2) $L = T = \frac{1}{2} m_1(\dot{r}_1^2 + r_1^2 \dot{\theta}_1^2) + \frac{1}{2} m_2 (\dot{r}_2^2 + r_2^2 \dot{\theta}_2^2)$

$$L_1 = m_1 r_1^2 \dot{\theta}_1 \qquad\qquad r_1 + r_2 = \ell$$
$$L_2 = m_2 r_2^2 \dot{\theta}_2 \qquad\qquad \dot{r}_1 = -\dot{r}_2$$

a) $$E = \underbrace{\frac{1}{2}(m_1+m_2)\dot{r}_1^2}_{\text{"T"}} + \underbrace{\frac{L_1^2}{2m_1 r_1^2} + \frac{L_2^2}{2m_2 r_2^2}}_{\text{"V"}}$$

Equilibrium when $\frac{\partial \text{"V"}}{\partial r_1} = 0$

$$\Rightarrow \frac{L_1^2}{m_1 r_1^3} = \frac{L_2^2}{m_2(\ell - r_1)^3}$$

$$\frac{r_1}{\ell - r_1} = \frac{L_1^2}{L_2^2}\frac{m_2}{m_1} \left(\equiv A \text{ for short}\right)$$

b) $$\boxed{r_1 = \frac{\ell A}{1+A}} \qquad r_2 = \frac{\ell}{1+A}$$

c) $$T = \frac{2\pi}{\omega}, \quad \omega = \sqrt{\frac{(\partial^2 \text{"V"}/\partial r_1^2)}{m_1 + m_2}}.$$

$$\frac{\partial^2 \text{"V"}}{\partial r_1^2} = \frac{3L_1^2}{m_1 r_1^4} + \frac{3L_2^2}{m_2(\ell - r_1)^4}$$
$$= \frac{3L_1^2}{m_1}\left[\left(\frac{1+A}{A\ell}\right)^4 + \left(\frac{1+A}{\ell}\right)^4\right]$$

d) $$\tau = \frac{L_1^2}{m_1 r_1^3} = \frac{L_2^2}{m_2(\ell - r_1)^3} = \frac{L_1^2}{m_1 \ell^3}\left(\frac{1+A}{A}\right)^3$$

e) $$\vec{F} = \vec{F}_1 + \vec{F}_2 = 2\tau \sin\left(\frac{\theta_1 - \theta_2}{2}\right)$$