COMPREHENSIVE EXAMINATION PROBLEMS 2007
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E & M PROBLEM 1

A point charge **q** is brought to a position a distance **d** away from an infinite plane conductor held at zero potential. Using the method of images, find:

(a) the surface-charge density induced on the plane, and plot it;

(b) the force between the plane and the charge by using Coulomb's law for the force between the charge and its image;

(c) the total force acting on the plane by integrating $2\pi\sigma^2$ over whole plane;

(d) the work necessary to remove the charge **q** from its position to infinity;

(e) the potential energy between the charge **q** and its image [compare the answer to **(d)** and discuss].

E & M PROBLEM 2

A point charger **q** is located in free space a distance **d** from the center of a dielectric sphere of radius **a** (**a** < **d**) and dielectic constant ϵ.

(a) Find the potential at all points in space as an expansion in spherical harmonics.

(b) Calculate the rectangular components of the electric field *near* the center of the sphere.

(c) Verify that, in the limit of $\epsilon \to \infty$, your result is the same as that for the conducting sphere.

E & M PROBLEM 3

(a) A spherical shell of radius **a** has an uniform charge density σ. The shell rotates with angular velocity Ω about a diameter. Write the scalar potential both inside and outside the shell as a sum of Legendre polynomials and apply the approximate boundary conditions to find the coefficients.

(b) Find the magnetic field **B** and sketch it both inside and outside the shell.

(c) Calculate the vector potential **A** for the above shell.

(d) Calculate the magnetic field $\mathbf{B} = \bigtriangledown$ x **A** <u>inside</u> and <u>outside</u> the sphere and compare with the above **B** in **(b)**.

MATH PHYSICS PROBLEM 1

A half cylinder of metal of length **a** and radius **a** is initially at temperature T_o. At time $t = 0$ the metal is immersed in a bath which maintains the surface at temperature $T = 0$. Find an approximate expression for $T(\rho, \theta,$ z, $t)$ for large t .