ELEMENTS OF GRAND UNIFICATION

ϕ 519 : 8/24/92

Unified field theory (ultimate reduction) $\longleftrightarrow$ <u>One</u> master set of field eqtns, describing <u>all</u> interactions : Maxwell's $\mathbb{E} \rightleftarrows \mathbb{B}$, + gravity & nuclear.

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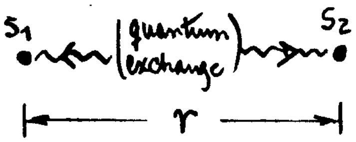
(1) Simple picture: describe interactions between source points by <u>forces</u>...*



$\Big\}$ $\mathbb{F}(r) = [\alpha f(r)]\,\hat{r}$ $\int$ $\alpha \leftrightarrow$ source strength $\propto s_1 s_2$, $f(r) \leftrightarrow$ geometry, quantum mass.

<u>Eg.</u> gravity : $F(r) = \dfrac{G m_1 m_2}{r^2}$ $\int$ so// $\alpha = G m_1 m_2$, $f(r) = 1/r^2$ (in 3D);

Coulomb : $F(r) = \dfrac{k q_1 q_2}{r^2}$ $\int$ so// $\alpha = k q_1 q_2$, $f(r) = 1/r^2$ (in 3D).

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(2) $f(r) = 1/r^2$ is a <u>geometric</u> factor (in 3D) $\propto$ absorption probability for a <u>massless</u> quantum. If the quantum has mass μ, then QM requires:

$\longrightarrow f(r) = \dfrac{1}{r^2} e^{-r/\lambda}$, $\underline{\underline{\lambda}} = \hbar/\mu c$ $\Big\{$ $f(r)$ has "range" λ; if $\mu \to 0$, $f(r)$ has ∞ range.

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(3) In these terms, there are only <u>four</u> fundamentally distinct interactions:

FORCE (coupling)	α	quantum : mass	range : λ	$f(r)$	Sources
gravity (tensor)	10^{-40}	graviton : 0	∞	$1/r^2$	all masses
electromagnetic (vector)	10^{-2}	photon : 0	∞	$1/r^2$	all charges
weak (pseudoscalar)	10^{-13}	boson : $80\, m_p$	10^{-15} cm	$\frac{1}{r^2} e^{-r/\lambda}$	nucleons (radioactivity)
strong (vector)	~ 10	pion : $0.2\, m_p$	10^{-12} cm	$\frac{1}{r^2} e^{-r/\lambda}$	nucleons (binding)

<u>Enormous</u> differences in strengths, ranges and character $\Rightarrow$ these F's will be <u>difficult</u> to unify.

* Interactions usually described by Lagrange Eqtns rather than force eqtns.

(4) Characteristics of an acceptable unified field theory...

A. COVARIANT { obeys laws of special relativity (Lorentz transform, etc.); can be applied at all velocities $\leq c$.

B. GAUGE INVARIANT { forces $\leftrightarrow$ fields describable by potential functions; sources specified by "charges" & "currents.

C. QUANTIZED { obeys laws of QM (uncertainty principle, etc.); [can theory explain quantized charges & masses] ?

D. RENORMALIZABLE { all quantities in the theory can be made finite; [e.g. self-energy (energy-of-assembly) of point sources].

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(5) Until 1970, only usable theory showing all these characteristics was "**QED**" (quantum electrodynamics)-- a quantized, fully relativistic, gauge invariant and renormalizable theory of EM interactions. At $\gtrsim$1970, "QCD" (quantum chromodynamics) emerged as a similar theoretical framework for strong interactions. Still, no real unification of the four fields was accomplished until the early 70's, when the EM & weak interactions were combined into an "electroweak" theory (Weinberg, Salam, Glashow: Nobel Prize (1979)). The structure of electroweak theory resembles Maxwell's EM. Current status is:

FIELD	COV'NT	GAUGE INV.	QNTZD	R'NORM
gravity	yes	—	no	no
EM	yes	yes	yes	yes
weak	yes	yes	yes	yes
strong	(yes)	(yes)	(yes)	(yes)
electroweak	yes	yes	yes	yes

Prescription for your 1ST Nobel Prize:
1) change the "no's" to "yesses";
2) write the Whole Theory down;
3) publish, with suitable audacity;
4) accept all calls from Sweden;
5) buy airline tickets early.