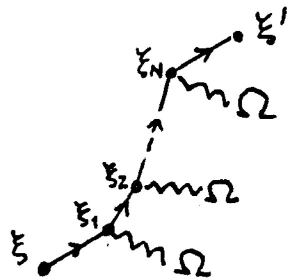
④ It is easy to generalize these results to N (∞smal) scatterings. The i^{th} scattering takes place at ξ_i and lasts for time Δt_i, $1 \leq i \leq N$. With time ordering implicit, i.e. $t < t_1 < t_2 < \cdots < t_N < t'$, $\psi(\xi')$ in (31) picks up new terms...



Generalization to N scatterings and continuous scattering.

$\psi(\xi') = i\int dx\, G(\xi',\xi)\psi_0(\xi)$,

w/

$$G(\xi',\xi) = G_0(\xi',\xi) + \sum_{i=1}^{N}\int dx_i\,\Delta t_i\, G_0(\xi',\xi_i)\underbrace{\Omega(\xi_i)}_{\Theta(\Omega)} G_0(\xi_i,\xi) +$$

$$+\sum_{j=1}^{N}\sum_{i=1}^{N}\int dx_j\,\Delta t_j\int dx_i\,\Delta t_i\, G_0(\xi',\xi_j)\Omega(\xi_j)G_0(\xi_j,\xi_i)\underbrace{\Omega(\xi_i)}_{\Theta(\Omega^2)}G_0(\xi_i,\xi) +$$

$$\sum_{k=1}^{N}\sum_{j=1}^{N}\sum_{i=1}^{N}\int dx_k\,\Delta t_k\int dx_j\,\Delta t_j\int dx_i\,\Delta t_i\, G_0(\xi',\xi_k)\Omega(\xi_k)G_0(\xi_k,\xi_j)\cdot$$

$$\cdot\,\Omega(\xi_j)G_0(\xi_j,\xi_i)\underbrace{\Omega(\xi_i)}_{\Theta(\Omega^3)}G_0(\xi_i,\xi) + \ldots \qquad (33)$$

And so on. We get a series of terms, containing higher and higher powers of the interaction Ω, and representing higher-order multiple scatterings of $\psi_0(\xi)$ enroute from $\xi=(x,t)$ to $\xi'=(x',t')$.

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⑤ The scatterings represented by the terms in $G(\xi',\xi)$ of (33) are <u>discrete</u>-- scattering at ξ_i takes place in Δt_i at t_i (with $t<t_i<t'$), and there could be a period of time from t_i to the next interaction at t_{i+1} when there is no interaction at all. To fill in the gaps, we pass to the limit of a <u>continuous</u> interaction by Ω over the entire path $\xi\to\xi'$. As follows...

Keep initial and final points ξ and ξ' fixed. Let $N\to\infty$ in Eq.(33). Then $\Delta t_i \to dt_i$ (time ∞smal), and $\sum_i \Delta t_i \to \int dt_i$. Use notation: $\sum_i \int dx_i\,\Delta t_i \to \int dx_i \int dt_i = \int d\xi_i$. Replace indices $i,j,k\ldots$ by $1,2,3,\ldots$

$$\boxed{\begin{aligned} G(\xi',\xi) &= G_0(\xi',\xi) + \int d\xi_1\, G_0(\xi',\xi_1)\Omega(\xi_1)G_0(\xi_1,\xi) + \\ &+\int d\xi_2\int d\xi_1\, G_0(\xi',\xi_2)\Omega(\xi_2)G_0(\xi_2,\xi_1)\Omega(\xi_1)G_0(\xi_1,\xi) + \\ &+\int d\xi_3\int d\xi_2\int d\xi_1\, G_0(\xi',\xi_3)\Omega(\xi_3)G_0(\xi_3,\xi_2)\Omega(\xi_2)G_0(\xi_2,\xi_1)\Omega(\xi_1)G_0(\xi_1,\xi) + \\ &+\ldots\ \Theta(\Omega^4),\ \text{etc.}\end{aligned}} \qquad (34)$$

Each integral obeys the time-ordering $t\leq t_1\leq t_2\leq\ldots\leq t_n\leq t'$.

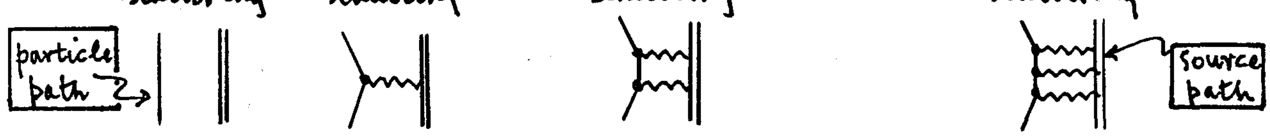
$\boxed{\text{REMARKS}}$ on propagator $G(\xi',\xi)$ of Eq. (34).

$\boxed{1}$ The "scattered" wavefcn: $\psi(\xi') = i\int dx\, G(\xi',\xi)\,\psi_0(\xi)$, together with $G(\xi',\xi)$ of Eq. (34), is a solution to the Schrödinger problem: $(\mathcal{H}_0' + \hbar\Omega')\psi(\xi') = i\hbar\frac{\partial}{\partial t'}\psi(\xi')$. All the dynamics of the operator $\{\mathcal{H}_0(\text{free}) + \hbar\Omega - i\hbar\frac{\partial}{\partial t}\}$ is incorporated in the propagator $G(\xi',\xi)$. Then, remarkably, the solution $\psi(\xi')$ can be generated from a <u>free</u>-particle state $\psi_0(\xi)$, with $G(\xi',\xi)$ expressible in terms of the interaction $\Omega(\xi)$ and <u>free</u>-particle propagators $G_0(\xi';\xi)$.*

$\boxed{2}$ The series for G in Eq. (34) can be written symbolically as...

$$G = G_0 + \int G_0\Omega G_0 + \iint G_0\Omega G_0\Omega G_0 + \iiint G_0\Omega G_0\Omega G_0\Omega G_0 + \cdots \qquad (35)$$

0th order scattering — 1st order scattering — 2nd order scattering — 3rd order scattering



(no exchange) — (exchange of 1 quantum) — (exchange of 2 quanta) — (exchange of 3 quanta)

The diagrams are elementary forms of the celebrated "Feynman diagrams", where the wavy lines each represent one coupling via $\Omega(\xi)$, during which the particle and source can exchange an interaction quantum (for EM interactions, the quantum is a photon). The exchanges are <u>not</u> localized at one space-time point -- the integrals $\int \leftrightarrow \int d\xi = \int dx \int dt$ add up contributions all along the particle's path.

$\boxed{3}$ When G of (35) is used in $\psi = i\int dx\, G\,\psi_0$, we get a series of terms for ψ, in powers Ω^n, with $n = 0, 1, 2, 3, \ldots$. This is, in effect, a <u>perturbation series</u> for ψ, with the n^{th} order dgm in (35) $\leftrightarrow$ n^{th} order perturbation. If Ω is "weak" (e.g. if $\Omega \ll$ particle energy), then the series will converge, and just the first few terms ought to give a good approximation to ψ.

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* $G(\xi',\xi)$ of Eq. (34) replaces solving Eq. (15) [pt. source eqtn] or evaluating Eq. (A5) [sum over states].