Dirac's Postulates for a "Reasonable" Theory of QM.

[1] All physical observables are represented by linear, Hermitian operators. For example, $p_k = -i\hbar\,\partial/\partial q_k$ is the k^{th} component of the (canonical) momentum for coordinate q_k.

[2] A system wavefcn $\psi = \psi(q,s,t)$ gives all possible information on the state of the system [w/ t = time, $q = (\mathbf{r}, \mathbf{p})$ = classical coordinates, s = additional QM coordinates (spin, parity, etc.) as needed]. $|\psi|^2 = \psi^*\psi \geqslant 0$ is finite everywhere, and is proportional to the probability of the system having coordinates (q,s) at time t.

[3] The wavefcn ψ obeys a wave eqtn of the form: $\boxed{i\hbar\,\partial\psi/\partial t = \mathcal{H}\psi}$, w/ $\mathcal{H}$ = system Hamiltonian operator. $\mathcal{H}$ is a linear Hermitian operator, allowing a superposition principle for solutions ψ. $\mathcal{H}$ Hermitian $\Rightarrow$ real expectation values $\langle\mathcal{H}\rangle$ = total system energy, and also $\Rightarrow$ conservation of the total system probability: $(d/dt)\sum_s \int |\psi|^2\, dq = 0$.

[4] The QM system is in an eigenstate ψ_n of a general operator Ω if: $\Omega\psi_n = \omega_n\psi_n$, where ω_n = cnst is the n^{th} eigenvalue of Ω. If Ω is Hermitian, then the ω_n are real numbers.

[5] An arbitrary state Ψ of the QM system can be written as a superposition: $\Psi = \sum_n a_n\psi_n$, w/ the $\{\psi_n\}$ an orthonormal and complete set† of system eigenfcns (as defined by an appropriate set of commuting operators $\{\Omega\}$; usu. including $\mathcal{H}$). For the given Ψ, the probability that the system will be found in eigenstate ψ_n is $|a_n|^2$, w/ $\langle\Psi|\Psi\rangle = \sum_n |a_n|^2 = 1$.

[6] If $\Psi = \sum_n a_n\psi_n$, with $\Omega\psi_n = \omega_n\psi_n$, then a measurement of the observable Ω in state Ψ yields the eigenvalue ω_n with probability $|a_n|^2$. The average value of Ω for a large number of measurement on Ψ is: $\langle\Omega\rangle = \langle\Psi|\Omega\Psi\rangle = \sum_n \omega_n |a_n|^2$.

Dirac abstracted these postulates from the non-relativistic wave mechanics of Schrödinger, and considered them to be sufficient to build any reasonable QM theory. In particular, Dirac constructed his relativistic version of Schrödinger's theory using the above postulates [1]-[6].

The reasons for and contents of postulates [1]-[4] are clear from the points made in the SUMMARY, pp. Prop. 25-26. Postulates [5] & [6] (known as the Expansion Postulates) need more elaboration. Briefly, for [5], when we have no prior knowledge that the system is in a particular eigenstate ψ_n, we use $\Psi = \sum_n a_n\psi_n$ to represent the general state -- with the coefficients a_n to be found. Then, with the ψ_n orthonormal, the amplitude of the k^{th} eigenstate in Ψ is the "projection" $\langle\psi_k|\Psi\rangle = \sum_n a_n\langle\psi_k|\psi_n\rangle = \sum_n a_n\delta_{kn} = a_k$, and $|a_k|^2$ is the probability of finding ψ_k upon performing a measurement on Ψ. Now [6] follows from inserting $\Psi = \sum_n a_n\psi_n$ into the definition $\langle\Omega\rangle = \langle\Psi|\Omega\Psi\rangle$, for $\Omega\psi_n = \omega_n\psi_n$.

† Orthonormal $\Rightarrow \langle\psi_m|\psi_n\rangle = \sum_s \int \psi_m^*(q,s,t)\,\psi_n(q,s,t)\,dq = \delta_{mn}$. The completeness requirement is that: $\sum_n \psi_n^*(q',s',t)\,\psi_n(q,s,t) = \delta_{ss'}\,\delta(q-q')$, this enables the expansion $\Psi = \sum_n a_n\psi_n$.