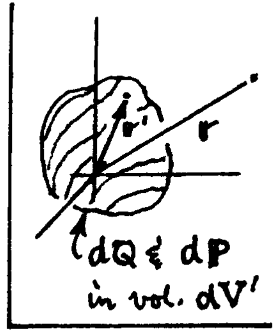
6) We can sculpt Eq. (8) to describe a charge assembly called "matter":



$$\rightarrow\ d\phi(\mathbf{r}) = \frac{1}{R}\,\overset{\text{charge}}{dQ} + \frac{\mathbb{R}}{R^3}\cdot \overset{\text{dipole moment}}{d\mathbb{p}},$$

potential generated by dQ & $d\mathbb{P}$ in dV', through dipole order.

w/ $\mathbb{R} = \mathbf{r} - \mathbf{r}'$, $R = |\mathbf{r} - \mathbf{r}'|$. (24)

Suppose the volume dV' contains a "large" # N_i (per unit volume) of molecules, each bearing <u>charge $\langle e_i\rangle$ & dipole moment $\langle \mathbb{p}_i\rangle$</u>. The $\langle\ \rangle$ imply "macroscopic" averages. These molecules are the <u>source</u> of the above dQ & $d\mathbb{p}$, so we assign...

$$\begin{cases} dQ = \rho(\mathbf{r}')\,dV', & \rho(\mathbf{r}') = \sum_i N_i\langle e_i\rangle + \rho(\mathbf{r}')\big|_{\text{free}} \\ d\mathbb{p} = \mathbb{P}\,dV', & \mathbb{P} = \sum_i N_i\langle \mathbb{p}_i\rangle \end{cases}$$

"usual" charge per unit volume;

dipole moment / unit vol. called "<u>polarization</u>";

So,

$$d\phi(\mathbf{r}) = \left[\frac{1}{R}\,\rho(\mathbf{r}') + \left(\frac{\mathbb{R}}{R^3}\right)\cdot\mathbb{P}(\mathbf{r}')\right] dV' \qquad (25)$$

put $\mathbb{R}/R^3 = (+)\nabla'\left(\frac{1}{R}\right)$

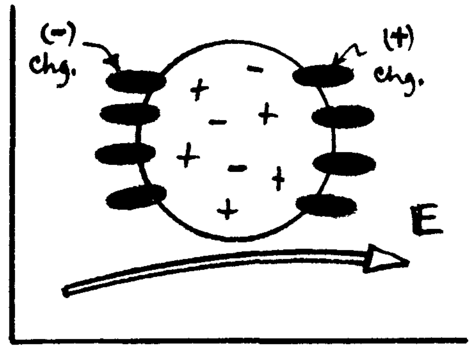
(next page)

and/

$$\phi(\mathbf{r}) = \int_{\text{material}} dV' \left[\frac{1}{R}\rho(\mathbf{r}') + \mathbb{P}(\mathbf{r}') \cdot \nabla'\left(\frac{1}{R}\right)\right] \tag{26}$$

Use $\nabla' \cdot (\mathbb{P}/R) = \mathbb{P} \cdot \nabla'(1/R) + \frac{1}{R}(\nabla' \cdot \mathbb{P})$, and the Divergence Theorem to transform: $\int dV' \, \nabla' \cdot (\mathbb{P}/R) = \oint d\mathbf{S} \cdot (\mathbb{P}/R) \rightarrow 0$. Then...

$$\phi(\mathbf{r}) = \int_{\text{material}} dV' \frac{1}{R} \underbrace{\left[\rho(\mathbf{r}') - \nabla' \cdot \mathbb{P}\right]}_{\rho_{eff} = \rho_{real} - \nabla' \cdot \mathbb{P}} \tag{27}$$



The polarization $\mathbb{P}$ changes the "real" charge in a small volume of material, if it is not uniform.
This is a potentially important material effect, and forces a redefinition of Maxwell's first equation, as follows...

in material-free space $\}$ $\nabla \cdot \mathbb{E} = 4\pi\rho_{real}$ ‖ in presence of material $\}$ $\nabla \cdot \mathbb{E} = 4\pi\rho_{eff}$

So// $\nabla \cdot \mathbb{E} = 4\pi\left[\rho_{real} - \nabla \cdot \mathbb{P}\right]$, or// $\boxed{\nabla \cdot \mathbb{D} = 4\pi\rho_{real}}$ (28)

Where: $\underline{\underline{\mathbb{D} = \underset{\text{applied}}{\mathbb{E}} + 4\pi \underset{\text{induced}}{\mathbb{P}}}}$. ← called "electric displacement."

From the way this is done, clearly $\mathbb{P}$ should represent all effective fields generated in the material by application of $\mathbb{E}$, not only the induced dipoles, but also the quadrupole & higher-order corrections.

REMARKS

1. Often the induced $\mathbb{P}$ is proportional to the $\mathbb{E}$ which caused it, i.e.

$\mathbb{P} = \alpha\,\mathbb{E}$, α = "polarizability" $\begin{cases} \alpha = \text{cnst, for simple linear materials;} \\ \alpha = \text{tensor (direction-dept) in general;} \end{cases}$ (29)

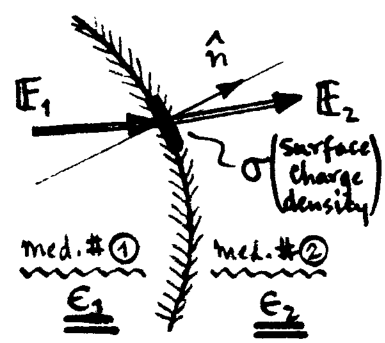
So// $\boxed{\mathbb{D} = \epsilon\,\mathbb{E}}$, w/ $\epsilon = 1 + 4\pi\alpha$ ← dielectric cnst $\begin{cases} \epsilon(\text{vacuum}) \equiv 1,\ \epsilon(\text{air}) = 1.0006, \\ \epsilon(\text{plastic}) \simeq 3,\ \epsilon(\text{water}) = 80. \end{cases}$

Multipoles & Dielectrics (cont'd)

The dielectric "cnst" ϵ [which is actually a strong fcn of the frequency of the applied field $\mathbb{E}$]* plays a pivotal role in describing the EM interactions of material media. E.g.

$$\left.\begin{array}{l} \text{a) Jk}^{\text{n}}\text{ Sec.(7.3): index of refraction: } n(\omega) = \sqrt{\mathrm{Re}\,\epsilon(\omega)}\,; \\ \text{b) Jk}^{\text{n}}\text{ Sec.(7.5): electrical conductivity: } \sigma(\omega) = \frac{\omega}{4\pi}\,\mathrm{Im}\,\epsilon(\omega)\,; \\ \text{c) Jk}^{\text{n}}\text{ Sec.(7.7): Skin depth: } \delta \simeq \frac{\lambda}{2\pi}\Big/\sqrt{\tfrac{1}{2}\,\mathrm{Im}\,\epsilon(\omega)}\,. \end{array}\right\} \quad (30)$$

2. Recall the B.C. at an interface (see Jk$^{\text{n}}$ Sec. I5, pp. 17-22)...



$$\nabla\cdot\mathbb{D} = 4\pi\rho_{real} \Rightarrow \underline{\underline{(\mathbb{D}_2-\mathbb{D}_1)\cdot\hat{n} = 4\pi\sigma_{real}}}\,;$$

possible jump discontinuity in normal $\mathbb{D}$

$$(\epsilon_2\mathbb{E}_2 - \epsilon_1\mathbb{E}_1)\cdot\hat{n} = 4\pi\sigma_{real}. \quad (31)$$

$$\nabla\times\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t} \Rightarrow \underline{\underline{(\mathbb{E}_2-\mathbb{E}_1)\times\hat{n} = 0}}.$$

(tangential comp. of $\mathbb{E}$ is conserved; $\dot{\mathbb{B}}$ or no). (32)

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* There is a simple but very useful dynamical model of a medium's dielectric "cnst" $\epsilon(\omega)$, and its frequency-dependence, described in Jackson's Sec. 7.5(a), p. 284. $\epsilon(\omega)$ is generally complex and shows resonances at certain frequencies characteristic of the medium.

We will outline this so-called simple harmonic oscillator model of $\epsilon(\omega)$ next page. Later, we will have many uses of the SHO model for $\epsilon(\omega)$.