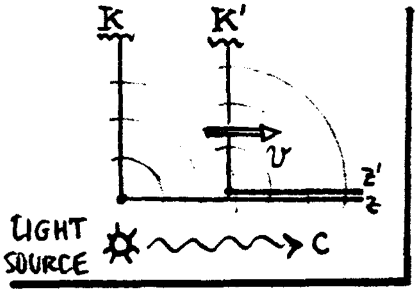
**3)** If $c$ = cnst, then the GT does not work for EM. The transformation which does work -- called the Lorentz Transformation (LT) -- guaranteeing that $c$ = same cnst in both systems $K$ & $K'$, can be found by the following analysis...




$z$ & $z'$ axes of systems $K$ & $K'$ are ∥. $K'$ moves down $z$-axis of $K$ at velocity $v$. At $t = t' = 0$, when $K$ & $K'$ origins coincide, a light source flashes a pulse. The pulse must propagate as a spherical wave in both $K$ & $K'$, in order to be independent of the motion. So:

$$\rightarrow \underbrace{(ct)^2 - (x^2+y^2+z^2) = (ct')^2 - (x'^2+y'^2+z'^2)}_{\text{Same lightspeed } c}. \qquad (11)$$

Assume the $K$ & $K'$ coordinates at pulse front, viz. $(t; x, y, z)$ & $(t'; x', y', z')$, are related by a linear transformation. Where $\gamma$ is an as-yet-unknown fcn of $|v|$:

$$\begin{cases} z' = \gamma(z - vt), \quad y' = y, \quad x' = x \leftarrow \text{just a modified GT;} \\ t' = \gamma\left(t - \frac{vz}{c^2}\right) \leftarrow \text{gives up idea of absolute time (!): } t' \neq t. \end{cases} \qquad (12)$$

Giving up the idea of (Newton's) absolute time is the BIG STEP. Now plug Eqs. (12) into Eq. (11) and equate like powers of $(t; x, y, z)$ to get:

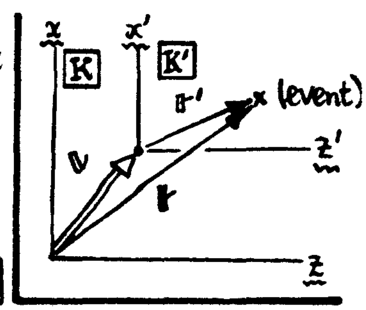
$$\boxed{\begin{aligned} x_0' &= \gamma(x_0 - \beta x_1); \\ x_1' &= \gamma(x_1 - \beta x_0); \\ x_2' &= x_2, \; x_3' = x_3 \end{aligned}} \qquad (13)$$

← **LORENTZ TRANSFORM**: per Jkⁿ Eq. (11.16); for $K \rightarrow K'$, $K'$ moving down $z$-axis of $K$ @ $v$.

NOTATION: $x_0 = ct$; $x_1 = z$, $x_2 = x$, $x_3 = y$; $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$.

The inverse transform $K' \rightarrow K$ is found by interchanging $\left\{\begin{matrix}\text{primed}\\ \text{unprimed}\end{matrix}\right\}$ variables, and $\beta \rightarrow (-)\beta$.

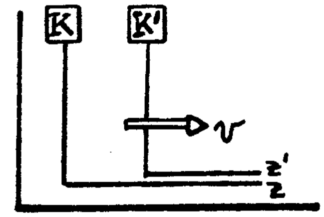
The LT can be generalized to the case where the $K$ & $K'$ axes are still parallel, but $\mathbf{v}$ is not necessarily along the $z$-axis:




$$\boxed{\begin{aligned} x_0' &= \gamma(x_0 - \boldsymbol{\beta}\cdot\mathbf{r}); \\ \mathbf{r}' &= \gamma(\mathbf{r} - \boldsymbol{\beta} x_0) - (\gamma - 1)\left[\mathbf{r} - \frac{1}{\beta^2}(\boldsymbol{\beta}\cdot\mathbf{r})\boldsymbol{\beta}\right]. \end{aligned}} \qquad (14)$$

[Jkⁿ Eq. (11.19)]

# SRT Introdⁿ — Consequences of LT.



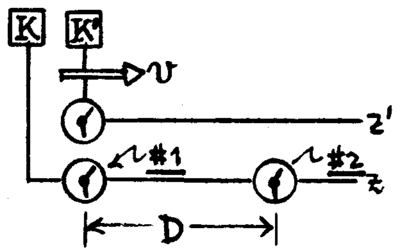

**REMARKS** on Lorentz Transform (LT).

1. The immediate consequences of the LT are to change the "absolute" character of measurements of length and time for K & K'. Some elementary results are:

Ⓐ Length of stick $\Delta L$ ... $\Delta L\binom{\text{at rest}}{\text{in } K} \rightarrow \boxed{\Delta L'\binom{\text{moving}}{\text{w.r.t. } K'} = \Delta L\binom{\text{at rest}}{\text{in } K}\sqrt{1-\beta^2}}$ length contraction (15)

Ⓑ Duration of event $\Delta T$ ... $\Delta T\binom{\text{same pt.}}{\text{in } K} \rightarrow \boxed{\Delta T'\binom{\text{two pts}}{\text{in } K'} = \Delta T\binom{\text{same pt.}}{\text{in } K}\Big/\sqrt{1-\beta^2}}$ time dilation (16)

Ⓒ Simultaneity: events simultaneous in K are not simultaneous in K', and vice versa.




K clocks synchronized by K.

Two clocks, located on z-axis of K, are synchronized by K. K', passing by at velocity $v$, checks the synchrony by one clock situated at K' origin. K' finds the clocks in K are not synchronized: to K', clock #2 shows a later time than #1 by an amount: $\boxed{\Delta t = vD/c^2}$ (in K-time). (17)

2. The "warping" of $\Delta L \rightarrow \Delta L'$ & $\Delta T \rightarrow \Delta T'$ measurements for K & K'... more precisely the dependence of lengths & times on the relative motion of K & K'... is demanded by the constancy of lightspeed c. $\Delta L$'s & $\Delta T$'s are no longer absolute quantities independent of the relative motion of the observer. Only the ratio $\Delta L/\Delta T = c$, in the tracking of a light pulse, must be constant.

It is worth noting that Newtonian mechanics contains no intrinsic scale constants... it scales arbitrarily with M (mass), L (length), and T (time). By contrast, Maxwell's E&M generates the constant c = lightspeed, intrinsic to the whole theory of E&M. Then L & T scales cannot be fixed independently... you always have to adjust L = cT for phenomena involving EM radiation.

$c = \text{cnst} < \infty \Rightarrow$ E&M obeys the LT; $\hbar = \text{cnst} > 0 \Rightarrow$ QM obeys quantization.
Do all fund^l cnsts ($e, m_e$, etc.) $\Rightarrow$ some radical condition on the "right" theory?

**REMARKS** on LT (cont'd)

3. The essence of the LT is that it preserves the "space-time interval", i.e. $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$, for all possible events measured by K & K'... not just observation of a light pulse (per Eq.(11) above), but all events. Consider...

$$\boxed{\text{EVENT}}\left\{\begin{array}{l|l|l|l|l} & \text{Starts at} & \text{finishes at} & \text{temporal duration} & \text{spatial separation} \\ \text{viewed by K}: & (x_1, t_1) & (x_2, t_2) & \Delta x_0 = c(t_2 - t_1) & \Delta x = (x_2 - x_1) \\ \text{viewed by K}': & (x_1', t_1') & (x_2', t_2') & \Delta x_0' = c(t_2' - t_1') & \Delta x' = (x_2' - x_1') \end{array}\right\} \quad (18)$$

Now, construct : $(\Delta s')^2 = (\Delta x_0')^2 - (\Delta x')^2$ ← event spacetime interval for K'.

Plug in LT intervals : $\Delta x_0' = \gamma(\Delta x_0 - \beta\Delta x)$, $\Delta x' = \gamma(\Delta x - \beta \Delta x_0)$.

Find (w/ some algebra) ; $\underline{c^2(\Delta t')^2 - (\Delta x')^2 \equiv c^2(\Delta t)^2 - (\Delta x)^2}$ ← event spacetime interval for K.

Soll $\boxed{(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2}$, is invariant under a LT. (19)

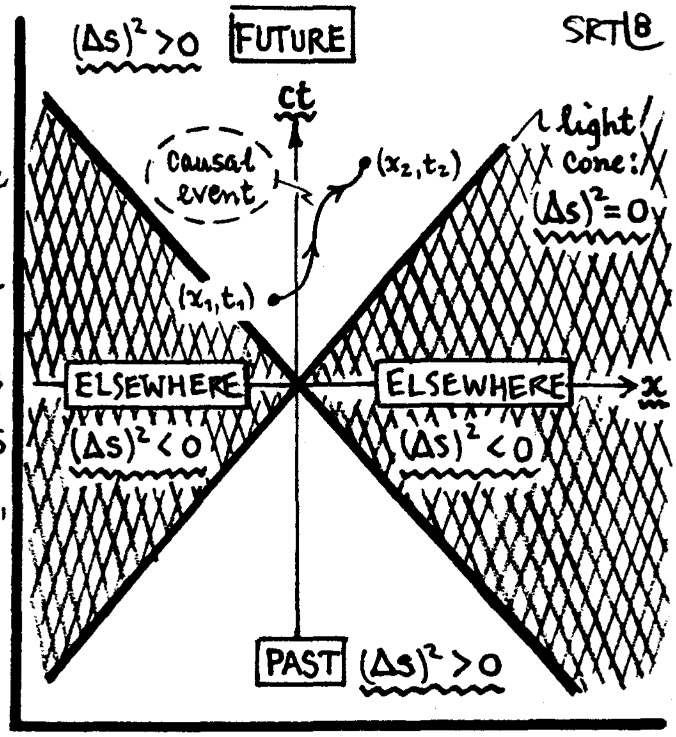
This invariance contains most of the physics of the LT. E.g. we can derive the time dilation formula quoted in Eq.(16) above. As follows...

Let K measure an event of duration $\Delta t$, on one clock fixed at one place in his system; $\Delta x = 0$. For the same event, the relatively moving observer K' measures $\Delta t'$ on his (one) clock, but the start & finish of the event are separated in space by $\Delta x' = v\Delta t'$. Invariance of the spacetime interval permits us to write:

$$\longrightarrow (c\Delta t')^2 - (v\Delta t')^2 = (c\Delta t)^2 - (\cancel{\Delta x})^2, \quad \text{Soll } \boxed{\Delta t' = \Delta t/\sqrt{1-\beta^2}} \int \text{time dilation} \quad (20)$$

**EXERCISE** : Derive length contraction [Eq.(15)] by considering invariance of $(\Delta s)^2$.

We note that $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ can be (+)ve, (−)ve, or zero... depending on the relative sizes of the space-interval $\Delta x$ & time-interval between the start & finish of the event. Further, the sign of $(\Delta s)^2$ is a Lorentz invariant.




**4)** The Lorentz invariance of the size & sign of the spacetime interval $(\Delta s)^2$ provides a scheme for Classifying events according to whether $(\Delta s)^2$ is (+)ve, (−)ve, or zero. This classification puts Causality on a quantitative basis. For EVENTS specified by: $(x_1, t_1)$[start] & $(x_2, t_2)$[finish], we consider the interval:

$$\boxed{(\Delta s)^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 = \text{cnst}} \quad (21)$$

Three cases are possible:

light cone
**1** $(\Delta s)^2 = 0$, so $\left(\frac{x_2 - x_1}{t_2 - t_1}\right) = \pm c$. { EVENTS 1 & 2 can only concern broadcast ↔ reception of a light signal; this defines "light cone" in above dgm. **(22a)**

past & future
**2** $(\Delta s)^2 > 0$, so $\left|\frac{x_2 - x_1}{t_2 - t_1}\right| < c$. { EVENTS 1 & 2 can be connected causally by a light signal; events w/ $(\Delta s)^2 > 0$ lie inside the light cone, and they define the PAST & FUTURE as indicated.

can transform to inertial frame @ $v = \frac{x_2 - x_1}{t_2 - t_1}$ ⇒ each $x' = \gamma(x - vt) = 0$, so $\boxed{(\Delta s)^2 = c^2(t_2' - t_1')^2}$ { this $(\Delta s)^2 > 0$ is called TIME-LIKE. (EVENT' occur at same $x'$, different $t'$s.) **(22b)**

elsewhere
**3** $(\Delta s)^2 < 0$, so $\left|\frac{x_2 - x_1}{t_2 - t_1}\right| > c$. { EVENTS 1 & 2 cannot be connected causally by a light signal; events w/ $(\Delta s)^2 < 0$ lie outside the light cone, in a place called (with engaging whimsy) ELSEWHERE.

can transform to inertial frame @ $\frac{v}{c} = \frac{c(t_2 - t_1)}{x_2 - x_1}$ ⇒ each $t' = \gamma\left(t - \frac{vx}{c^2}\right) = 0$, so $\boxed{(\Delta s)^2 = -(x_2' - x_1')^2}$ { this $(\Delta s)^2 < 0$ is SPACE-LIKE. (EVENT' is at same $t'$, different $x'$s). **(22c)**

In the real world, causally connected events occur inside or on the light cone, so long as c = cnst is the limiting signal velocity in the universe. ELSEWHERE is the realm of coincidence, chance, and the current home of John Sununu.