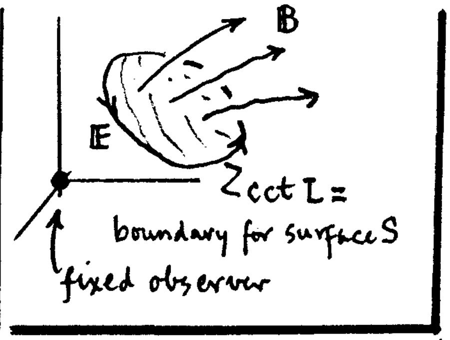
Comment on Faraday's Law

1) $\nabla \times \mathbb{E} = -\frac{1}{c}\frac{\partial \mathbb{B}}{\partial t}$

Apply this Maxwell Eq. to a cct problem involving EM induction.

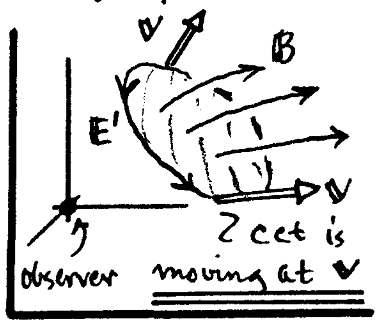


Use of Stokes Thm ⇒

$$\underbrace{\oint_L \mathbb{E}\cdot d\mathbb{l}}_{\text{cct EMF}} = -\frac{1}{c}\frac{\partial}{\partial t}\underbrace{\int_S \mathbb{B}\cdot d\mathbb{S}}_{\Delta\text{ flux linkage}}$$

cct is fixed w.r.t. observer

2) What happens if the cct (or observer) is moving? The magnetic flux through the cct may change not only because $\mathbb{B}$ is changing in time but also because the cct is changing orientation w.r.t. $\mathbb{B}$ (e.g. rotating, flipping, etc.). To account for this:



$$\oint_{cct} \mathbb{E}'\cdot d\mathbb{l}' = -\frac{1}{c}\frac{d}{dt}\int_S \mathbb{B}\cdot d\mathbb{S} = -\frac{1}{c}\int_S\left(\frac{d\mathbb{B}}{dt}\right)\cdot d\mathbb{S}.$$

↑ fld in cct ↑ total time derivative

The total time derivative accounts for $\mathbb{B}$'s convection† as well as $\dot{\mathbb{B}}$...

$$\frac{d\mathbb{B}}{dt} = \frac{\partial \mathbb{B}}{\partial t} + (\mathbb{v}\cdot\nabla)\mathbb{B} \quad \leftarrow \text{convective term}$$

$$\ldots \text{use}: \nabla\times(\mathbb{v}\times\mathbb{B}) = \mathbb{v}\,\cancel{(\nabla\cdot\mathbb{B})}^{\,0} - \underbrace{\mathbb{B}\,\cancel{(\nabla\cdot\mathbb{v})}^{\,0} - \cancel{(\mathbb{B}\cdot\nabla)\mathbb{v}}^{\,0}}_{\mathbb{v}\neq\text{ fcn of space}} - (\mathbb{v}\cdot\nabla)\mathbb{B}\ldots$$

So // $\frac{d\mathbb{B}}{dt} = \frac{\partial \mathbb{B}}{\partial t} - \nabla\times(\mathbb{v}\times\mathbb{B})$ ← put back into Faraday's integral form

$$\boxed{\oint_{cct}\left(\mathbb{E}' - \frac{\mathbb{v}}{c}\times\mathbb{B}\right)\cdot d\mathbb{l}' = -\frac{1}{c}\int_S(\partial\mathbb{B}/\partial t)\cdot d\mathbb{S}}$$

FARADAY LAW for moving cct.

( ) is $\mathbb{E}$ for the fixed observer } $\mathbb{E}'\left(\substack{\text{moving}\\ \text{obs.}}\right) = \mathbb{E}\left(\substack{\text{fixed}\\ \text{obs.}}\right) + \frac{\mathbb{v}}{c}\times\mathbb{B}\left(\substack{\text{fixed}\\ \text{obs.}}\right)$. Lorentz correction.

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† Simple Chain Rule: $f = f(t; x, y\ldots) \Rightarrow \frac{df}{dt} = \frac{\partial f}{\partial t} + \left(\frac{dx}{dt}\right)\frac{\partial f}{\partial x} + \left(\frac{dy}{dt}\right)\frac{\partial f}{\partial y} + \ldots$