods 197, 567 (June 1982).