DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE / PH. D. QUALIFYING EXAMINATION

APRIL 1, 1985

DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE and PH.D. QUALIFYING EXAM

MONDAY, 1 APRIL 1985, 8 AM-12 NOON

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Answer each of the following eight (8) questions.

All questions are of equal weight.

Begin your answer to each question on a <u>new</u> sheet of paper. Solutions to different questions must <u>not</u> appear on the same sheet of paper.

Label each page of your answer sheets as follows:

A. Your name in upper left-hand corner.

B. Problem number, and page number <u>for that problem</u>, in upper right-hand corner.

1. A one-dimensional quantum system is described by a Hamiltonian H and a wave function $\psi(x,t)$. There are two other interesting physical variables A and B represented by operators $\hat{A}$ and $\hat{B}$. ψ is determined by the Schrödinger equation

$$H\psi = i\hbar \frac{\partial \psi}{\partial t}$$

Assume that $[\hat{H},\hat{A}] = 0$, $[\hat{A},\hat{B}] \neq 0$, $[\hat{H},\hat{B}] \neq 0$ and that you have solved the eigenvalue problems for $\hat{H}$, $\hat{A}$ and $\hat{B}$; i.e.,

$$\hat{H}u_n = E_n u_n, \quad \hat{A}\phi_i = a_i \phi_I, \quad \hat{B}X_r = b_r X_r.$$

(a) What is the physical import of E_n, a_i, b_r?

(b) At t=0, the energy of the system was measured and found to be E_3. What is the wave function at $t=T_1$?

(c) At $t=T_1$, you plan to measure B. Before the measurement, what can you predict about the result you will find?

(d) What will the wavefunction be immediately after the measurement of B?

2. A μ-meson of mass μ is bound to a lead nucleus of uniformly distributed charge Ze and radius R. Compute the meson's ground state energy, assuming

(a) The meson wavefunction is negligible outside the nucleus; and

(b) The meson-nucleon interaction is purely electrostatic.

3. Consider a spinless particle described by the wavefunction at t=0

$$\psi = K[x+y+2z]e^{-\alpha r}, \text{ where } r = \sqrt{x^2+y^2+z^2}$$

and K and α are real constants. K is chosen to normalize the wave function.

(a) What is the total angular momentum of the particle?

(b) Find K in terms of a radial integral. What is the expectation value of the z-component of angular momentum?

(c) If the z-component of angular momentum were measured, what is the probability that the result would be $+\hbar$?

(d) What is the probability of finding the particle in solid angle $d\Omega$ at θ, ϕ? (Do not spend a lot of time on algebra.)

The first few spherical harmonics are

$$Y_0^0 = \frac{1}{\sqrt{4\pi}} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \; e^{\pm i\phi}$$

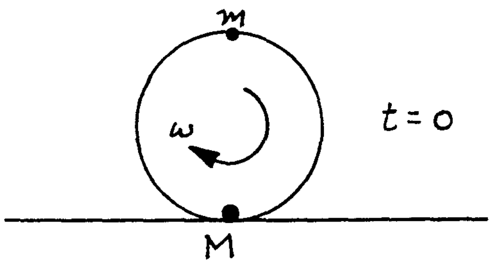
$$Y_2^0 = \sqrt{\frac{5}{16\pi}} (3 \cos^2\theta - 1)$$

$$Y_2^{\pm} = \mp\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta \; e^{\pm i\phi}$$

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2\theta \; e^{\pm 2i\phi}$$

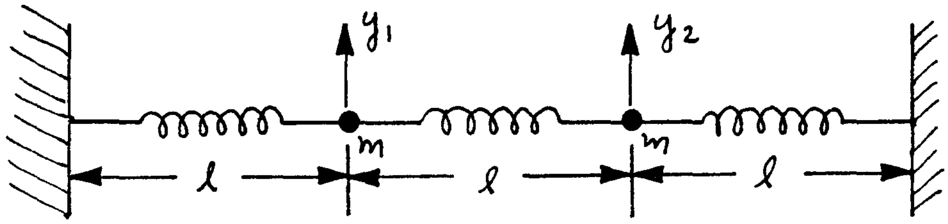
4. A hoop of mass M_o and radius R has point masses m and M (M>m) attached to the rim of the hoop 180^o apart. The hoop rolls without slipping.

(a) What angular velocity must the hoop be given when M is in contact with floor so that the hoop rolls without stopping and reversing directions?



(b) Calculate the frequency of small amplitude oscillations about the illustrated equilibrium.

5. Two equal masses M are held by three equal springs as shown



The masses are constrained to move transversely as indicated. The springs are such that, when the system is in equilibrium as shown, they store energy $1/2kl^2$ each (i.e., the relaxed length of the springs is zero). If the masses are released at t=0 with $y_1(t=0) = 0$, $y_2(t=0) = 0$, $\dot{y}_1(t=0) = V$ and $\dot{y}_2(t=0) = 0$, what is their subsequent motion?

6. Two observers agree to test time dilation. They use identical clocks and one observer in frame S′ moves with speed $v = 0.6c$ relative to the other observer in frame S. When their origins coincide, they start their clocks. They agree to send a signal when their clocks read 60 min and to send a confirmation signal when each receives the other's signal.

(a) When does the observer in S receive the first signal from the observer in S′?

(b) When does he receive the confirmation signal?

(c) Make a table showing the times in S when the observer sent the first signal, received the first signal, and received the confirmation signal. How does this table compare with one constructed by the observer in S′?

7. A binary star system consists of a primary star of two solar masses and one solar radius and a neutron star of one solar mass and 10 km radius, separated by a distance of 0.01 AU and revolving mutually about their common center of mass. (1 AU = 1.496×10^8 km.)

(a) By applying Kepler's third law, find the binary period of the system (the time it takes to complete one revolution). Note that when a pair of 0.5 solar mass stars in a binary system are separated by 1 AU, the binary period will be 1 year. Assume that we are looking at this system head-on.

(b) It was found that the surface temperature of the neutron star is 10^5 K. Assuming that stars radiate like a blackbody, find the total energy output (luminosity) of this neutron star in units of the solar luminosity. (the surface temperature of the sun is 6000 K.)

(c) If you weigh 150 pounds on the earth, how much would you weigh on this neutron star?

(d) Suppose you are trying to escape from this neutron star by getting your space ship ejected vertically by a powerful rocket. How fast should your ship be ejected? (Note: the escape velocity from the earth = 11.19 km/sec.)

The following information my be used:
Diameter of the earth = 12756 km,
Mass of the earth = 5.98×10^{24} km,
Mass of the sun = 2×10^{28} km,
Radius of the sun = 6.95×10^5 km,

8. Consider a large number N>>>>1 of independent, identical particles. The kth particle has energy e_k, and the total energy

$$E = \sum_k e_k$$

is conserved. Let p(e) be the probability that any given particle has energy e. Then since two particles are independent of each other, the probability that they have energies e_1 and e_2, respectively, can be assumed:

a) $p(e_1,e_2) = p(e_1)\ p(e_2)$

b) $p(e_1,e_2) = f(e_1+e_2)$

since the N-2 remaining particles only "care" how much total energy is removed from their reservoir. From a) and b) deduce the functional form of p(e). Hint: let $e_1 = e_1 + de_1$, $e_2 = e_2 + de_2$ such that $de_1 + de_2 = 0$. obtain differential equations for p(e) using a) + b).

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9. A heat engine works in a cycle between reservoirs at 400 and 200 K. The engine absorbs 1000 J of heat from the hot reservoir and does 200 J of work in each cycle.

(a) What is the efficiency of this engine?

(b) Find the entropy change of the engine, each reservoir, and of the universe for each cycle.

(c) What is the efficiency of a Carnot engine working between the same two reservoirs? How much work could be done by a Carnot engine in each cycle if it absorbed 1000 J from the hot reservoir?

(d) Show that the difference in the work done by the Carnot engine and the original engine is $T_c \Delta S_u$, where ΔS_u is that calculated in part (b), as the entropy change of the universe.

10. Consider a system of 6 particles, each of which can be in one of three possible energy states, with respective energies +1, −1, or 0 units of energy. The total energy of the system as +4 units.

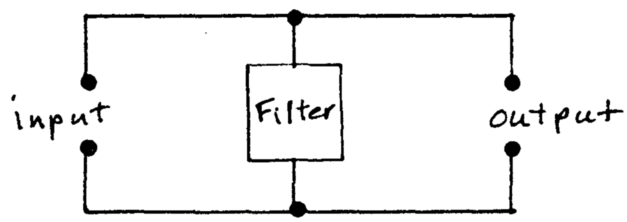
(a) Find the entropy of the system (in units of Boltzmann's constant) if the particles are distinguishable.

(b) If the particles are distinguishable, what is the likelihood that at least one particle has energy zero?

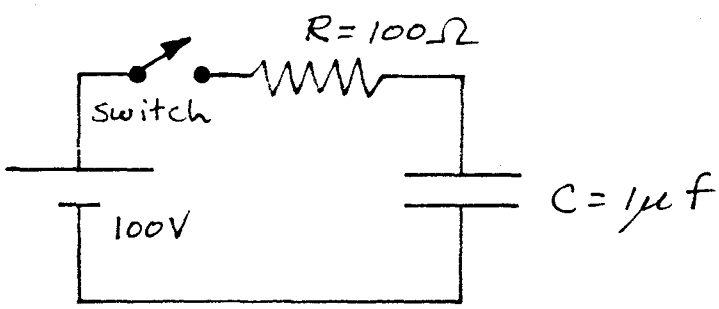
(c) Find the entropy of the system if the particles are identical spinless bosons.

11. One day it started snowing at a steady rate. At noon a snowplow started out, going 2 miles the 1st hour and 1 mile the 2nd hour. Its speed was inversely proportional to the depth of the snow. When did it start snowing?

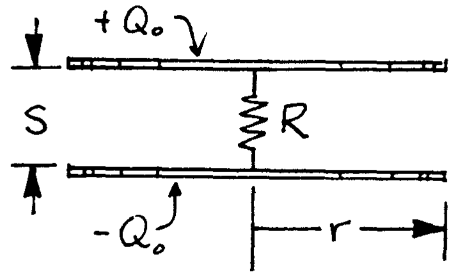
12. (a) Find a simple filter containing an inductor L and a capacitor C that will filter the 60 Hz ripple in the circuit below. What values of the circuit elements are necessary?



(b) A capacitor C is suddenly connected to 100V in the circuit shown. How long will it take for the capacitor to reach 50V?



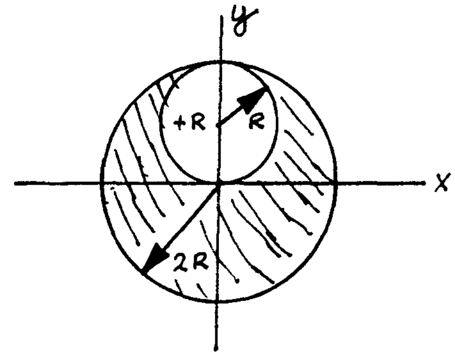
13. A parallel plate capacitor consists of two circular plates of radius r separated by a small gap s<<r. Charges + Q_0 and $-Q_0$ are put on the plates, and at time t = 0 their centers are connected by a thin straight wire of resistance R. Assume R is very large, so that at t>0 the current flow is "slow", the electric field between the plates remains uniform in space, and inductance can be neglected.



(a) Calculate the charge on the plates as a function of time for $t \geq 0$.

(b) Calculate the magnetic field between the plates as a function of time for $t \geq 0$, at radial distance $\rho < r$ from the center of the capacitor. Hint: Don't forget the displacement current density.

14. Consider a nonconducting sphere of radius 2R centered at the origin with a spherical cavity of radius R located at $(x, y, z) = (0, R, 0)$. Let the sphere have a uniform, positive volume charge density ρ.



(a) Find the electric field vector $\vec{E}$ on the y axis for $0 < y < 2R$.

(b) Now assume that the charged sphere suddenly turns into a conductor. After electrostatic equilibrium is reached, what is the $\vec{E}$ field vector at points on the y-axis?

15. (a) Write down Maxwell's Equations.

(b) Consider TEM (Transverse ElectroMagnetic wave) solutions of these equations of the form

$$\vec{E}=\hat{i}E_o \exp[i(kz-\omega t)], \ \vec{H}=\hat{j}H_o \exp[i(kz-\omega t)].$$

Consider the motion of an electron of charge $-e$ and mass m oscillating in this electric field. Solve for its velocity $\vec{v}(t)$ in the complex notation used above for the fields. (We assume the Lorentz force can be neglected.)

(c) Now assume N such electrons per unit volume, and write down the expression for the current density $J_f=-Nev$. (We assume a positive ion charge density $+Ne$ of ions too heavy to move appreciably in the field, so that there is no net charge density.)

(d) Consider E_o and ω as given, and solve for H_o and k.

(e) From your solution for (d), find the Plasma Frequency ω_p <u>below</u> which electromagnetic waves of the form given in (b) cannot propagate.

16. For a diffraction grating one is interested not only in resolving power $\lambda/|\Delta\lambda|$, but also in the dispersion

$$D = \Delta\theta_m/\Delta\lambda,$$

where $\Delta\theta_m$ is the angular separation of two m^{th}-order maxima corresponding to a wavelength difference $\Delta\lambda$.

(a) Write an expression for D in terms of m, λ, and d, where d is the slit spacing of the grating.

(b) If a diffraction grating with 2000 slits per cm is to resolve the two sodium yellow lines (wavelengths 589.0 and 589.6 nm) in second order, how many slits must be illuminated by the beam?

(c) What would the separation be between these resolved yellow lines if the pattern were viewed on a screen 4m from the grating?