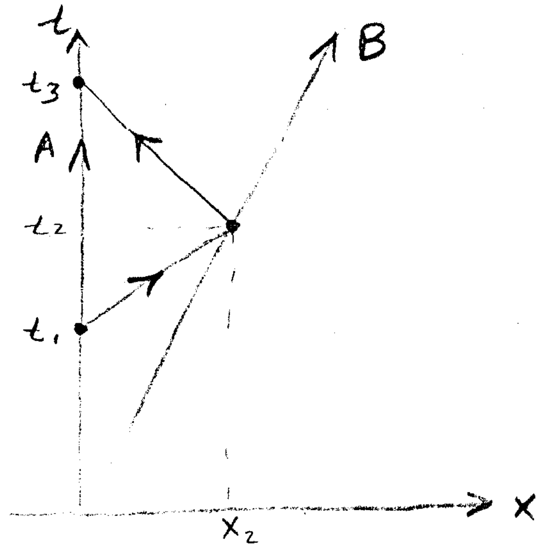
8)

View the system in the rest frame of A. The velocity of B is the relative velocity

$$v = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2} = \frac{3/5 + 4/5}{1 + 12/25} = \frac{35}{37} c$$



$t_1 = 1$ sec.

To find the location of (x_2, t_2):

$$\begin{cases} x_2 = v t_2 \\ x_2 = c(t_2 - 1) \end{cases}$$

$$\Rightarrow \quad t_2 = \frac{1}{1 - v/c} \qquad x_2 = \frac{v}{1 - v/c}$$

In the rest frame of B, the proper time is less than t_2 by a factor $1/\gamma$

$$t_2' = t_2 \sqrt{1 - \frac{v^2}{c^2}} = \boxed{\sqrt{\frac{1 - v/c}{1 + v/c}} = \frac{1}{6} \text{ sec}} \quad (a)$$

$$t_3 - t_2 = t_2 - t_1 \Rightarrow t_3 = t_1 + 2(t_2 - t_1)$$

$$t_3 = \boxed{\frac{c+v}{c-v} = 36 \text{ sec}} \qquad (c)$$

B sees the light redshifted by:

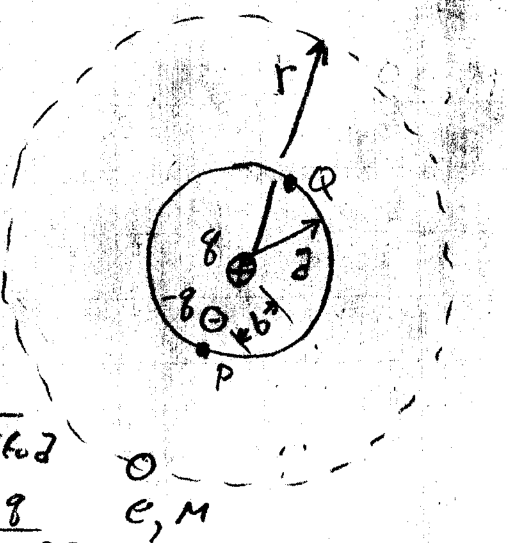
$$f_1 = f_0 \sqrt{\frac{c-v}{c+v}} = \boxed{\frac{f_0}{6}} \qquad (b)$$

The returning pulse is again redshifted by the same factor:

$$f_2 = f_0 \left(\frac{c-v}{c+v}\right) = \boxed{\frac{f_0}{36}} \qquad (d)$$

?H5 Solution B

(a) To make surface of sphere an equipotential, it is enough to have points P and Q at the same potential:



$$V_P = \frac{e}{4\pi\epsilon_0(r-a)} - \frac{q}{4\pi\epsilon_0(a-b)} + \frac{q}{4\pi\epsilon_0 a}$$

$$= V_Q = \frac{e}{4\pi\epsilon_0(r+a)} - \frac{q}{4\pi\epsilon_0(a+b)} + \frac{q}{4\pi\epsilon_0 a}$$

Can set $V_P = V_Q = \frac{q}{4\pi\epsilon_0 a}$, so then $q = \frac{(a-b)}{(r-a)}e = \frac{a+b}{r+a}e$

or $(a-b)(r+a) = (a+b)(r-a)$, or $b(r-a+r+a) = a(r+a-r+a)$,
or $\boxed{b = a^2/r}$

$$q = \left(\frac{a-b}{r-a}\right)e = \left(\frac{a-\frac{a^2}{r}}{r-a}\right)e = \frac{a(r-a)}{r(r-a)}e = \boxed{\frac{a}{r}e = q}$$

(b) $$F = \frac{Mv^2}{r} = Mr\omega^2 = \frac{qe}{4\pi\epsilon_0}\left[\frac{1}{(r-b)^2} - \frac{1}{r^2}\right]$$

$$= \frac{e^2 a}{4\pi\epsilon_0 r}\left[\frac{r^2-(r^2-2br+b^2)}{r^2(r-b)^2}\right]$$

? RJS

$$= \frac{e^2 a}{4\pi\epsilon_0 r}\left[\frac{2a^2 + a^4/r^2}{r^2\left(\frac{r^2-a^2}{r}\right)^2}\right] = \frac{e^2 a\,(2a^2 + a^4/r^2)}{4\pi\epsilon_0 r\,(r^2-a^2)^2}$$

$$\boxed{\omega = \frac{e}{r(r^2-a^2)}\sqrt{\frac{2a^3 + a^5/r^2}{4\pi\epsilon_0 M}}}$$

(c) Not inverse-square-law force, so won't obey Kepler's Laws.

Ph. D. Qual. Spr. 1984 (12) 3/84

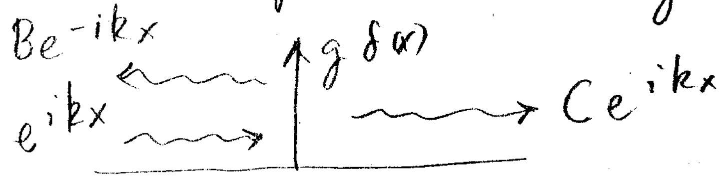
1. Q. Mech. J. Hermanson

Determine the transmission and reflection coefficients for 1D scattering of a particle of mass m from the potential

$$V(x) = g\,\delta(x) \quad ; \quad g = \text{real constant.}$$

[Hint: note the slope discontinuity of the WF at x=0]

Sol'n:



At $x=0$,

a) match value: $1+B=C$ (3) (1)

b) match slope but include discontinuity $\frac{2mg}{\hbar^2}\psi(0)$ —

$$ik(1-B) = ikC - \frac{2mg}{\hbar^2}C \quad (2)\ (3)$$

$$= (ik-\alpha)C \quad , \ \alpha \equiv 2mg/\hbar^2$$

$$= (ik-\alpha)(1+B) \quad \text{From (1)}$$

$$(2ik-\alpha)B = ik - ik + \alpha$$

OK KW

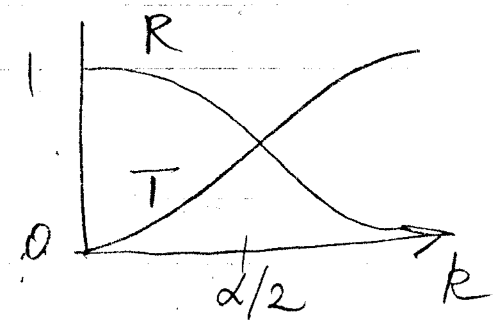
OK RC, perhaps needed hint

$$\begin{cases} B = \dfrac{\alpha}{2ik-\alpha} \\ C = 1+B = \dfrac{2ik}{2ik-\alpha} \end{cases}$$

coefficients

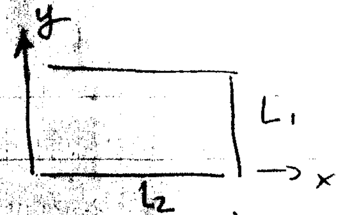
$$R = |B|^2 = \frac{\alpha^2}{4k^2+\alpha^2}$$

$$T = |C|^2 = \frac{4k^2}{4k^2+\alpha^2}$$



(4)

13 Box of dimensions $L_1 \times L_2$



Periodic Boundary condition with zero at wall means

$$\psi(x,y) = A\sin\left(\frac{n\pi x}{L_2}\right)\sin\left(\frac{m\pi y}{L_1}\right)$$

Free particle energy is $-\frac{\hbar^2}{2M}\nabla^2\psi = E_{n,m}\psi$

$$E_{n,m} = \hbar^2/2m\left(\left(\frac{n\pi}{L_2}\right)^2+\left(\frac{m\pi}{L_1}\right)^2\right)$$

$$= \frac{\hbar^2\pi^2}{2m}\left(\left(n/L_2\right)^2+\left(m/L_1\right)^2\right)$$

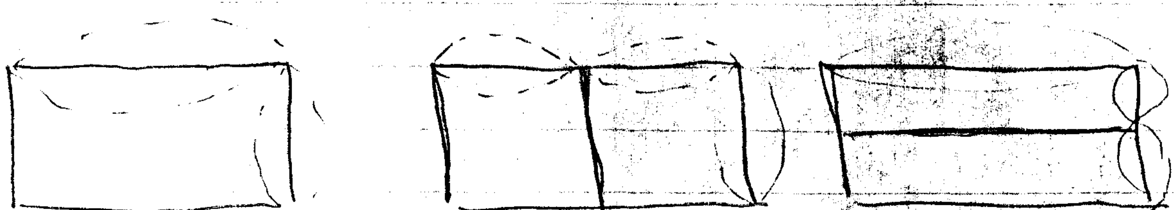
Let $L_2 = \sqrt{5/3}\,L_1 \Rightarrow E_{n,m} = (\text{const})\left(\frac{3}{5}n^2+m^2\right)$

(a)
$E_{1,1} \sim 3/5+1 = 1.6$ ⇐ Ground State
$E_{1,2} \sim 3/5+4 = 4.6$ ⇐ 2st Excited State
$E_{2,1} = 3/5\cdot 4+1 = 3.4$ ⇐ 1st Excited State

$E_{1,3} = 3/5+9 = 9.6$

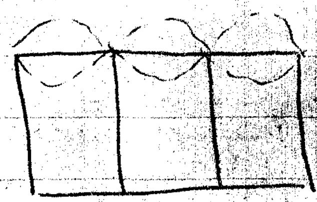
(c)
$E_{3,1} = 3/5\cdot 9+1 = 6.4$ ⇐
$E_{2,2} = 3/5\cdot 4+4 = 6.4$ ⇐ Degenerate 3rd Excited State

(b)

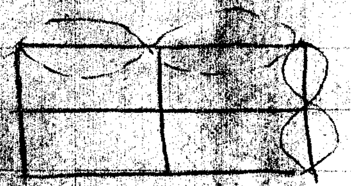


1,1   2,1   1,2

(c) cont'd

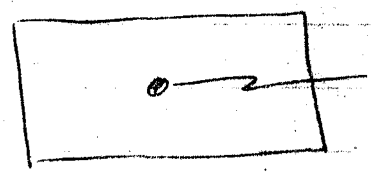


3,1



2,2

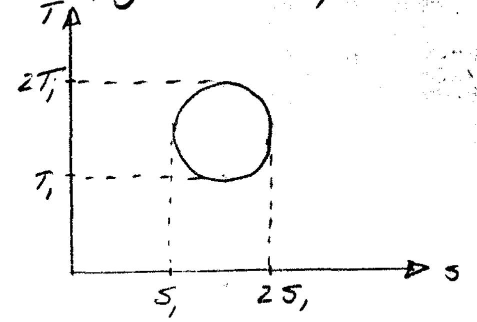
(d) repulsive barrier at center



$V = \infty$

at a node for 2,2 but at an anti node for 3,1 so 3,1 is disturbed most.

Consider a cyclic thermodynamic process in which the working substance traces out the circular path in the T-S (temperature - entropy) plane, shown below



OK KW
OK RC

a) What direction does the system follow around the path if it operates as an engine? (State your reasoning.)

b) What is the net work done in one cycle of the engine, in terms of T_1 and S_1

c) Calculate the thermodynamic efficiency of this engine.

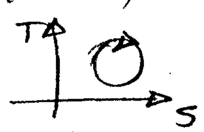
d) Compare your result for c) with the thermodynamic efficiency of a Carnot engine operating between heat reservoirs at $2T_1$ and T_1.

Answers:

a) An engine does positive net work W, where

$$W = \oint p\,dV = \oint (T\,dS - dU) = \oint T\,dS$$ since U (int'l energy) is a state function

For $\oint T\,dS$ to be positive, the direction of travel must be clockwise



b) $W = \oint T\,dS$ = area of circle. Bearing in mind the fact that the horiz & vertical axes have different dimensions,

we find
$$W = \pi \frac{S_1}{2} \cdot \frac{T_1}{2} = \frac{\pi S_1 T_1}{4}$$

c) efficiency $\eta = \dfrac{W}{Q_{in}}$

Now Q_{in} is just the integral $\int T\,dS$ taken over the part of the path for which S is increasing, so

$$Q_{in} = \frac{3T_1}{2} S_1 + \frac{1}{2}\left(\pi \frac{S_1 T_1}{4}\right) = T_1 S_1 \left(\frac{3}{2} + \frac{\pi}{8}\right)$$

$$\eta = \frac{W}{Q_{in}} = \frac{\pi/4 \; S_1 T_1}{S_1 T_1 \left(\frac{3}{2} + \frac{\pi}{8}\right)} = \frac{\pi}{6 + \frac{\pi}{2}} = 0.415$$

d) Carnot effic. $\eta_c = \dfrac{T_{hot} - T_{cold}}{T_{hot}} = \dfrac{2T_1 - T_1}{2T_1} = \dfrac{1}{2}$

$$\frac{\eta}{\eta_{car}} = \underline{0.83}$$

MSU M.S. Comp. -- Nov. 83 #15 3/84 OK Ken ✓ OK - RC 15 Nov. 83

(R. Robiscoe)

Arithmetic Problem

" Sum the infinite series

$$S = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$$ "

Solution

Can write : $S = \sum_{n=1}^{\infty} 1/n(n+1)$.

Consider : $S(x) = \sum_{n=1}^{\infty} x^n/n(n+1)$. Want $S(1) = S$.

Multiply thru by x and differentiate...

$$\frac{d}{dx}[x\,S(x)] = \frac{d}{dx}\sum_{n=1}^{\infty}\frac{x^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty}\frac{x^n}{n} = -\ln(1-x)$$

so $S(x) = -\frac{1}{x}\int \ln(1-x)\,dx + \text{cnst}$

The cnst $= 0$, since $x\,S(x)$ vanishes as $x \to 0$. Then...

$$S(x) = -\frac{1}{x}\int_0^x \ln(1-\xi)\,d\xi \Rightarrow S(1) = -\int_0^1 \ln(1-\xi)\,d\xi$$

or $S = S(1) = -\int_0^1 \ln u\,du = (u - u\ln u)\Big|_{u=0}^{u=1} = 1$

tabulated

so $$\boxed{\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = 1}$$

Dick Smith

Exam #1

KN good

RC ✓ [omit hint?]

Consider a gas of N non-interacting spin-1/2 particles (Fermi gas) in a cube of side L, volume $V = L^3$. Derive the energy distribution of states (density of states) for this gas and determine the energy, E_F, of the most energetic particle at temperature T = 0 (Fermi energy) as a function of (N/V).

?. (Hint: Use periodic boundary conditions on a plane wave representation)

— " —

Free particle : $\psi = Ce^{i\bar{k}\cdot\bar{x}}$

$$\text{PBC} \Rightarrow \quad k_x = \frac{2\pi}{L} n_x \qquad n_x = \text{integer}$$
$$k_y = \frac{2\pi}{L} n_y \qquad n_y = \text{''}$$
$$k_z = 2\pi/L\, n_z \qquad n_z = \text{''}$$

So volume associated with one state is

K-space volume per state $\dfrac{1}{(2\pi/L)^3} \Rightarrow g(k) = (L/2\pi)^3 = \dfrac{V}{(2\pi)^3} \times \underset{\text{spin}}{\underset{\uparrow}{2}}$

Free particle $\Rightarrow E = \dfrac{\hbar^2 k^2}{2m}$

Energy distribution of states: $g(\epsilon)\,d\epsilon = g(k) \underbrace{d^3k}_{\substack{\text{volume}\\ \text{at appropriate } k}} = g(k) \underbrace{4\pi k^2 dk}_{\text{volume of shell, radius } k}$

$$= g(k)\, 4\pi k^2 \frac{dk}{d\epsilon}\, d\epsilon = \frac{V}{(2\pi)^3} 2\, 4\pi k^2 \left(\frac{m}{\hbar^2 k}\right) d\epsilon$$

$$g(\epsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)\frac{1}{k} = \boxed{\frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \epsilon^{1/2}}$$

#1 cont'd

Fill up the states to E_F at $T=0$

$$N = \# \text{ particles} = \int_0^{E_F} g(\epsilon)\,d\epsilon = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{3/2} \cdot 2/3$$

$$\boxed{E_F = \left(3\pi^2 N/V\right)^{2/3} \frac{\hbar^2}{2m}}$$