A turbine is used to reduce the gas pressure in a chamber (the "low pressure" region). A separate vacuum pump keeps the "high pressure" region at 7.6×10^{-3} torr. The turbine consists of a series of thin, diagonal, 5 mm blades moving at speed v_f (see sketch). Let $T = 300\,\mathrm{K}$ everywhere.[1]

A. Show that the mean free path of the gas in the high pressure region is greater than the dimensions of the turbine blades (thus collisions between molecules can be ignored).

B. Molecules in the high pressure region have a Maxwellian distribution,

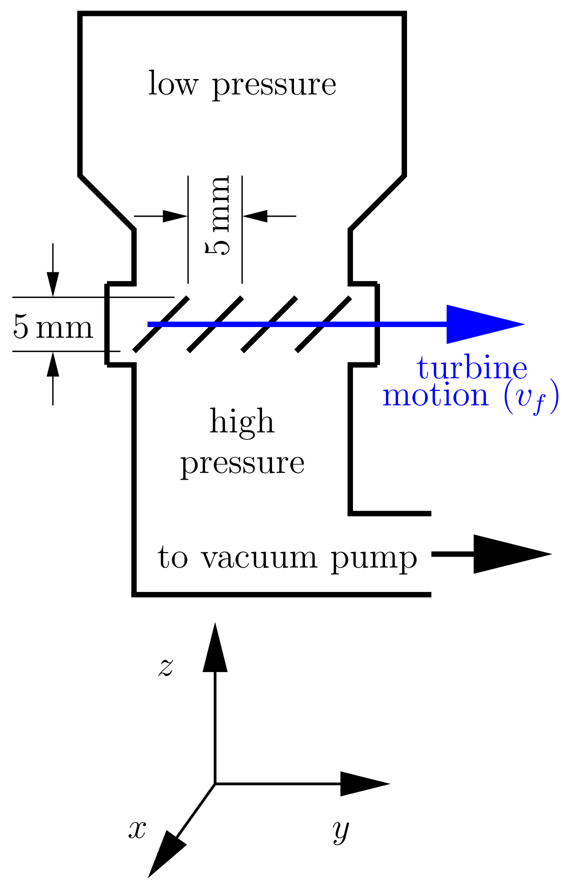
$$\frac{dn_H}{dv_z} = \frac{n_H}{c\sqrt{2\pi}}\, e^{-v_z^2/2c^2},$$

where n_H is the molar density ($\mathrm{mol\,m^{-3}}$) of molecules and c is the thermal speed. Express the rate of molecules incident on a unit area of the turbine ($\mathrm{mol\,m^{-2}\,s^{-1}}$) from the high pressure region in the velocity range v_z to $v_z + dv_z$, in terms of the above variables.

C. Let $v_f = 10^3\,\mathrm{m/s}$. Assume that all molecules with $v_z > v_f$ will "backstream" (cross over from the high to low pressure region), and calculate the total rate ($\mathrm{mol\,m^{-2}\,s^{-1}}$).

D. Assume that *all* molecules incident on the turbine from the low pressure region (which has molar density n_L) pass "forward" through the turbine. Express this forward streaming rate in terms of n_L.

E. In a steady state, the forward and backstreaming rates balance. Calculate the pressure (in torr) of the low pressure region.



[1] One mole of air at 300 K, 760 torr has a volume of ~22.4 liters and masses 0.029 kg.