Midterm 2 Equations	PHSX519 Electron	nagnetic Theory I	Roy Smart
$\nabla \cdot \mathbf{D} = \rho, \nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{F}$	$\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ Maxwell's Equations (6.6)	$\mathbf{M} = \frac{3}{\mu_0} \left(\frac{\mu - \mu_0}{\mu + 2\mu_0} \right) \mathbf{B}_0$	Permeable sphere in uniform magnetic field ${\bf B}_0$ (5.115)
$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{ \mathbf{x} - \mathbf{x}' ^3} d^3$	3x' Coulomb's Law (1.5)	$F = \int_{S} \mathbf{B} \cdot \mathbf{n} \ da = \oint \mathbf{A} \cdot d\boldsymbol{\ell}$	Magnetic flux (5.133)
$\delta(f(x)) = \sum_{i} \frac{1}{\left \frac{\mathrm{d}f}{\mathrm{d}x}(x_{i})\right } \delta(x - x_{i})$		$\mathscr{E} = \oint_C \mathbf{E}' \cdot d\mathbf{l}$	Electromotive force (5.134)
$i \mid \overline{\mathrm{d}x}(x_i) $ $\mathbf{E} = -\nabla\Phi$	Electric field in terms of scalar potential (1.16)	$\mathscr{E} = -k \frac{\mathrm{d}F}{\mathrm{d}t}$	Faraday's Law (5.135)
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{ \mathbf{x} - \mathbf{x}' } d^3x'$	Scalar potential in terms of charge density (1.17)	$W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} \ d^3 x$	Energy to ramp current from zero (4.83)::(5.149)
$\nabla^2 \Phi = -\rho/\epsilon_0$	Poisson Equation (1.28)	$W = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \ d^3x$	Magnetic energy in fields (4.89)::(5.148)
$q = \int \rho(\mathbf{x}') \ d^3x'$	Monopole (4.4)	$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \mathbf{H} \cdot \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \ d^3x$	Power in magetic field (5.147)
$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') \ d^3 x'$	Dipole (4.8)		
$Q_{ij} = \int (3x_i'x_j' - r'\delta_{ij})\rho(\mathbf{x}')d^3s$	$x' = 3M_{ij} - \text{Tr}(\mathbf{M}\delta_{ij})$ Quadrupole (4.9)	$\Delta W = \frac{1}{2} \int_{V_1} \mathbf{M} \cdot \mathbf{B}_0 \ d^3 x$	Energy to place permeable object in ${f B}_0$ (4.93)::(5.150)
$M_{ij} = \int x_i' x_j' \rho(x') \ d^3x$	Dana definition	$\frac{\mathrm{d}W}{\mathrm{d}t} = -\int \mathbf{J} \cdot \mathbf{E} \ d^3x$	Change in energy due to EMF
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} \right]$	$Q_{ij} \frac{x_i x_j}{r^5} + \dots$ Multipole Expansion (4.10)	$W = \frac{\mu_0}{2} \int \mathbf{H} ^2 d^3x = \frac{\mu_0}{2} \sum_{i=1}^{N}$	$\sum_{1}^{N} \int \Phi_{M_{j}}(\mathbf{x}) \rho_{M_{i}}(\mathbf{x}) \ d^{3}x \qquad N \text{ ferromagnets (HW 8.2b)}$
$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$	Electric displacement(4.34) Induced polarization (4.36)	$W = \frac{1}{2} \sum_{i=1}^{N} L_i I_i^2 + \sum_{i=1}^{N} \sum_{j>i}^{N} M_j$	$ij I_i I_j$ Inductive energy (5.152)
$\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$ $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$	Better expression for polarization	$M_{ij} = \frac{1}{I} F_{ij}$	Mutual inductance (5.156)
$\mathbf{D} = \epsilon \mathbf{E}$ $\epsilon = \epsilon_0 (1 + \chi_e)$	Electric displacement (4.37) Electric permittivity (4.38)	- <i>j</i>	uo m
$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}, \rho_b = -\nabla \cdot \mathbf{P}$	Electric bound charge density (G. 4.11)	$\mathbf{A}_{\mathrm{dip}} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \Rightarrow \mathbf{B}_{\mathrm{dip}} =$	
$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \end{cases}$	Boundary conditions (4.40)	$\mathbf{B}(z) = \frac{\mu_0 I}{2} \left[\frac{a^2}{(a^2 + z^2)^{3/2}} \right] \hat{\mathbf{z}}$	On-axis magnetic field of current loop (G. 5.41)
$\begin{cases} \Phi_{\rm in} = -\left(\frac{3}{\epsilon/\epsilon_0 + 2}\right) E_0 r \cos \theta \\ \Phi_{\rm out} = -E_0 r \cos \theta + \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) \end{cases}$	Dielectric sphere in $\mathbf{E} = E_0 \hat{\mathbf{z}}$ (4.54)	$\mathbf{B} = rac{\mu_0 NI}{L} \hat{\mathbf{z}}$	Magnetic field inside solenoid (5.59)
$W = \int \rho(\mathbf{x}) \Phi(\mathbf{x}) \ d^3x = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{I}$		$\mathbf{B}=rac{\mu_0NI}{2\pi ho}oldsymbol{\hat{\phi}}$	Magnetic field inside toroidal coil (G. 5.60)
2 0		$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$	Continuity Equation (6.3)
$\Delta W = -\frac{1}{2} \int_{V_1} \mathbf{P} \cdot \mathbf{E}_0 \ d^3 x$	Dielectric placed in \mathbf{E}_0 (4.93)	$\mathbf{E} = -\mathbf{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t}$	Potentials in dynamic systems (6.9)
$W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_{i} \sum_{i}$	$Q_{ij} \frac{\partial E_j}{\partial x_i}(0) + \dots$ Work multipole expsn. (4.24)	2	
$oldsymbol{ au} = \mathbf{p} imes \mathbf{E}$	Torque on electric dipole (G. 4.4)	$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -\rho/\epsilon_0$	Inhomogenous wave equation in Φ (6.15)
$\mathbf{F} = (\mathbf{P} \cdot \nabla)\mathbf{E}$	Force on electric dipole (G. 4.5)	$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$	Inhomogenous wave equation in ${\bf A}$ (6.16)
$\Phi = -E_0 \left(r - \frac{a^3}{r^2} \right)$	Electric potential of conducting sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (2.14)	$\nabla \cdot \mathbf{A}' + \frac{1}{c^2} \frac{\partial \Phi}{\partial t} = 0$	Lorenz gauge condition (6.17)
$E_r = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}, E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$	Electric dipole at origin in $\hat{\mathbf{z}}$ (4.12)	$u = \frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	Total energy density (6.106)
$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0 \mathbf{x} - \mathbf{x}_0 ^3}$	E -field due to dipole \mathbf{p} (4.13)	$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$	Differential continuity equation (6.108)
$ au=\mathbf{m} imes\mathbf{B}$	Torque on magnetic dipole moment (5.1)	$S = E \times H$	Poynting vector definition (6.109)
$\nabla \cdot \mathbf{J} = 0$ $d\mathbf{l} \times \mathbf{x}$	Condition of magnetostatics (5.3)	$\mathbf{g} = \frac{1}{c^2} (\mathbf{E} \times \mathbf{H})$	Electromagnetic momentum density (6.118)
$d\mathbf{B} = kI \frac{d1 \times \mathbf{x}}{ \mathbf{x} ^3}$	Biot-Savart Law (5.4)	$\mathbf{J} = \sigma \mathbf{E}, V = IR$	Ohm's Law (G. 7.3,4)
$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$	Ampère's law (5.25)	$P = IV = I^2R$	Joule heating law (G. 7.7)
$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x})$	Magnetic vector potential (5.27)	$\frac{\partial^2 V}{\partial t^2} = \frac{1}{\mathcal{L}\mathcal{C}} \frac{\partial^2 V}{\partial z^2} = c^2 \frac{\partial^2 V}{\partial z^2}$	Wave formulation of telegrapher's equations (L. 23)
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{ \mathbf{x} - \mathbf{x}' } d^3x'$	Magnetic vector potential of current distribution (5.32)	V(z,t) = f(t - z/c) + g(t + z)	
$\mathbf{m} = \frac{1}{2} \int \mathbf{x'} \times \mathbf{J}(\mathbf{x'}) \ d^3 x$	Magnetic moment definition (5.54)		+z/c)] Current solutions to telegrapher's equations (L. 30)
$\mathbf{m} = \frac{I}{2} \oint \mathbf{x} \times d\mathbf{l}$	Magnetic moment of closed circuit (J. pg. 186)	$Z = c\mathcal{L} = \sqrt{\mathcal{L}/\mathcal{C}}$	Impedance definition (L. 31) $ R-Z$
$ \mathbf{m} = I \times (Area)$	Magnetic moment of plane loop (5.57)	$V(\ell, t) = RI(\ell, t) \Rightarrow g(t + \ell/c)$	10 2
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{ \mathbf{x} ^3}$	Dipole vector potential (5.55)		$-\delta$] Sinusoidal solutions to telegrapher's equation (L. 53) $c(n-1)$ $n-2$ 2
$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[\frac{3\mathbf{n}(\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{ \mathbf{x} ^3} \right]$	Dipole induction (5.56)	$(a+x)^n = a^n + na^{n-1}x + \frac{n}{2}$	2!
$4\pi \left[\mathbf{x} ^3 \right]$ $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$	Force on dipole (5.69)	$f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) +$	$\frac{f''(a)}{2!}(x-a)^2 + \dots$ Taylor Series
$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$	Magnetic field (5.81)	$\begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \end{pmatrix}$	$ \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix}, \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} $
μ_0 $\mathbf{M} = (\mu/\mu_0 - 1)\mathbf{H}$	Magnetization in linear media (G. 6.29)		
$\mathbf{B} = \mu \mathbf{H}$ $\int (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0$	Linear condition (5.84)	$\begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{z}} \end{pmatrix}, \begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{z}} \end{pmatrix} = \begin{pmatrix} \rho/\sqrt{\rho^2 + z^2} & z/\sqrt{\rho^2 + z^2} & 0 \\ 0 & 0 & 1 \\ z/\sqrt{\rho^2 + z^2} & -\rho/\sqrt{\rho^2 + z^2} & 0 \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix}$
$\left(\mathbf{n}\times(\mathbf{H}_2-\mathbf{H}_1)=\mathbf{K}_f\right.$	Interface BC (5.86)	,	(-1 V P 1 2 P) V P 1 2 0) ***
$\mathbf{H} = -\nabla \Phi_{11}$	Magnetic scalar potential (5.93)	1	

Magnetic scalar potential (5.93)

Bound current density (G. 6.13,14)

Total magnetic moment (J. pg. 197)

Effective magnetic charge density (5.96,99)

Magnetic scalar potential of dipole (J. pg. 196)

Sphere with $\mathbf{M} = M_0 \mathbf{\hat{z}} \ [(r_{<}, r_{>}), \ (r, a)] \ (5.104)$

Poisson equation in terms of magnetic vector potential (5.101)

 $\mathbf{H} = -\boldsymbol{\nabla}\Phi_{M}$

 $\Phi_M(\mathbf{x}) = \frac{\mathbf{m} \cdot \mathbf{x}}{4\pi r^3}$ $\mathbf{m} = \int \mathbf{M} \ d^3 x$

 $\mathbf{J}_M = \mathbf{\nabla} \times \mathbf{M}, \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{n}$

 $\rho_M = -\nabla \cdot \mathbf{M}, \quad \boldsymbol{\sigma}_M = \mathbf{n} \cdot \mathbf{M}$

 $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_M \qquad \mathbf{I}$ $\Phi_M(r,\theta) = \frac{1}{3} M_0 a^2 \frac{r <}{r_+^2} \cos \theta$

	Wiggly	Decaying
x,y,z	$e^{\pm ik_n x}$, $A\cos(k_n x) + B\sin(k_n x)$	$e^{\pm k_n x}$, $A \cosh(k_n x) + B \sinh(k_n x)$
$ ho,\phi,z$	$e^{im\phi}, AJ_m(k_n\rho) + BY_m(k_n\rho)$	$AI_{m}(k_{n}\rho) + BK_{m}(k_{n}\rho)$
ρ, ϕ	$e^{im\phi}$	$A_0 + B_0 \ln \rho + \sum A_m \rho^m + B_m \rho^{-m}$
r , θ	$P_{\ell}(\cos \theta)$	$A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$
r, θ, ϕ	$Y_{\ell m}(\theta,\phi)$	$A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$