$\boldsymbol{\nabla}\cdot\mathbf{D}=\rho, \quad \boldsymbol{\nabla}\cdot\mathbf{B}=0, \quad \boldsymbol{\nabla}\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}, \quad \boldsymbol{\nabla}\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$ Maxwell's Eqns. (6.6)

General Equations of Electrostatics

$\oint_S \mathbf{E}\cdot\mathbf{n}\,da=\frac{1}{\epsilon_0}\int_V \rho(\mathbf{x})d^3x$ Gauss' Law (1.11)

$\mathbf{E}=-\boldsymbol{\nabla}\Phi, \quad \Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x'$ Scalar potential (1.16, 1.17)

$\nabla^2\Phi=-\rho/\epsilon_0$ Electrostatic Poisson Eequation (1.28)

$V_i=\sum_{j=1}^{n}p_{ij}q_j, \qquad Q_i=\sum_{j=1}^{n}C_{ij}V_j$ Capactiance matrices (1.61)

$q'=-\frac{a}{r}q, \quad r'=\frac{a^2}{r}$ Magnitude and position of image charge on sphere (2.4)

$q=\int\rho(\mathbf{x}')\,d^3x'$ Electric Monopole (4.4)

$\mathbf{p}=\int\mathbf{x}'\rho(\mathbf{x}')\,d^3x'$ Electric dipole (4.8)

$Q_{ij}=\int(3x_i'x_j'-r'\delta_{ij})\rho(\mathbf{x}')d^3x'=3M_{ij}-\mathrm{Tr}(\mathbf{M}\delta_{ij})$ Electric Quadrupole (4.9)

$M_{ij}=\int x_i'x_j'\rho(x')\,d^3x$ Dana definition of **M** matrix

$\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}+\frac{\mathbf{p}\cdot\mathbf{x}}{r^3}+\frac{1}{2}\sum_{i,j}Q_{ij}\frac{x_ix_j}{r^5}+\ldots\right]$ Electric multipole Expansion (4.10)

$\mathbf{E}(\mathbf{x})=\frac{3\mathbf{n}(\mathbf{p}\cdot\mathbf{n})-\mathbf{p}}{4\pi\epsilon_0|\mathbf{x}-\mathbf{x}_0|^3}$ **E**-field due to dipole **p** (4.13)

$\boldsymbol{\tau}=\mathbf{p}\times\mathbf{E}, \quad \mathbf{F}=(\mathbf{p}\cdot\boldsymbol{\nabla})\mathbf{E}$ Torque, force on electric dipole (G. 4.4, 4.5)

$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ Electric displacement(4.34)

$\mathbf{D}=\epsilon\mathbf{E}$ Electric displacement (linear materials) (4.37)

$\mathbf{P}=\epsilon_0\chi_e\mathbf{E}=(\epsilon-\epsilon_0)\mathbf{E}$ Induced polarization (linear materials) (4.36)

$\epsilon=\epsilon_0(1+\chi_e)$ Electric permittivity (linear materials) (4.38)

$\sigma_b=\mathbf{P}\cdot\hat{\mathbf{n}}, \quad \rho_b=-\boldsymbol{\nabla}\cdot\mathbf{P}$ Electric bound charge density (G. 4.11)

$\begin{cases}(\mathbf{D}_2-\mathbf{D}_1)\cdot\mathbf{n}_{21}=\sigma\\(\mathbf{E}_2-\mathbf{E}_1)\times\mathbf{n}_{21}=0\Rightarrow\Phi_2=\Phi_1=V\end{cases}$ Electric JC's (evaluate at boundary) (4.40)

$W=\int\rho(\mathbf{x})\Phi(\mathbf{x})\,d^3x=\frac{1}{2}\int\mathbf{E}\cdot\mathbf{D}\,d^3x$ Energy to bring charges from ∞ (4.83,89)

$W=\frac{1}{2}\sum_{i=1}^{n}Q_iV_i=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}V_iV_j$ Potential energy of capacitor system (1.62)

$W=q\Phi(0)-\mathbf{p}\cdot\mathbf{E}(0)-\frac{1}{6}\sum_i\sum_jQ_{ij}\frac{\partial E_j}{\partial x_i}(0)+\ldots$ Work multipole expansion (4.24)

Specific Cases in Electrostatics

$\begin{cases}\Phi_{\text{in}}=-\left(\frac{3}{\epsilon/\epsilon_0+2}\right)E_0r\cos\theta\\\Phi_{\text{out}}=-E_0r\cos\theta+\left(\frac{\epsilon/\epsilon_0-1}{\epsilon/\epsilon_0+2}\right)E_0\frac{a^3}{r^2}\cos\theta\end{cases}$ Dielectric sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (4.54)

$\Phi=-E_0\left(r-\frac{a^3}{r^2}\right)\cos\theta$ Electric potential of conducting sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (2.14)

$\mathbf{E}=\frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$ **E** of parallel-plate capacitor ($\hat{\mathbf{n}}$ points from pos. to neg.) (G. Ex. 2.6)

$\mathbf{E}=\frac{p}{4\pi\epsilon_0r^3}\left(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\boldsymbol{\theta}}\right)$ Electic dipole at origin pointing in $\hat{\mathbf{z}}$ (4.12)

General Equations of Magnetostatics

$\oint_C\mathbf{B}\cdot d\mathbf{l}=\mu_0I$ Ampère's law (5.25)

$\nabla\cdot\mathbf{J}=0$ Condition of magnetostatics (5.3)

$\nabla^2\mathbf{A}=-\mu_0\mathbf{J}_M$ Poisson equation in terms of magnetic vector potential (5.101)

$\mathbf{B}(\mathbf{r})=\frac{\mu_0I}{4\pi}\int\frac{d\mathbf{l}'\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}$ Biot-Savart law (G. 5.34)

$\mathbf{B}(\mathbf{x})=\nabla\times\mathbf{A}(\mathbf{x}), \quad \mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x'$ Magnetic vector potential (5.27,5.32)

$\mathbf{m}=\frac{1}{2}\int\mathbf{x}'\times\mathbf{J}(\mathbf{x}')\,d^3x'$ Magnetic dipole (5.54)

$\mathbf{m}=IA\hat{\mathbf{n}}$ Magnetic moment of plane loop (5.57)

$\mathbf{m}=\int\mathbf{M}\,d^3x$ Total magnetic moment (J. pg. 197)

$\Phi_M(\mathbf{x})=\frac{\mathbf{m}\cdot\mathbf{x}}{4\pi r^3}, \quad \mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$ Magnetic potentials of a dipole (5.55)

$\mathbf{B}(\mathbf{x})=\frac{\mu_0}{4\pi}\left[\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{m})-\mathbf{m}}{|\mathbf{x}|^3}\right]$ Magnetic field of dipole (**n** parallel to **x**) (5.56)

$\boldsymbol{\tau}=\mathbf{m}\times\mathbf{B}, \quad \mathbf{F}=\boldsymbol{\nabla}(\mathbf{m}\cdot\mathbf{B})$ Torque and force on magnetic dipole (5.1, 5.69)

$\mathbf{H}=-\boldsymbol{\nabla}\Phi_M$ Magnetic scalar potential (valid if $\mathbf{J}_f=0$) (5.93)

$\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$ Definition of **H** (5.81)

$\mathbf{B}=\mu\mathbf{H}$ Property of linear permeable materials (5.84)

$\begin{cases}(\mathbf{B}_2-\mathbf{B}_1)\cdot\mathbf{n}=0\\\mathbf{n}\times(\mathbf{H}_2-\mathbf{H}_1)=\mathbf{K}_f, \ \mathbf{K}_f=0\Rightarrow\Phi_1=\Phi_2\end{cases}$ Magnetic JC's (eval. at boundary) (5.86)

$\mathbf{J}_M=\boldsymbol{\nabla}\times\mathbf{M}, \quad \mathbf{K}_b=\mathbf{M}\times\mathbf{n}$ Bound current density (G. 6.13,14)

$\rho_M=-\boldsymbol{\nabla}\cdot\mathbf{M}, \quad \sigma_M=\mathbf{n}\cdot\mathbf{M}$ Effective magnetic charge density (5.96,99)

$F=\int_S\mathbf{B}\cdot\mathbf{n}\,da=\oint\mathbf{A}\cdot d\mathbf{l}$ Magnetic flux (5.133)

$\mathscr{E}=-\frac{dF}{dt}=\oint\mathbf{E}\cdot d\mathbf{l}=-\int\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{a}$ EMF due to Faraday's Law (5.135)

$M_{ij}=\frac{1}{I_j}F_{ij}$ Mutual inductance (5.156)

$\mathcal{L}=\frac{1}{I^2}\int_{\mathcal{C}}\mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})\,d^3x=\frac{1}{I^2}\int\frac{\mathbf{B}\cdot\mathbf{B}}{\mu}\,d^3x$ Formulae for inductance (5.154, 5.157)

$W=\frac{1}{2}\int\mathbf{J}\cdot\mathbf{A}\,d^3x=\frac{1}{2}\int\mathbf{H}\cdot\mathbf{B}\,d^3x$ Energy in magnetic field (5.149,5.148)

$W=\frac{1}{2}\sum_{i=1}^{N}\mathcal{L}_iI_i^2+\sum_{i=1}^{N}\sum_{j>i}^{N}M_{ij}I_iI_j$ Potential energy in inductor system(5.152)

Specific Cases of Magnetostatics

$\mathbf{A}_{\text{dip}}=\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\Rightarrow\mathbf{B}_{\text{dip}}=\frac{\mu_0m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\boldsymbol{\theta}})$ Field of dipole (G 5.87,88)

$\mathbf{B}(z)=\frac{\mu_0I}{2}\left[\frac{a^2}{(a^2+z^2)^{3/2}}\right]\hat{\mathbf{z}}$ On-axis magnetic field of current loop (G. 5.41)

$\mathbf{B}=\frac{\mu_0NI}{L}\hat{\mathbf{z}}, \quad \mathbf{B}=\frac{\mu_0NI}{2\pi\rho}\hat{\boldsymbol{\phi}}$ Magnetic field inside (solenoid, toroidal coil) (G. 5.59,60)

$\Phi_M(r,\theta)=\frac{1}{3}M_0a^2\frac{r_<}{r_>^2}\cos\theta$ Sphere with uniform $\mathbf{M}=M_0\hat{\mathbf{z}}$ $[(r_<,r_>),(r,a)]$ (5.104)

$\mathbf{M}=\frac{3}{\mu_0}\left(\frac{\mu-\mu_0}{\mu+2\mu_0}\right)\mathbf{B}_0$ Permeable sphere in uniform magnetic field $\mathbf{B}_0$ (5.115)

$\mathcal{L}=\frac{\mu_0\pi a^2N^2}{L}$ Inductance of a solenoid with N turns per unit length L (L. HW9 Pr. 2)

Electrodynamics

$\mathbf{E}=-\boldsymbol{\nabla}\Phi-\frac{\partial\mathbf{A}}{\partial t}, \quad \boldsymbol{\nabla}\cdot\mathbf{A}'+\frac{1}{c^2}\frac{\partial\Phi'}{\partial t}=0$ Lorenz gauge (6.9, 6.17)

$\boldsymbol{\nabla}\cdot\mathbf{J}+\frac{\partial\rho}{\partial t}=0, \quad \frac{\partial u}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{S}=-\mathbf{J}\cdot\mathbf{E}$ Continuity Equations (6.3, 6.108)

$\nabla^2\Phi-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=-\rho/\epsilon_0, \quad \nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=-\mu_0\mathbf{J}$ EM wave equations (6.15,6.16)

$\Phi(\mathbf{x},t)=\frac{1}{4\pi\epsilon_0}\int\frac{[\rho(\mathbf{x}',t')]_{\text{ret}}}{|\mathbf{x}-\mathbf{x}'|}\,d^3x$

$\mathbf{A}(\mathbf{x},t)=\frac{\mu_0}{4\pi}\int\frac{[\mathbf{J}(\mathbf{x}',t')]_{\text{ret}}}{|\mathbf{x}-\mathbf{x}'|}\,d^3x'$

Retarded potentials where $t'=t-|\mathbf{x}-\mathbf{x}'|/c$ (6.48)

$r\gg c\tau$ (radiation zone), $\quad r\ll c\tau$ (Static zone) (G. 11.10, 11.13)

$u=\frac{1}{2}(\mathbf{E}\cdot\mathbf{D}+\mathbf{B}\cdot\mathbf{H})$ Total electromagnetic energy density (6.106)

$\mathbf{S}=\mathbf{E}\times\mathbf{H}$ Poynting vector definition (6.109)

$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$ Lorentz force law (6.113)

$\mathbf{g}=\frac{1}{c^2}(\mathbf{E}\times\mathbf{H})$ Electromagnetic momentum density (6.118)

$\mathbf{J}=\sigma\mathbf{E}, \quad V=IR; \qquad P=IV=I^2R$ Ohm's Law, Joule Heating Law (G. 7.3, 7.4; 7.7)

$R=\rho\ell/A, \quad \sigma=1/\rho$ Pouillet's Law (ℓ is length, A is cross-sectional area) (Wikipedia)

Telegrapher's Equations

$\left.\begin{aligned}\frac{\partial I}{\partial t}&=-\frac{1}{\mathcal{L}}\frac{\partial V}{\partial z}\\\frac{\partial V}{\partial t}&=-\frac{1}{\mathcal{C}}\frac{\partial I}{\partial z}\end{aligned}\right\}\Rightarrow\frac{\partial^2V}{\partial t^2}=\frac{1}{\mathcal{L}\mathcal{C}}\frac{\partial^2V}{\partial z^2}=c^2\frac{\partial^2V}{\partial z^2}$ Telegrapher's equations (L. 21, 22, 23)

$\mathcal{C}=\frac{2\pi\epsilon}{\ln(b/a)}, \quad \mathcal{L}=\frac{\mu}{2\pi}\ln(b/a)$ Capacitance and inductance of coaxial cable (L. 17, 19)

$V(z,t)=f(t-z/c)+g(t+z/c), \ I(z,t)=\frac{1}{Z}\left[f(t-z/c)-g(t+z/c)\right]$ (L. 25, 30)

$Z=c\mathcal{L}=\sqrt{\mathcal{L}/\mathcal{C}}, \quad k=\pm\omega/c$ Impedance definiton, wave vector value (L. 31)

$V(\ell,t)=RI(\ell,t)\Rightarrow g(t+\ell/c)=\frac{R-Z}{R+Z}f(t-\ell/c)$ Resistive BC (L. 37,39)

$V(z,t)=\Re[\tilde{A}(z)e^{-i\omega t}]\Rightarrow\frac{d^2\tilde{A}}{dz^2}=-(w/c)^2\tilde{A}\Rightarrow\tilde{A}(z)=\tilde{A}_0e^{ikz}$ (L. 48, 49, 50)

$V(z,t)=|\tilde{A}_0|\cos[\omega(t\mp z/c)-\delta]$ Sinusoidal solutions to telegrapher's equation (L. 53)

$V(z,t)=\frac{V_0}{\sin(\omega\ell/c)}\cos(\omega t)\sin[\omega(\ell-z)/c]$ Short circuit, $V(0,t)=V_0\cos(\omega t)$ (L. 62)

Potentially Useful Mathematical Identities

$\delta(f(x))=\sum_i\frac{1}{\left|\frac{df}{dx}(x_i)\right|}\delta(x-x_i)$ Jackson Dirac delta function Rule 5

$(a+x)^n\approx a^n+na^{n-1}x+\ldots, \quad f(x)\approx f(a)+\frac{f'(a)}{1!}(x-a)+\ldots$ Taylor Expansions

$J_m(k\rho)\propto(k\rho)^m, \ Y_m(k\rho)\propto(k\rho)^{-m}, \ I_m(k\rho)\propto(k\rho)^m, \ K_m(k\rho)\propto(k\rho)^{-m}$, As $\rho\to0$

$$\begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix}=\begin{pmatrix}\cos\phi&\sin\phi&0\\-\sin\phi&\cos\phi&0\\0&0&1\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\\hat{\mathbf{y}}\\\hat{\mathbf{z}}\end{pmatrix}, \quad \begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi&\sin\theta\sin\phi&\cos\theta\\\cos\theta\cos\phi&\cos\theta\sin\phi&-\sin\theta\\-\sin\phi&\cos\phi&0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\\hat{\mathbf{y}}\\\hat{\mathbf{z}}\end{pmatrix}$$

$$\begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}=\begin{pmatrix}\sin\theta&0&\cos\theta\\\cos\theta&0&-\sin\theta\\0&1&0\end{pmatrix}\begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix}, \quad \begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix}=\begin{pmatrix}\rho/\sqrt{\rho^2+z^2}&z/\sqrt{\rho^2+z^2}&0\\0&0&1\\z/\sqrt{\rho^2+z^2}&-\rho/\sqrt{\rho^2+z^2}&0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}$$

	Wiggly	Decaying
x, y, z	$e^{\pm ik_nx}, \ A\cos(k_nx)+B\sin(k_nx)$	$e^{\pm k_nx}, \ A\cosh(k_nx)+B\sinh(k_nx)$
ρ, ϕ, z	$e^{im\phi}, \ AJ_m(k_n\rho)+BY_m(k_n\rho)$	$AI_m(k_n\rho)+BK_m(k_n\rho)$
ρ, ϕ	$e^{im\phi}$	$A_0+B_0\ln\rho+\sum A_m\rho^m+B_m\rho^{-m}$
r, θ	$P_\ell(\cos\theta)$	$A\left(\frac{r}{a}\right)^\ell+B\left(\frac{r}{a}\right)^{-(\ell+1)}$
r, θ, ϕ	$Y_{\ell m}(\theta,\phi)$	$A\left(\frac{r}{a}\right)^\ell+B\left(\frac{r}{a}\right)^{-(\ell+1)}$