Then, conservation of energy demands: $\frac{dK}{dt} = -P$, or...

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2\right] = -\mathcal{E}^2/R \qquad \text{(2)}$$

later... $\tau = \frac{4\rho c^2}{\sigma B^2}$

or// $$\frac{1}{2}mr^2\omega\,\frac{d\omega}{dt} = -\frac{1}{R}\left(\pi r^2 B/c\right)^2 \omega^2 \sin^2\omega t$$

or// $$\boxed{\frac{d\omega}{dt} = -\frac{\omega}{\tau}\left[2\sin^2\omega t\right]}, \quad \text{w/ } \tau = mRc^2/(\pi r B)^2. \qquad \text{(3)}$$

B. If the loop rotates several times before stopping, a time-average of Eq. (3) yields $\langle 2\sin^2\omega t\rangle = 1$ on the RHS, so that approximately:

$$\frac{d\omega}{dt} \simeq -\frac{\omega}{\tau} \Rightarrow \boxed{\omega(t) \simeq \Omega e^{-t/\tau}}, \quad \tau \text{ given in Eq. (3).} \qquad \text{(4)}$$

Same effect produced by time-averaging the Joule loss $\frac{\mathcal{E}^2}{R}$ in Eq. (2).

C. Turning ∡ between $t=0$ & loop stop is

$$\theta = \int_0^\infty \omega(t)\,dt = \Omega\tau. \qquad \text{(5)}$$

Now τ involves $\left\{\begin{array}{l} m = 2\pi r A\rho \\ R = 2\pi r/\sigma A \end{array}\right\} mR = 4\pi^2 r^2 \rho/\sigma$, i.e.

$$\left.\begin{array}{l} \tau = mRc^2/(\pi r B)^2 = 4\rho c^2/\sigma B^2 \\ \text{So// } \boxed{\theta = \Omega\tau = 4\Omega\rho c^2/\sigma B^2} \end{array}\right\} \text{indpt of loop dimensions} \qquad \text{(6)}$$

Special Relativity | Hiscock | #11

A photon (γ) scattered by an electron (e^-), may if energetic enough, produce an electron-positron pair as follows:

$$\gamma + e^- \rightarrow 2e^- + e^+ .$$

Find the minimum photon energy, in the rest frame of the initial electron, for this reaction to occur.

Solution: Use conservation of 4-momentum; $c = 1$

Before:

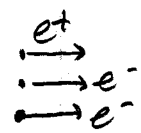
$p_\gamma^\mu = (E_\gamma, p_\gamma, 0, 0)$

$p_e^\mu = (m_e, 0, 0, 0)$

photon massless $\Rightarrow$

$E_\gamma = p_\gamma \quad (0)$

After:



At threshold, the 3 particles will move off together (this is obvious if one thinks of the process in the CM frame).

$p_3^\mu = (E_3, p_3, 0, 0)$

Normalization:

$m_3^2 = (3m_e)^2 = E_3^2 - p_3^2 \quad (1)$

Conserve 4-momentum:

energy: (before = after)

$$E_\gamma + m_e = E_3 \quad (2)$$

momentum

$$p_\gamma = p_3 \quad (3)$$

Square (2): $E_\gamma^2 + 2E_\gamma m_e + m_e^2 = E_3^2$ ← replace using (1)

$E_\gamma^2 + 2E_\gamma m_e + m_e^2 = 9m_e^2 + p_3^2$ ← replace using (3) & (0)

$\cancel{E_\gamma^2} + 2E_\gamma m_e + \cancel{m_e^2} = \underset{8}{\cancel{9}}m_e^2 + \cancel{E_\gamma^2}$

$\Rightarrow$

$2E_\gamma m_e = 8m_e^2 \rightarrow \boxed{E_\gamma = 4m_e} \boxed{\simeq 2.044\ \text{MeV}}$ ✓