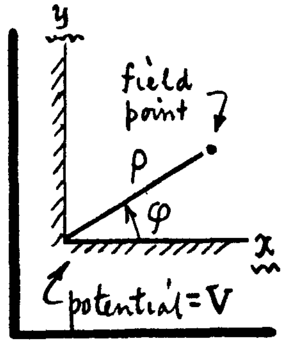
(17) Try solving the 2D wedge problem [Jk$^{\underline{n}}$ Sec. (2.11)] in rectangular cds $(x, y)$.




1) For $\nabla^2\phi = 0$ in 2D rectangular cds $(x, y)$, we have only one free separation cnst available; call it $\alpha$. Then solutions for the $x$ & $y$ variation go as $\left\{\begin{matrix}\sin\alpha x\\ \cos\alpha x\end{matrix}\right\}$ & $\left\{\begin{matrix}\sinh\alpha y\\ \cosh\alpha y\end{matrix}\right\}$; we can as well take solutions $\left\{\begin{matrix}\sinh\alpha x\\ \cosh\alpha x\end{matrix}\right\}$ & $\left\{\begin{matrix}\sin\alpha y\\ \cos\alpha y\end{matrix}\right\}$, if we are doing a problem where $x$ & $y$ are treated equivalently, and are thus interchangeable. For a 2D wedge with opening $\measuredangle\, \beta = \frac{\pi}{2}$, where the potential $\phi = V = $ cnst on both the planes $y = 0$ & $x = 0$, we can therefore consider a solution

$$\rightarrow \phi(x, y) = V + \sum_\alpha A_\alpha [\sin\alpha x \sinh\alpha y + \sinh\alpha x \sin\alpha y], \quad A_\alpha = \text{cnst}. \qquad (1)$$

In fact this $\phi$ satisfies: $\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = 0$ (for all $\alpha$ & $A_\alpha$), and it will obey the "in-close" B.C: $\phi(0, y) = 0$, $\phi(x, 0) = 0$.†

2) The cnsts $\alpha$ & $A_\alpha$ will be fixed by distant B.C. We can get information for the behavior of $\phi$ near the vertex by assuming that the "lowest" values of $(\alpha, A_\alpha)$ do not vanish, and expanding: $\sin z \simeq z$ & $\sinh z \simeq z$. Thus, as $(x, y) \rightarrow (0, 0)$, the leading term in the potential goes as...

$$\underline{\underline{\phi(x, y) \simeq V + 2V_1 xy}}, \quad \text{w/} \quad V_1 = \alpha^2 A_\alpha = \text{cnst}. \qquad (2)$$

This is the result in rectangular cds $(x, y)$. To compare with polar cds, put $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, so: $2xy = \rho^2 \cdot 2\cos\varphi \sin\phi = \rho^2 \sin 2\varphi$. Then...

$$\rightarrow \phi(\rho, \varphi) \simeq V + V_1 \rho^2 \sin 2\varphi, \quad \text{as} \quad \rho = \sqrt{x^2 + y^2} \rightarrow 0. \qquad (3)$$

This recovers Jackson's result in his Eq. (2.73) for a wedge with $\beta = \frac{\pi}{2}$. The rect. cd. solution in (1) would be enormously more complicated if we had $\beta \neq$ multiple of $\pi/2$

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† The sin sinh & sinh sin terms enter with equal weight because $x$ & $y$ are coequal.