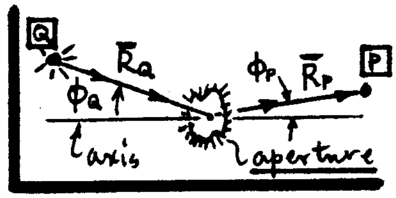
Fraunhofer & Fresnel Diffraction from a plane aperture.

4) For plane apertures, we have by now reduced the diffraction solution to...

$$\left[\begin{array}{l} \psi_k(P) = -ik\,\Theta(\phi_P,\phi_Q)\,\dfrac{1}{\bar{R}_P \bar{R}_Q}\,e^{ik(\bar{R}_P+\bar{R}_Q)}\cdot \underline{\underline{\mathcal{K}_{PQ}}}, \\ \text{w/}\ \underline{\underline{\mathcal{K}_{PQ}}} = \displaystyle\int_{\text{aperture}} \rho\,d\rho\, e^{-ik\Delta(P,Q)},\quad \Delta_{PQ} = a_{PQ}\rho - \tfrac{1}{2}b_{PQ}\rho^2; \\ \text{where}:\ \underline{a_{PQ}} = \sin\phi_P + \sin\phi_Q,\quad \underline{b_{PQ}} = \dfrac{1}{\bar{R}_P}\cos^2\phi_P + \dfrac{1}{\bar{R}_Q}\cos^2\phi_Q,\ \text{\&/}\ \rho \ll R_{P,Q}. \end{array}\right. \quad (16)$$

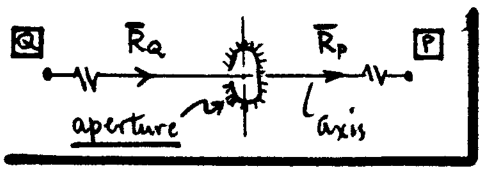


Evidently, Kirchoff's integral $\mathcal{K}_{PQ}$ is more or less complicated depending on whether or not we can ignore the order ρ^2 term in the phase $\Delta(P,Q)$. There are two cases of interest, which depend on the detailed geometry...

① **FRAUNHOFER DIFFRACTION** $\left\{\begin{array}{l} a_{PQ} \neq 0,\ \text{order}\ (\rho/\bar{R})^2 \to 0\ (\text{for}\ \bar{R}\to\infty), \\ \text{so/}\ \Delta(P,Q) \simeq a_{PQ}\rho\ ; \end{array}\right.$ (17A)

② **FRESNEL DIFFRACTION** $\left\{\begin{array}{l} a_{PQ} = 0,\ \text{or order}\ (\rho/\bar{R})^2\ \underline{\text{not}}\ \text{negligible}, \\ \text{so/}\ \Delta(P,Q) \simeq a_{PQ}\rho - \tfrac{1}{2}b_{PQ}\rho^2. \end{array}\right.$ (17B)

$\mathcal{K}_{PQ}$ is less complicated to handle in the Fraunhofer case, but there is an important example where Fresnel <u>can't</u> be avoided. This is the <u>axial problem</u>...



pts P & Q <u>on axis</u> $\Rightarrow$ ϕ_P & $\phi_Q = 0$, so/ $a_{PQ} = 0$ & $b_{PQ} = \dfrac{1}{\bar{R}_P} + \dfrac{1}{\bar{R}_Q}$,

and/ $\mathcal{K}_{PQ} = \displaystyle\int_{\text{aperture}} \rho\,d\rho\, e^{ik(\frac{1}{2}b_{PQ}\rho^2)}$, in Eq. (16). (18)

For $\mathcal{K}_{PQ}$ in (18), let $u^2 = \frac{1}{2}k\,b_{PQ}\rho^2$. Then, for the axial problem, have...

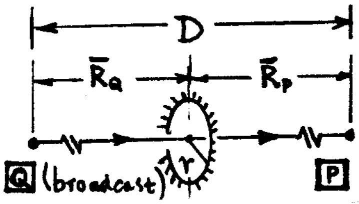
$$\rightarrow \mathcal{K}_{PQ} = \frac{2}{k}\left(\frac{\bar{R}_P\bar{R}_Q}{\bar{R}_P+\bar{R}_Q}\right)\int_{\text{aperture}} u\,du\,e^{iu^2},\quad \text{so/}\quad \boxed{\psi_k(P) = -2i\left[\frac{e^{ik(\bar{R}_P+\bar{R}_Q)}}{\bar{R}_P+\bar{R}_Q}\right]\int_{\text{aperture}} u\,du\,e^{iu^2}} \quad (19)$$

For simplicity, assume a <u>circular aperture</u> here, for the axial problem. Then...

$$\left[\begin{array}{l} 0 \le \rho \le r \Rightarrow 0 \le u^2 \le \tfrac{1}{2}k\,b_{PQ}r^2 = \underline{\underline{u_0^2}},\ \text{circular aperture of radius } r, \\ \text{and/}\ \displaystyle\int_{\text{aperture}} u\,du\,e^{iu^2} = \frac{1}{2}\int_0^{u_0^2} dx\,e^{ix} = -\frac{1}{2i}\left(1-e^{iu_0^2}\right), \\ \text{so/}\ \psi_k(P) = \underbrace{\left(1-e^{iu_0^2}\right)}_{\text{diffractive effect of aperture}}\underbrace{\left[\dfrac{e^{ik(\bar{R}_P+\bar{R}_Q)}}{\bar{R}_P+\bar{R}_Q}\right]}_{\text{freely propagating spherical wave}} \quad (20) \end{array}\right.$$

The aperture has the effect of modifying the otherwise freely propagating wave by the factor $(1-e^{iu_0^2})$ indicated.

Diffractive Effect of a Circular Aperture.



Q & P both on axis.
Circ. aperture : rad. r.
Let : $\bar{R}_p + \bar{R}_a = D$.

5) Look at Eq.(20) in more detail, to better understand the "diffractive effect of aperture". Intensity at P is:

$$\rightarrow I_p \propto |\psi_k(P)|^2 = \frac{1}{D^2}\left|1 - e^{i\underbrace{u_0^2}_{\text{aperture}}}\right|^2, \quad \text{w/} \quad u_0^2 = \frac{k}{2}\left(\frac{Dr^2}{\bar{R}_p \bar{R}_a}\right)$$

or/

$$\underline{\underline{I_p \propto \frac{4}{D^2}\sin^2\left[\frac{k}{4}(Dr^2/\bar{R}_p\bar{R}_a)\right]}} \sqrt{\;} \begin{array}{l} r \ll \bar{R}, \\ k\bar{R} \ll 1. \end{array} \qquad (21)$$

Check some limits for physical sense...

1. no aperture : $r \to 0 \Rightarrow I_P \to 0$... makes sense (P entirely screened from Q).

2. large distances : with $\bar{R}_a \gg r \gg \lambda$ fixed, let $\bar{R}_p \to$ large. Put $k = \frac{2\pi}{\lambda}$.

So/ $$\underline{\underline{I_p \sim \frac{4}{D^2}\left[\frac{k}{4}(Dr^2/\bar{R}_p\bar{R}_a)\right]^2}} = \underbrace{\left(\frac{\pi^2}{\bar{R}_p^2}\right)}_{①}\underbrace{\left(\frac{r^2}{\bar{R}_a^2}\right)}_{②}\underbrace{\left(\frac{r}{\lambda}\right)^2}_{③}. \qquad (22)$$

① factor for normal (geometric) diminution of spherical wave propagating to P;
② factor measuring fraction of wave from Q actually passing thru aperture;
③ <u>diffraction factor</u> (a new toy)... depends on relative size of λ(wave) & r(aperture).

Eq.(22) shows the <u>geometry</u> of diffraction. Its connection w/ <u>interference</u> goes as...

$$\underline{I_p \propto (4/D^2)\sin^2\phi}, \quad \text{w/} \quad \phi = \frac{k}{4}(Dr^2/\bar{R}_p\bar{R}_a) = \left[\frac{\pi r^2}{2\lambda \bar{R}_a}\right]\left(1 + \frac{\bar{R}_a}{\bar{R}_p}\right). \qquad (23)$$

... assume wavelength λ & broadcast location $\bar{R}_a$ are <u>fixed</u>...
... <u>vary</u> reception pt. location $\bar{R}_p$ (along axis). <u>NOTE</u> : $\bar{R}_p = \bar{R}_a / \left[\left(\frac{\lambda \bar{R}_a}{r^2}\right)\frac{2\phi}{\pi} - 1\right]$.

$$\left[\begin{array}{l} \text{get bright spot at P when : } \phi = (n + \tfrac{1}{2})\pi, \\ \;\text{"}\quad \text{dark} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad : \phi = n\pi, \end{array}\right\rangle \underline{n = 0, 1, 2, \ldots}$$

$$\left\{\begin{array}{l} \text{pt. P is bright when : } \bar{R}_p = \bar{R}_a / |(2n+1)\delta - 1|; \\ \text{pt. P is dark when : } \bar{R}_p = \bar{R}_a / |2n\delta - 1|; \end{array}\right\rangle \boxed{\delta = \left(\frac{\bar{R}_a}{r}\right)\frac{\lambda}{r}}. \qquad (24)$$

This intensity alternation at P when $\bar{R}_p$ is varied is certainly an interference effect. BUT, not so simple as just $(\bar{R}_p + \bar{R}_a)$ = integral or half-integral # of λ's. The aperture generates the factor $\delta \propto \lambda/r$, which governs the interference.

ϕ520 : Coming Attractions

21 Feb. 94

DATE	LECTURE	ASSIGNMENT
Mon. 21 Feb.	$\boxed{\text{HOLIDAY}}$ (President's Day)	-
Wed. 23 "	Charged Particle Collisions I (Jk^{n} Ch.13).	-
Fri. 25 "	Charged Particle Collisions II. ↓	#7 Probs. 16- [#6 due].
Mon. 28 Feb.	Charged Particle Collisions III.	-
Wed. 2 Mar.	SRT & Covariance (review) I (Jk^{n} Ch.11).	-
Fri. 4 "	SRT & Covariance (review) II. ↓	no assignment [#7 due].
Mon. 7 Mar.	SRT & Covariance (review) III.	-
Wed. 9 "	SRT & Covariance (review) IV.	-
Fri. 11 "	$\boxed{\text{MID-TERM EXAM}}$ (in class, 2 hrs. open book & notes)*	#8 (due 25 Mar.)
Mon. 14 Mar.	SPRING BREAK	-
Wed. 16 "	" "	-
Fri. 18 "	" "	no assignment
Mon. 21 Mar.	Covariance of Maxwell Eqs.	
Wed. 23 "	Relativistic L & H for EM (Jk^{n} Ch.12)	
Fri. 25 "	Rel^{y}tic L & H for EM : II ↓	#9 (due 1 Apr.) [#8 due].

* The MID-TERM will cover material through lecture of 4 Mar.