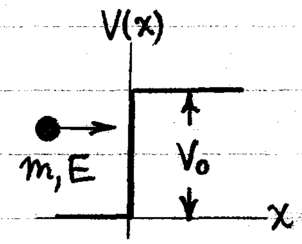
Physics 505 Final Exam

12/15/71

① Use the Bohr-Sommerfeld quantization rule, $\oint p(x)\,dx = nh$, to calculate the allowed energy levels of a ball of mass m bouncing elastically in a vertical direction in a uniform gravitational field of acceleration g.

② Solve the one-dimensional Schrödinger equation for a particle of mass m in an attractive delta-function potential, i.e.: $V(x) = -C\,\delta(x)$, C = const. Show that there is only one bound state, and calculate its energy.



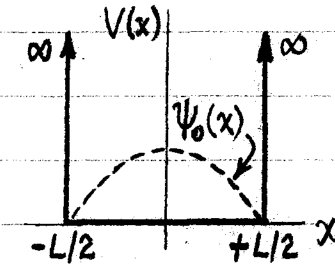
③ A particle of mass m and energy $E>0$ moves along the x-axis and encounters a step-function potential at the origin: $V(x)=0$ for $x<0$, $V(x)=V_0$ for $x>0$. Calculate the reflection coefficient R for the encounter, and sketch R versus E for $E<V_0$ <u>and</u> $E>V_0$.

④ Consider the operator $\Lambda = a^\dagger a$, where a and $a^\dagger$ obey the anti-commutation rule: $aa^\dagger + a^\dagger a = 1$. Assume a set of orthonormal eigenstates $|\lambda\rangle$ such that $\Lambda|\lambda\rangle = \lambda|\lambda\rangle$. By calculating $a|\lambda\rangle$ and $a^\dagger|\lambda\rangle$, show that there are only two eigenstates of Λ. What are the allowed eigenvalues of Λ?

⑤ A particle is in a one-dimensional harmonic oscillator potential. At time $t=0$, it is entirely localized at the origin, i.e.: $\Psi(x,0) \propto \delta(x)$. What is the probability, at some later time t, of finding the particle in the n^{th} state of the oscillator?

⑥ Let the stationary states of a system be specified by eigenfunctions $u_\alpha(x)$, with energy eigenvalues E_α (i.e. if H is the system Hamiltonian: $Hu_\alpha = E_\alpha u_\alpha$). If p is the momentum operator for a particle of mass m, and x is the conjugate position, prove the identity

$$\langle u_\alpha | p | u_\beta \rangle = \frac{im}{\hbar}(E_\alpha - E_\beta)\langle u_\alpha | x | u_\beta \rangle.$$



⑦ A particle is in the ground state of an infinitely deep one-dimensional square well potential of width L as shown. Calculate the probability distribution function for the various values of the particle momentum in this state. Sketch a rough graph of this function versus the momentum, clearly indicating the zeroes and the maxima. What is the most probable value of the momentum?

⑧ Given a complete set of eigenstates $\psi_m(x)$, and arbitrary operators A and B, use the closure relation to establish the identity

$$\langle k | AB | l \rangle = \sum_m \langle k | A | m \rangle \langle m | B | l \rangle,$$

where $\langle k | Q | l \rangle$ is the matrix element $\int \psi_k^*(x)\, Q\, \psi_l(x)\, dx$, etc.