DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE / PH. D. QUALIFYING EXAMINATION

SATURDAY APRIL 2, 1986

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SATURDAY, APRIL 2, 8-12 AM ,1986

Answer each of the following questions, all of which carry equal weight. Begin your answer to each question on a new sheet of paper; solutions to different questions must <u>not</u> appear on the same sheet. Each sheet of paper must be labeled with your name and the problem number, in the upper right hand corner. When more than one sheet is submitted for a problem, be sure the pages are ordered properly. [Problems 1-8].

1. The Coriolis force is given by $\vec{F} = -2m\vec{\omega} \times \vec{v}$.

a) Discuss physically what this expression means.

b) The water in a river flowing to the south in the northern hemisphere will have the water level higher on one side than the other. Which side, east or west, will be higher?

c) For the Mississippi river, estimate that height.

Prel-lim Mechanics Drumheller

1. The Coriolis force is given by $\vec{F} = -2m\vec{\omega} \times \vec{v}$.

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c.) For the Mississippi river, estimate that height.

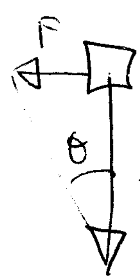
Sol'n

a.) The Coriolis force occurs in a non-inertial reference frame. ω is the angular velocity with respect to the inertial frame and v is the velocity within the rotating frame.

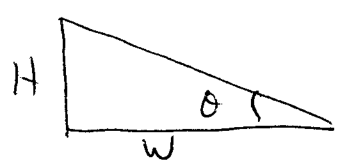
It can be thought of as conservation of angular momentum.

b.) To the west. The west bank will be higher.

c.)



$F = -2m|\bar{\omega}\times\bar{v}| = 2m\omega v \sin\lambda$



$$\tan\theta = \frac{2m\omega v \sin\lambda}{mg} = \frac{2\omega v \sin\lambda}{g}$$

$$H = W\tan\theta = W\,\frac{2\omega v \sin\lambda}{g}$$

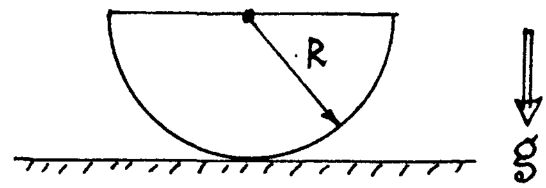
$\lambda = 45°$

$$= \frac{2(10^3)\,m\,(2)\left(2\pi\,\frac{rad}{day}\,\frac{day}{24\,hr}\,\frac{hr}{3600\,sec}\right)\frac{1}{\sqrt{2}}\,\frac{m}{s}}{10\,\frac{m}{s^2}}$$

$= 0.02\ m = 2\ cm.$

2. A solid glass hemisphere sits on a horizontal surface. Find its frequency for small oscillations.

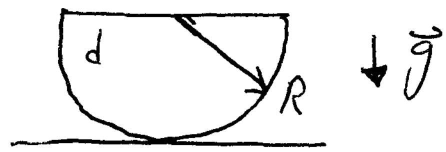
Note: $I_{cm} = \frac{2}{5} MR^2$ for solid sphere.



hemisphere has uniform density d.

Classical Mechanics - Problem 2/45

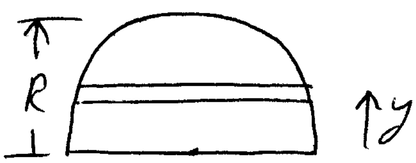
2\. A solid glass hemisphere sits on a horizontal surface. Find its frequency for small oscillations.



Note - $I_{cm} = \frac{2}{5} m R^2$ for solid sphere.

Method
1) Find c.m.
2) Find P.E. for small displacement from equilibrium.
3) Equate this to KE when passing thru equilibrium position.
4) Find frequency from KE.

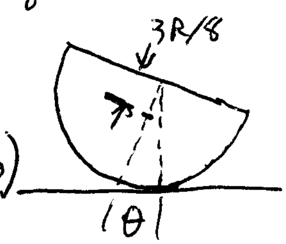
1) $y_{cm} = \int y\,dV / \int dV$



$= \int_0^R y\pi(R^2-y^2)dy \Big/ \frac{2}{3}\pi R^3$

$= \left(-\pi\frac{R^4}{4} + \pi\frac{R^4}{2}\right) \Big/ \frac{2}{3}\pi R^3 = \frac{3}{8}R$

2) $PE = mg(h-h_0)$



$= \frac{2}{3}\pi R^3 d g\left(-\frac{5}{8}R + R - \frac{3}{8}R\cos\theta\right)$

$= mg\frac{3}{16}R\theta^2$

3) $KE = \frac{1}{2}I_{cm}\dot{\theta}^2 + \frac{1}{2}mv_{cm}^2 \qquad v_{cm} = \frac{5}{8}R\dot{\theta}$

$I_{center} = \frac{1}{5}mR^2 = I_{cm} + m\left(\frac{3}{8}R\right)^2 = I_{cm} + \frac{9}{64}mR^2$

$I_{cm} = \frac{64-45}{320}mR^2 = \frac{19}{320}mR^2$

$KE = \frac{19}{640}mR^2\dot{\theta}^2 + \frac{25}{128}mR^2\dot{\theta}^2 = \frac{144}{640}mR^2\dot{\theta}^2 = \frac{9}{40}mR^2\dot{\theta}^2$

4) $\theta = \theta_0 \sin \omega t$ $\quad \dot{\theta} = \omega \theta_0 \cos \omega t$

$\Rightarrow \omega \theta_0$ at equilibrium

PE = KE; $\frac{3}{16} mgR\theta_0^2 = \frac{9}{40} mR^2\omega^2\theta_0^2$

$$\omega = \sqrt{\frac{\frac{3}{16}g}{\frac{9}{40}R}} = \sqrt{\frac{120}{144}\frac{g}{R}} = \boxed{\sqrt{\frac{5}{6}\frac{g}{R}}}$$

Or, using Lagrangian approach,

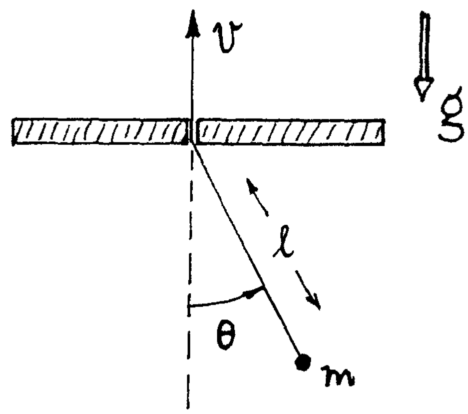
$$L = T - V = \frac{9}{40} mR^2\dot{\theta}^2 - \frac{3}{16} mgR\theta^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{9}{20} mR^2\ddot{\theta} + \frac{3}{8} mgR\theta = 0$$

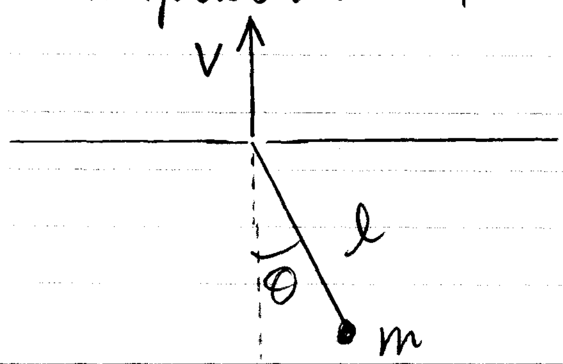
$$\ddot{\theta} + \frac{5}{6}\frac{g}{R}\theta = 0 \qquad \theta = \theta_0 \sin \omega t$$

$$\omega = \sqrt{\frac{5}{6}\frac{g}{R}}$$

3. A point particle of mass m is attached to the end of a massless string which is pulled at a constant velocity v through a small hole in a table. The mass moves in a vertical plane, with an oscillating angular displacement $\theta \ll 1$ with respect to the vertical. At t=0 the maximum displacement is θ_0, and the length of string below the hole is ℓ. Determine the maximum displacement for <u>small</u> positive t.



3. A point particle of mass m is attached to the end of a massless string which is pulled at a constant velocity v through a small hole in a table. The mass moves in a vertical plane, with an oscillating angular displacement $\theta \ll 1$ with respect to the vertical. At $t=0$ the maximum displacement is θ_0, and the length of string below the hole is l. Determine the maximum displacement for small positive t.



Sol'n: $L = \frac{1}{2}m\left(v^2 + (l-vt)^2\dot\theta^2\right) + mg(l-vt)\cos\theta$

Lagrange's Eqn's $\Rightarrow$ $\frac{d^2}{dt^2}\left[(l-vt)\theta\right] + g\sin\theta = 0$

For small θ, $\frac{d^2}{dt^2}\left[(l-vt)\theta\right] + g\theta = 0$

or $(l-vt)\ddot\theta - 2v\dot\theta + g\theta = 0$

For small t, $l-vt \approx l$, and

$$\ddot\theta - \frac{2v}{l}\dot\theta + \frac{g}{l}\theta = 0$$

or $\boxed{\ddot\theta - b\dot\theta + \omega_0^2\theta = 0}$ $b = \frac{2v}{l}$, $\omega_0 = \sqrt{g/l}$

negative damping!

Assume $\theta \sim e^{(a+i\omega)t}$

$$(a+i\omega)^2 - b(a+i\omega) + \omega_0^2 = 0$$

$$a^2 + 2ia\omega - \omega^2 - ba - ib\omega + \omega_0^2 = 0$$

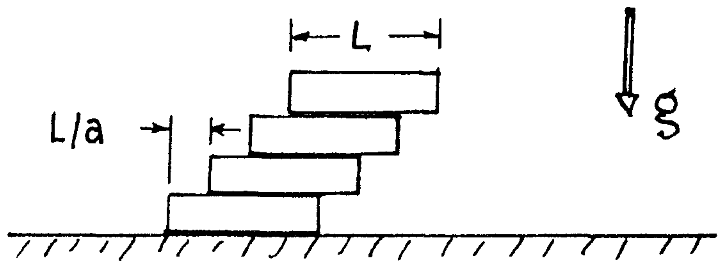
$$\text{Re, Im} = 0 \begin{cases} 2a\omega - b\omega = 0 \Rightarrow \boxed{a = \frac{b}{2}} \\ a^2 - \omega^2 + ab + \omega_0^2 = 0 \Rightarrow \omega = \sqrt{\omega_0^2 - \frac{b^2}{4}} \end{cases}$$

Thus θ_{max} increases exponentially —

$$\theta \sim e^{bt/2} e^{i\omega t}$$

$$\boxed{\theta_{max} = \theta_0 e^{bt/2} = \theta_0 e^{vt/l}}$$

4. A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are piled as shown, so that the ends are offset at each brick by L/a, where a is an integer (the other vertical sides form continous planes). How many bricks can be stacked in this manner before the pile collapses?



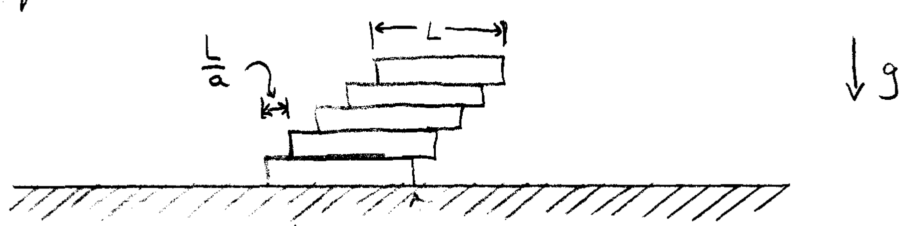
The old Frank Lloyd Wright problem. Do students know enough architecture? // RTR

Classical Mechanics — Bill Hiscock

A Pile of Bricks

Too easy? yes. Hogan

4. A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are piled as shown, so that the ends are offset at each brick by L/a, where a is an integer (the other vertical sides form continuous planes). How many bricks can be stacked in this manner before the pile collapses?



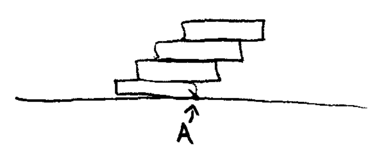
Is it necessary? no

Hint: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

OK WWS

Solution:

Evaluate the torque about the point A. Clockwise torques are negative, counterclockwise positive.



Torque of ith brick about A:

$$\tau_i = mg\left[-\frac{L}{2}+(i-1)\frac{L}{a}\right]$$

mass of single brick $= m$

Pile will collapse when total torque about A becomes zero

$$0 = \sum_{i=1}^{n} \tau_i = \sum_{i=1}^{n} \frac{mgL}{a}\left[-\frac{a}{2}+(i-1)\right]$$

$\Rightarrow$

$$0 = \sum_{i=1}^{n} -\frac{a}{2} + i - 1 = -n\left(\frac{a}{2}+1\right) + \sum_{i=1}^{n} i$$

or

$$\sum_{i=1}^{n} i = n\left(\frac{a}{2}+1\right) \qquad \text{but} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

so

$$\frac{\cancel{n}(n+1)}{\cancel{2}} = \frac{\cancel{n}(a+2)}{\cancel{2}} \qquad \boxed{n = a+1}$$ if bricks are glued together

$\boxed{n = a}$ if bricks are "loose"

(so that bottom brick does not move

5. A delton is a one-dimensional system in which an electron interacts with an attractive delta function potential whose strength is $-g$.

a) Solve Schrodinger's equation to determine the bound states of the delton.

b) Discuss qualitatively under what conditions the delton "negative ion" exists, i.e. a delton having two electrons.

W07

QM.

Deltons

5. A delton is a one-dimensional system in which an electron interacts with an attractive delta function potential whose strength is $-g$.

~~(a) Discuss the eigenvalue eqn and quantum mechanical bound states of the delton by solving Schrödinger's equation.~~

(a) Solve Schrödinger's equation ~~for the delton~~ to determine the bound states ~~quantum~~ of the delton.

(b) Discuss qualitatively under what conditions the delton "negative ion" exists, i.e., a delton having two electrons.

Soln $\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi(x) = E\psi(x)$

Let $\hbar^2/2m = 1$ and $V(x) = -g\delta(x)$.

$$(1) \quad \left(\frac{d^2}{dx^2} + E \right) \psi(x) = -g\,\delta(x)\,\psi(x)$$

Look for bound states. The homogeneous solns are

$$\left(\frac{d^2}{dx^2} + E \right) \psi(x) = 0$$

$$\psi(x) = \begin{cases} Ae^{-\kappa x} & x > 0 \\ Ae^{\kappa x} & x < 0 \end{cases}$$

Assume κ is real for b.s.

$$(2) \quad \kappa^2 + E = 0 \quad \text{or} \quad E = -\kappa^2 < 0$$

Inhomo. eqn.

$$\left(\frac{d^2}{dx^2} + E \right) \psi(x) = -g\,\delta(x)\,\psi(x)$$

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon}^{\varepsilon} \left(\frac{d^2\psi}{dx^2} + E\psi \right) dx = -g\,\psi(0)$$

LHS: $\psi_+' - \psi_-' + E\cancel{(\psi_+ - \psi_-)}^{0} = -g\,\psi(0)$

$(3) \quad \psi_+' = -\kappa\,\psi(0)$

$\psi_-' = \kappa\,\psi(0)$

Thus, $\quad -2\kappa\psi(0) = -g\psi(0)$

or $\quad \kappa = \dfrac{g}{2}$

(2) $\quad E = -\kappa^2 = -\dfrac{g^2}{4} \left(= \dfrac{\hbar^2}{2m} \dfrac{1}{4} \left[\dfrac{g}{\hbar^2/2m} \right]^2 \right\} = -\dfrac{mg^2}{2\hbar^2} \Big)$

There is only one.

(b) To lowest order, the Coulomb repulsion energy of the electrons

(3) must be weaker than the sum of the binding energies.

6. A particle of mass m moves in one dimension in the potential

$$V(x) = \begin{cases} V_o \cos (\pi/a)x & , o < x < a \\ \infty, \text{ otherwise} \end{cases}$$

Find the ground state energy to second order in V_o, and the ground state wave function to first order in V_o.

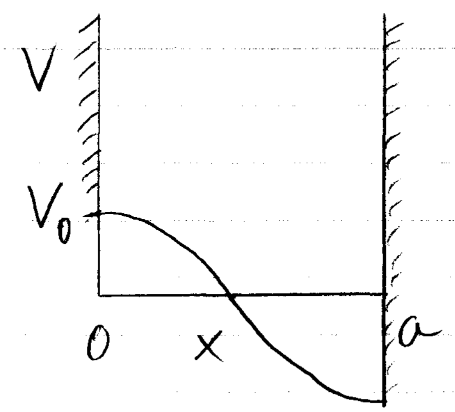
Quantum Mechanics J. Hermanson

6. A particle of mass m moves in one dimension in the potential

$$V(x) = \begin{cases} V_0 \cos \frac{\pi}{a} x , & 0 < x < a \\ \infty , & \text{otherwise} \end{cases}$$

Find the ground state energy to second order in V_0, and the ground state wavefunction to first order in V_0.

Sol'n:



$V = V_0 \cos kx \quad , k \equiv \frac{\pi}{a}$

$\psi_1 = \sqrt{\frac{2}{a}} \sin kx \qquad \psi_2 = \sqrt{\frac{2}{a}} \sin 2kx$

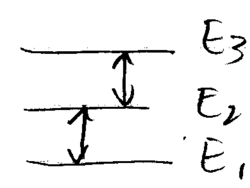
$E_1 = \frac{\hbar^2 \pi^2}{2ma^2} \qquad E_2 = 4E_1$

The perturbation $V = V_0 \dfrac{e^{ikx} + e^{-ikx}}{2} \Rightarrow \Delta k = \pm k$ selection rule

The unperturbed states $\psi_n \sim \sin nkx$

$$= \frac{e^{inkx} - e^{-inkx}}{2i}$$

Thus $\psi_1 \longleftrightarrow \psi_2 \longleftrightarrow \psi_3$ etc.



The non-vanishing matrix elements are

$$V_{12} = V_{21} = V_0 \cdot \frac{2}{a} \int_0^a \sin 2kx \cos kx \sin kx \, dx$$

$$= V_0 \cdot \frac{1}{a} \int_0^a \sin^2 2kx \, dx = \frac{1}{2} V_0$$

$[V_{11} = 0 = V_{22}]$

To 2nd order,

$$E_0 = E_1 + V_{11} + \frac{|V_{21}|^2}{E_1 - E_2} = E_1 - \frac{V_0^2}{12E_1}$$

$$\boxed{E_0 = \frac{\pi^2 \hbar^2}{2ma^2} - \frac{ma^2 V_0^2}{6\pi^2 \hbar^2}}$$

To 1st order,

$$\Psi_0 = \psi_1 + \frac{V_{21}}{E_1 - E_2}\psi_2 = \psi_1 - \frac{V_0}{6E_1}\psi_2$$

$$\boxed{\Psi_0 = \sqrt{\frac{2}{a}} \sin\frac{\pi}{a}x - \sqrt{\frac{2}{a}}\,\frac{ma^2 V_0}{3\pi^2 \hbar^2}\sin\frac{2\pi}{a}x}$$

7. Classical electromagnetic fields interact with a quantum mechanical particle according to the Schrodinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{1}{2m} \left(\frac{\hbar}{i} \vec{\nabla} - \frac{e}{c} \vec{A}(\vec{x},t) \right)^2 + e\phi(\vec{x},t) \right] \psi(\vec{x},t)$$

Since the wave function Ψ depends on the electromagnetic potential $(\phi,\vec{A})$, it must change if we change the potential by a gauge transformation:

$$\vec{A}'(\vec{x},t) = \vec{A}(\vec{x},t) + \vec{\nabla} f(\vec{x},t)$$
$$\phi'(\vec{x},t) = \phi(\vec{x},t) - \frac{1}{c}\frac{\partial f}{\partial t}$$

(the electromagnetic fields, $\vec{E}$ and $\vec{B}$, are of course, unchanged by this transformation), where $f(\vec{x},t)$ is an arbitrary function of $\vec{x}$ and t. The wave function Ψ in the new gauge obeys

$$i\hbar \frac{\partial \psi'}{\partial t} = \left[\frac{1}{2m} \left(\frac{\hbar}{i} \vec{\nabla} - \frac{e}{c} \vec{A}' \right)^2 + e\phi' \right] \psi'(\vec{x},t),$$

$$\text{where} \quad \psi' = \exp\left[\frac{ie}{\hbar c} f(\vec{x},t) \right] \psi$$

a) Compare $\langle\Psi| \vec{x} |\Psi\rangle$ and $\langle\Psi'| \vec{x} |\Psi'\rangle$. Are they equal?

b) Compare $\langle\Psi| \vec{p} |\Psi\rangle$ and $\langle\Psi'| \vec{p} |\Psi'\rangle$. Show that they are not equal.

c) Does (b) imply that quantum mechanics is not gauge invariant? Is $\vec{p}$ an observable? (Hint: consider the operator equation of motion for $\vec{x}$).

7. Classical electromagnetic fields interact with a quantum mechanical particle according to the Schrodinger equation:

I'll do equations

$$\left\{ i\hbar \frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e}{c}\vec{A}(\vec{x},t)\right)^2 + e\,\phi(\vec{x},t)\right] \Psi(\vec{x},t) \right\}$$

Since the wave function Ψ depends on the electromagnetic potentials $(\phi, \vec{A})$, it must change if we change the potentials by a gauge transformation:

$$\vec{A}'(\vec{x},t) = A(\vec{x},t) + \vec{\nabla} f(\vec{x},t)$$
$$\phi'(\vec{x},t) = \phi(\vec{x},t) - \frac{1}{c}\frac{\partial f}{\partial t}$$

(The electromagnetic fields, $\vec{E}$ and $\vec{B}$, are of course, unchanged by this transformation), where $f(\vec{x},t)$ is an arbitrary function of $\vec{x}$ and t. The wave function Ψ' in the new gauge obeys

$$i\hbar \frac{\partial \Psi'}{\partial t} = \left[\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e}{c}\vec{A}'\right)^2 + e\,\phi'\right] \Psi'(\vec{x},t) \quad .$$

$\Psi'(\vec{x},t)$ is related to $\Psi(\vec{x},t)$ by:

$$\Psi' = \exp\left[\frac{ie}{\hbar c} f(\vec{x},t)\right] \Psi$$

(a) Compare $\langle \Psi | \vec{x} | \Psi \rangle$ and $\langle \Psi' | \vec{x} | \Psi' \rangle$. Are they equal?

(b) Compare $\langle \Psi | \vec{p} | \Psi \rangle$ and $\langle \Psi' | \vec{p} | \Psi' \rangle$. Show that they are <u>not</u> equal.

(c) Does (b) imply that quantum mechanics is not gauge invariant? Is $\vec{p}$ an observable? (Hint: consider the operator equation of motion for $\vec{x}$).

Solution:

(a) $\langle\psi|\vec{x}|\psi\rangle = \int d^3x \; \psi^* \vec{x}\, \psi$

$$\langle\psi'|\vec{x}|\psi'\rangle = \int d^3x \; \psi^* e^{-\frac{ie}{\hbar c}f} \,\vec{x}\, e^{\frac{ie}{\hbar c}f}\psi = \int d^3x\, \psi^* \vec{x}\, \psi = \langle\psi|\vec{x}|\psi\rangle$$

They are the same: $\langle\vec{x}\rangle$ is gauge invariant

(b) $\langle\psi|\vec{p}|\psi\rangle = \int d^3x\, \psi^*(-i\hbar\vec{\nabla})\psi$

$$\langle\psi'|\vec{p}|\psi'\rangle = \int d^3x \; \psi^* e^{-\frac{ie}{\hbar c}f}(-i\hbar\vec{\nabla})\, e^{\frac{ie}{\hbar c}f}\psi$$

$$= \int d^3x\, \psi^*(-i\hbar\vec{\nabla})\psi + \int d^3x\, \psi^*\left(\frac{e}{c}\vec{\nabla}f\right)\psi$$

$$= \langle\psi|\vec{p}|\psi\rangle + \int d^3x\; \psi^*\left(\frac{e}{c}\vec{\nabla}f\right)\psi$$

They are not equal! $\langle\vec{p}\rangle$ is not gauge invariant! However, this does not mean quantum mechanics is not gauge invariant: gauge invariance only requires that all observable physical quantities must be gauge invariant. The physically observable momentum is $m\, d\vec{x}/dt$! From the equation of motion for $\vec{x}$:

$$i\hbar \frac{d\vec{x}}{dt} = [\vec{x}, H] = \frac{-1}{m}\frac{\hbar}{i}\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A}\right) = +\frac{i\hbar}{m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)$$

$\Rightarrow$

$$\boxed{m\frac{d\vec{x}}{dt} = \left(\vec{p} - \frac{e}{c}\vec{A}\right)}$$ as in classical mechanics

Note that

$$\langle\psi|\vec{p} - \tfrac{e}{c}\vec{A}|\psi\rangle = \langle\psi'|\vec{p} - \tfrac{e}{c}\vec{A}'|\psi'\rangle$$ so the observable quantity is gauge invariant.

and

$$\langle \nu_e | \Psi(t,x)\rangle = \exp\left(-i\frac{E_0 t}{\hbar}\right)\left\{\cos^2\theta \exp\left(\frac{i\sqrt{2m_1E_0}}{\hbar}x\right) + \sin^2\theta \exp\left(\frac{i\sqrt{2m_2E_0}}{\hbar}x\right)\right\}$$

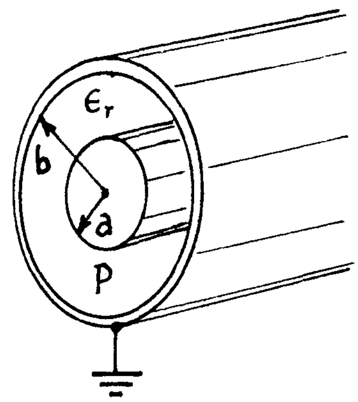
$$|\langle \nu_e|\Psi\rangle|^2 = \cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\left\{\exp\left[i\frac{\sqrt{2E_0}}{\hbar}(\sqrt{m_1}-\sqrt{m_2})x\right] - \exp\left[-i\frac{\sqrt{2E_0}}{\hbar}(\sqrt{m_1}-\sqrt{m_2})x\right]\right\}$$

$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\left[\frac{\sqrt{2E_0}}{\hbar}(\sqrt{m_1}-\sqrt{m_2})x\right]$$

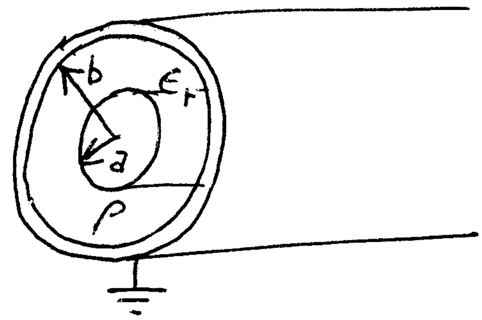
$$\boxed{|\langle \nu_e|\Psi\rangle|^2 = 1 + \tfrac{1}{2}\sin 2\theta\left\{\cos\left[\frac{\sqrt{2E_0}}{\hbar}(\sqrt{m_1}-\sqrt{m_2})x\right] - 1\right\}}$$

Note that if either $\theta \to 0$ <u>or</u> $m_1 \to m_2$, the oscillations vanish.

8. A coaxial cable of inner conductor radius a, outer conductor inside radius b, and relative dielectric permittivity ε_r is open-circuited with initial voltage 0 on the inner conductor. It receives a burst of ionizing radiation which leaves the dielectric with a uniform free volume charge density P. Find the voltage on the inner conductor if the outer conductor is grounded.



<u>E&M</u> Problem 1/75



8. A coaxial cable of ~~inside~~ inner conductor radius a, grounded outer conductor inside radius b, and ~~dielectric~~ relative dielectric permittivity ϵ_r is open-circuited with initial voltage 0 on the inner conductor. It receives a burst of ionizing radiation which leaves the dielectric with (a uniform free volume charge density ρ. Find the voltage on the inner conductor.

E+M Solution

Use Gauss' Law: $\frac{\int_{encl} dq}{\epsilon_0 \epsilon_r} = \int \vec{E} \cdot d\vec{a}$

$$\frac{\rho \pi (r^2 - a^2) \ell}{\epsilon_0 \epsilon_r} = 2\pi r \ell E \qquad E = \frac{\pi \rho (r^2 - a^2) \ell}{2\pi \epsilon_0 \epsilon_r r \ell}$$

$$\vec{E} = \frac{\rho (r^2 - a^2)}{2\epsilon_0 \epsilon_r r} \hat{r} \qquad V = -\int_b^a \vec{E} \cdot d\vec{r} = +\int_a^b E\, dr$$

$$V = \frac{\rho}{2\epsilon_0 \epsilon_r} \int_a^b \left(r - \frac{a^2}{r}\right) dr = \boxed{\frac{\rho}{2\epsilon_0 \epsilon_r}\left(\frac{b^2 - a^2}{2} - a^2 \ln \frac{b}{a}\right)}$$

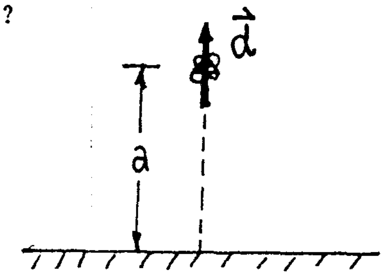
DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE/PH.D. QUALIFYING EXAMINATION

SATURDAY, APRIL 2, 1-5 PM / 1986

Answer each of the following questions, all of which carry equal weight. Begin your answer to each question on a new sheet of paper; solutions to different questions must not appear on the same sheet. Each sheet of paper must be labeled with your name and the problem number, in the upper right hand corner. When more than one sheet is submitted for a problem, be sure the pages are ordered properly. [Problems 9-16].

9. Consider a carbon monoxide molecule positioned a distance a above an ideal metal surface. The molecule has a dipole moment $\vec{d}$. Assuming that $\vec{d}$ is perpendicular to the surface and parallel to the surface normal, what, if any, force exists on the molecule?



E+M

good – maybe better for Ph.D. Comp. JED.

WJJ

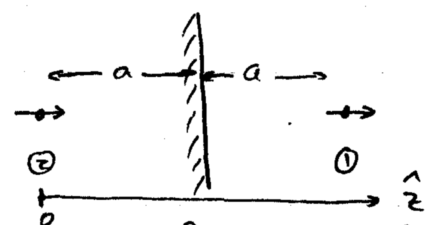
OK/WJJ 9. Consider a carbon monoxide molecule positioned a distance "a" above an ideal ~~gas~~ metal surface. The molecule has a dipole moment $\vec{d}$. Assuming that $\vec{d}$ is perpendicular to the surface and parallel to the surface normal, what, if any, force exists on the molecule?

Soln

General dipole potl: $\varphi = \dfrac{\vec{d}\cdot\vec{r}}{r^3} = \dfrac{d\cos\theta}{r^2}$

$\vec{E} = (-\nabla\varphi) = -\hat{e}_r \dfrac{\partial\varphi}{\partial r} - \dfrac{\hat{e}_\theta}{r}\dfrac{\partial\varphi}{\partial\theta}$ (axial symmetry)

$\vec{E} = \dfrac{2d\cos\theta}{r^3}\hat{e}_r + \dfrac{d\sin\theta}{r^3}\hat{e}_\theta$



Let E_2 be the image field. Then the interaction energy is

$$U_{int} = -\vec{p}_1\cdot\vec{E}_2$$

$$E_2 = \frac{2d_2}{z^3}\hat{e}_z$$

$$U_{int} = -\frac{2d_1d_2}{z^3}$$

$$F = (-\nabla U) = +\frac{2d^2}{z^4}(-3)\hat{e}_z = -\frac{6d^2}{z^4}\hat{e}_z$$

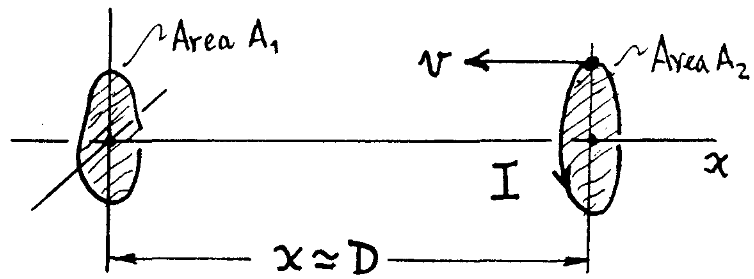
$$\vec{F} = -\frac{6}{2^4}\frac{d^2}{a^4}\hat{e}_z = -\frac{3}{8}\frac{d^2}{a^4}\hat{e}_z$$

10. Two coaxial, coplanar wire loops of areas A_1 & A_2 are situated on the x-axis as shown, and are initially separated by a "large" distance D (D>> linear dimension of either loop). Loop 1 remains fixed at the origin, while loop 2 carries a steady current I and is made to move at a known velocity v = dx/dt to the <u>left</u> (toward loop 1) along the x-axis.

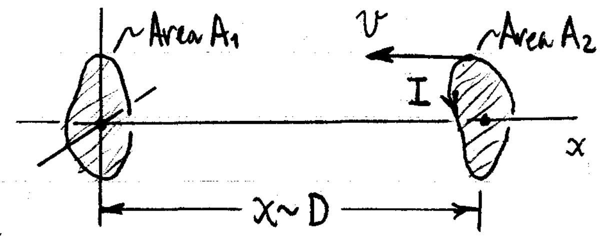
a) Calculate the emf induced in loop 1 by the movement of loop 2, so long as the loop separation x ~ D is large.

b) If the current flow is counterclockwise in loop 2, in what direction does the induced current flow in loop 1?

Hint: Use the dipole approximation.



M.S. Exam ; Apr. 1986 (Robiscoe) Elec. & Mag. } Magnetic Induction (4/10/86)



10. Two coaxial, coplanar wire loops of areas A_1 & A_2 are situated on the x-axis as shown, and are initially separated by a "large" distance D (D >> linear dimension of either loop). Loop 1 remains fixed at the origin, while loop 2 carries a steady current I and is made to move at a known velocity $v = dx/dt$ to the **left** (toward loop 1) along the x-axis.

A. Calculate the emf induced in loop 1 by the movement of loop 2, so long as the loop separation $x \sim D$ is "large".

B. If the current flow is counterclockwise in loop 2, in what direction does the induced current flow in loop 1?

~~optional~~

HINT: use the dipole approximation."

Solution

1. D → large ⇒ dipole approxn can be used for the $\mathbb{B}$-fld of loop # 2:

$$\mathbb{B} = \frac{1}{x^3}\left[3\mathbf{n}(\mathbf{n}\cdot\mathbf{m}) - \mathbf{m}\right] \quad \begin{cases} \mathbf{n} = (-)\hat{x}, \text{ unit vector from \#2 to \#1,} \\ \mathbf{m} = \left(\frac{I}{c}A_2\right)\hat{x}, \text{ mag. moment of \#2.} \end{cases} \quad (1)$$

(we are using cgs units, as any sensible person would do). For the given geometry, this gives the field at loop 1 due to loop 2...

$$\mathbb{B} = \left(\frac{2IA_2}{cx^3}\right)\hat{x} \leftarrow \text{points to the right, and increases as } x \text{ decreases.} \quad (2)$$

2. With $x \sim D$ → large (compared to loop # 1 dimensions), $\mathbb{B}$ is ~ cnst over the area A_1, so the flux through loop # 1 is

(over)

$$\phi = BA_1 = (2IA_1A_2/c)\frac{1}{x^3}, \tag{3}$$

Faraday's law then provides the induced emf...

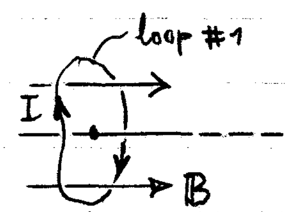
$$\mathcal{E} = -\frac{d\phi}{dt} = -\underbrace{\left(\frac{dx}{dt}\right)}_{v,\ \text{velocity of loop \#2}}\frac{d\phi}{dx} = -v\left(\frac{2IA_1A_2}{c}\right)\frac{d}{dx}\left(\frac{1}{x^3}\right)$$

or,

$$\boxed{\mathcal{E} = 6vIA_1A_2/cx^4.} \tag{4}$$

This is the required emf in #1 due to #2's motion, per part A.

3. As noted in Eq. (2), $\mathbb{B}$ is to the right at loop #1, and is increasing with time. By Lenz' law, the induced I will flow in a direction so as to oppose this change... I flows to create a $\mathbb{B}'$ in the opposite direction. So...



<u>If I flows CCW in loop #2, the induced I flows CW in loop #1.</u>

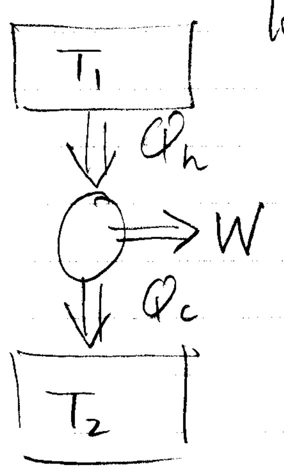
11. Two identical bodies with heat capacities C_p and C_v are at different initial temperatures T_1 and $T_2 < T_1$. While remaining at constant pressure the bodies are used as reservoirs for a heat engine. What is the maximum amount of work obtainable?

Hint: allow the two bodies to come to a common final temperature.

Thermal Physics J. Hermanson

11. Two identical bodies with heat capacities C_p and C_V are at different initial temperatures T_1 and $T_2 < T_1$. While remaining at constant pressure the bodies are used as reservoirs for a heat engine. What is the maximum amount of work obtainable? Hint: allow the two bodies to come to a common final temperature T_f.

Sol'n:



Here $T_1 > T_f > T_2$

$$W = Q_h - Q_c$$

$$= C_p(T_1 - T_f) - C_p(T_f - T_2)$$

$$= C_p(T_1 + T_2 - 2T_f)$$

What is T_f for max W?

The maximum efficiency (max W) is obtained when $\Delta S = 0$ for the complete cycle.

$$\text{Now } \Delta S = \overset{<0}{C_p \ln \frac{T_f}{T_1}} + \overset{>0}{C_p \ln \frac{T_f}{T_2}} \quad \text{for a cycle}$$

$$= C_p \ln \frac{T_f^2}{T_1 T_2} = 0 \text{ when } T_f = \sqrt{T_1 T_2}$$

Thus $\boxed{W_{max} = C_p\left(T_1 + T_2 - 2\sqrt{T_1 T_2}\right)}$

12. A careless experimenter left the valve on a tank of helium gas slightly open over the weekend. The gas, originally at 100 atm. slowly escaped isothermally at 300 K. What was the change in entropy per kilogram of helium?

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Thermal Physics Bill Hiscock

12. A careless experimenter left the valve on a tank of helium gas slightly open over the weekend. The gas, originally at 100 atm. slowly escaped isothermally at 300K. What was the change in entropy per kilogram of helium?

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

Solution:

$$\Delta S = \int_a^b \frac{dQ}{T} = \frac{\Delta Q}{T} \quad \text{since isothermal}$$

isothermal expansion $\Rightarrow dU = 0$ $\quad \Delta Q = \int_a^b P\,dV = -\int_{P_1}^{P_2} V\,dP = -\int_{P_1}^{P_2} \frac{NkT\,dP}{P}$

$$\Delta Q = NkT \log\left(\frac{P_1}{P_2}\right) \Rightarrow \Delta S = Nk \log\left(\frac{P_1}{P_2}\right)$$

$$\frac{\Delta S}{kg} = N_{kg}\, k \log\left(\frac{P_1}{P_2}\right)$$

1 kg He = 250 moles $\simeq 1.5 \times 10^{26}$ He atoms

$$\frac{\Delta S}{kg} \simeq (1.5 \times 10^{26})\left(1.38 \times 10^{-23}\ \frac{J}{K}\right) \log_e(100)$$

$$\boxed{\frac{\Delta S}{kg} = 9.5 \times 10^{3}\ J/K}$$

13. T(a) is a <u>translation</u> <u>operator</u> which converts $\psi(x)$ to $\psi(x+a)$;

$$T(a)\,\psi(x) = \psi(x+a) \quad .$$

Show that $T(a) = \exp(iap_x)$, where p_x is the quantum mechanical momentum operator,

$$p_x = -\,i(d/dx) \quad .$$

Math Physics Bill Hiscock

13. $T(a)$ is a <u>translation operator</u> which converts $\psi(x)$ to $\psi(x+a)$;

$$T(a)\,\psi(x) = \psi(x+a)$$

Show that $T(a) = \exp(i a p_x)$, where p_x is the quantum mechanical momentum operator,

$$p_x = -i\, d/dx \, .$$

Solution:

$$T(a) = \exp(i a p_x) = \exp\left(a \frac{d}{dx}\right) = 1 + a\frac{d}{dx} + \frac{1}{2}a^2\left(\frac{d}{dx}\right)^2 + \frac{1}{3!}a^3\left(\frac{d}{dx}\right)^3 + \cdots$$

$$T(a)\,\psi(x) = \psi(x) + a\frac{d\psi}{dx} + \frac{1}{2}a^2\frac{d^2\psi}{dx^2} + \frac{1}{3!}a^3\frac{d^3\psi}{dx^3} + \cdots$$

$$= \psi(x+a) \quad \blacksquare$$

14. A proton-antiproton pair may be created in the absorption of a photon (γ) by a proton at rest:

$$\gamma + p \to p + p + \bar{p}$$

The threshold energy for this reaction corresponds to the three particles on the right moving off together as a single particle of rest mass $3m_p$. What is the threshold energy E_γ of the photon (in terms of $m_p c^2$)?

Stfwd, but OK. // RTR

standard: see mirror problem more.

OK HHS

OK WES

Special Relativity - Easy

Bill Hiscock

14. A proton - antiproton pair may be created in the absorption of a photon (γ) by a proton at rest :

$$\gamma + p \rightarrow p + p + \bar{p}$$

The threshold energy for this reaction corresponds to the three particles on the right moving off together as a single particle of rest mass $3m_p$. What is the threshold energy E_γ of the photon (in terms of $m_p c^2$)?

<u>Solution :</u>

conservation of momentum

$$p_\gamma = p_{3p} \qquad (1)$$

Conservation of energy

$$E_\gamma + m_p c^2 = E_{3p} \qquad (2)$$

but $\quad E_\gamma = p_\gamma c \qquad p_\gamma = E_\gamma / c \overset{(1)}{\Rightarrow} p_{3p} = E_\gamma / c$

and

$$E_{3p}^2 = (3m_p c^2)^2 + (p_{3p} c)^2 \overset{(2)}{=} E_\gamma^2 + 2E_\gamma m_p c^2 + m_p^2 c^4$$

$$9m_p^2 c^4 + \cancel{E_\gamma^2} = \cancel{E_\gamma^2} + 2E_\gamma m_p c^2 + m_p^2 c^4$$

$$\Rightarrow \boxed{E_\gamma = 4 m_p c^2}$$

15. Describe how you would generate and measure electromagnetic radiation in all different frequency ranges.

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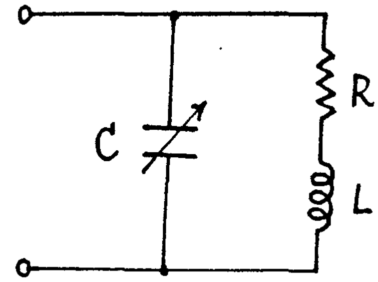
	generator	rec'r
Audio	transistors	tuned circuits
r.f.	"	"
uhf	"	diodes
microwave	Klystrons	diodes, thermistors
far i.r.		
ir	heat	thermistors, film
visible	filament, gaseous discharge	eye, film, "
u.v.		
X-ray	hi-voltage electrons on metal	film
γ-ray	nuclear excitation	secondary electrons

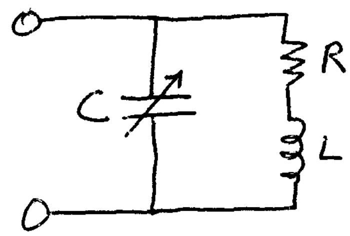
16. To what value must C be adjusted to achieve maximum impedance at angular frequency ω?



Experimental Problem

2/f5

16. To what value must C be adjusted to achieve maximum impedance at angular frequency ω?



Experimental solution

$$Z_{RL} = R + j\omega L$$

$$Y_{RL} = \frac{1}{Z_{RC}} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$Y = Y_{RL} + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

Y is smallest when real, so $\boxed{C = \frac{L}{R^2 + \omega^2 L^2}}$