In 1960, physicist Freeman Dyson theorized that an advanced civilization might build a rigid sphere enclosing a star, to capture all the energy from the star.

A. Suppose we built a Dyson sphere of radius $R = 200$ million km around our own Sun, which has luminosity $L_\odot = 3.8 \times 10^{26}$ W. Assuming that $L_\odot$ remains constant and that the outside surface of the sphere is a perfect absorber, what is the equilibrium temperature of the outside of the Dyson sphere?

B. In 1970, Larry Niven's novel Ringworld described a rigid ring built around a star. MIT students at the 1971 World Science Fiction Convention chanted, "The Ringworld is unstable!"

   i. Sketch the potential energy of the Dyson sphere of radius R when its center is displaced a distance r from the Sun.

   ii. The potential $U(r)$ for the Ringworld cannot be calculated analytically. To gain some insight, consider the case $r \sim R$, so that the ring can be treated as an infinitely long rod. Based on what you learn from this analogy, sketch $U(r)$ for the Ringworld.

   iii. If the Dyson sphere is displaced slightly ($r/R = \epsilon \ll 1$), and is left at rest, does the perturbation tend to grow? What about for the Ringworld?

   iv. Does the rotation of the Ringworld affect its stability?