Experimental Basis of QM

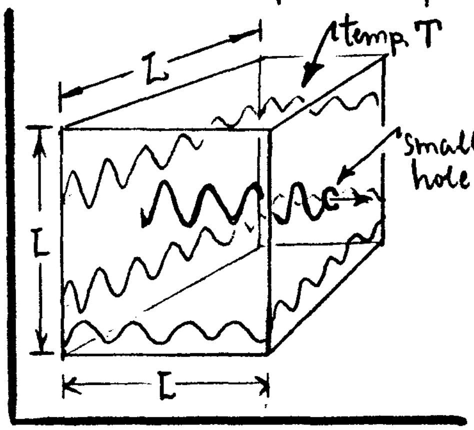
Introduction: Why QM?

Most theories in physics begin as attempts to explain puzzling experimental results. QM is no exception -- it was "invented" to deal with certain experiments on microscopic systems (electrons, atoms & molecules, and their interaction with light, etc), when it was found that dynamical quantities such as the system energies were observable only in discrete units (e.g. line spectra of excited atoms). Ascribing a discrete rather than continuous character to the dynamical variables of classical mechanics (position, momentum, energy, etc.) is called "quantization", and the science of how this is done is called "quantum mechanics" (abbrev. QM).

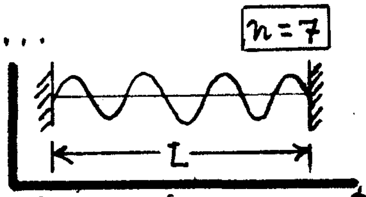
To begin, we will describe 3 expts which require QM notions for their explication: BlackBody Radiation, PhotoElectric Effect, Compton Effect. These expts. mainly concern the quantum nature of light -- e.g. they establish the existence of the photon -- but the QM ideas involved are ~ easily extended to all of matter.

BlackBody Radiation

1) A "blackbody" is an enclosure in thermodynamic equilibrium at a uniform temperature T. The energy distribution of (heat) radiation in the interior is independent of the shape & material of the walls so long as the enclosure is large enough -- the distribution depends only on T and on the radiation frequency ν. The radiation inside can be observed through a small hole in the enclosure wall -- the hole acts as a (nearly) perfect absorber for radiation incident from outside, since that radiation enters, reflects randomly off the walls, and in effect becomes trapped inside with vanishingly small chance of getting back out. The hole itself thus shows the (near) perfect absorption character of a classical blackbody.



2) The radiation inside the BB enclosure consists of <u>standing</u> EM waves, as in a cavity resonator. Not all wavelengths (& frequencies) are allowed -- a given wave must "fit" inside the box, obeying specific boundary conditions at the walls: in particular, an integral # of half wavelengths λ must "fit" along each dimension (say of length L) so that the standing wave intensity looks the <u>same</u> at each end of the box. This is the condition...



$\rightarrow n\frac{\lambda}{2} = L$, in 1D, w/ $n = 1, 2, 3, \ldots$ (1)

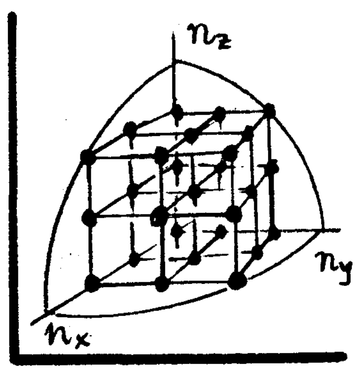
From this, we can find the # waves permitted as a fcn of frequency. Define a wave-number: $k = 2\pi/\lambda$ (# waves per unit length). In 3D, k is a vector...

$\rightarrow \mathbf{k} = (k_x, k_y, k_z)$, for an EM plane wave $e^{i\mathbf{k}\cdot\mathbf{r}}$;

... but : $k_i = 2\pi/\lambda_i = 2\pi/(2L/n_i) = \pi n_i/L$, w/ $i = x, y, z$;

... so/ $\underline{\underline{\mathbf{k} = (\pi/L)\mathbf{n}}}$, w/ $\mathbf{n} = (n_x, n_y, n_z)$, the n_i = integers. (2)

These integer triplets generate a set of lattice points in an (n_x, n_y, n_z) space as sketched. We can find the # of points for a given $n = |\mathbf{n}|$, with $n \rightarrow$ large, as follows...



$\rightarrow n = \sqrt{n_x^2 + n_y^2 + n_z^2}$, n = radius of sphere in $\mathbf{n}$-space;

... NOTE: n = <u>same</u> for $(\pm n_x, \pm n_y, \pm n_z) \Rightarrow 8$ "degenerate" points;

$\left.\begin{array}{l}\text{\# distinct points in spherical shell} \\ \quad \text{between } n \text{ \& } n+dn\end{array}\right\} \quad dN = \underbrace{(4\pi n^2 dn)}_{\text{shell volume}} \times \underbrace{\frac{1}{8}}_{\text{degeneracy factor}}$

$\left.\begin{array}{l}\text{... but : } n = Lk/\pi \text{, so...} \\ \text{\# EM waves between } k \text{ \& } k+dk\end{array}\right\} \quad \underline{\underline{dN = 4\pi \times (L/2\pi)^3 \times k^2 dk}}.$ (3)

Multiply dN by 2×, because there are 2 independent polarizations for each wave. Also, convert to frequency ν via : $\lambda\nu = c \Rightarrow \underline{k = 2\pi\nu/c}$, w/ c = speed of light. Then we arrive at a key result, viz...

BB Radiation: Mode Counting

$$\left[\begin{array}{l}\text{\# EM waves (or "modes") per unit volume}\\ \text{with frequency in the interval } \nu \text{ to } \nu+d\nu\end{array}\right\} \boxed{d\rho = 2\times\frac{dN}{L^3} = \frac{8\pi}{c^3}\nu^2 d\nu}. \qquad (4)$$

REMARKS On the classical BB energy distribution.

1. As we have calculated it, $d\rho$ in Eq. (4) is the density of allowed EM waves in the cavity (enclosure); as noted, this is sometimes called the "mode density". One may also speak of the (differential) number of "degrees-of-freedom" for excitation of the cavity by EM oscillations (in freq. interval ν to $\nu+d\nu$).

2. From statistical mechanics, energy can be stored in each of the EM modes or degrees-of-freedom just noted. At a given (equilibrium) temperature T, the probability of exciting a mode with energy in the range E to $E+dE$ is proportional to the Boltzmann factor, viz...

$$\left[\begin{array}{l}\text{probability of exciting a mode}\\ \text{with energy in } E \text{ to } E+dE\end{array}\right\} \propto e^{-E/kT}dE, \quad k = \text{Boltzmann cnst.} \qquad (5)$$

From this, the average excitation energy per EM mode in the cavity is...

$$\rightarrow \bar{E} = \int_0^\infty E e^{-E/kT}dE \Big/ \int_0^\infty e^{-E/kT}dE = kT. \qquad (6)$$

This is the Classical Equipartition Theorem: "A thermodynamic energy kT can be associated with each degree-of-freedom of a mechanical system."

3. Eq. (4) gives the EM mode density in the cavity, and (6) gives the average energy per mode, so we can find the average EM energy density at frequency ν:

$$\left[\begin{array}{l}\text{average EM energy density}\\ \text{in freq. range } \nu \text{ to } \nu+d\nu\end{array}\right\} \underline{U_\nu d\nu = \bar{E}\,d\rho = (8\pi kT/c^3)\nu^2 d\nu}. \qquad (7)$$

This is the classical result for U_ν -- called the Rayleigh-Jeans Law. It violently disagrees with experiment, as indicated at right, and it also predicts the absurd result that the total energy density in the cavity is infinite...