A point charge ***q*** is located in free space at distance ***d*** from the center of a dielectric sphere of radius ***a*** (***a***<***d***) and dielectric constant ε.

(a) Use *appropriate boundary conditions* to verify which one of the following functions is a suitable Green's function describing the electrostatic potential in this problem (Only one function is correct – all others are false).

$$G^{(1)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y_{lm}^{*}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\text{outside} & \dfrac{r_<^l}{r_>^{l+1}}-\dfrac{l(\varepsilon-1)}{[l(\varepsilon+1)-2]}\dfrac{a^{2l+1}}{\left(r_>r_<\right)^{l+1}}\\ \text{inside } (r<a, r'>a) & \dfrac{2(l-1)}{[l(\varepsilon+1)-2]}\dfrac{r'^l}{r^{l+1}}\end{cases}$$

$$G^{(2)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y_{lm}^{*}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\text{outside} & \dfrac{r_<^l}{r_>^{l+1}}-\dfrac{l(\varepsilon-1)}{[l(\varepsilon+1)-1]}\dfrac{a^{2l+1}}{\left(r_>r_<\right)^{l+1}}\\ \text{inside } (r<a, r'>a) & \dfrac{2l-1}{[l(\varepsilon+1)-1]}\dfrac{r^l}{r'^{l+1}}\end{cases}$$

$$G^{(3)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y_{lm}^{*}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\text{outside} & \dfrac{r_<^l}{r_>^{l+1}}-\dfrac{l(\varepsilon-1)}{[l(\varepsilon+1)+1]}\dfrac{a^{2l+1}}{\left(r_>r_<\right)^{l+1}}\\ \text{inside } (r<a, r'>a) & \dfrac{2l+1}{[l(\varepsilon+1)+1]}\dfrac{r^l}{r'^{l+1}}\end{cases}$$

$$G^{(4)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y_{lm}^{*}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\text{outside} & \dfrac{r_<^l}{r_>^{l+1}}-\dfrac{l(\varepsilon+1)}{[l(\varepsilon-1)-1]}\dfrac{a^{2l+1}}{\left(r_>r_<\right)^{l+1}}\\ \text{inside } (r<a, r'>a) & \dfrac{2l+1}{[1-l(\varepsilon-1)]}\dfrac{r^l}{r'^{l+1}}\end{cases}$$

(b) Calculate the electrostatic field near the center of the sphere up to and including the $l=1$ terms. Verify your answer in the limits $\varepsilon\to 1$ and $\varepsilon\to\infty$. (Hint: $\theta'=0$)

(c) Calculate the *l=2* correction to the el. field in Cartesian coordinates.