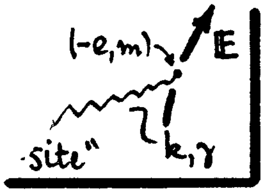
Simple Model for Dielectric Constant $\epsilon(\omega)$ [ref. Jk$^{\underline{n}}$ Sec. 7.5(a), p.284]



Electron $(-e,m)$ is bound to a "site" by spring of spring cnst $\underline{k=m\omega_0^2}$ (ω_0 = natural freq. for e's oscillation). By interaction with nearby sites & nearby electrons, its motion is <u>damped</u> by an effective viscous force characterized by a damping cnst γ. When experiencing a time-varying external electric field $\mathbb{E}$, the electron's equation-of-motion will be...

$$\rightarrow m(\underbrace{\ddot{\mathbf{r}} + \gamma\dot{\mathbf{r}}}_{\text{damping}} + \underbrace{\omega_0^2\,\mathbf{r}}_{\text{binding}}) = \underbrace{-e\,\mathbb{E}(\mathbf{r},t)}_{\text{driving force}}$$

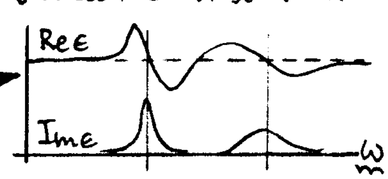
Suppose $\mathbb{E}$ is cnst in space over the extent of e's motion, and $\mathbb{E}$ is harmonic in time. So:

$$\rightarrow \mathbb{E}(\mathbf{r},t) = \mathbb{E}_0 e^{-i\omega t} \Rightarrow \mathbf{r}(t) = \mathbf{r}_0 e^{-i\omega t},\ ^{w/}\ \mathbf{r}_0 = \left[\frac{e}{m}\Big/(\omega^2-\omega_0^2+i\gamma\omega)\right]\mathbb{E}_0 \int \text{steady state}$$

and// induced dipole moment for <u>one</u> e $\Big\}$ $\mathbb{p} = -e\mathbf{r}_0 = \underbrace{\left[\frac{e^2}{m}\Big/(\omega_0^2-\omega^2-i\gamma\omega)\right]}_{\propto\,\alpha,\ \text{polarizability for one electron.}}\mathbb{E}_0.$

Suppose there are N molecules/unit volume, w/ Z electrons/molecule, and of the Z e's, a number f_j have binding frequencies ω_j & damping cnsts γ_j. Then (with $\sum_j f_j = Z$), the above polarizability is $\alpha = (Ne^2/m)\sum_j f_j/(\omega_j^2-\omega^2-i\gamma_j\omega)$, and the dielectric constant is:

$$\boxed{\epsilon(\omega) = 1+4\pi\alpha(\omega) = 1+\left(\frac{4\pi Ne^2}{m}\right)\sum_j f_j\big/(\omega_j^2-\omega^2-i\gamma_j\omega)}$$



This simple (damped SHO) model will be a workhorse in future.

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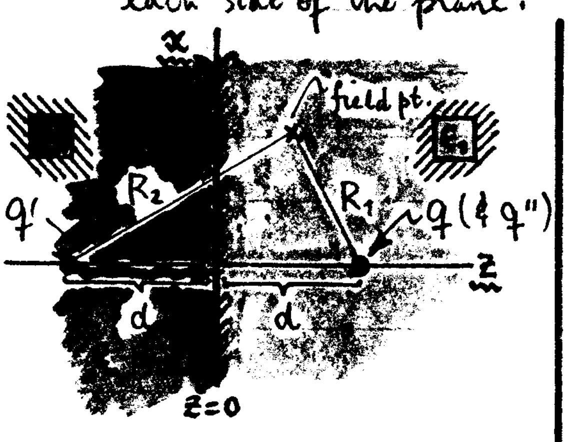
7) A good example to see how the transition $\mathbb{E} \rightarrow \mathbb{D} = \epsilon\mathbb{E}$ affects things is to do the pointcharge-plane problem, but now with different media on each side of the plane.



The plane $z=0$ is xy-plane.

1. No σ_{real} on boundary plane $z=0$, by assumption, so// $\left[\!\left[\epsilon_1 E_z^{(1)} = \epsilon_2 E_z^{(2)},\ \text{and } E_x \ \&\ E_y \text{ cont}^s \text{ at } z=0.\right]\!\right]$

2. The $z=0$ plane is <u>not</u> an equipotential.

3. To meet above B.C. @ $z=0$, insert image q's. So...

$$\left.\begin{aligned} &\rightarrow \phi(z\geqslant 0) = \frac{1}{\epsilon_1}\left(\frac{q}{R_1}+\frac{q'}{R_2}\right), \\ &^{w/}\ R_{1,2} = \sqrt{r^2+(d\mp z)^2},\ r \text{ in } xy \text{ plane.}\end{aligned}\right\}\ (33)$$

4. The proferred $\phi(z \geqslant 0)$ will certainly satisfy $\nabla^2\phi = -4\pi q\,\delta(\mathbf{r} - \hat{z}d)$ in the right half-space. For the left half-space, write...

$$\rightarrow \phi(z \leqslant 0) = \frac{1}{\epsilon_2}\,\frac{q''}{R_1}, \quad q'' \equiv \text{shielded } q \text{ seen at } z \leqslant 0. \tag{34}$$

This ϕ satisfies $\nabla^2\phi = 0$ in left half-space, as needed. Now have:

$$\left[\begin{aligned} \phi(z \geqslant 0) &= \frac{1}{\epsilon_1}\left(\frac{q}{R_1} + \frac{q'}{R_2}\right) \\ \phi(z \leqslant 0) &= \frac{1}{\epsilon_2}\left(\frac{q''}{R_1}\right) \end{aligned}\right] \Bigg\| \quad \text{B.C.} \left\{\begin{aligned} &\epsilon_1 \frac{\partial\phi(z=0+)}{\partial z} = \epsilon_2 \frac{\partial\phi(z=0-)}{\partial z} \Rightarrow \underline{q - q' = q''}, \\ &\text{and: } \frac{\partial\phi(z=0+)}{\partial r} = \frac{\partial\phi(z=0-)}{\partial r} \Rightarrow \underline{\frac{q+q'}{\epsilon_1} = \frac{q''}{\epsilon_2}}; \end{aligned}\right.$$

so $\boxed{q' = -\left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}\right) q, \quad q'' = \left(\frac{2\epsilon_2}{\epsilon_1 + \epsilon_2}\right) q}$ (35)

and $$\left[\begin{aligned} \phi(z \geqslant 0) &= \frac{q}{\epsilon_1}\left[\frac{1}{R_1} - \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}\right)\frac{1}{R_2}\right], \\ \phi(z \leqslant 0) &= \frac{q}{R_1}\cdot\left(\frac{2}{\epsilon_1 + \epsilon_2}\right). \end{aligned}\right]$$

Note that when $\epsilon_2 = \epsilon_1 = \epsilon$, the bndy plane term vanishes, and: $\phi = \frac{1}{\epsilon}\,q/R_1$, everywhere. Right!*