A point charge ***q*** is located in free space at distance ***d*** from the center of a dielectric sphere of radius ***a*** (***a***>***d***) and dielectric constant ε.

(a) Which one of the following three functions is a suitable Green's function, describing the electrostatic potential? (Only one function is correct – all others are fake). Explain.

$$G^{(1)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\dfrac{r^l_<}{r^{l+1}_<} - \dfrac{(\varepsilon-1)l}{[(\varepsilon+1)l+1]}\dfrac{a^{2l+1}}{\left(r_> r_<\right)^{l+1}} & \text{outside the sphere}\\[2ex] \dfrac{1}{[(\varepsilon+1)l+1]}\dfrac{r^l}{r'^{l+1}} & \text{inside } (r<a, r'>a)\end{cases}$$

$$G^{(2)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\dfrac{r^{2l+1}_<}{r^{l+1}_>} - \dfrac{(\varepsilon-1)l}{[(\varepsilon+1)l+1]}\dfrac{a^{2l+1}}{r^{l+1}_<} & \text{outside the sphere}\\[2ex] \dfrac{1}{[(\varepsilon+1)l+1]}\dfrac{r^l}{r'^{l+1}} & \text{inside } (r<a, r'>a)\end{cases}$$

$$G^{(3)}(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y^*_{lm}(\theta',\phi')Y_{lm}(\theta,\phi)}{(2l+1)}\times\begin{cases}\dfrac{(\varepsilon-1)l}{(\varepsilon+1)l+1}\left[\dfrac{r^l_<}{r^{l+1}_<} - \dfrac{a^{2l+1}}{\left(r_> r_<\right)^{l+1}}\right] & \text{outside the sphere}\\[2ex] \dfrac{1}{[(\varepsilon+1)l+1]}\dfrac{r'^l}{r^{l+1}} & \text{inside } (r<a, r'>a)\end{cases}$$

(b) Calculate the field near the center of the sphere. Verify that, in the limit of $\varepsilon\to\infty$ your result is the same as that for the conducting sphere.

(c) Calculate something else?