# DEPARTMENT OF PHYSICS

## M.S. COMPREHENSIVE / PH. D. QUALIFYING EXAMINATION

## DECEMBER 2, 1985

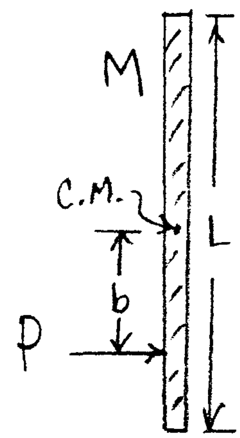
DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE/PH.D. QUALIFYING EXAMINATION

MONDAY, DECEMBER 2, 1985 8-12 AM

Answer each of the following questions, all of which carry equal weight. Begin your answer to each question on a new sheet of paper; solutions to different questions must not appear on the same sheet. Each sheet of paper must be labeled with you name and the problem number, in the upper right-hand corner. When more than one sheet is submitted for a problem, be sure the pages are ordered properly.

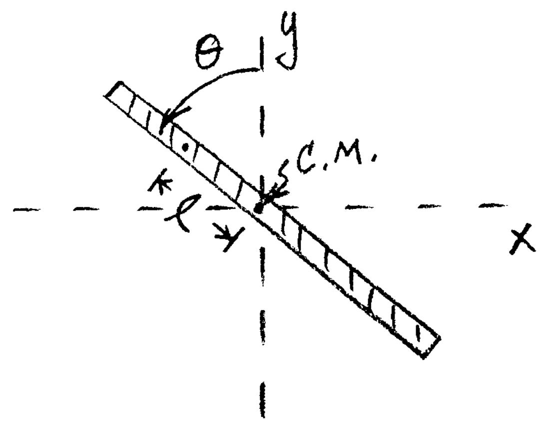
1. A long, uniform, straight stick of mass M and length L is initially at rest on a horizontal, smooth (frictionless) surface. At time $t = 0$, the stick suffers a sharp blow at right angles to its length and at distance b below its CM (center-of-mass), with: $0 < b < L/2$. The sharp blow can be described by an <u>impulse</u>: $P = \int F dt$, acting directly to the right.




A. At $t = 0+$, in what direction is the stick's CM moving?

B. At $t = 0+$, find the translational velocity V of the stick's CM. Also, find the rotational velocity $\omega$ of the stick about its CM.

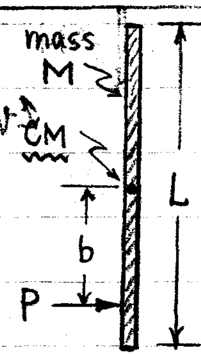
C. Calculate the total energy E of the stick's motion after the impulse. At what value of b are the translational and rotational energies equal?



M.S. Exam : Dec. 1985 (Robiscoe)

CM Problem : Translation-Rotation for a Struck Body. 11/16/85

1\. A long, uniform, straight stick of mass M and length L is initially at rest on a horizontal, smooth (frictionless) surface. At time $t=0$, the stick suffers a sharp blow at right angles to its length and at distance b below its CM (center-of-mass), with : $0 \leq b \leq L/2$. The sharp blow can be described by an <u>impulse</u> : $P = \int F\,dt$, acting directly to the right.

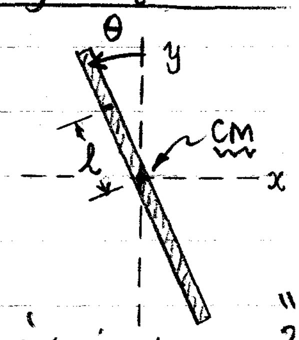



**A.** At $t=0+$, in what direction is the stick's CM moving?

**B.** At $t=0+$, find the translational velocity V of the stick's CM. Also, find the rotational velocity ω of the stick about its CM.

**C.** Calculate the total energy E of the stick's motion after the impulse. At what value of b are the translational and rotational energies equal?

**D.** Define a coordinate system with y-axis along the original direction of the stick, and x-axis along the original direction of the impulse P. After the stick has turned through an angle θ ($0 \leq \theta < \pi/2$), find the speed $v$, <u>relative to the surface</u> over which the stick moves, of a point at distance $l$ above the CM, as shown. Under what conditions does $v=0$?




## Solution

**A.** CM moves according to impressed external force : $M\dot{\mathbb{V}} = \mathbb{F} \Rightarrow M\mathbb{V} = \int \mathbb{F}\,dt = \mathbb{P}$. So <u>CM moves exactly along impulse $\mathbb{P}$</u>, i.e. directly to right. This motion, initiated at $t=0+$, continues ever afterward (on a <u>smooth</u> surface).

**B.** From part A, the CM velocity is : $\underline{\underline{V = P/M}}$, to right. The angular impulse is $bP$, and if $I$ = stick's moment-of-inertia about its CM, this results in an ∢ velocity ω such that : $I\omega = bP \Rightarrow \underline{\underline{\omega = bP/I}}$.

(over)

**C.** The total energy of the motion after impulse is

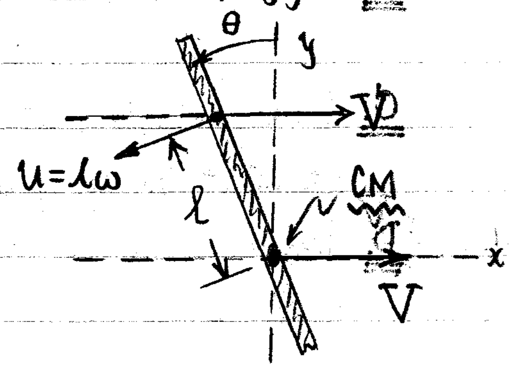
$$\boxed{E = \underbrace{\tfrac{1}{2}MV^2}_{\text{translational}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{rotational}} = \frac{P^2}{2M}\Big[\underbrace{1}_{\text{trans.}} + \underbrace{(Mb^2/I)}_{\text{rotn}}\Big]}.$$

The rotational energy and translational energy are equal when

$$Mb^2/I = 1 \Rightarrow \underline{\underline{b = \sqrt{I/M}}}.$$

For a uniform stick of length $L$ & mass $M$: $I_{cm} = \frac{1}{12}ML^2$, and this relation yields: $b = L/2\sqrt{3}$ = stick's radius of gyration.

**D.** All points on the stick move over the surface with at least the CM velocity $V = P/M$. The points which are rotating, such as point at $\ell$ as shown, also have a tangential velocity $\underline{u = \ell\omega}$, whose components w.r.t. surface are




$$\left.\begin{aligned} v_x &= V - u\cos\theta \\ v_y &= -u\sin\theta \end{aligned}\right\} \text{ so: } v = \sqrt{v_x^2 + v_y^2} = \left[V^2 + u^2 - 2Vu\cos\theta\right]^{\frac{1}{2}}$$

or// $$\boxed{v = \left[(V-u)^2 + 2Vu(1-\cos\theta)\right]^{\frac{1}{2}}}$$

When $\theta \to 0$ (at $t = 0+$), have $v = V - u$, and can have $v = 0$...

if// $V = u \Rightarrow \frac{P}{M} = \ell\frac{bP}{I} \Rightarrow \underline{\underline{\ell b = I/M}}.$

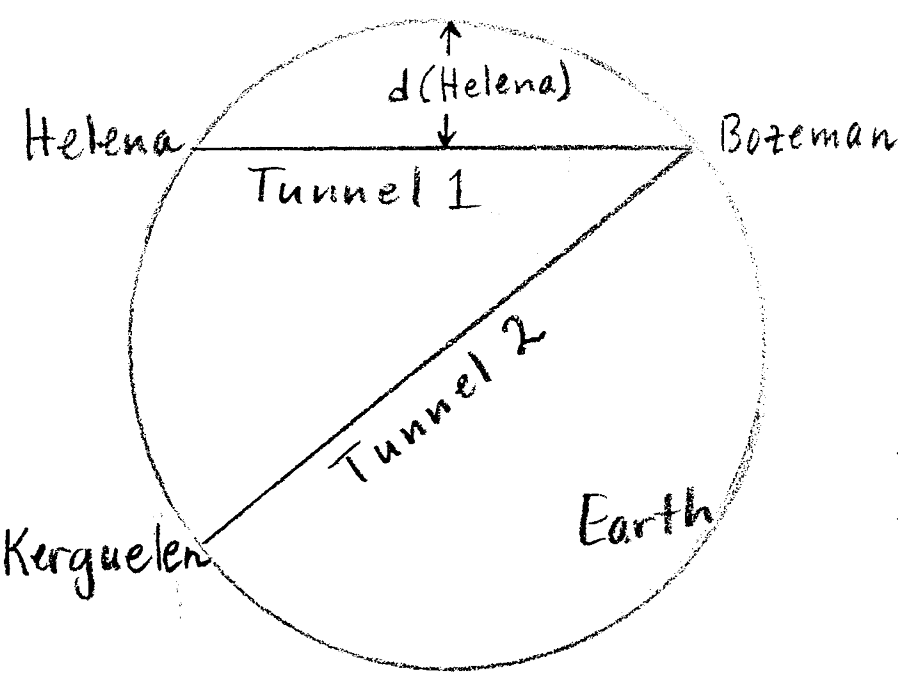
2. The Bozeman city commission is considering building an advanced subway system. Two tunnels will be drilled, along chords (straight lines) through the Earth, one to link Bozeman to Helena, the other to travel through the center of the Earth and on to the Island of Kerguelen in the Southern Indian Ocean (the antipodal point of Bozeman on the Earth's surface). The tunnels will be evacuated (vacuum) and the subway cars will be magnetically levitated using super conductors on their rails. The system is thus entirely frictionless, and driven by gravity: a subway car falls towards the center of the Earth, then glides back to the surface with no expenditure of energy. Assume the Earth is a uniform density sphere; ignore the Earth's rotation.

Evaluate the one-way travel time from Bozeman to:

a) Helena; and

b) Kerguelen

in terms of $\rho$, the density of the Earth, R, the radius of the Earth, and d, the greatest depth the tunnel achieves.




d(Kerguelen) = R

d(Helena) = $10^{-6}$R

Which trip takes less time?

Why?

Classical Mechanics — Bill Hiscock

Bozeman Subway System

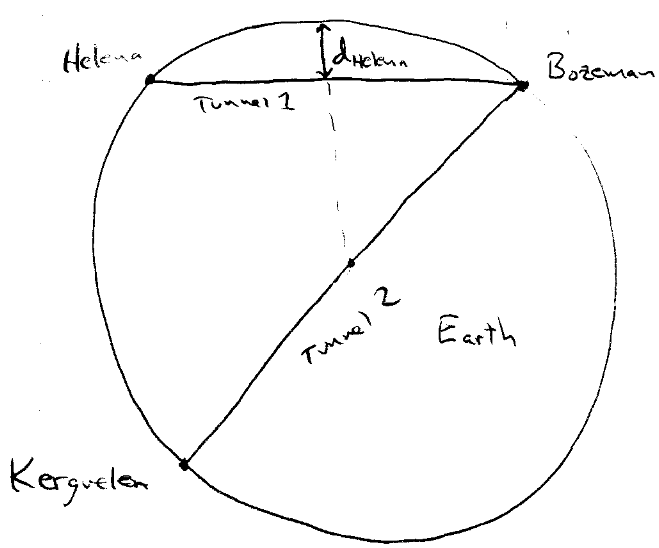
2. The Bozeman city commission is considering building an advanced subway system. Two tunnels will be drilled, along chords (straight lines) through the Earth, one to link Bozeman to Helena, the other to travel through the center of the Earth and on to the island of Kerguelen in the Southern Indian Ocean (the antipodal point of Bozeman on the Earth's surface). The tunnels will be evacuated (vacuum) and the subway cars will be magnetically levitated using superconductors on their rails. The system is thus entirely frictionless, and driven by gravity: a subway car falls toward the center of the Earth, then glides back to the surface with no expenditure of energy. Assume the Earth is a uniform density sphere; ignore the Earth's rotation.

Evaluate the one way travel time from Bozeman to:

(a) Helena; and

(b) Kerguelen

in terms of $\rho$, the density of the Earth, $R$, the radius of the Earth, and $d$, the greatest depth the tunnel achieves.

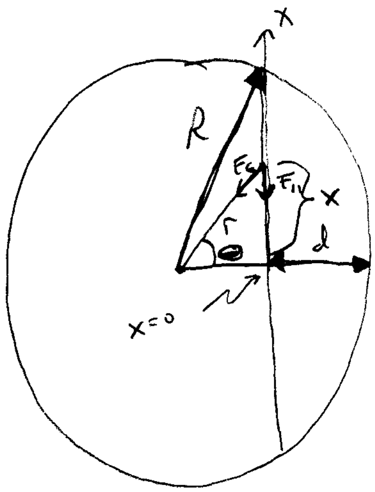



$d_{Kerguelen} = R$

$d_{Helena} = 10^{-6} R$

Which trip takes less time? Why?

Solution




Treat a general tunnel with arbitrary d

subway car mass $= m$

gravitational force on car at radius $r$:

$$F_G = \frac{GM(r)\,m}{r^2} \qquad M(r) = \frac{4}{3}\pi\rho r^3$$

$$= \frac{4}{3}\pi G\rho m\, r$$

component of force along tunnel:

$$F_{\parallel} = F_G\cos\theta = \frac{4}{3}\pi G\rho m\, r\sin\theta$$

Let $x$ be the distance along the tunnel from the deepest point. Then

$$x = r\sin\theta \qquad r = x/\sin\theta$$

$$F_x = -F_{\parallel} = -\frac{4}{3}\pi G\rho \cancel{m}\, x = \cancel{m}\ddot{x} \qquad \ddot{x} = -\frac{4}{3}\pi G\rho\, x$$ <u>Harmonic oscillator</u>

$$x = x_0\cos\omega t \qquad \omega = \sqrt{\frac{4}{3}\pi G\rho}$$

one way trip: $\omega t$ goes from 0 to $\pi$: $\omega T = \pi$;

$$T = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{\frac{4}{3}\pi G\rho}} = \boxed{\sqrt{\frac{3\pi}{4G\rho}} = T}$$

Like any harmonic oscillator, <u>the period is independent of the amplitude</u>; hence a trip to Helena takes the same time as a trip to Kerguelen, or a trip to anywhere else.

3. A point particle of mass m moves in an attractive inverse cube central force:

$$\vec{F} = -\frac{k}{r^3}\hat{r} \qquad . \qquad (k > 0)$$

a) What is the maximum (absolute) value of angular momentum that a particle can have and still reach $r = 0$?

b) For this maximum value, explicitly integrate the equations of motion and find $r(t)$, $\theta(t)$, and the orbit equation, $r(\theta)$.

Hint: Use the effective potential.

Classical Mechanics  Bill Hiscock

---

3. A point particle of mass $m$ moves in an attractive inverse cube central force:

$$\vec{F} = -\frac{k}{r^3}\hat{r} \qquad (k>0)$$

(a) What is the maximum (absolute) value of angular momentum that a particle can have and still reach $r=0$?

(b) For this maximum value, explicitly integrate the equations of motion and find $r(t)$, $\theta(t)$, and the orbit equation, $r(\theta)$.

Hint: Use ~~conservation of energy~~ and the effective potential.

---

Solution:

As in any central force problem, the motion is planar and angular momentum is conserved; in plane polar coordinates

$$\boxed{l = mr^2\dot{\theta}} \; ; \quad \frac{dl}{dt} = 0 \quad \left[\text{One of the Euler-Lagrange equations}\right]$$

Conservation of energy:

$$E = T + V \qquad \frac{dE}{dt} = 0 \qquad V = -\int \vec{F}\cdot d\vec{r} = \int \frac{k}{r^3}\,dr = \frac{-k}{2r^2}$$

$$\boxed{E = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{k}{2r^2}}$$

but

$$\dot{\theta} = l/mr^2$$

$$E = \tfrac{1}{2}m\dot{r}^2 + \tfrac{1}{2}\cancel{mr^2}\frac{l^2}{m^2\cancel{r^4}} - \frac{k}{2r^2} = \tfrac{1}{2}\left[m\dot{r}^2 + \left(\frac{l^2}{m} - k\right)\frac{1}{r^2}\right]$$

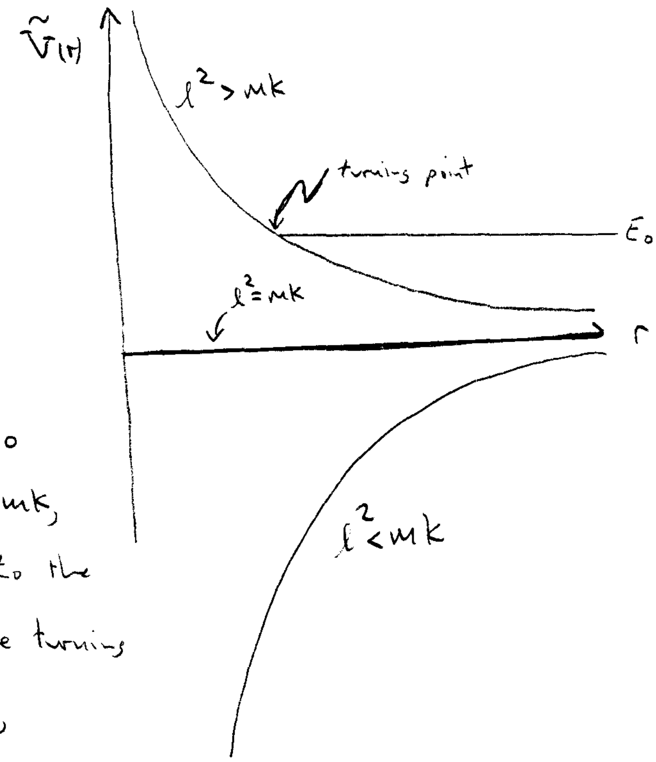
The effective potential, $\tilde{V}(r)$, is

$$\tilde{V}(r) = V(r) + \frac{l^2}{2mr^2} = -\frac{1}{2}\left(k - \frac{l^2}{m}\right)\frac{1}{r^2}$$

Then

$$\tfrac{1}{2}m\dot r^2 = E - \tilde V(r)$$

$$\tilde V(r) = \tfrac{1}{2}\left(\frac{\ell^2}{m} - k\right) r^{-2}$$




(a) As the graph of $\tilde V(r)$ clearly shows, particles can reach $r=0$ only if $\underline{\ell^2 \le mk}$. If $\ell^2 > mk$, then the particle is confined to the region $r \ge r_0$, where $r_0$ is the turning point for a particle of energy $E_0$,

$$r_0 \equiv \left[\frac{\ell^2 - km}{2mE_0}\right]^{\frac{1}{2}}$$

The maximum angular momentum a particle can possess and still reach $r=0$ is then

$$\boxed{\ell^2 = mk \quad \text{or} \quad \ell = \pm\sqrt{mk}}$$

(b) $\ell^2 = mk \Rightarrow \tilde V(r) = 0$

$$\Rightarrow \tfrac{1}{2}m\dot r^2 = E \qquad \dot r = 2E/m \qquad dr = \sqrt{\frac{2E}{m}}\,dt$$

$$\boxed{r = \sqrt{\frac{2E}{m}}(t-t_0)}$$

now integrate the angular momentum equation

$$mr^2\dot\theta = \ell = +\sqrt{mk}$$

$$\cancel{m}\left(\frac{2E}{\cancel{m}}\right)(t-t_0)^2\dot\theta = \sqrt{mk} \qquad d\theta = \frac{\sqrt{mk}}{2E}\frac{dt}{(t-t_0)^2}$$

$$\boxed{\theta = \theta_0 - \frac{\sqrt{mk}}{2E}\frac{1}{(t-t_0)}}$$

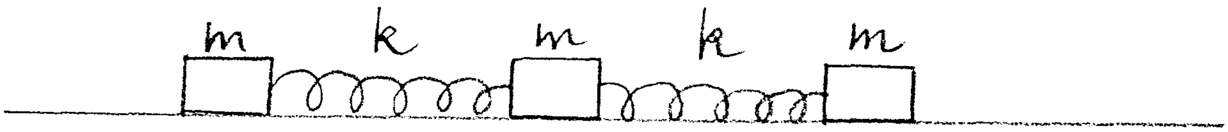
For the orbit equation, choose $\theta_0 = 0$ for convenience

then $\theta = \frac{\sqrt{mk}}{2E}\frac{1}{t-t_0} \Rightarrow t - t_0 = \frac{\sqrt{mk}}{2E}\frac{1}{\theta} \Rightarrow$

$$r = \sqrt{\frac{2E}{\cancel{m}}}\frac{\sqrt{\cancel{m}k}}{2E}\frac{1}{\theta} \Rightarrow \boxed{r = \sqrt{\frac{k}{2E}}\frac{1}{\theta}}$$

or $\boxed{r\theta = \sqrt{\frac{k}{2E}}}$ $\underline{\text{a spiral orbit}}$

4. Three masses are connected to two springs on a frictionless, horizontal track as shown below. If the mass on the left is given an initial displacement A at t = 0, determine the displacement of all masses at t > 0.



Mechanics J. Hermanson

---

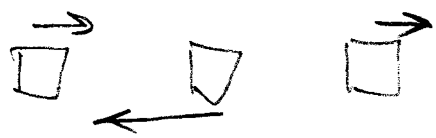
4. Three masses are connected to two springs on a frictionless, horizontal track as shown below. If the mass on the left is given an initial displacement $A$ at $t=0$, determine the displacement of all masses at $t>0$.


---

Sol'n: normal modes are 1)

$$X = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad \omega = \sqrt{\frac{k}{m}}$$

2)



$$X = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \qquad \omega' = \sqrt{\frac{3k}{m}} = 3\omega$$

stretch = $3x$
force = $-3kx$

3) uniform mode $X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\omega = 0$

In gen'l
$$X = a\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\cos\omega t + b\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}\cos\sqrt{3}\,\omega t + c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

[cos chosen so $\dot{X}(0) = 0$]

Then
$$X(0) = \begin{pmatrix} a+b+c \\ -2b+c \\ -a+b+c \end{pmatrix} = A\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\left\{ \begin{aligned} a+b+c &= A \\ c &= 2b \\ a &= 3b \end{aligned} \right.$$

Thus $a = \frac{A}{2}$ $b = \frac{A}{6}$ $c = \frac{A}{3}$

$$\frac{X(t)}{A} = \frac{1}{2}\begin{pmatrix}1\\0\\-1\end{pmatrix}\cos\omega t + \frac{1}{6}\begin{pmatrix}1\\-2\\1\end{pmatrix}\cos\sqrt{3}\,\omega t + \frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\boxed{\begin{aligned}\frac{x_1}{A} &= \frac{1}{2}\cos\omega t + \frac{1}{6}\cos\sqrt{3}\,\omega t + \frac{1}{3}\\ \frac{x_2}{A} &= \qquad\qquad\quad -\frac{1}{3}\cos\sqrt{3}\,\omega t + \frac{1}{3}\\ \frac{x_3}{A} &= -\frac{1}{2}\cos\omega t + \frac{1}{6}\cos\sqrt{3}\,\omega t + \frac{1}{3}\end{aligned}}$$

ch: $x_1 + x_2 + x_3 = A$

$$\bar{x} = \frac{m x_1 + m x_2 + m x_3}{3m} = \frac{A}{3}$$

So the c.m. remains at its initial value $\frac{A}{3}$ ✓

5. Three matrices $M_x$, $M_y$, $M_z$, each with 64 rows and columns, are known to obey the commutation rules $[M_x, M_y] = iM_z$ (with cyclic permutations of x,y,z). The eigenvalues of one matrix, say $M_x$, are $\pm 2$ (each occurs once), $\pm$ 3/2 (each occurs four times), $\pm$ 1 (each occurs nine times), $\pm$ 1/2 (each occurs nine times), and 0 (occurs 18 times). What are the 64 eigenvalues of the matrix $M^2 = M_x^2 + M_y^2 + M_z^2$?

Pre-lim. Quantum Mechanics Drumheller

5. Three matrices $M_x, M_y, M_z$, each with 64 rows and columns, are known to obey the commutation rules $[M_x, M_y] = i M_z$ (with cyclic permutations of $x, y, z$). The eigenvalues of one matrix, say $M_x$, are $\pm 2$ (each occurs once), $\pm 3/2$ (each occurs four times), $\pm 1$ (each occurs nine times), $\pm 1/2$ (each occurs nine times), and $0$ (occurs 18 times). What are the 64 eigenvalues of the matrix $M^2 = M_x^2 + M_y^2 + M_z^2$?

18

Sol'n

Because of the commutation rules, M's are angular momentum matrices.

Take the highest eigenvalue of $M_z$, viz. ~~3~~ 2. It must belong to $J=2$. There is only one.

It also is part of the group $M_x = \pm 1$. Take it away and there are 8 left. These belong to $J=1$.

Take the $J=2$ and $J=1$ M's away from $M_x=0$ and there are 9 left so there are nine $J=0$'s.

Similarly there are 4 $J=\frac{3}{2}$'s and 5 $J=\frac{1}{2}$'s.

Table.

| $J$ | # of each | degeneracy $2J+1$ | # of $M=M_J$ | $M^2=M_J^2$ | $J(J+1)$ |
|---|---|---|---|---|---|
| 2 | 1 | 5 | 5 | 4 | 5 |
| $\frac{3}{2}$ | 4 | 4 | 16 | $\frac{9}{4}$ | $\frac{15}{4}$ |
| 1 | 8 | 3 | 24 | 1 | 2 |
| $\frac{1}{2}$ | 5 | 2 | 10 | $\frac{1}{4}$ | $\frac{3}{4}$ |
| 0 | 9 | 1 | 9 | 0 | 0 |
| | | | 64 | | |

answer

value of $M^2$

Jack?

6. A three-level system is described by the Hamiltonian

$$H = \begin{pmatrix} 0 & 0 & \varepsilon \\ 0 & 0 & 0 \\ \varepsilon & 0 & 0 \end{pmatrix}$$ in the

basis $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ . If the system is known to be in the

state $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ at $t = 0$, what is its state at $t > 0$? What is its energy?

Q Mechanics J. Hermanson

6. A three-level system is described by the Hamiltonian $H = \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & 0 \\ \epsilon & 0 & 0 \end{pmatrix}$ in the basis $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. If the system is known to be in the state $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ at $t=0$, what is its state at $t>0$? What is its energy?

Sol'n: Let $\psi = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\dot{\psi} = -\frac{i}{\hbar} H\psi = -\frac{i}{\hbar}\begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & 0 \\ \epsilon & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

or $$\begin{pmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} -i\frac{\epsilon}{\hbar}\, c \\ 0 \\ -\frac{i\epsilon}{\hbar}\, a \end{pmatrix}$$ with $\begin{pmatrix} a(0) \\ b(0) \\ c(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Clearly, $\boxed{b = \text{const} = 0}$

Now $$\ddot{a} = -\frac{i\epsilon}{\hbar}\dot{c} = -\frac{i\epsilon}{\hbar}\left(-\frac{i\epsilon}{\hbar}a\right)$$

$$\ddot{a} = -\frac{\epsilon^2}{\hbar^2}a$$

$\Rightarrow \boxed{a = \cos\frac{\epsilon}{\hbar}t}$ satisfies $a(0) = 1$ ✓

Also $\dot{c} = -\frac{i\epsilon}{\hbar} a = -\frac{i\epsilon}{\hbar} \cos\frac{\epsilon}{\hbar}t$

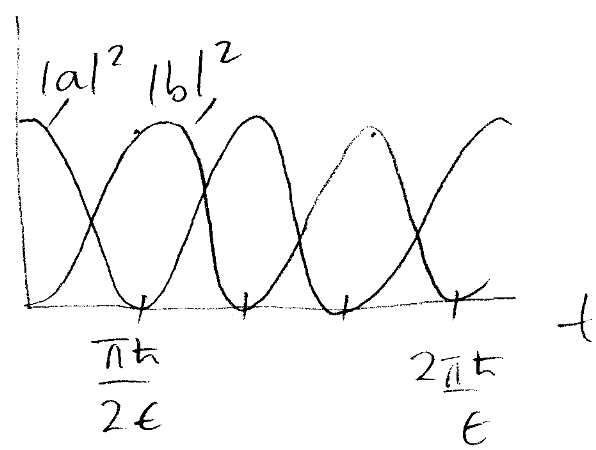
$$\boxed{c = -i \sin \frac{\epsilon}{\hbar} t}$$ satisfies $c(0) = 0$ ✓

Thus $$\boxed{\psi = \begin{pmatrix} \cos\frac{\epsilon}{\hbar}t \\ 0 \\ -i\sin\frac{\epsilon}{\hbar}t \end{pmatrix}}$$ , normalized ✓ $\psi^\dagger \psi = 1$

$$E = \psi^\dagger H \psi = \psi(0)^\dagger H \psi(0) \quad \underline{\underline{t\text{-indep}}}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^\dagger \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & 0 \\ \epsilon & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\boxed{E = 0}$$



7. A wave function $\psi = \psi(\vec{r}, t)$ obeys the Schrödinger equation

$$i\hbar(\partial\psi/\partial t) = [-(\hbar^2/2m)\nabla^2 + V(\vec{r})]\psi,$$

for a particle of mass m in a real potential $V(\vec{r})$. Show that the probability density $\rho = \psi^*\psi$ then obeys a continuity equation

$$\partial\rho/\partial t + \vec{\nabla}\cdot\vec{J} = 0,$$

where $\vec{J}$ is a "probability current" density. Find $\vec{J}$ explicitly in terms of $\psi$ and its derivatives. <u>Also</u>: comment on what happens to $\vec{J}$ when $\psi$ is a purely real function. What kind of quantum-mechanical problems can be done with $\psi$ = pure real? Evaluate $\vec{J} = \rho\vec{v}$ for a one-dimensional ~~plange~~ plane wave. Is $\vec{v}$ the phase velocity of the $\psi$-waves, the group velocity, or neither?

Math Physics

Bill Hiscock

Consider the free particle Schrodinger equation:

$$i\frac{\partial\psi}{\partial t}-\frac{\hbar}{2m}\nabla^2\psi=0 \qquad (1)$$

Show that the probability density, $\rho=\psi^*\psi$, satisfies a fluid-like continuity equation

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0$$

Dirac - note

Give an expression for $\vec{v}$ in terms of $\psi$. Evaluate $\vec{v}$ for a one-dimensional (plane wave) solution to (1). Is $\vec{v}$ the phase velocity of the $\psi$-waves, the group velocity, or neither?

<u>Solution</u>

Schrodinger eqn

$$\frac{\partial\psi}{\partial t}=-i\frac{\hbar}{2m}\nabla^2\psi$$

(Schrodinger eqn)*

$$\frac{\partial\psi^*}{\partial t}=i\frac{\hbar}{2m}\nabla^2\psi^*$$

$$\frac{\partial\rho}{\partial t}=\frac{\partial(\psi^*\psi)}{\partial t}=\psi^*\frac{\partial\psi}{\partial t}+\psi\frac{\partial\psi^*}{\partial t}=-i\frac{\hbar}{2m}\psi^*\nabla^2\psi+i\frac{\hbar}{2m}\psi\nabla^2\psi^*$$

$$=-i\frac{\hbar}{2m}\left(\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right)=-i\frac{\hbar}{2m}\vec{\nabla}\cdot\left[\psi^*\vec{\nabla}\psi+\psi\vec{\nabla}\psi^*\right]$$

so

$$\boxed{\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot\left[i\frac{\hbar}{2m}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)\right]=0}$$

$$\Rightarrow \rho\vec{v}=i\frac{\hbar}{2m}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)$$

$$\boxed{\vec{v}=\frac{i\hbar}{2m(\psi^*\psi)}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)}$$

&

$$\boxed{\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0} \quad ✓$$

Now consider plane wave solutions to (1) : $\psi = \psi(t,x)$

$$i \frac{\partial \psi}{\partial t} - \frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial x^2} = 0 \qquad \text{Let } \psi = \underset{\uparrow \text{ constant}}{\psi_0} e^{i(kx + \omega t)}$$

$$i(i\omega)\psi - \frac{\hbar}{2m}(-k^2)\psi = 0 \qquad \boxed{\omega = \frac{\hbar k^2}{2m}}$$

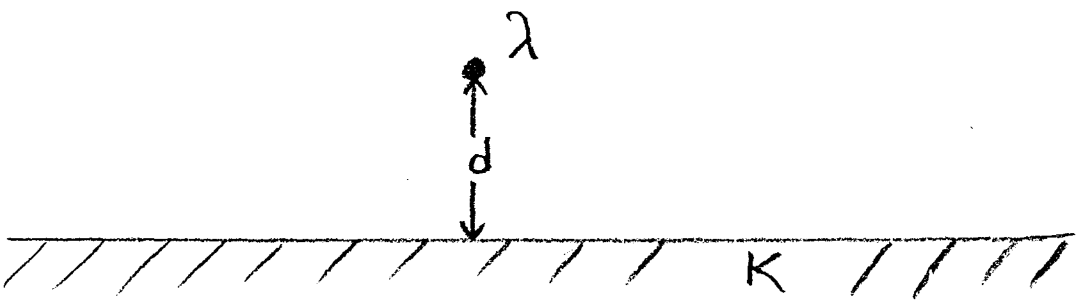
$$V_x = \frac{i\hbar}{2m(\cancel{\psi_0^* \psi_0})}\left\{-ik \cancel{\psi_0^* \psi_0} - ik \cancel{\psi_0^* \psi_0}\right\}$$

$$\boxed{V_x = \frac{\hbar k}{m}} \qquad \text{cf} \quad \text{phase velocity } \frac{\omega}{k} = \frac{\hbar k}{2m}$$

$$\boxed{\text{group velocity } \frac{d\omega}{dk} = \frac{\hbar k}{m}}$$

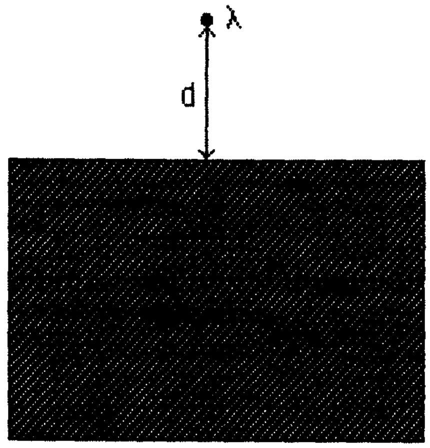
The velocity of probability is the same as the group velocity of the wave function $\psi$.

8. An infinite line of charge with linear charge density $\lambda$ is placed in a vacuum at a distance d above a semi-infinite dielectric block of dielectric constant K. The block fills the lower half-space. Find an expression for the electrostatic potential field in the vacuum and in the dielectric.



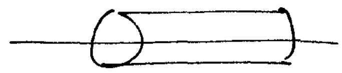
## 3. **A Line Charge Near a Dielectric**

8. An infinite line of charge with linear charge density $\lambda$ is placed in a vacuum at a distance d above a semi-infinite dielectric block of dielectric constant K. The block fills the lower half-space. Find an expression for the electrostatic potential field in the vacuum and in the dielectric.




Solution: see attachment.

③ The field of a line charge is derived simply from Gauss's Law



$$\int E \cdot da = Q/\varepsilon_0$$

$$(2\pi r) L E_r = \lambda L/\varepsilon_0$$

$$E_r = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}$$

By symmetry $\quad \vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0}\dfrac{1}{r}\,\hat{r}.$

Near a dielectric one has two potentials.

$$\phi_V = A \ln r + B \ln r'$$

$$\phi_D = C \ln r$$

where $\quad A = -\dfrac{\lambda}{2\pi\varepsilon_0}, \quad r' = \sqrt{(x+d)^2+y^2}, \quad r = \sqrt{(x-d)^2+y^2}.$

Here we use the <u>method of images</u>.

At $x=0$, $r=r'$ [at the interface]. Then

(i) $\quad \phi_V = \phi_D$

(ii) $\quad D_V = D_D \quad$ (no charge present)

(3-cont) By (*)

$$A + B = C$$

By (**)

$$(\varepsilon_0 E_V)_x = (\varepsilon E_D)_x$$

$$(\vec{E}_V)_x = (-\vec{\nabla}\phi_V)_x = -\frac{\partial}{\partial x}\phi_V = -\frac{\partial r}{\partial x}\frac{d\phi_V}{dr}$$

$$= -\frac{1}{2}\frac{1}{r}\,2(x-d)\,\frac{A}{r} - \frac{1}{2}\frac{1}{r'}\,2(x+d)\,\frac{B}{r'}$$

At $x=0$,

$$(\vec{E}_V)_x = Ad\frac{1}{r^2} - Bd\frac{1}{r^2}$$

$$(\vec{E}_D)_x = -\frac{\partial r}{\partial x}\frac{d\phi_D}{dr} = -\frac{1}{2}\frac{1}{r}\,2(x-d)\cdot\frac{C}{r}$$

At $x=0$

$$(\vec{E}_D)_x = \frac{Cd}{r^2}$$

$$\therefore \quad \varepsilon_0(Ad - Bd) = \varepsilon Cd$$

or

$$A - B = KC$$

$$2A = (K+1)C$$

$$\frac{2}{(K+1)} A = C = -2 \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{(K+1)}$$

$$B = C - A = -2 \frac{\lambda}{2\pi\varepsilon_0} \frac{1}{K+1} + \frac{\lambda}{2\pi\varepsilon_0}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \left[ \frac{2}{K+1} - 1 \right]$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \left[ \frac{1-K}{1+K} \right]$$

$\therefore$

$$\phi_V = -\frac{\lambda}{2\pi\varepsilon_0} \left[ \ln r - \frac{K-1}{K+1} \ln r' \right]$$

$$\phi_D = -\frac{2\lambda}{K+1} \frac{1}{2\pi\varepsilon_0} \ln r$$

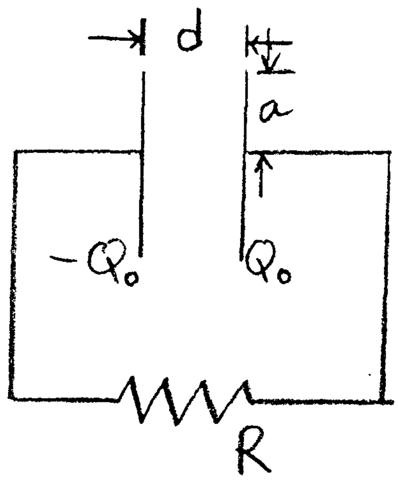
DEPARTMENT OF PHYSICS

M.S. COMPREHENSIVE/PH.D. QUALIFYING EXAMINATION

MONDAY, DECEMBER 2, 1985 1-5 PM

Answer each of the following questions, all of which carry equal weight. Begin your answer to each question on a new sheet of paper; solutions to different questions must not appear on the same sheet. Each sheet of paper must be labeled with you name and the problem number, in the upper right-hand corner. When more than one sheet is submitted for a problem, be sure the pages are ordered properly.

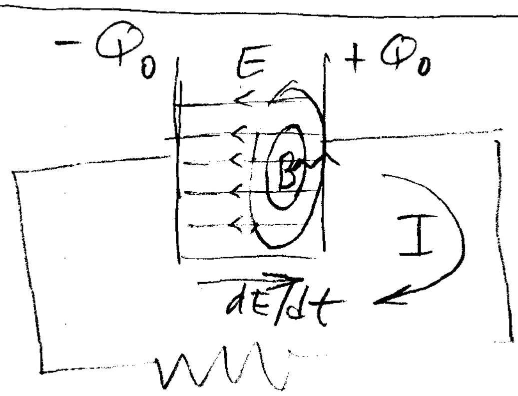
9. A capacitor with parallel, circular plates of radius a, separation d, and initial charge $\pm Q_0$ discharges through a resistor of resistance R. Using SI units, determine the magnetic field anywhere inside the capacitor for $t > 0$ (neglect fringing).



EM Theory  J Hermanson

9. A capacitor with parallel, circular plates of radius a, separation d, and initial charge $\pm Q_0$ discharges through a resistor of resistance R. Using SI units, determine the magnetic field anywhere inside the capacitor for $t>0$ (neglect fringing)

Sol'n:




From $\oint B \cdot dl = \mu_0 (I + I_d)$

$$B \cdot 2\pi r = \mu_0 I_d \quad \text{inside}$$

$$B = \frac{\mu_0 I_d}{2\pi r}$$

But $I_d$ depends on r —

$$I_d = J_d(\pi r^2) = \epsilon_0 \frac{dE}{dt}(\pi r^2)$$

$$= \cancel{\epsilon_0} \frac{d}{dt} \frac{Q}{\cancel{\epsilon_0} A}(\pi r^2) = I \frac{\pi r^2}{A}$$

$$\Rightarrow \quad I_d(r) = I\left(\frac{r^2}{a^2}\right)$$

$$B = \frac{\mu_0 I \, r^{\cancel{2}}/a^2}{2\pi \cancel{r}} = = \frac{\mu_0 I r}{2\pi a^2}$$

Now $I = I_0 \, e^{-t/\tau}$ $\left\{ \begin{array}{l} I_0 = \dfrac{V_0}{R} = \dfrac{Q_0}{RC} \\ \tau = RC \end{array} \right.$

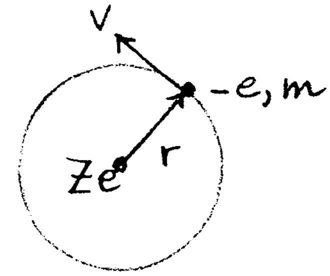
$$= \frac{Q_0}{RC} e^{-t/RC}, \quad C = \frac{\epsilon_0 (\pi a^2)}{d}$$

$$B = \frac{\mu_0 \frac{Q_0}{RC} r e^{-t/RC}}{2\pi a^2}$$

$$= \frac{\mu_0 Q_0 r e^{-t/RC}}{2\pi a^2 R \frac{\epsilon_0 \pi a^2}{d}}$$

$$\boxed{B = \frac{\mu_0 Q_0 d r e^{-t/RC}}{2\pi^2 \epsilon_0 R a^4}}$$

10. An electron (mass m, charge $-e$) is in a circular orbit of radius r about a stationary nucleus of charge Ze. Treat this system classically, and assume the electron velocity $v << c$.



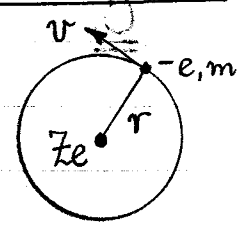

A. Neglect radiative losses, and find the total orbit energy E as a function of r.

B. Now assume the electron radiates an amount of energy $\Delta E$ <u>per orbit</u>, such that $\Delta E << |E|$. Find the radiated power P as a function of r, [assuming, according to the Larmor formula, that: $P = (2e^2/3c^3)|a|^2$, where a is the acceleration of the electron.]

C. Equate P of part B to a rate of loss or orbit energy E of part A. In this way, obtain a differential equation for the rate of decrease of the orbit radius r due to radiation. Calculate the total time for the electron to spiral into the nucleus if it starts from $r = R$.

M. S. Exam : Dec. 1985 (Robiscoe)

E & M Problem : Radiative Collapse of Classical Atom. 11/16/85




10. An electron (mass m, charge $-e$) is in a circular orbit of radius $r$ about a stationary nucleus of charge $Ze$. Treat this system classically, and assume the electron velocity $v \ll c$.

**A.** Neglect radiative losses, and find the total orbit energy $E$ as a function of $r$.

**B.** Now assume the electron radiates an amount of energy $\Delta E$ per orbit such that $\Delta E \ll |E|$. Find the radiated power $P$ as a function of $r$, [assuming, according to the Larmor formula, that : $P = (2e^2/3c^3)|a|^2$, where $a$ is the acceleration of the electron.]*

**C.** Equate $P$ of part B to a rate of loss of orbit energy $E$ of part A. In this way, obtain a differential equation for the rate of decrease of the orbit radius $r$ due to radiation. Calculate the total time for the electron to spiral into the nucleus if it starts from $r = R$.

## Solution

**A.** Centripetal = Coulomb $\Rightarrow \frac{mv^2}{r} = \frac{Ze^2}{r^2} \Rightarrow$ K.E. : $K = \frac{1}{2}mv^2 = \frac{Ze^2}{r}$ ; 2 Right! (ETR)

P.E. = Coulomb potential : $V(r) = (-) Ze^2/r$ ;

So $K + V(r) = \boxed{-Ze^2/2r = E}$, total orbit energy.

**B.** If $\Delta E$ (per orbit) $\ll |E|$, the radius $r$ is ~ cnst in a given orbit, and the electron's centripetal acceleration : $a = v^2/r$ ~ cnst. Then loss rate :

$$\left[\!\left[ P = \frac{2e^2}{3c^3}|a|^2 = \frac{2e^2}{3c^2}\left|\frac{v^2}{r}\right|^2 = \frac{2}{3}\left[\frac{e^2(Ze^2)^2}{m^2c^3}\right]\frac{1}{r^4}. \right]\!\right]$$

We have used $v^2/r = Ze^2/mr^2$ from part A.

(over)

* Specifying the Larmor formula is optional.

C. Per instructions...

$$\underbrace{-\frac{2}{3}\left[\frac{e^2(Ze^2)^2}{m^2c^3}\right]\frac{1}{r^4}}_{\text{radiation loss rate}} = \underbrace{\frac{d}{dt}\left(-\frac{Ze^2}{2r}\right)}_{\text{orbit energy loss rate}} \Rightarrow \boxed{\frac{dr}{dt} = -\frac{4}{3}\left(\frac{Ze^4}{m^2c^3}\right)\frac{1}{r^2}}$$

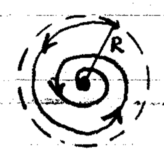
ORBIT DECAY EQTN

Again, this holds when $\Delta E$(per orbit) $\ll |E|$. Integrate to get orbit lifetime from $r=R$ to $r=0$ (at nucleus)...

$$\left[\mathcal{T}(\text{orbit}) = \int_{r=R}^{r=0} dt = \frac{3}{4}\left(\frac{m^2c^3}{Ze^4}\right)\int_0^R r^2\,dr = \frac{1}{4Z}\left(\frac{m^2c^3}{e^4}\right)R^3\right]$$

If we define the classical electron radius : $r_0 = \frac{e^2}{mc^2} = 2.8\times10^{-13}$ cm,

then,
$$\boxed{\mathcal{T}(\text{orbit}) = \frac{1}{4Z}\,\frac{r_0}{c}\left(\frac{R}{r_0}\right)^3}$$




Typically, this is a very small # for classical atom orbits... for hydrogen ($Z=1$, $R \simeq 0.53\times10^{-8}$ cm) ; $\mathcal{T}(\text{orbit}) \simeq 1.6\times10^{-11}$ sec. Life is short!

11. Clarify the following as scalars, pseudoscalars, vectors, and pseudovectors by explicitly demonstrating how the quantity (or components) transform under a coordinate inversion ($x \rightarrow -x$, $y \rightarrow -y$, $z \rightarrow -z$). You may assume that the electrostatic potential $\phi$ is a scalar and the vector potential $\vec{A}$ is a vector.

a. $\vec{E}$, the electric field

b. $\vec{B}$, the magnetic field

c. $\vec{E} \times \vec{B}$, the Poynting vector

d. $\vec{E} \cdot \vec{B}$

Math Physics Bill Hiscock

---

11. Classify the following as scalars, pseudoscalars, vectors, and pseudovectors by explicitly ~~examining~~ demonstrating how the quantity (or components) transform under a coordinate inversion ($x \to -x$, $y \to -y$, $z \to -z$). You may assume that the electrostatic potential $\Phi$ is a scalar and the vector potential $\vec{A}$ is a vector.

(a) $\vec{E}$, the electric field
(b) $\vec{B}$ the magnetic field
(c) $\vec{E} \times \vec{B}$, the Poynting vector
(d) $\vec{E} \cdot \vec{B}$

---

Solution:

(a) $\vec{E} \equiv -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t}$ examine $E_x = -\frac{\partial}{\partial x}\Phi - \frac{\partial A_x}{\partial t}$

under inversion $x' = -x$

$$\frac{\partial}{\partial x} = -\frac{\partial}{\partial x'} \qquad A_x = -A_{x'}$$

$$E_{x'} = -\frac{\partial}{\partial x'}\Phi - \frac{\partial A_{x'}}{\partial t} = \frac{\partial}{\partial x}\Phi + \frac{\partial A_x}{\partial t} = -E_x$$

similarly for $E_y$; $E_z$ : so $\boxed{E_{i'} = -E_i}$

$\Rightarrow$ $\boxed{\vec{E} \text{ is a vector}}$

(b) $\vec{B} = \vec{\nabla} \times \vec{A}$ $\qquad B_x = \frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y$

inversion $\frac{\partial}{\partial y} = -\frac{\partial}{\partial y'}$ $\quad A_z = -A_{z'}$

$\frac{\partial}{\partial z} = -\frac{\partial}{\partial z'}$ $\quad A_y = -A_{y'}$

$$B_{x'} = \frac{\partial}{\partial y'}A_{z'} - \frac{\partial}{\partial z'}A_{y'} = \left(-\frac{\partial}{\partial y}\right)(-A_z) - \left(-\frac{\partial}{\partial z}\right)(-A_y) = \frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y = B_x$$

so $\boxed{B_i = B_{i'}}$ $\Rightarrow$ $\boxed{\vec{B} \text{ is a pseudovector}}$

(c) $\vec{E}\times\vec{B}$ $\qquad (\vec{E}\times\vec{B})_x = E_y B_z - E_z B_y$

$$(\vec{E}\times\vec{B})_{x'} = E_{y'} B_{z'} - E_{z'} B_{y'}$$

from (a),(b) $\quad E_{y'} = -E_y \qquad B_{y'} = B_y$

$$E_{z'} = -E_z \qquad B_{z'} = B_z$$

$$\Rightarrow (\vec{E}\times\vec{B})_{x'} = (-E_y)B_z - (-E_z)B_y = -E_y B_z + E_z B_y = -(\vec{E}\times\vec{B})_x$$

so $\boxed{(\vec{E}\times\vec{B})_{i'} = -(\vec{E}\times\vec{B})_i}$

The Poynting vector is a vector

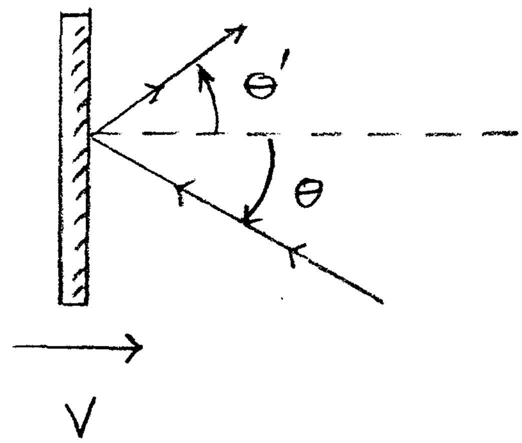
(d) $(\vec{E}\cdot\vec{B}) = E_x B_x + E_y B_y + E_z B_z$

$$(\vec{E}\cdot\vec{B})' = E_{x'} B_{x'} + E_{y'} B_{y'} + E_{z'} B_{z'}$$

using (a) & (b) $\quad (\vec{E}\cdot\vec{B})' = -E_x B_x - E_y B_y - E_z B_z = -(\vec{E}\cdot\vec{B})$

$\Rightarrow \quad \boxed{\vec{E}\cdot\vec{B} \text{ is a pseudoscalar}}$

12. A mirror moves perpendicular to its plane with velocity V in the lab frame. A photon strikes the mirror at an angle $\theta$ to the normal in the lab frame. At what angle $\theta'$ to the normal is the photon reflected in the lab frame? What is the value of $\theta'$ in the limit $V \rightarrow C$?



But not, I think, hard: just 2 Lorentz transforms and a bit of reasoning

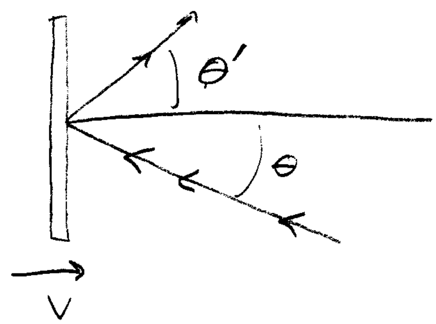
Special Relativity - Harder

Relativistic Law of Reflection

Bill Hiscock

12. A mirror moves perpendicular to its plane with velocity $V$ in the lab frame. A photon strikes the mirror at an angle $\theta$ to the normal in the lab frame. At what angle $\theta'$ to the normal is the photon reflected in the lab frame?

What is the value of $\theta'$ in the limit $V \to c$?

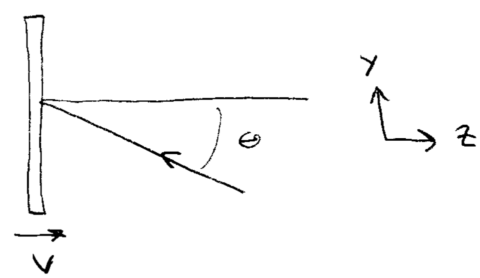



should we provide "hint" that Newtonian law of reflection holds in mirror rest frame?

<u>Solution</u> set up coordinates as follows:

mirror in x-y plane

photon in y-z plane




$\vec{V} = V\vec{e}_z$

Set $c = 1$

Four momentum of photon in lab frame $(t, x, y, z)$

$$\vec{p}_{in} = (E,\ 0,\ E\sin\theta,\ -E\cos\theta)$$

Lorentz Transform to mirror frame $(t', x, y, z')$

$$\vec{p}_{in} = [\gamma(E + VE\cos\theta),\ 0,\ E\sin\theta,\ \gamma(-E\cos\theta - VE)]$$

In the <u>Mirror frame</u>, the law of reflection holds: $\theta_{out} = \theta_{in}$, or

$p_z^{out} = -p_z^{in}$ ; so

$$\vec{p}_{out} = [\gamma E(1 + V\cos\theta),\ 0,\ E\sin\theta,\ \gamma E(\cos\theta + V)]$$ in mirror frame

now transform back to the lab frame:

$$\vec{p}_{out} = \{ \gamma^2 E[(1+v\cos\theta) + v(\cos\theta+v)],\ 0,\ E\sin\theta,$$
$$\gamma^2 E[(\cos\theta+v) + v(1+v\cos\theta)] \}$$

This is also equal to

$$\vec{p}_{out} = (E',\ 0,\ E'\sin\theta',\ E'\cos\theta')$$

so
$$\frac{p^z_{out}}{p^t_{out}} = \cos\theta' = \frac{\cos\theta+v+v(1+v\cos\theta)}{(1-v\cos\theta)+v(\cos\theta+v)}$$

$$\boxed{\cos\theta' = \frac{(1+v^2)\cos\theta+2v}{1+2v\cos\theta+v^2}}$$

As $v \to 1$ (i.e., $v \to c$)

$$\cos\theta' \to \frac{2\cos\theta+2}{2+2\cos\theta} = 1 \qquad \boxed{\theta' \to 0}$$

13. One mole of an ideal gas expands quasistatically and isothermally at room temperature from 40 liters to 80 liters.

a. What is the entropy change in the gas?

b. What is the entropy change in the universe?

c. Suppose the process were not quasi-static. Discuss both the entropy change of the gas <u>and</u> of the universe.

Ideal gas constant $R = 8.314$ J/mole-K.

13. One mole of an ideal gas expands quasi-statically and isothermally at ~~the~~ room temperature from 40 l to 80 l.

a./ What is the entropy change in the gas?

b./ What is the entropy change in the universe?

c./ Suppose the process were not quasi-static. Discuss both the entropy change of the gas and of the universe.

Ideal gas constant R= 8.314 J/mole-K.

Sol'n.

a./ $S_2 - S_1 = \frac{Q}{T} = \frac{1}{T} W = \frac{1}{T}\int P\,dV$ for isothermal

(since $\Delta Q = \Delta U + \Delta W$ but $\Delta U = 0$ for isothermal)

Ideal gas: $PV = nRT$

$$S_2 - S_1 = \frac{1}{T}\int_1^2 nRT\,\frac{dV}{V} = nR\ln\frac{V_2}{V_1} \qquad n = 1$$

$$S_2 - S_1 = R\ln\frac{V_2}{V_1} = 8.314\,\frac{J}{mol\cdot K}\,1\,mole\,\ln 2$$

$$= 5.763\,\frac{J}{K} \quad \text{for gas}$$

b./ no change in entropy of the universe for a quasi-static process as the system remains in equilibrium throughout. Quasi-static implies reversible process.

c./ Since the final and initial states are the same and since entropy is a state function and therefore depends only on the initial and final states, the entropy of the gas is the same as in (a.), i.e., $5.76\,\frac{J}{K}$.
But, since equilibrium is no longer assumed, the entropy of the universe must have increased.

14. a. The deuteron has total angular momentum $J = 1$. Based on this fact derive its possible spin wave functions.

b. The deuteron has a parity $\eta = +1$. Does this imply anything for its spin wave function? If so, what?

c. What can be said about the electrostatic multipole moments of the deuteron based on parts (a) and (b)?

## 1. The Deuteron

14. (a) The deuteron has total angular momentum **J** = 1. Based on this fact derive its possible spin wave functions.

(b) The deuteron has a parity $\eta = +1$. Does this imply anything for its spin wave function? If so, what.

(c) What can be said about the electrostatic multipole moments of the deuteron based on parts (a) and (b)?

---

Solution:

(a) $J = S_p + S_n + L_{pn}$. $S_p = 1/2$. $S_n = 1/2$. $S = S_n + S_p = 0$ or 1.

$J = 1 \Rightarrow L=0$ and $S = 1$, or $L=1$ and $S = 0$, or $L = 2$ and $S = 1$.

| | | |
|---|---|---|
| S = 0 | $\{ \lvert\uparrow\downarrow\rangle - \lvert\downarrow\uparrow\rangle \}/2$ | Anti-symmetric |
| S = 1 | $\lvert\uparrow\uparrow\rangle$ | Symmetric |
| | $\{ \lvert\uparrow\downarrow\rangle + \lvert\downarrow\uparrow\rangle \}/2$ | |
| | $\lvert\downarrow\downarrow\rangle$ | |

(b) The spatial w.f. parity is $(-1)^L$. $\eta = +1$ implies a symmetric spatial wave function. Thus L = 1 is not allowed and only the S = 1 w.f. above is permitted by the Pauli principle.

(c) No L=1 term implies there should be no dipole moment. The monopole moment is the charge and is not zero. The quadrupole moment ~~may~~ also ~~be~~ is nonzero.

No proof! Keep working.

15. a) Derive the MKS units for thermal conductivity.

b) Discuss briefly the principal mechanism for thermal conductivity in diamond at room temperature.

c) Repeat b), but for copper.

d) Assume that for copper, the conduction electron density N is $7 \times 10^{28}/m^3$, the Fermi velocity $V_F$ is $10^6 m/s$, and the mean free path L is $2.5 \times 10^{-8}m$. If the Fermi temperaure $T_F$ is 45000K and room temperature is taken as 300K, estimate the thermal conductivity of copper at room temperature.

Double Space

Solid State - Hugo Schmidt

---

15. a) Derive the MKS units for thermal conductivity.

b) Discuss briefly the principal mechanism for ~~thepmal~~ thermal conductivity in diamond at room temperature.

c) Repeat b), but for copper.

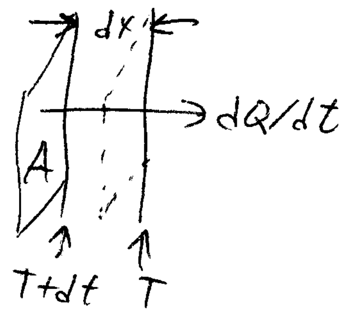
d) Assume that for copper, the ~~free~~ conduction electron density $N$ is $7\times10^{28}/m^3$, the Fermi velocity $v_F$ is $10^6$ m/s, and the mean free path L is $2.5\times10^{-8}$ m. If the Fermi temperature ~~is~~ $T_F$ is 45000K and room temperature is taken as 300K, estimate the thermal conductivity of copper at room temperature.

i.e. "vee"

---

Solid State - Solution - Hugo Schmidt

a) thermal conductivity $= \dfrac{dQ/dt}{A\,dT/dx}$




3 Units $\dfrac{J/s}{m^2\ K/m} = \boxed{\dfrac{J}{m\text{-}s\text{-}K}}$

or $\dfrac{kg\,m^2}{s^2\ m\text{-}s\text{-}k} = \boxed{\dfrac{kg\,m}{s^3 K}}$

2 b) Diamond is an insulator with no free electrons, so propagation and scattering of phonons dominates the thermal conductivity.

2 c) In copper, the thermal conductivity from the propagation and scattering of free electrons is dominant.

3 d) A fraction $\frac{300}{45000} = \frac{1}{150}$ of the electrons will be effective in transferring thermal energy. The mean free time is $\frac{L}{v_F} = \frac{2.5\times10^{-8}m}{10^6 m/s} = 2.5\times10^{-14}s$

The number of electrons effectively transferring energy in a mean free path per $m^2$ area is $\frac{7\times10^{28}}{m^3} \times \frac{1}{150} \times 2.5\times10^{-8} m$ $= 1.17\times10^{19}/m^2$. The energy transfer per electron is $k\Delta T = 1.38\times10^{-23}\frac{J}{K}\frac{dT}{dx}2.5\times10^{-8}m = 3.45\times10^{-31}\frac{Jm}{K}\frac{dT}{dx}$

$$\frac{dQ/dt}{m^2} = \frac{1.17\times10^{19}}{m^2}\times\frac{3.45\times10^{-31}Jm}{K}\frac{dT}{dx}\frac{1}{2.5\times10^{-14}s} = 161\ \frac{W}{m\,K}\frac{dT}{dx}$$

$= K\frac{dT}{dx}$ so $\boxed{K = 161\ \frac{W}{m\text{-}K}}$

(Tabulated value, HB Chem Phys 51st Ed P. E-10 is $398\ \frac{W}{m\text{-}K}$)

16. 25 years ago, Richard Feynman offered a \$1,000 prize to anyone who could store an encyclopedia in a volume the size of a pinhead (1 $mm^3$). Someone just claimed the prize.

a) Estimate the volume available to store each bit of information. Show in detail how you arrived at that estimate.

b) Is this volume adequate to store one bit of information? Explain.

c) Describe at least one method used to store information, completely.

Exp. Q - Hugo

16. 25 years ago, Richard Feynman offered a $1000 prize to anyone who could store an encyclopedia in a volume the size of a pinhead. (1 mm³). Someone just claimed the prize.

a) Estimate the volume available to store each bit of information. Show in detail how you arrived at that estimate.

b) ~~Do you consider~~ Is this volume adequate to store one bit of information? Explain.

c) Describe at least ~~two~~ one method~~s~~ used to store information compactly.

## Exp. θ – Higa – Solutión

4 a) How many bits in encyclopedia?

$$10^4 \text{ pages} \times 10^2 \frac{\text{lines}}{\text{page}} \times 10^2 \frac{\text{char.}}{\text{line}} \times 6 \frac{\text{bits}}{\text{char.}} = 6 \times 10^8 \text{ bits}$$

$1 \text{ mm} = 10^7 A^\circ$ so have $\dfrac{10^{21} A^{\circ 3}}{6 \times 10^8 \text{ bits}} = 0.17 \times 10^{13} \dfrac{A^{\circ 3}}{\text{bit}}$

or $\sim \boxed{(1.2 \times 10^4 A^\circ)^3 / \text{bit}}$

3 b) This is adequate as it is volume of $\sim 3000$ atoms on a side.

3 c) 1) Semiconductor storage – small diodes turned on or off – talking of layers $\sim 100\ A^\circ$ thick for such devices

2) Magnetic bubbles – regions polarized one way or the other.