

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho, \quad \boldsymbol{\nabla} \cdot \mathbf{B} = 0, \quad \boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Maxwell's Eqns. (6.6)

General Equations of Electrostatics

$$\mathbf{E} = -\boldsymbol{\nabla}\Phi - \frac{\partial \mathbf{A}}{\partial t}$$

Electric field in terms of potentials (6.9)

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Scalar potential in terms of charge density (1.17)

$$\nabla^2 \Phi = -\rho/\epsilon_0$$

Electrostatic Poisson Equation (1.28)

$$V_i = \sum_{j=1}^n p_{ij} q_j, \quad Q_i = \sum_{j=1}^n C_{ij} V_j$$

Capactiance matrix (1.61)

$$q' = -\frac{a}{r}q, \quad r' = \frac{a^2}{r}$$

Magnitude and position of image charge on sphere (2.4)

$$q = \int \rho(\mathbf{x}') \, d^3x'$$

Electric Monopole (4.4)

$$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') \, d^3x'$$

Electric dipole (4.8)

$$Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\mathbf{x}') d^3x' = 3M_{ij} - \text{Tr}(\mathbf{M} \delta_{ij})$$

Electric Quadrupole (4.9)

$$M_{ij} = \int x'_i x'_j \rho(x') \, d^3x$$

Dana definition of **M** matrix

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \dots \right]$$

Electric multipole Expansion (4.10)

$$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0 |\mathbf{x} - \mathbf{x}_0|^3}$$

**E**-field due to dipole **p** (4.13)

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}, \quad \mathbf{F} = (\mathbf{p} \cdot \boldsymbol{\nabla}) \mathbf{E}$$

Torque, force on electric dipole (G. 4.4, 4.5)

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

Electric displacement(4.34)

$$\mathbf{D} = \epsilon \mathbf{E}$$

Electric displacement (linear materials) (4.37)

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} = (\epsilon - \epsilon_0) \mathbf{E}$$

Induced polarization (linear materials) (4.36)

$$\epsilon = \epsilon_0 (1 + \chi_e)$$

Electric permittivity (linear materials) (4.38)

$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}, \quad \rho_b = -\boldsymbol{\nabla} \cdot \mathbf{P}$$

Electric bound charge density (G. 4.11)

$$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \Rightarrow \Phi_1 = \Phi_2 \end{cases}$$

Electric JC's (evaluate at boundary) (4.40)

$$W = \int \rho(\mathbf{x}) \Phi(\mathbf{x}) \, d^3x = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{D} \, d^3x$$

Energy to bring charges from  $\infty$  (4.83,89)

$$W = \frac{1}{2} \sum_{i=1}^n Q_i V_i = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ij} V_i V_j$$

Potential energy of capacitor system (1.62)

$$W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_i \sum_j Q_{ij} \frac{\partial E_j}{\partial x_i}(0) + \dots$$

Work multipole expsn. (4.24)

Specific Cases in Electrostatics

$$\begin{cases} \Phi_{\text{in}} = -\left(\frac{3}{\epsilon/\epsilon_0+2}\right) E_0 r \cos \theta \\ \Phi_{\text{out}} = -E_0 r \cos \theta + \left(\frac{\epsilon/\epsilon_0-1}{\epsilon/\epsilon_0+2}\right) E_0 \frac{a^3}{r^2} \cos \theta \end{cases}$$

Dielectric sphere in  $\mathbf{E} = E_0 \hat{\mathbf{z}}$  (4.54)

$$\Phi = -E_0 \left( r - \frac{a^3}{r^2} \right)$$

Electric potential of conducting sphere in  $\mathbf{E} = E_0 \hat{\mathbf{z}}$  (2.14)

$$\mathbf{E} = \frac{\sigma}{\epsilon_0} \hat{\mathbf{n}}$$

**E** of parallel-plate capacitor ( $\hat{\mathbf{n}}$  points from pos. to neg.) (G. Ex. 2.6)

$$\mathbf{E} = \frac{P}{4\pi\epsilon_0 r^3} \left( 2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}} \right)$$

Electric dipole at origin pointing in  $\hat{\mathbf{z}}$  (4.12)

General Equations of Magnetostatics

$$\boldsymbol{\nabla} \cdot \mathbf{J} = 0$$

Condition of magnetostatics (5.3)

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_M$$

Poisson equation in terms of magnetic vector potential (5.101)

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3}$$

Biot-Savart law (G. 5.34)

$$\mathbf{B}(\mathbf{x}) = \boldsymbol{\nabla} \times \mathbf{A}(\mathbf{x}), \quad \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Magnetic vector potential (5.27,5.32)

$$\mathbf{m} = \frac{1}{2} \int \mathbf{x}' \times \mathbf{J}(\mathbf{x}') \, d^3x'$$

Magnetic dipole (5.54)

$$\mathbf{m} = I A \hat{\mathbf{n}}$$

Magnetic moment of plane loop (5.57)

$$\mathbf{m} = \int \mathbf{M} \, d^3x$$

Total magnetic moment (J. pg. 197)

$$\Phi_M(\mathbf{x}) = \frac{\mathbf{m} \cdot \mathbf{x}}{4\pi r^3}, \quad \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{|\mathbf{x}|^3}$$

Magnetic potentials of a dipole (5.55)

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[ \frac{3\mathbf{n}(\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{|\mathbf{x}|^3} \right]$$

Magnetic field of dipole (5.56)

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}, \quad \mathbf{F} = \boldsymbol{\nabla}(\mathbf{m} \cdot \mathbf{B})$$

Torque and force on magnetic dipole (5.1, 5.69)

$$\mathbf{H} = -\boldsymbol{\nabla} \Phi_M$$

Magnetic scalar potential (valid if  $\mathbf{J}_f = 0$ ) (5.93)

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

Definition of **H** (5.81)

$$\mathbf{B} = \mu \mathbf{H}$$

Property of linear permeable materials (5.84)

$$\begin{cases} (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f, \, \mathbf{K}_f = 0 \Rightarrow \Phi_1 = \Phi_2 \end{cases}$$

Magnetic JC's (eval. at boundary) (5.86)

$$\mathbf{J}_M = \boldsymbol{\nabla} \times \mathbf{M}, \quad \mathbf{K}_b = \mathbf{M} \times \mathbf{n}$$

Bound current density (G. 6.13,14)

$$\rho_M = -\boldsymbol{\nabla} \cdot \mathbf{M}, \quad \sigma_M = \mathbf{n} \cdot \mathbf{M}$$

Effective magnetic charge density (5.96,99)

$$F = \int_S \mathbf{B} \cdot \mathbf{n} \, da = \oint \mathbf{A} \cdot d\mathbf{l}$$

Magnetic flux (5.133)

$$\mathcal{E} = -\frac{dF}{dt} = \oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t}$$

EMF due to Faraday's Law (5.135)

$$M_{ij} = \frac{1}{I_j} F_{ij}$$

Mutual inductance (5.156)

$$\mathcal{L} = \frac{1}{I^2} \int_C \mathbf{J}(\mathbf{x}) \cdot \mathbf{A}(\mathbf{x}) \, d^3x$$

Neumann's formula for self-inductance (5.154)

$$W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} \, d^3x = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \, d^3x$$

Energy in magnetic field (5.149,5.148)

$$W = \frac{1}{2} \sum_{i=1}^N \mathcal{L}_i I_i^2 + \sum_{i=1}^N \sum_{j>i}^N M_{ij} I_i I_j$$

Inductive energy (5.152)

Specific Cases of Magnetostatics

$$\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \Rightarrow \mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

Field of dipole (G 5.87,88)

$$\mathbf{B}(z) = \frac{\mu_0 I}{2} \left[ \frac{a^2}{(a^2 + z^2)^{3/2}} \right] \hat{\mathbf{z}}$$

On-axis magnetic field of current loop (G. 5.41)

$$\mathbf{B} = \frac{\mu_0 N I}{L} \hat{\mathbf{z}}, \quad \mathbf{B} = \frac{\mu_0 N I}{2\pi \rho} \hat{\boldsymbol{\phi}}$$

Magnetic field inside (solenoid, toroidal coil) (G. 5.59,60)

$$\Phi_M(r, \theta) = \frac{1}{3} M_0 a^2 \frac{r_{<}}{r_{>}^2} \cos \theta$$

Sphere with uniform  $\mathbf{M} = M_0 \hat{\mathbf{z}}$  [ $(r_{<}, r_{>})$ ,  $(r, a)$ ] (5.104)

$$\mathbf{M} = \frac{3}{\mu_0} \left( \frac{\mu - \mu_0}{\mu + 2\mu_0} \right) \mathbf{B}_0$$

Permeable sphere in uniform magnetic field  $\mathbf{B}_0$  (5.115)

$$\mathcal{L} = \frac{\mu_0 \pi a^2 N^2}{L}$$

Inductance of a solenoid with N turns per unit length L (L. HW9 Pr. 2)

Electrodynamics

$$\boldsymbol{\nabla} \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0, \quad \frac{\partial u}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$$

Continuity Equations (6.3, 6.108)

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -\rho/\epsilon_0, \quad \nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$$

EM wave equations (6.15,6.16)

$$\boldsymbol{\nabla} \cdot \mathbf{A}' + \frac{1}{c^2} \frac{\partial \Phi'}{\partial t} = 0$$

Lorenz gauge condition (6.17)

$$\Phi(\mathbf{x}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{[\rho(\mathbf{x}', t')]_{\text{ret}}}{|\mathbf{x} - \mathbf{x}'|} d^3x$$

Retarded potentials (6.48)

$$\mathbf{A}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \int \frac{[\mathbf{J}(\mathbf{x}', t')]_{\text{ret}}}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

Retarded time (J. pg. 246)

$$r \gg c\tau \text{ (radiation zone)}, \quad r \ll c\tau \text{ (Static zone)}$$

(G. 11.10, 11.13)

$$u = \frac{1}{2} (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$$

Total electromagnetic energy density (6.106)

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Poynting vector definition (6.109)

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Lorentz force law (6.113)

$$\mathbf{g} = \frac{1}{c^2} (\mathbf{E} \times \mathbf{H})$$

Electromagnetic momentum density (6.118)

$$\mathbf{J} = \sigma \mathbf{E}, \quad V = IR; \quad P = IV = I^2 R$$

Ohm's Law, Joule Heating Law (G. 7.3, 7.4; 7.7)

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{\mathcal{L}C} \frac{\partial^2 V}{\partial z^2} = c^2 \frac{\partial^2 V}{\partial z^2}$$

Wave formulation of telegrapher's equations (L. 23)

$$V(z, t) = f(t - z/c) + g(t + z/c)$$

Voltage solutions to telegrapher's equations (L. 25)

$$I(z, t) = \frac{1}{Z} [f(t - z/c) - g(t + z/c)]$$

Current solutions to telegrapher's equations (L. 30)

$$Z = c\mathcal{L} = \sqrt{\mathcal{L}/\mathcal{C}}$$

Impedance definition (L. 31)

$$V(\ell, t) = RI(\ell, t) \Rightarrow g(t + \ell/c) = \frac{R - Z}{R + Z} f(t - \ell/c)$$

Resistive BC (L. 37,39)

$$V(z, t) = |\tilde{A}_0| \cos[\omega(t \mp z/c) - \delta]$$

Sinusoidal solutions to telegrapher's equation (L. 53)

Potentially Useful Mathematical Identities

$$\delta(f(x)) = \sum_i \frac{1}{\left| \frac{df}{dx}(x_i) \right|} \delta(x - x_i)$$

Jackson Dirac delta function Rule 5

$$(a+x)^n \approx a^n + na^{n-1}x + \frac{n(n-1)}{2!} a^{n-2}x^2 + \dots =$$

Binomial Expansion

$$f(x) \approx f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots$$

Taylor Series

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \frac{L}{2} \delta_{nm}$$

Trig function normalization

$$\int_0^{2\pi} d\phi \int_0^L dz \, e^{i(m-n')\phi} e^{i\frac{2\pi}{L}(n-n')z} = 2\pi L \delta_{m'm} \delta_{n'n}$$

Exponential normalization

$$J_m(k\rho) \propto (k\rho)^m, \, Y_m(k\rho) \propto (k\rho)^{-m}, \, I_m(k\rho) \propto (k\rho)^m, \, K_m(k\rho) \propto (k\rho)^{-m}, \, \text{As } \rho \rightarrow 0$$

$$\begin{pmatrix} \hat{r} \\ \hat{\phi} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix}, \quad \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix}$$

$$\begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{r} \\ \hat{\phi} \\ \hat{z} \end{pmatrix}, \quad \begin{pmatrix} \hat{r} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \rho/\sqrt{\rho^2+z^2} & z/\sqrt{\rho^2+z^2} & 0 \\ 0 & 0 & 1 \\ z/\sqrt{\rho^2+z^2} & -\rho/\sqrt{\rho^2+z^2} & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix}$$

	Wiggly	Decaying
$x, y, z$	$e^{\pm i k_n x}, \, A \cos(k_n x) + B \sin(k_n x)$	$e^{\pm k_n x}, \, A \cosh(k_n x) + B \sinh(k_n x)$
$\rho, \phi, z$	$e^{im\phi}, \, A J_m(k_n \rho) + B Y_m(k_n \rho)$	$A I_m(k_n \rho) + B K_m(k_n \rho)$
$\rho, \phi$	$e^{im\phi}$	$A_0 + B_0 \ln \rho + \sum A_m \rho^m + B_m \rho^{-m}$
$r, \theta$	$P_\ell(\cos \theta)$	$A \left(\frac{r}{a}\right)^\ell + B \left(\frac{r}{a}\right)^{-(\ell+1)}$
$r, \theta, \phi$	$Y_{\ell m}(\theta, \phi)$	$A \left(\frac{r}{a}\right)^\ell + B \left(\frac{r}{a}\right)^{-(\ell+1)}$