CCK Special Topics Problem: Whack-a-Mol
2007-Jul-26

A turbine is used to reduce the gas pressure in a volume (the "low pressure" region). A separate vacuum pump keeps the "high pressure" region at 7.6×10^{-3} torr. The turbine will be idealized as a series of thin, diagonal 1 cm blades in linear motion (see sketch).

A. Let $T = 300\,\mathrm{K}$ everywhere.[1] Show that the mean free path of the gas in the high pressure region is longer than the dimensions of the turbine blades (thus collisions between molecules can be ignored).

B. A molecule of velocity $v_z\hat{z}$ is incident on the turbine from the high pressure region. The turbine blades move at speed v_f. Treat collisions with the blades as elastic scattering of a point particle from a flat surface. Show that for $v_z > v_f$, the molecule will cross over ("backstream") to the low pressure region.

C. Molecules in the high pressure region have a Maxwellian distribution,

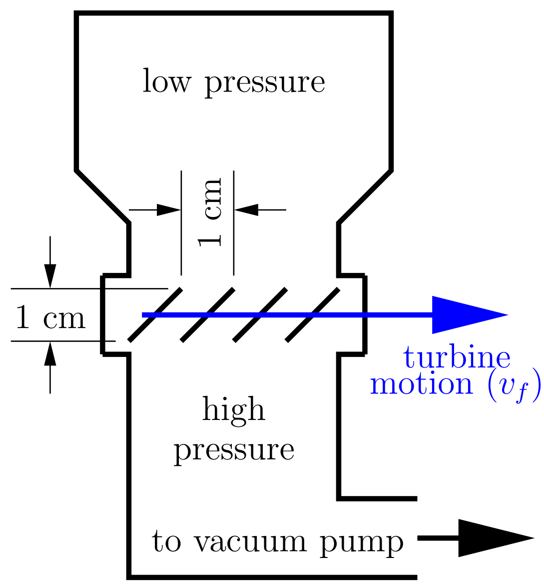
$$\frac{dn_H}{dv_z} = \frac{n_H}{c\sqrt{2\pi}}\, e^{-v_z^2/2c^2},$$

where n_H is the molar density (mol m^{-3}) of molecules and c is the thermal speed. Express the rate of molecules (mol $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$) incident on the turbine from the high pressure region in the velocity range v_z to $v_z + dv_z$, in terms of n_H.

D. Let $v_f = 10^4\,\mathrm{m/s}$. Assume that all molecules with $v_z > v_f$ will backstream, and estimate the total rate (mol $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$).

E. Assume that *all* molecules incident on the turbine from the low pressure region (which has number density n_L) pass "forward" through the turbine. Express the forward rate (mol $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$) in terms of n_L.

F. In a steady state, the forward and backstreaming rates balance. Calculate numerical values for the molar density N_L (mol m^{-3}) and pressure (torr) of the low pressure region.



[1] Gas at 300 K, 760 torr has a molar volume of ~22.4 liters.