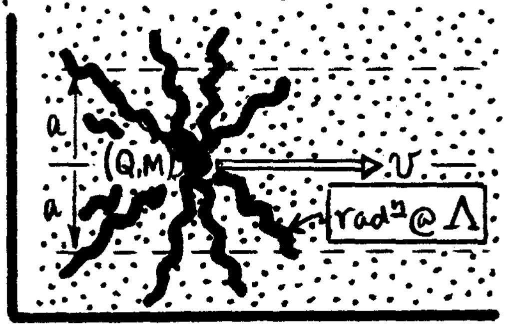
8) We now look at Fermi's formula, Eq. (36), w.r.t. the <u>wavelengths</u> of the fields by which (Q, M) couples to the medium and by which it loses energy. First, note that the arguments of the K-fcns in Eq. (36) for $(dE/dx)_{b\geqslant a}$ are...

$$\rightarrow \lambda a = \frac{\omega a}{v}\left[1-\beta^2\epsilon(\omega)\right]^{1/2} = \frac{2\pi}{\beta}\left(\frac{a}{\Lambda}\right)\left[1-\beta^2\epsilon(\omega)\right]^{1/2}, \tag{37}$$

w/ $\underline{\Lambda} = \frac{2\pi c}{\omega}$ = <u>wavelength</u> of field(s) contributing to loss $\left(\frac{dE}{dx}\right)_{b\geqslant a}$,

&/ a = scale distance for (Q,M)'s energy loss,

$\left(\frac{dE}{dx}\right)_{b\geqslant a}$ = energy loss for all interactions (collisions) at distances $\geqslant a$.



Fermi's formula, Eq. (36), has in its integrand...

$$\rightarrow \underline{F(z) = \left(\sqrt{2z^*/\pi}\, K_1(z^*)\right)\left(\sqrt{2z/\pi}\, K_0(z)\right)}, \quad \text{w/}\ z = \lambda a \propto \frac{a}{\Lambda};$$

so/

$$F(z) \rightarrow \begin{cases} \frac{2}{\pi}\sqrt{\lambda/\lambda^*}\left[\ln(2/z) - C\right], & \text{when } a \ll \Lambda \quad (C = 0.57722\ldots,\ \text{Euler cnst}), \\ e^{-(z+z^*)} = e^{-2\,\mathrm{Re}(\lambda a)}, & \text{when } a \gg \Lambda. \end{cases} \tag{38}$$

Using these asymptotics, we can track the loss for fields at wavelength Λ...

$$\left(\frac{dE}{dx}\right)_{b\geqslant a} \rightarrow \begin{cases} \frac{2Q^2}{\pi c^2}\,\mathrm{Re}\int_0^\infty (i\omega)\left[\frac{1}{\beta^2\epsilon(\omega)} - 1\right]\left[\ln(2/\lambda a) - C\right] d\omega, & \text{① points close to track: } a \ll \Lambda; \\ \frac{Q^2}{c^2}\,\mathrm{Re}\int_0^\infty \left(i\omega\sqrt{\frac{\lambda^*}{\lambda}}\right)\left[\frac{1}{\beta^2\epsilon(\omega)} - 1\right]\left[e^{-2\,\mathrm{Re}(\lambda a)}\right] d\omega, & \text{② points far from track: } a \gg \Lambda. \end{cases} \tag{39}$$

Normally: ① ≫ ②, i.e. the "close-in" collisions contribute most to (dE/dx). <u>BUT</u>, suppose that in a non-lossy medium ($\mathrm{Im}\,\epsilon(\omega) \to 0$), the velocity v of (Q, M) <u>exceeds</u> the phase velocity $v_p(\omega)$ of EM rad^n in the medium at that freq. Then, by Eq. (34): $\lambda = i\frac{\omega}{v}\left[(v/v_p(\omega))^2 - 1\right]^{1/2}$, is pure imaginary, and the distant loss ② ⇒

Jk^n Eq. (19)

$$\boxed{\left(\frac{dE}{dx}\right)_{\text{radn}} = \frac{Q^2}{c^2}\int_{\Delta\omega} \omega\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right] d\omega.} \tag{40}$$

The integration is over freq. intervals $\Delta\omega$ such that $\beta^2\epsilon(\omega) \geqslant 1$, i.e. $v > v_p(\omega)$. <u>NOTE</u>: (dE/dx) is now indpt of "a", and is a <u>radiation</u> <u>loss</u> because the contributing EM fields propagate to ∞. Eq. (40) ⇒ <u>Cerenkov Radiation</u>.

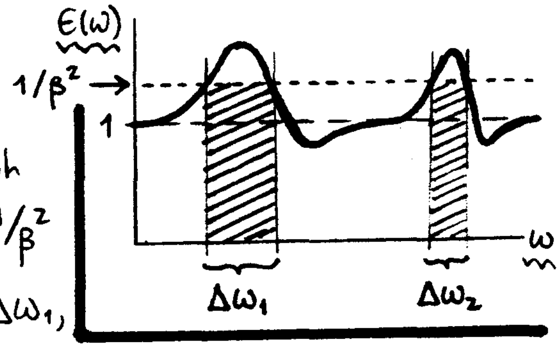
The nature of Cerenkov radiation. Principle of Cerenkov Detector.

REMARKS on Cerenkov Radⁿ: $\left(\frac{dE}{dx}\right)_{radn} = \frac{Q^2}{c^2}\int_{\Delta\omega} \omega \{1 - [1/\beta^2 \epsilon(\omega)]\} d\omega$.

1. Again, the loss here is due to radiation, because the fields involved are no longer damped at large impact parameters b -- they become oscillatory and carry off energy [e.g. in Eq.(30): $E_z(\omega, b) \rightarrow \frac{Q/c}{\beta\epsilon(\omega)}\sqrt{i|\lambda|/b}\, e^{-i|\lambda|b}$, at large b.]

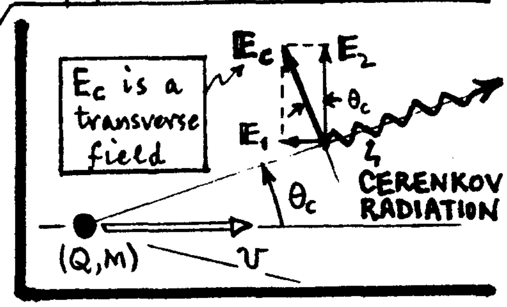
2. The integration range $\Delta\omega$ indicated in the Cerenkov integral is defined by:

$\rightarrow \Delta\omega$ is such that: $\boxed{\epsilon(\omega) > 1/\beta^2}$. (41)



For highly relativistic (Q,M)'s, $\beta \rightarrow 1$, the graph of $\epsilon(\omega)$ vs. ω [$\epsilon(\omega) \sim$ real] shows that $\epsilon(\omega) > 1/\beta^2$ may be satisfied over several frequency bands $\Delta\omega_1$, $\Delta\omega_2$, etc. So Cerenkov radiation can be multi-colored... the colors are characteristic of the resonant frequencies ω_k of the medium.* As (Q,M) slows down, $1/\beta^2$ increases, and the colors gradually fade & disappear.

3. A useful feature of Cerenkov Radⁿ is that (at each freq. band $\Delta\omega$), it is emitted at a specific ∡ $\theta_c(\omega)$ relative to the direction of motion of Q. We can see this as follows. To actually be radiation, the net Cerenkov field $\mathbb{E}_c = \mathbb{E}_1$(long^l nal) + $\mathbb{E}_2$(transverse) must be ⊥ emission direction (i.e. $\mathbb{E}_c$ is itself a transverse radⁿ field). This requires...



$$\rightarrow \tan\theta_c = \left|\frac{E_1}{E_2}\right| = \left|\frac{v}{\omega}\lambda \frac{K_0(\lambda a)}{K_1(\lambda a)}\right| = [\beta^2 \mathrm{Re}\,\epsilon(\omega) - 1]^{1/2}, \text{ or } \boxed{\cos\theta_c(\omega) = \frac{1}{\beta\sqrt{\mathrm{Re}\,\epsilon(\omega)}}} \quad (42)$$

use Eq.(30); $\frac{K_0(\lambda a)}{K_1(\lambda a)} \rightarrow 1$, for large a; Jkⁿ Eq.(13.81)

The light at ω is emitted into a cone of apex ∡ $\theta_c(\omega)$. If Q enters a medium w/ $\epsilon(\omega)$ known, then β can be measured by reading off $\theta_c(\omega)$. A Cerenkov detector!

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* For SHO model in Eq.(27): $\mathrm{Re}\,\epsilon(\omega) = 1 + (\omega_p^2/Z)\sum_k f_k(\omega_k^2 - \omega^2)/[(\omega_k^2-\omega^2)^2 + (\Gamma_k\omega)^2]$.