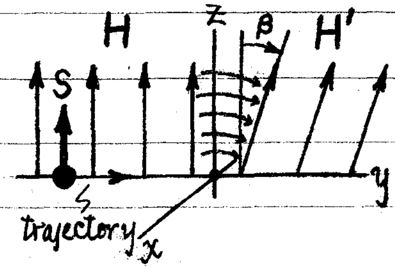
Physics 507 Hour Exam — 16 May 1972



① A particle with spin $S=\frac{1}{2}$ is traveling down the y-axis of the cd. system shown. To the left of the origin, there is a magnetic field $\vec{H}$ along the z-axis; here the spin is aligned with the field (i.e. spin "up", or $m_s=+\frac{1}{2}$ state). Near the origin, the field suddenly rotates to $\vec{H}'$, which is inclined at ∡ β w.r.t. the z-axis. Calculate the probability that the spin "flips" due to the field rotation (i.e. that one finds spin "down", or a $m_s=-\frac{1}{2}$ state to the right).

② Carbon is a six-electron atom with the "configuration" $(1s)^2(2s)^2(2p)^2$, which means there are two electrons in each of the 1s and 2s orbitals (forming closed shells with total spin and ∡ momentum $\equiv 0$), and two "valence" electrons in 2p orbitals. The latter electrons each have spin $\frac{1}{2}$ and orbital ∡ momentum 1; they obey Russell-Saunders coupling to form a total spin $\vec{S}=\vec{S}_1+\vec{S}_2$ and total orbital ∡ momentum $\vec{L}=\vec{L}_1+\vec{L}_2$, which in turn couple to form a total ∡ momentum $\vec{J}=\vec{L}+\vec{S}$.

a) What are the possible spectroscopic states formed by the coupling of the two equivalent 2p electrons? Denote these states by ${}^{2S+1}(L)_J$, with $L=0 \Rightarrow$ S state, $L=1 \Rightarrow$ P state, etc. E.g. is ${}^1P_1$ possible?

b) Of the states formed in part a, which must be ruled out on the grounds of improper exchange symmetry?

c) Of the states remaining from part b, which will be the ground state (i.e. state of lowest energy), and why? What J value will the ground state have, and why?

③ Initially, a QM system has three levels, of which two are degenerate, with energy E_1, and the third has energy $E_2 > E_1$. In the presence of an external field of strength K, the system Ham$^{\underline{n}}$ is the matrix

$$\underset{\sim}{\mathcal{H}} = \begin{pmatrix} E_1 & 0 & a \\ 0 & E_1 & b \\ a^* & b^* & E_2 \end{pmatrix} \qquad \left\{ \begin{array}{l} a = \alpha K \\ b = \beta K \end{array} \right. \text{ matrix elements of the external field.}$$

a) Find the system eigenenergies $\mathcal{E}_i$ for $K > 0$.
b) Draw a graph of the behaviour of the $\mathcal{E}_i$ as fcns of K.

④ For a QM system governed by a Ham$^{\underline{n}}$ H, suppose a general state of the system is written $|\alpha\rangle = \sum_n c_n |n\rangle$. Here the fcns $|n\rangle$ are a compleat, orthonormal basis -- which are <u>not</u> necessarily eigenfcns of H. In general, the expansion coefficients c_n may depend on time. It is "easy" to shew that the expectation value of a QM operator Q in the state α is then given by $\langle Q \rangle = \mathrm{tr}(\underset{\sim}{\rho}\, \underset{\sim}{Q})$, where $\underset{\sim}{Q}$ is a matrix of entries $Q_{mn} = \langle m | Q | n \rangle$, and $\underset{\sim}{\rho}$ is called the "density matrix".

a) Find the elements ρ_{kl} of the density matrix in terms of the expansion coefficients c_n. What is the physical significance of the diagonal entries ρ_{kk}? How about the off-diagonal elements?
b) From the fact that $|\alpha\rangle$ obeys the S. eqtn, $H|\alpha\rangle = i\hbar \frac{\partial}{\partial t} |\alpha\rangle$, shew that $\underset{\sim}{\rho}$ obeys the "eqtn of motion": $i\hbar \frac{\partial}{\partial t} \underset{\sim}{\rho} = [\underset{\sim}{H}, \underset{\sim}{\rho}]$. Here $\underset{\sim}{H}$ is the matrix rep$^{\underline{n}}$ of H w.r.t. the basis $|n\rangle$.
c) Suppose the state $|\alpha\rangle$ corresponds to a wavefcn $\psi_\alpha(x)$ in the coordinate rep$^{\underline{n}}$. What is $\underset{\sim}{\rho}$ in the coordinate rep$^{\underline{n}}$? (Hint: the form of $\underset{\sim}{\rho}$ in cd. rep$^{\underline{n}}$ is what gives ρ its name).