Summary of Jackson's "Introd$^{\underline{n}}$ & Survey" (pp. 1-25)

Above citation ⇒ general remarks on features of EM itself, worth repeating.

Sec. I.1 : Max. Eqs., Fields & Sources

1. Intrinsic velocity scale : $c = 3\times10^{10}\,\frac{cm}{sec}$ (± 1 part$/10^9$). Indpt of $\nu\,[0.1-10^{24}\,Hz]$. ← remarkable!
2. In Max. EM, $\mathbb{E}$ & $\mathbb{B}$ are smooth fcns of $\mathbb{r}$ & t ⇒ "many" photons/charges must be present. Theory is modified at quantum level of few photons/charges.
3. Fund$^{\underline{l}}$ charge : $e = 4.8\times10^{-10}\,\frac{e}{s}{}_{u}$ ($\pm 1/10^6$). $\left|\frac{e(\text{electron})}{e(\text{proton})}\right| = 1 \pm (1/10^{23})$. ← remarkable!

Sec. I.2 : Inverse Sq. Law & Photon Mass m_γ

1. $\left.\begin{array}{l} m_\gamma < 4\times10^{-48}\,gm\ (2\times10^{-15}\,eV) \\ \lambda_\gamma = \hbar/m_\gamma c > 10^{10}\,cm. \end{array}\right\}$ from dipole structure of $\mathbb{B}$(earth).
2. $F_{coul} \equiv 1/r^2$ is "exact" over $10^{-15}\,cm < r < 10^{10}\,cm$.
   $10^{-15}\,cm \sim 0.01\times$ nucleon radius is impressive ; $10^{10}\,cm \sim 10\times$ earth radius is not.
   NB : 1 l.y. = 10^{18} cm

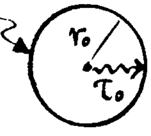
Sec. I.3 : Superposition

1. For vectors $\mathbb{E}_1+\mathbb{E}_2+\mathbb{E}_3+\ldots=\mathbb{E}$, presence of $\mathbb{E}_2$ should not affect size or functional dependence of $\mathbb{E}_1$ or $\mathbb{E}_3$, i.e. $\mathbb{E}_i \neq$ fcn$(\mathbb{E}_{k\neq i})$. Theory should be linear.
2. ∃ no unexpected nonlinear effects in EM up to $E\sim e/r_0^2 \sim 10^{18}\,V/cm$. Here $r_0 = e^2/m_e c^2$ is "class$^{\underline{l}}$ electron rad.", and $E\sim$ that at surface of proton.

ASIDE If (e, m_e) is elementary EM charge, and c = intrinsic velocity of EM theory, then -- from just these 3 csts -- can devise characteristic length & time...

SCALE LENGTH : $r_0 = e^2/m_e c^2 = 2.8\times10^{-13}$ cm (~ size of elem. particles)

SCALE TIME : $\tau_0 = r_0/c = 0.9\times10^{-23}$ sec. (~ min. interaction time)

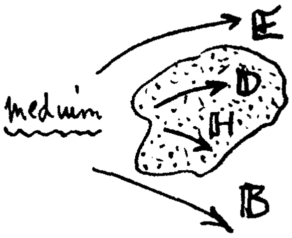
e) 

$\left\{\begin{array}{l} \text{Size} \sim r_0 \\ \text{Signal speed} \sim \tau_0 \end{array}\right.$ ‖ at distances $< r_0$, and times $< \tau_0$, the particle structure must be important, and EM fails unless structure is known.

Jackson I Summary (cont'd)

Sec. I.4 : Max. Eqs. in Media

1. When $\mathbb{E}$ & $\mathbb{B}$ applied to space containing matter (atoms & molecules, intrinsic EM systems), get changes:



$\mathbb{E} \to \mathbb{D} = \underset{\sim}{\epsilon}\,\mathbb{E}$ ∫ $\underset{\sim}{\epsilon}$ = electric permittivity tensor (due to polarization of medium);

$\mathbb{B} \to \mathbb{H} = \underset{\sim}{\mu}'\,\mathbb{B}$ ∫ $\underset{\sim}{\mu}'$ = (inverse) permeability tensor (due to magnetization of medium).

$\underset{\sim}{\epsilon}$ & $\underset{\sim}{\mu}'$ ∝ medium-dpt. In linear approxn, they are indpt. of $\mathbb{E}$ and/or $\mathbb{B}$.

2. The Maxwell Eqs. involving <u>sources</u> ρ & $\mathbb{J}$ change...

free-space:

$$\underbrace{\nabla\cdot\mathbb{E} = 4\pi\rho,\quad \nabla\times\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t},\quad \nabla\cdot\mathbb{B} = 0,\quad \nabla\times\mathbb{B} = \frac{1}{c}\frac{\partial\mathbb{E}}{\partial t} + (4\pi/c)\mathbb{J};}$$

with ρ & $\mathbb{J}$ specified and $\mathbb{E}$ & $\mathbb{B}$ to be found } 8 eqs in 6 unknowns

$\to$ in medium:

$$\underbrace{\nabla\cdot\underline{\mathbb{D}} = 4\pi\rho,\quad \nabla\times\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t},\quad \nabla\cdot\mathbb{B} = 0,\quad \nabla\times\underline{\mathbb{H}} = \frac{1}{c}\frac{\partial\underline{\mathbb{D}}}{\partial t} + (4\pi/c)\mathbb{J},}$$

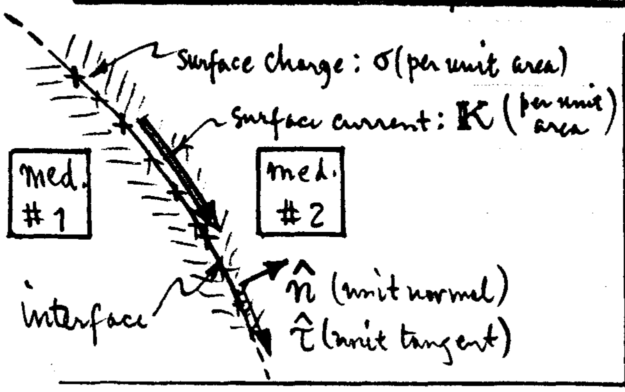
ρ & $\mathbb{J}$ specied and $\mathbb{E}$ & $\mathbb{B}$, $\mathbb{D}$ & $\mathbb{H}$ to be found } <u>8 eqs. in 12 unknowns</u>

Solution undetermined <u>unless</u> we have relations like $\left\{\begin{array}{l}\mathbb{D} = \underset{\sim}{\epsilon}\,\mathbb{E},\\ \mathbb{H} = \underset{\sim}{\mu}'\,\mathbb{B}.\end{array}\right.$

Often possible (& convenient) to assume Ohm's Law : $\mathbb{J} = \underset{\sim}{\sigma}\,\mathbb{E}$ ($\underset{\sim}{\sigma}$: conductivity tensor)

called "Constitutive relations"

Sec. I.5 : Boundary Conditions on Fields



1. σ & $\mathbb{K}$ on the interface will <u>change</u> the fields ($\mathbb{E}$ & $\mathbb{B}$, $\mathbb{D}$ & $\mathbb{H}$) as we go from medium #1 to #2. In fact, they generate <u>discontinuities</u> at bndy:

NORMAL COMPONENTS

$$\begin{array}{l}\nabla\cdot\mathbb{D} = 4\pi\rho\\ \nabla\cdot\mathbb{B} = 0\end{array} \Rightarrow \boxed{\begin{array}{l}\hat{n}\cdot(\mathbb{D}_2 - \mathbb{D}_1) = 4\pi\sigma\\ \hat{n}\cdot(\mathbb{B}_2 - \mathbb{B}_1) = 0\end{array}}$$

σ generates electric fields $\perp$ interface;
$\mathbb{K}$ " magnetic " $\parallel$ " .
So discontinuities are $\perp$ & $\parallel$, resp.

TANGENTIAL COMPONENTS

$$\begin{array}{l}\nabla\times\mathbb{E} = -\frac{1}{c}(\partial\mathbb{B}/\partial t)\\ \nabla\times\mathbb{H} = +\frac{1}{c}\frac{\partial\mathbb{D}}{\partial t} + \frac{4\pi}{c}\mathbb{J}\end{array} \Rightarrow \boxed{\begin{array}{l}\hat{n}\times(\mathbb{E}_2 - \mathbb{E}_1) = 0\\ \hat{n}\times(\mathbb{H}_2 - \mathbb{H}_1) = \frac{4\pi}{c}\mathbb{K}\end{array}}$$