Wave Eqtns for $\mathbb{E}$ & $\mathbb{B}$: Motion of the EM Field.

Poynting's Energy Theorem (Ref. Jackson Sec. 6.8).

1) Now that we've added t-variation, all the quantities on the last page ($\mathbb{E}$, $\mathbb{B}$, ϕ, $\mathbb{A}$, etc.) can "travel"; the scale velocity is $c = 3\times 10^{10}$ cm/sec. To see this...

ASIDE

1. Assume empty space $\left(\begin{matrix}\rho=0\\ \mathbb{J}=0\end{matrix}\right) \Rightarrow$ MAXWELL'S EQUATIONS (in vacuo)

$$\left\{\begin{array}{ll} ① \ \nabla\cdot\mathbb{E}=0, & ② \ \nabla\cdot\mathbb{B}=0, \\ ③ \ \nabla\times\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t}, & ④ \ \nabla\times\mathbb{B} = +\frac{1}{c}\frac{\partial\mathbb{E}}{\partial t}. \end{array}\right. \quad (9)$$

2. $\nabla\times\{\text{Eq.}③\} \Rightarrow$

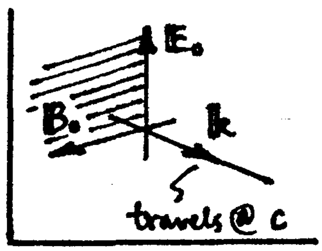
$$\underbrace{\nabla\times(\nabla\times\mathbb{E})}_{\nabla(\nabla\cdot\mathbb{E}) - \nabla^2\mathbb{E}} = -\frac{1}{c}\frac{\partial}{\partial t}\underbrace{(\nabla\times\mathbb{B})}_{\text{use Eq.}④}$$

$\nabla\cdot\mathbb{E} \to 0$, by Eq. ①

i.e. $0 - \nabla^2\mathbb{E} = -\frac{1}{c^2}\frac{\partial^2\mathbb{E}}{\partial t^2}$, w/

$$\boxed{\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbb{E}(\mathbf{r},t) = 0}. \quad (10)$$

3. Eq. (10) is a wave equation for $\mathbb{E}$. Since the "in vacuo" Maxwell Eqtns, in Eq. (9), are invariant under $(\mathbb{E},\mathbb{B}) \to (\mathbb{B},-\mathbb{E})$, we can derive the same eqtn for $\mathbb{B}$, viz. $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbb{B}(\mathbf{r},t) = 0$. Prototype solutions to such eqtns are: $(\mathbb{E},\mathbb{B}) = (\mathbb{E}_0,\mathbb{B}_0)\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, with $k=\omega/c$, and $\mathbb{E}_0$ & $\mathbb{B}_0$ cnst vectors. The solutions are "plane waves" at frequency ω, advancing in direction $\mathbb{k}$ at cnst phase velocity $c=\omega/k$.



4. Max. Eq. ③ relates the amplitudes $\mathbb{E}_0$ & $\mathbb{B}_0$ by: $\mathbb{k}\times\mathbb{E}_0 = (\omega/c)\,\mathbb{B}_0$. So we have a transverse wave of $\mathbb{E}$ & $\mathbb{B}$ fields traveling as shown. LET THERE BE LIGHT!

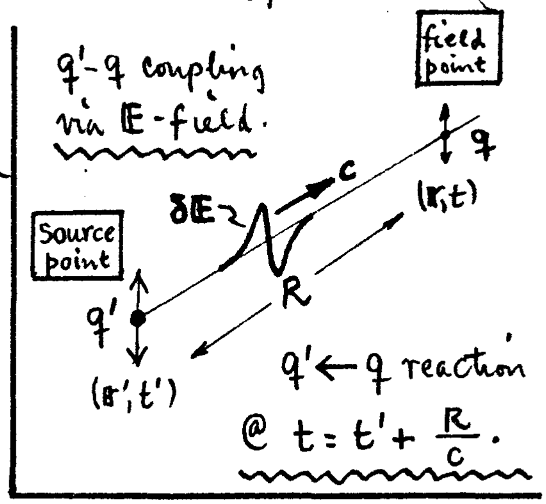
5. The light wave just invented is transporting a field energy density: $u = \frac{1}{8\pi}\langle E_0^2 + B_0^2\rangle_{avg} = \frac{1}{8\pi}E_0^2$, and therefore an effective mass density $\frac{u}{c^2}$, and therefore an effective momentum density $\sim u/c$.

... SO ... Traveling fields $\mathbb{E}$ & $\mathbb{B}$ transport momentum as well as energy.

END ASIDE

Poynting (cont'd). Notion of fields carrying energy & momentum. (1884)

2) Poynting's Theorems will establish energy & momentum conservation laws for EM systems which involve time-varying $\mathbb{E}$ & $\mathbb{B}$ fields, and they will do so by directly investing the fields themselves with energy & momentum.



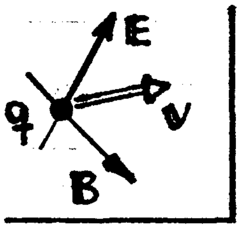
But first, there is a simpler way to look at why the fields carry momentum. For q'-q coupled by an $\mathbb{E}$-field, when q' is "jiggled", q will begin "jiggling" after a time delay $(t-t') = R/c$. The delay is due to the fact that the $\mathbb{E}$-field disturbance $\delta\mathbb{E}$, which relays to q the fact that q' has moved, travels over the separation distance R at finite speed c. During the interval $(t-t')$ the $q' \rightarrow q$ energy & momentum transfer resides in neither charge -- the energy & momentum transfer is transported (at c) by the fields themselves. This will be true so long as c = any finite #. Also, any statement regarding energy/momentum conservation must necessarily involve the fields as well as the particles. This is what Poynting realized.

This is an astonishing idea. Conceptually, it is a very large step from characterizing energy/momentum & conservation laws for localized, discrete, Newtonian-type particles, to carrying out the same program for non-localized, diffuse Maxwellian-type fields. In fact, this step was the first time that classical physics began to give up on the previously sharp distinction between particles & fields -- these entities could now share similar properties (in fact they had to). This "blurring" of particle (point-like) and field (wave-like) characteristics set the stage for the invention and acceptance of QM, some 40 years later, where the main message is... it is a Big Mistake to call anything purely a particle or purely a wave.

Poynting (cont'd). Conservation of Energy for Particles & Fields.

3) Now for the nuts & bolts of Poynting's Theorem on conservation of energy.

1. For a single q in fields $\mathbb{E}$ & $\mathbb{B}$...



Fields do work on q at a rate $\Big\}$ $\dfrac{dW}{dt} = \mathbb{v}\cdot\mathbb{F}$, w/ $\mathbb{F} = q\left(\mathbb{E} + \frac{\mathbb{v}}{c}\times\mathbb{B}\right)$;

so// $$\frac{dW}{dt} = q\Big(\mathbb{v}\cdot\mathbb{E} + \frac{1}{c}\underbrace{\cancel{\mathbb{v}\cdot(\mathbb{v}\times\mathbb{B})}}_{\equiv 0\ (\mathbb{B}\text{ does no work at all})}\Big) = (q\mathbb{v})\cdot\mathbb{E} \qquad (11)$$

Many q's $\Rightarrow$ $q\mathbb{v} \to \int(\underbrace{n}_{\#q\text{'s/vol.}}q\mathbb{v})\,d^3x$, so// $\boxed{\dfrac{dW}{dt} = \int_V \mathbb{J}\cdot\mathbb{E}\,d^3x}$. $\qquad$ (12)

and $nq\mathbb{v} \to \mathbb{J}$.

This energy change relates directly to the mechanical motion of the q's comprising the current density $\mathbb{J}$ -- it is really just $\frac{d}{dt}$(kinetic energy) of that motion. We are not accounting explicitly for any radiation energy lost by $\mathbb{J}$ during any accelerations it may undergo.*

2. We want to balance the energy gained by $\mathbb{J}$ in Eq. (12) with the energy lost (supplied) by the fields. Accordingly, express $\mathbb{J}\cdot\mathbb{E}$ wholly in terms of fields:

$$\left(\nabla\times\mathbb{H} = \frac{1}{c}\frac{\partial\mathbb{D}}{\partial t} + \frac{4\pi}{c}\mathbb{J}\right)\cdot\mathbb{E} \Rightarrow \mathbb{J}\cdot\mathbb{E} = \frac{1}{4\pi}\left[c\,\underline{\mathbb{E}\cdot(\nabla\times\mathbb{H})} - \mathbb{E}\cdot\frac{\partial\mathbb{D}}{\partial t}\right]$$

$$= \mathbb{H}\cdot\underbrace{(\nabla\times\mathbb{E})}_{=(-)\frac{1}{c}(\partial\mathbb{B}/\partial t),\ \text{by Faraday's Law.}} - \nabla\cdot(\mathbb{E}\times\mathbb{H}), \text{ by a vector identity}$$

... (next page) ...

---

* For nonrelativistic motion, a single q loses energy to EM radiation at a rate given by the Larmor formula [Jk$^{\underline{n}}$ Eq. (14.22)]: $(dW/dt)_{rad} = \frac{2}{3}(q^2/c^3)|\dot{\mathbb{v}}|^2$. This energy loss is not accounted for in dW/dt of Eq. (12), or in the Lorentz force law, unless $\mathbb{E}$ includes the so-called "radiation fields" in Jk$^{\underline{n}}$ Eqs. (14.13) & (14.14) [the rad$^{\underline{n}}$ fields are those parts of $\mathbb{E}$ & $\mathbb{B}$ generated by q itself which depend on $\dot{\mathbb{v}}$ and fall off with distance as $1/R$]. If the $\mathbb{E}$ & $\mathbb{B}$ appearing in Eqs. (11)-(15) here are thought of as external fields which exclude q's self-fields, then Poynting's Theorem in Eq. (14) is correct only up to the neglect of q's radiation. It is not clear that q's self-fields have ever been satisfactorily included.

Poynting (cont'd). Conservation of Total Energy.

So//

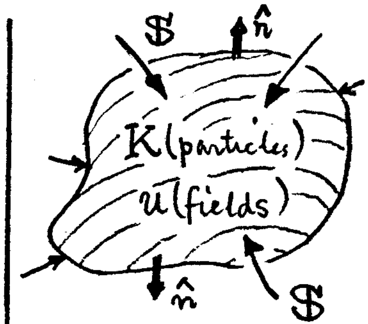
$$\left.\begin{aligned} \mathbb{J}\cdot\mathbb{E} &= -\nabla\cdot\$ - \frac{1}{4\pi}\left(\mathbb{E}\cdot\frac{\partial\mathbb{D}}{\partial t} + \mathbb{H}\cdot\frac{\partial\mathbb{B}}{\partial t}\right), \\ \text{where: } \underline{\underline{\$}} &= \frac{c}{4\pi}(\mathbb{E}\times\mathbb{H}) \leftarrow \text{called "Poynting Vector".} \end{aligned}\right\} \quad (13)$$

If the medium is <u>linear</u> (e.g. $\mathbb{D}=\epsilon\mathbb{E}$ & $\mathbb{B}=\mu\mathbb{H}$, with ϵ and μ indpt of fields), then: $\mathbb{E}\cdot\frac{\partial\mathbb{D}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\mathbb{E}\cdot\mathbb{D})$, and: $\mathbb{H}\cdot\frac{\partial\mathbb{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\mathbb{B}\cdot\mathbb{H})$, so (13) becomes...

$$\boxed{\frac{\partial u}{\partial t} + \nabla\cdot\$ = -\mathbb{J}\cdot\mathbb{E}}, \quad \text{w/} \quad \underline{\underline{u}} = \frac{1}{8\pi}(\mathbb{E}\cdot\mathbb{D} + \mathbb{B}\cdot\mathbb{H}) \Big\} \text{ total field energy density.} \quad (14)$$

<u>This is one form of Poynting's Theorem on energy conservation</u> for fields + particles [Jk^n Eq. (6.108)] ... it balances energy flow <u>out of</u> the field sector on the LHS of the eqtn against energy flow <u>into</u> the particle sector on the RHS.

3. Integrate Eq. (14) over a volume V enclosed by surface S. Assume no particles leave V, but that the particle kinetic energy K inside and field energy density u in V may change because of the motions. Then (14) $\Rightarrow$



<u>Inward</u> flow of $\$$ increases K & u inside.

$$\int_V \mathbb{J}\cdot\mathbb{E}\, d^3x + \frac{\partial}{\partial t}\int_V u\, d^3x = -\oint_S \$\cdot\hat{n}\, da$$

$$\uparrow \frac{dK}{dt}\text{, particles} \qquad \uparrow \frac{dU}{dt}\text{, fields}$$

i.e.//

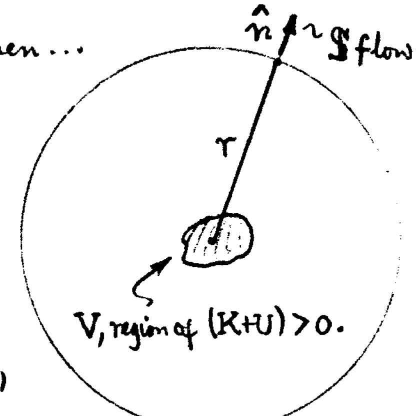
$$\boxed{\frac{d}{dt}\left[K\left(\begin{smallmatrix}\text{particle}\\ \text{K.E.}\end{smallmatrix}\right) + U\left(\begin{smallmatrix}\text{field}\\ \text{energy}\end{smallmatrix}\right)\right] = (-)\,\mathcal{P}} \Big\} \ \mathcal{P} = \oint_S \$\cdot\hat{n}\, da \text{ is the } \underline{\text{flux}} \text{ of } \$ \text{ thru surface.} \quad (15)$$

Whether or not (K+U) increases in V depends on whether or not $\$ = \frac{c}{4\pi}(\mathbb{E}\times\mathbb{H})$ is flowing <u>in</u> across the surface. This is another form of Poynting's Energy Theorem -- it suggests that all changes in the total system energy (K+U) are accompanied by an inward (or outward) flow of $\$$.

<u>NOTE</u>: If surface $S\to\infty$, and $\mathbb{E}$ & $\mathbb{H}$ vanish there, then (15) $\Rightarrow \frac{d}{dt}(K+U)=0$, or: $K+U=\text{cnst}$. Then any ΔK results in a $\Delta U = (-)\Delta K$, etc.

Poynting (cont'd) Remarks on the Energy Theorem.

REMARKS on Poynting Energy Theorem.



4. If, in Eq. (15), we let the surface recede to ∞, then...

$$\rightarrow \frac{d}{dt}(K+U) = -\frac{c}{4\pi}\oint_\infty (\mathbb{E}\times\mathbb{H})\cdot\hat{n}\,da$$

$$= -\frac{c}{4\pi}\lim_{r\to\infty}\oint(\mathbb{E}\times\mathbb{H})_r\, r^2 d\Omega. \quad (16)$$

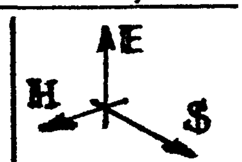
"Normal" fields $\mathbb{E}$ fall off as $\frac{1}{r^2}$ (monopole) or faster, and $\mathbb{H}$ falls off as $\frac{1}{r^3}$ (dipole) or faster. For such fields $(\mathbb{E}\times\mathbb{H})_r r^2$ falls off as $\frac{1}{r^3}$ or faster, and the $\$$ flow vanishes at ∞.

BUT, THERE IS AN EXCEPTION. If the charges in V are accelerated, they generate "radiation fields" $\mathbb{E}$ & $\mathbb{H}$ transverse to $\hat{r}$, which (each) fall off as $\frac{1}{r}$. Then $(\mathbb{E}\times\mathbb{H})_r r^2 = \text{cnst}$ as $r\to\infty$, and the RHS of (16) is finite.

For radiation fields, $\$$ flow at ∞ is finite, and energy is lost to the system.*

---

5. The Poynting vector $\$ = \frac{c}{4\pi}(\mathbb{E}\times\mathbb{H})$ enters the theory [in the energy eqtn: $\mathbb{J}\cdot\mathbb{E} + (\partial u/\partial t) = -\nabla\cdot\$$] as a field energy transport per unit time & area. We have also suggested that traveling fields transport momentum. $\$$ accommodates this notion also, by means of the following picture...



$\$$ is in direction of propagation

A) Assume $\$$ is a radiant energy transport (not transmitted by massive particles).

B) Let energy transported by $\$$ be in form of massless "photons" hitting area A:

n photons/unit vol., traveling at speed c, each photon energy = $\mathcal{E}$

photons incident/sec = nAc,

energy transport per unit time & area: $S = \frac{(nAc)\mathcal{E}}{A} = cn\mathcal{E}$. (17)

C) But $\mathcal{E} = pc$ for photons, so: $S = c^2 np = c^2 \times \left(\begin{smallmatrix}\text{momentum}\\ \text{density}\end{smallmatrix}\text{ of incident radiation}\right)$.

This exercise thus connects S with momentum transport as well as energy flux.

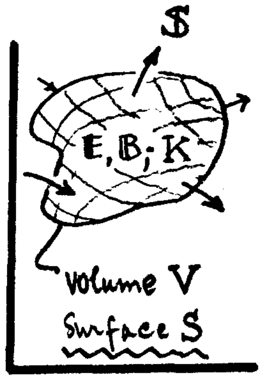
---

* We will study the peculiarities of radiation fields later, in Jackson's Chap. 9, 14 & 15.

Poynting's Momentum Theorem (Ref. Jackson Sec. 6.8).

1) Having derived a statement re energy transfer between particles & EM fields... ★

$$\left[\begin{array}{l} \frac{d}{dt}(\mathcal{E}_{mech} + \mathcal{E}_{field}) = -\oint_S \mathbb{S}\cdot\hat{n}\,da, \quad \text{w/ } \mathbb{S} = \frac{c}{4\pi}(\mathbb{E}\times\mathbb{H}); \\ \text{where: } \dot{\mathcal{E}}_{mech} = \dot{K}(\text{particle K.E.}) = \int_V (\mathbb{J}\cdot\mathbb{E})\,dV, \qquad (18) \\ \text{and: } \dot{\mathcal{E}}_{field} = \dot{U}(\text{field energy}) = \int_V \frac{1}{8\pi}\left[\frac{\partial}{\partial t}(\mathbb{E}\cdot\mathbb{D} + \mathbb{B}\cdot\mathbb{H})\right]dV; \end{array}\right.$$



We expect to find a similar statement re momentum transfer, e.g.

$$\rightarrow \frac{d}{dt}(\mathbb{P}_{mech} + \mathbb{P}_{field}) = (\text{vector})\text{ flux of something.} \qquad (19)$$

This is in fact true... it is part II of Poynting's Theorem. We have reason to believe that $\mathbb{S}$ has something to do with the field momentum; in fact it turns out that $\mathbb{P}_{field} = \int_V (\mathbb{S}/c^2)\,dV$. The hard part of (19) is the RHS.

---

2) The LHS of (19) is a <u>force</u>, so start from Lorentz' Law...

$$\underbrace{\frac{d\mathbf{p}}{dt}}_{\text{single } q} = q\left(\mathbb{E} + \frac{\mathbb{v}}{c}\times\mathbb{B}\right) \longrightarrow \underbrace{\frac{d\mathbb{P}_{mech}}{dt}}_{\text{many } q\text{'s}} = \int\left(\rho\,\mathbb{E} + \frac{1}{c}\mathbb{J}\times\mathbb{B}\right)dV. \qquad (20)$$

$$① \; \frac{1}{4\pi}\nabla\cdot\mathbb{E}, \quad ② \; \frac{1}{4\pi}\left(c\nabla\times\mathbb{B} - \frac{\partial\mathbb{E}}{\partial t}\right).$$

This serves to <u>define</u> the particle forces $\dot{\mathbb{P}}_{mech}$. As before, we re-express the elements of $\dot{\mathbb{P}}_{mech}$ in terms of fields alone: put in $\underline{\rho = \frac{1}{4\pi}\nabla\cdot\mathbb{E}}$, and eliminate

---

★ There is a fuzziness introduced here, in going from a time & space <u>local</u> energy statement: $\underline{\mathbb{J}\cdot\mathbb{E} + (\partial u/\partial t) = -\nabla\cdot\mathbb{S}}$, to the <u>global</u> (integral) claim of Eq. (18), particularly in moving the energy flux term in $\mathbb{S}$ out of the volume V an on to a distant surface S. The reason is that points in V separated by distance Δr have a built-in time delay $\Delta r/c$ for EM signals passing between them. So if the $\mathbb{E}$ & $\mathbb{H}$ fields in V are the <u>source</u> of $\mathbb{S}$ passing through the surface, $\mathbb{S}$ does not even appear there until a suitable time has elapsed. The LHS & RHS of: $\frac{d}{dt}(\mathcal{E}_{mech} + \mathcal{E}_{field}) = -\oint_S \mathbb{S}\cdot\hat{n}\,da$, are then running on different time scales.

Poynting (cont'd)

$\mathbb{J} = \frac{1}{4\pi}(c\nabla\times\mathbb{B} - \partial\mathbb{E}/\partial t)$ ... we are assuming a "non-medium", with $\epsilon = 1$ & $\mu = 1$.
With these substitutions and a few vector identities, we find that Eq. (21) yields:

$$\left[\!\left[\frac{d}{dt}\left[\mathbb{P}_{mech} + \int_V (\mathbb{S}/c^2)\,dV\right] = \int_V \mathfrak{F}\,dV\right]\!\right], \text{ w/ } \mathbb{S} = \frac{c}{4\pi}(\mathbb{E}\times\mathbb{H});$$

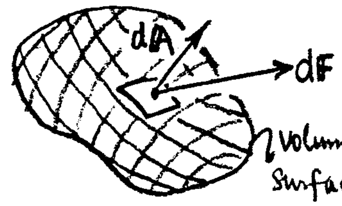
$$\text{and/ } \mathfrak{F} = \frac{1}{4\pi}\left[\mathbb{E}(\nabla\cdot\mathbb{E}) - \mathbb{E}\times(\nabla\times\mathbb{E}) + \mathbb{B}(\nabla\cdot\mathbb{B}) - \mathbb{B}\times(\nabla\times\mathbb{B})\right]. \qquad (21)$$

The LHS of (21) is readily interpreted... $\dot{\mathbb{P}}_{mech} = \int_V (\rho\mathbb{E} + \frac{1}{c}\mathbb{J}\times\mathbb{B})\,dV$ is clearly OK for the force exerted by the fields on the particles (represented by ρ & $\mathbb{J}$), while $\dot{\mathbb{P}}_{field} = \frac{d}{dt}\int_V (\mathbb{S}/c^2)\,dV$ will do for field momentum changes. What remains is to write $\mathfrak{F}$ in a more palatable form... in fact $\mathfrak{F}$ is the divergence of a "stress tensor".

ASIDE General definition of a stress tensor $\underset{\sim}{T}$.

For a force $d\mathbb{F}$ acting thru surface area element $d\mathbb{A}$ on volume V, write

$$\left[\!\left[d\mathbb{F} = \underset{\sim}{T}\,d\mathbb{A}, \text{ or: } dF_i = \sum_k T_{ik}\,dA_k, \right.\right.$$
$$\left.\left. \text{where: } \underset{\sim}{T} = (T_{ik}) \text{ is the "stress tensor".}\right]\!\right] \qquad (24)$$



Volume V, Surface S

Write: $dF_i = T_{ik}\,dA_k$ ← use "summation convention": sum over repeated indices.
This covers possible fact that $d\mathbb{F}$ does not act along $d\mathbb{A}$... in addition to compression at $d\mathbb{A}$, the force $d\mathbb{F}$ may also cause a shear. Now we can say...

$F_i = \oint_S T_{ik}\,dA_k$ ← i^{th} comp. of total $\mathbb{F}$ on V,

or/ $F_i = \int_V (\partial T_{ik}/\partial x_k)\,dV$ ← by use Gauss' Divergence Theorem,

So// $\underline{F_i = \int_V \mathfrak{F}_i\,dV}$, w/ i^{th} comp^t of $\frac{\text{force}}{\text{unit vol.}}$ in V is: $\boxed{\mathfrak{F}_i = \frac{\partial T_{ik}}{\partial x_k}}$. (25)

More compactly, the volume force is: $\mathfrak{F} = \text{div}\,\underset{\sim}{T}$, where $\text{div}\,\underset{\sim}{T}$ is a vector, whose i^{th} comp^t is: $(\text{div}\,\underset{\sim}{T})_i = \partial T_{ik}/\partial x_k$ (sum on k). Now, with some impunity, we can convert volume force integrals to surface stress integral via:

$$\boxed{\int_V \mathfrak{F}_i\,dV = \oint_S T_{ik}\,dA_k} \qquad (26)$$

NOTE: these definitions do not depend on the specific nature of the forces involved.

END of ASIDE

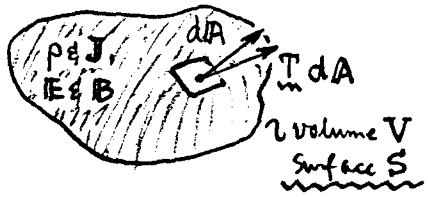
3) Some arithmetic [Jk^n Eqs. (6.119) & (6.120)] serves to show that for the $\mathcal{F}$ in Eq. (21):

$$\rightarrow \mathcal{F}_i = \frac{1}{4\pi}\left[\mathbb{E}(\nabla\cdot\mathbb{E}) - \mathbb{E}\times(\nabla\times\mathbb{E}) + \mathbb{B}(\nabla\cdot\mathbb{B}) - \mathbb{B}\times(\nabla\times\mathbb{B})\right]_i = \partial T_{ik}/\partial x_k$$

where: $\boxed{T_{ik} = \frac{1}{4\pi}(E_i E_k + B_i B_k) - u\,\delta_{ik}}$ ∫ δ_{ik} = Kronecker delta, $u = (E^2 + B^2)/8\pi$. (27)

(T_{ik}) is called the "Maxwell stress tensor". Momentum conservation is now written:

$$\boxed{\frac{d}{dt}\left(\mathbb{P}_{mech} + \mathbb{P}_{field}\right) = \oint_S \underset{\sim}{T}\cdot\hat{n}\,dA} \quad (28)$$



w/ $\dot{\mathbb{P}}_{mech} = \int_V (\rho\,\mathbb{E} + \frac{1}{c}\mathbf{J}\times\mathbb{B})\,dV$ ← mechanical (Lorentz) force on sources ρ & $\mathbf{J}$;

$\mathbb{P}_{field} = \int_V (\mathbb{S}/c^2)\,dV$, $\mathbb{S} = \frac{c}{4\pi}(\mathbb{E}\times\mathbb{B})$ ∫ $\mathbb{S}$ = Poynting vector, $\mathbb{S}/c^2$ = field momentum density;

$T_{ik} = \frac{1}{4\pi}(E_i E_k + B_i B_k) - u\,\delta_{ik}$ ← comp^{ts} of Maxwell stress tensor (in a non-medium).

---

4) In this picture, the total momentum of fields & particles in V does not change unless some $\underset{\sim}{T}$ flows in across the bounding surface S. This flow is measured by:

$\rightarrow T_{ik}\,dA_k = \Delta$(momentum) / unit time, in i^{th} direction across dA.
↳ the components T_{ik} have dimensions of: momentum / unit time & area. (29)

Notice that if the surface S recedes to ∞, where all nonradiative fields vanish faster than $1/R$, then $\oint_\infty \underset{\sim}{T}\,\hat{n}\,dA \rightarrow 0$, and $(\mathbb{P}_{mech} + \mathbb{P}_{field})$ = Cnst is conserved. We know, however, that this rule is "violated" for radiation fields.

[[Anyway, Poynting's Theorems make it clear that particles & fields must trade both momentum & energy during interactions. The fields really do have mechanical properties.]]

REMARKS

1. The global ← local caveat in footnote, p. ME 7, applies to Eq. (28) perforce.
2. Eq. (28) is done for a non-medium, where $\mathbb{D} = \mathbb{E}$, $\mathbb{B} = \mathbb{H}$. With matter present, the def^n of T_{ik} changes to: $T_{ik} = \frac{1}{4\pi}(E_i D_k + H_i B_k) - u\,\delta_{ik}$, $u = \frac{1}{8\pi}(\mathbb{E}\cdot\mathbb{D} + \mathbb{H}\cdot\mathbb{B})$. As discussed in Jk^n Sec. (6.9), there is some attendant interpretational flap on how to define the energy/momentum flow $\mathbb{S}$. We shall skip that discussion.