6. An electron in the Coulomb field of a proton is initially (t=0) in a state described by the wave function.

Corrections given at start of exam.

$$\psi(\vec{r},0) = 1/6\ [4\ \psi_{100} + 3\ \psi_{211} - \psi_{210} + \sqrt{10}\ \psi_{21\bar{1}}]$$

with ψ_{nlm}, the energy eigenfunctions, satisfying

$$\left(\frac{\vec{P}^{\,2}}{2m} - \frac{e^2}{r}\right)\psi_{nlm} = E_n \psi_{nlm} \text{ where}$$

ψ_{nlm} is normalized to 1

$$E_n = -\frac{R_\infty}{n^2} \quad \text{and } \bar{m} = -m$$

1) Is ~~ψ_{nlm}~~ $\psi(\vec{r},0)$ normalized? Give evidence for your answer. $\langle\psi|\psi\rangle = \frac{1}{36}[16+9+1+10] = 1$

2) What is the probability of measuring an eigenvalue of L^2 to be $2\hbar^2$. $\ell = 1$

$\frac{1}{36}[9+1+10] = \frac{5}{9}$

3) What is the probability of measuring an eigenvalue of L_z to be $-\hbar$?

$m=-1$ $\quad \frac{1}{36}[10] = \frac{5}{18}$

4) What is the expectation value of the energy? $\langle H\rangle = \frac{-1}{36}\left[16 + \frac{9+1+10}{4}\right]R_\infty = -\frac{7}{12}R_\infty$

5) What is the expectation value of L_z? $\langle L_z\rangle = \frac{1}{36}[9-10]\hbar = -\frac{1}{36}\hbar$

6) What is the probability of measuring an energy of $-(1/4)R_\infty$? $\frac{9+1+10}{36} = \frac{5}{9}$

7) What is the wave function at time t?

$$\psi(\vec{r},t) = \frac{1}{6}\left[4\psi_{100}\, e^{-\frac{iE_1 t}{\hbar}} + (3\psi_{211} - \psi_{210} + \sqrt{10}\,\psi_{21\bar{1}})\, e^{-\frac{iE_2 t}{\hbar}}\right]$$

8) Will any of the answers to parts 1 through 6 be different at time t? If so, which ones? Give your reasoning.

L_z, ... constants of motion

Note corrections