Remarks on QM Selection Rules

1) The CPT table just compiled has use in deciding what sort of elementary EM interactions can occur in nature. Consider an interaction energy U = coupling of a charge or current (ρ or $\mathbb{J}$) to an EM field ($\mathbb{E}$ or $\mathbb{B}$). From the definition of energy (e.g. $U = \int \mathbb{F}\cdot d\mathbb{r}$), U must have $CPT = (+1, +1, +1)$.

Suppose $U \propto \mathbb{J}\cdot\mathbb{B}$ were a candidate. Its CPT signature is $(+1, -1, +1)$, and it is ruled out on the grounds that it is a pseudoscalar. Similarly, coupling of the system ∡ momentum $\mathbb{L}$ to $\mathbb{E}$, i.e. $U \propto q\,\mathbb{L}\cdot\mathbb{E}$ has $CPT = (+,-,-)$ and is ruled out because it is a T-odd pseudoscalar.

---

2) For atoms, there are two basic couplings of charge/current to $\mathbb{E}/\mathbb{B}$. They are:

① <u>STARK EFFECT</u> : $U_S = e\,\mathbb{E}\cdot\mathbb{r}$ ← $CPT = (+,+,+)$; is acceptable.

so// $\langle U_S\rangle = \int_\infty d^3x\, \psi_f^*(\mathbb{r})\,[e\,\mathbb{E}\cdot\mathbb{r}]\,\psi_i(\mathbb{r})$ ... applied $\mathbb{E}$ = cnst over atomic dimensions,

i.e.// $\langle U_S\rangle = e\,\mathbb{E}\cdot\int_\infty d^3x\,[\mathbb{r}\,\psi_f^*(\mathbb{r})\,\psi_i(\mathbb{r})]$ ∫ $\mathbb{r}$ is P-odd, so if $\langle U_S\rangle \neq 0$, <u>must have $\psi_f^*(\mathbb{r})\psi_i(\mathbb{r})$ P-odd.</u>

⇒ <u>Selection Rule</u> : $\mathbb{E}$ connects $\psi_i \to \psi_f$ only if the states have <u>opposite</u> parity. I.e. $\psi_i \xrightarrow[\mathbb{E}]{} \psi_f$ involves ∡ momentum change: $\underline{\Delta J = \pm 1}$.

② <u>ZEEMAN EFFECT</u> : $U_Z = \mathbb{m}\cdot\mathbb{B}$ ← $CPT = (+,+,+)$; is acceptable.

so// $\langle U_Z\rangle = \int_\infty d^3x\, \psi_f^*(\mathbb{r})\,[\mathbb{m}\cdot\mathbb{B}]\,\psi_i(\mathbb{r})$ ... applied $\mathbb{B}$ = cnst over atomic dimensions,

i.e.// $\langle U_Z\rangle = \mathbb{B}\cdot\int_\infty d^3x\,[\mathbb{m}\,\psi_f^*(\mathbb{r})\,\psi_i(\mathbb{r})]$ ∫ $\mathbb{m}$ is P-even; if $\langle U_Z\rangle \neq 0$, <u>must have $\psi_f^*(\mathbb{r})\psi_i(\mathbb{r})$ P-even.</u>

⇒ <u>Selection Rule</u> : $\mathbb{B}$ connects $\psi_i \to \psi_f$ only if the states have <u>same</u> parity. I.e. $\psi_i \xrightarrow[\mathbb{B}]{} \psi_f$ involves <u>no</u> ∡ momentum change: $\underline{\Delta J = 0}$

Selection rules are <u>strict</u> so long as $\mathbb{E}$ is polar, $\mathbb{B}$ is axial, and P is a "good" quantum #.

Remarks on Parity-Violation in Atomic Physics

1) The selection rules we have just derived, viz

① **STARK**: $\langle U_s \rangle = e\mathbb{E} \cdot \int d^3x \, [\mathbf{r}\, \psi_f^* \psi_i]$ ∫ couples, or drives transitions $\psi_i \to \psi_f$, only when ψ_f is opposite parity to ψ_i (i.e. $\Delta J = \pm 1$); (1)

② **ZEEMAN**: $\langle U_Z \rangle = \mathbb{B} \cdot \int d^3x \, [\mathbf{m}\, \psi_f^* \psi_i]$ ∫ couples, or drives transitions $\psi_i \to \psi_f$, only when ψ_f is same parity as ψ_i (i.e. $\Delta J = 0$); (2)

depend on the assumptions: (1) $\mathbb{E}$ & $\mathbb{B}$ have definite parities (−) & (+) resp. [so then $\mathbf{r}$ & $\mathbf{m}$ have definite parities (−) & (+) resp.], (2) the quantum states ψ_i & ψ_f have definite parities [usu. $(-)^l$, for state of orbital ∡ momentum l].

---

2) Bound states ψ_i & ψ_f in atoms are generated principally by electromagnetic couplings (mainly Coulomb) between the proton (nucleus) and its electron(s). Then if parity P is a "good" (conserved) quantum # for EM couplings, P will also be good for the atomic states ψ_i & ψ_f, and above rules are absolute.

---

3) BUT, suppose the P-conserving EM coupling between proton & electron has a small admixture of a P-nonconserving interaction... this is the case for the modern "electroweak" theory (Weinberg, Glashow, Salam; 1979) which unifies EM & weak interactions into one (combined) field. Then parity P is almost, but not quite, a good quantum # for atomic states ψ, and ψ becomes a parity-mixed state. To lowest order in the parity-mixing, we write

$$\psi \to \tilde{\psi} = \psi + \kappa\varphi \quad \begin{cases} \psi \text{ has nominal state parity, } \varphi \text{ is opp. parity to } \psi; \\ \kappa = \text{parity-mixing parameter, w/ } |\kappa| \ll 1. \end{cases} \qquad (3)$$

Here $\kappa \sim$ (weak coupling strength)/(EM coupling strength) is very small; for a single proton – single electron interaction: $|\kappa| \sim 10^{-10}$. But $\kappa \neq 0$ violates parity for the state since: $P\tilde{\psi} = \pm(\tilde{\psi} - 2\kappa\varphi)$, when ψ has (±) parity.

4) Parity-violation in atoms can be searched for as follows. The wavefunction combinations which occur in the Stark & Zeeman matrix elements above are

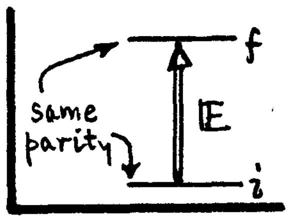
$$\psi_f^* \psi_i \to \tilde{\psi}_f^* \tilde{\psi}_i \simeq \psi_f^* \psi_i + \kappa_i [\psi_f^* \varphi_i] + \kappa_f^* [\varphi_f^* \psi_i], \text{ to } \mathcal{O}(\kappa). \tag{4}$$

these terms have parity opposite to $\psi_f^* \psi_i$

The Stark matrix element (w.r.t. parity-mixed $\tilde{\psi}$'s) picks up new terms...

$$\to \langle U_S \rangle = e\mathbb{E} \cdot \int d^3x\, [\mathbf{r}\, \psi_f^* \psi_i] + \kappa_i\, e\mathbb{E} \cdot \int d^3x\, [\mathbf{r}\, \psi_f^* \varphi_i] + \kappa_f^*\, e\mathbb{E} \cdot \int d^3x\, [\mathbf{r}\, \varphi_f^* \psi_i], \tag{5}$$

and likewise the Zeeman matrix element $\langle U_Z \rangle$ acquires terms in κ. Now, consider driving the transition $\psi_i \to \psi_f$ by an electric field $\mathbb{E}$, when the states i & f have the same "parity". For parity-pure states, such a transition is forbidden by the selection rule in Eq. (1); i.e. the matrix element $e\mathbb{E} \cdot \int d^3x\, [\mathbf{r}\, \psi_f^* \psi_i] \equiv 0$. But for the parity-mixed states, the terms in κ in Eq. (5) are non-zero, so we have



$$\to \langle U_S \rangle = e\mathbb{E} \cdot \left\{ \kappa_i \int d^3x\, [\mathbf{r}\, \psi_f^* \varphi_i] + \kappa_f^* \int d^3x\, [\mathbf{r}\, \varphi_f^* \psi_i] \right\}, \tag{6}$$

as a transition amplitude for an otherwise forbidden transition $i \to f$ by $\Delta J = 0$. So, if we see a violation of the selection rule $\Delta J = \pm 1$ for $\mathbb{E}$-field driven transitions (i.e. we detect a $\Delta J = 0$ transition driven by $\mathbb{E}$), we can blame it on electroweak parity-nonconserving (PNC) effects. Likewise, a violation of $\Delta J = 0$ for $\mathbb{B}$-field driven transitions implies PNC for an atom.

---

5) Atomic ϕ expts have in fact shown the existence of forbidden transitions and PNC effects in atoms. They are ferociously difficult, because the measurable violation rates go as $|\langle U \rangle|^2 \propto |\kappa|^2 \lll 1$. See R.T. Robiscoe & W.L. Williams, Nucl. Instr. Methods 197, 567 (June 1982).