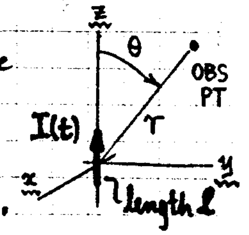
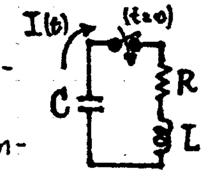
④⓪ [15 pts]. Consider a 1D arc discharge along the z-axis: a current pulse $I(t)$ -- which begins at time $t=0$ -- flows along a path of length ℓ. An observer, situated at position (r,θ) [with $r \gg \ell$] detects the arc radiation.



(A) Start from the arc's Poynting vector derived in class [notes of 1/29/91]. Show that the arc's frequency-angle spectrum at the observer pt. is:

$$\boxed{\frac{d^2 I}{d\omega\, d\Omega} = \left(\frac{\sin^2\theta}{8\pi^2 c^3}\right)\ell^2\omega^2\left|\int_0^\infty I(t)\,e^{-i\omega t}\,dt\right|^2}.$$

(B) A (crude) model of the arc's $I(t)$ is the discharge of a capacitor C (at voltage V_0 initially, and switched on at $t=0$) through a resistance-inductance combination R & L (both cnsts). Then: $I(t) = (V_0/L\Gamma)\,e^{-\gamma t}\sinh\Gamma t$, w/ $\gamma = \frac{R}{2L}$ & $\Gamma = \sqrt{\gamma^2 - (1/LC)}$, for the overdamped case. Sketch $I(t)$ vs. t, roughly indicating the pulse risetime & duration.



(C) Calculate the arc spectrum $d^2I/d\omega\, d\Omega$ for the model of part (B). Sketch the spectrum as a fcn of ω. Over what range of frequencies is the arc detectable?

(D) Calculate the total energy radiated by the arc. Compare it with $\int I^2 R\, dt$ = discharge energy.

HINT: see R. Robiscoe & Z. Shi, J. Appl. Phys. 64, 4364 (Nov. 1988).

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④① [10 pts: Jackson Prob. (15.1)]. Calculate the classical differential cross-section for production of photons (radiation) during the elastic scattering of a non-relativistic Q from a hard sphere of radius R. HINTS: (1) Start from Jackson's Eq. (15.4) for $d^3\sigma/d\Omega_p\, d\mathcal{E}_\gamma\, d\Omega_\gamma$, and integrate out the $d\Omega_p$ dependence, (2) note that $d\sigma/d\Omega_p = \frac{1}{4}R^2$ is isotropic for Q.

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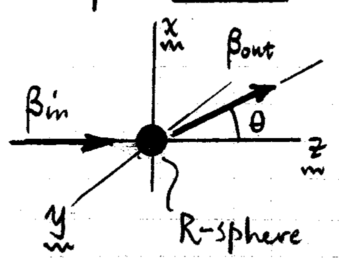
④② [20 pts: Jackson Prob. (15.2)]. Again find the photoemission cross-section for $Q \to$ hard sphere scattering, but when Q's approach velocity is relativistic. Show that:

$$\boxed{\frac{d^2\sigma}{d\mathcal{E}_\gamma\, d\Omega_\gamma} = \frac{R^2}{4\pi}\left(\frac{Q^2\beta^2}{\hbar c\, \mathcal{E}_\gamma}\right)\left[\frac{\sin^2\theta}{(1-\beta\cos\theta)^2} + \frac{1}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{2}{\beta^2}\right]}.$$



HINT: this is the longest Jackson problem you will ever do.