ϕ519 Introductory Remarks — Mon. 26 Aug. 91

1) A remarkable feature of Maxwell's EM is that it is ~ only theory in classical physics which survived intact into the 20^{th} century...

CLASSICAL ϕ	MODERN ϕ	ENDGAME ϕ
Newtonian Mech. (~1685)	Relativity (1905-15)	Σ(Rel., QM, EM) = QED (1950).
Thermodynamics (~1850)	Q. Mechanics (1920-30)	Serves as a model field theory.
Maxwell's EM (~1865)	(automatic) (add-on)	+ other couplings ⇒ QCD, GUTS, Electroweak, etc.

REMARKS
1) Advent of Rel. & QM ⇒ all classical ϕ needed redoing, except EM.
2) EM already compatible with SRT, ... could have been used to discover SRT.
J.C. Maxwell (1831-1879)
3) Re forces... EM replaced idea of action-at-a-distance with fields.
4) Other key EM notions, viz.

fields due to "charges" / "currents",
particle interactions via field "couplings",
field equations obey symmetries {Lorentz covariance, gauge transforms, etc.}

} Serve as paradigms for (all) modern field theories.

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2) Maxwell's EM is based (mainly) on just 4 deceptively simple-looking eqtns:

MAXWELL'S FIELD EQS: cgs units, in vacuum, w/ $c = 3\times 10^{10}$ cm/sec:

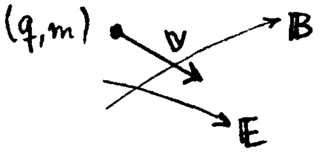
$$\boxed{\begin{array}{ll} \text{Gauss: } \nabla\cdot\mathbb{E} = 4\pi\rho, & \text{(Dirac): } \nabla\cdot\mathbb{B} = 0, \\ \text{Faraday: } \nabla\times\mathbb{E} = -\frac{1}{c}\,\partial\mathbb{B}/\partial t, & \text{Ampere-Maxwell: } \nabla\times\mathbb{B} = \frac{4\pi}{c}\mathbb{J} + \underbrace{\frac{1}{c}\,\partial\mathbb{E}/\partial t}_{\text{Maxwell's gem}}. \end{array}}$$

w/ [$\mathbb{E}$ & $\mathbb{B}$ = (vector) electric & magnetic fields;
ρ = (scalar) net electric charge/volume, $\mathbb{J}$ = (vector) current/area.

these eqtns ⇒
- { c = limit velocity in entire universe, ! light is "just" an EM wave traveling at c;
- ⋮
- { how to design a workable lightning rod, what makes an AC transformer work.

enormous range of information!

3) Maxwell's EM is a <u>vector</u> field theory, since the field eqtns are written in terms of vectors $\mathbb{E}$ & $\mathbb{B}$, and the dynamics of a particle are described by...



$$\boxed{\frac{d\mathbf{p}}{dt} = q\left(\mathbb{E} + \frac{\mathbf{v}}{c} \times \mathbb{B}\right)},$$ Lorentz force law.

Notice that Maxwell's Eqs <u>unifies</u> the forces due to $\mathbb{E}$ (electric) & $\mathbb{B}$ (magnetic) -- previously thought to be separate & distinct (what does static electricity have to do with a compass needle?) -- by prescribing how $\mathbb{B}$ is generated by $\mathbb{E}$, and vice-versa.

So Maxwell's EM was the first example of a "<u>unified field theory</u>", restric- to $\mathbb{E}$ & $\mathbb{B}$, but interrelating & combining those fields nevertheless.

$\boxed{\text{ASIDE}}$ Elements of Grand Unification.

Present pp. 3-4 following on overheads.

Coming Attractions

1. Brief summary of Jackson's "Introd$^{\underline{n}}$ & Survey", pp. 1-25.
2. Proof of Helmholtz Thm: $\mathbb{F}$ specified by $\nabla\cdot\mathbb{F}$ & $\nabla\times\mathbb{F}$.
3. Elements of potential theory: Jackson Ch. 1, pp. 27-53.

① $\nabla\cdot\mathbb{E} = 4\pi\rho$

② $\nabla\times\mathbb{E} = -\frac{1}{c}(\partial\mathbb{B}/\partial t)$

③ $\nabla\cdot\mathbb{B} = 0$

④ $\nabla\times\mathbb{B} = \frac{4\pi}{c}\mathbb{J} + \boxed{\frac{1}{c}\left(\frac{\partial\mathbb{E}}{\partial t}\right)}$

Maxwell's term

All Maxwell did was add the "displacement current" in $\partial\mathbb{E}/\partial t$. But

$$\frac{\partial}{\partial t}① \Rightarrow \frac{\partial\rho}{\partial t} = \frac{1}{4\pi}\nabla\cdot\left(\frac{\partial\mathbb{E}}{\partial t}\right)$$

from ④

$$\nabla\cdot[\,c\,\cancel{\nabla\times\mathbb{B}}^{\,0} - 4\pi\mathbb{J}\,] = -4\pi\nabla\cdot\mathbb{J}$$

So, $\boxed{\partial\rho/\partial t + \nabla\cdot\mathbb{J} = 0}$ CHARGE CONSERVATION.