# ~ All of Classical E & M, on One Page!

## Maxwell Equations: t-Variation & Conservation Laws

We now attack Jackson's Chap. 6. Our order-of-battle includes...

**MAXWELL EQUATIONS**

① $\nabla\cdot\mathbb{D} = 4\pi\rho$, ~free  ② $\nabla\cdot\mathbb{B} = 0$,

③ $\nabla\times\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{B}}{\partial t}$, ④ $\nabla\times\mathbb{H} = +\frac{1}{c}\frac{\partial\mathbb{D}}{\partial t}$ (~Maxwell addition) $+ \frac{4\pi}{c}\mathbb{J}$ (~free). (1)

**CONSERVATION of CHARGE** } $\nabla\cdot\{\text{Eq.}④\}$, and use of Eq. ① ⇒ $\boxed{\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbb{J} = 0}$. (2)

**LORENTZ FORCE LAW**

$\mathbb{F} = q(\mathbb{E} + \frac{1}{c}\mathbb{v}\times\mathbb{B})$, on single charge $q$;

$\mathcal{F} = \rho\mathbb{E} + \frac{1}{c}\mathbb{J}\times\mathbb{B}$, force/vol. on distributions $\rho$ & $\mathbb{J}$. (3)

**CONSTITUTIVE RELATIONS**

$\mathbb{D} = \mathbb{E} + 4\pi\mathbb{P} \rightarrow \epsilon\mathbb{E}$, $\mathbb{P}$ = (bound) polarization field;

$\mathbb{B} = \mathbb{H} + 4\pi\mathbb{M} \rightarrow \mu\mathbb{H}$, $\mathbb{M}$ = (bound) magnetization field. (4)

NOTE... In same vein, Ohm's Law often assumed: $\mathbb{J} = \sigma\mathbb{E}$, $\sigma$ = electrical conductivity. (5)

NOTE... For vacuum (non-interacting sources): $\epsilon = 1$ & $\mu = 1$, so $\mathbb{D} \equiv \mathbb{E}$, $\mathbb{B} \equiv \mathbb{H}$. (6)

**FIELD ENERGY DENSITIES**

Electric [linear medium: $\epsilon \neq \text{fcn}(E)$]: $u_E = \frac{1}{8\pi}\mathbb{E}\cdot\mathbb{D}$;

Magnetic [linear medium: $\mu \neq \text{fcn}(B)$]: $u_M = \frac{1}{8\pi}\mathbb{B}\cdot\mathbb{H}$.† (7)

**REPRESENTATION by POTENTIALS**

$\mathbb{B} = \nabla\times\mathbb{A}$, still OK by Max. Eq. ②;

$\mathbb{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\mathbb{A}}{\partial t}$, $\dot{\mathbb{A}}$ term req'd by Eq. ③. (8)

This is a powerful strike force. Virtually all of classical E & M appears here.

† $u_M$ not actually derived until Jk^n Sec. 6.2, Eq. (6.16).