MSU M.S. Comp. -- Nov. 83 #15 3/84 OK Ken ✓ OK - RC 15 Nov. 83

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Arithmetic Problem

" Sum the infinite series

$$S = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots$$ "

Solution

Can write : $S = \sum_{n=1}^{\infty} 1/n(n+1)$.

Consider : $S(x) = \sum_{n=1}^{\infty} x^n/n(n+1)$. Want $S(1) = S$.

Multiply thru by x and differentiate...

$$\frac{d}{dx}[x\,S(x)] = \frac{d}{dx}\sum_{n=1}^{\infty}\frac{x^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty}\frac{x^n}{n} = -\ln(1-x)$$

so// $S(x) = -\frac{1}{x}\int \ln(1-x)\,dx + \text{cnst}$

The cnst = 0, since $x\,S(x)$ vanishes as $x \to 0$. Then...

$$S(x) = -\frac{1}{x}\int_0^x \ln(1-\xi)\,d\xi \Rightarrow S(1) = -\int_0^1 \ln(1-\xi)\,d\xi$$

or// $S = S(1) = -\int_0^1 \ln u\,du = (u - u\ln u)\Big|_{u=0}^{u=1} = 1$

↑ tabulated

so// $\boxed{\sum_{n=1}^{\infty}\frac{1}{n(n+1)} = 1}$