Midterm 2 Equations	PHSX519 Electron
$\nabla \cdot \mathbf{D} = \rho, \nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{H} = \mathbf{J}$	$\mathbf{F} + \frac{\partial \mathbf{D}}{\partial t}, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \text{Maxwell's Equations (6.6)}$
$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{ \mathbf{x} - \mathbf{x}' ^3} d^3x'$	Coulomb's Law (1.5)
$\delta(f(x)) = \sum_{i} \frac{1}{\left \frac{\mathrm{d}f}{\mathrm{d}x}(x_{i})\right } \delta(x - x_{i})$	Delta function Rule 5
$i \mid \frac{\vec{d}x}{\vec{d}x}(x_i) \mid$ $\mathbf{E} = -\nabla \Phi$	Electric field in terms of scalar potential (1.16)
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x'})}{ \mathbf{x} - \mathbf{x'} } d^3x'$	Scalar potential in terms of charge density (1.17)
$\nabla^2 \Phi = -\rho/\epsilon_0$	Poisson Equation (1.28)
$q = \int \rho(\mathbf{x'}) \ d^3x'$	Monopole (4.4)
$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') \ d^3 x'$	Dipole (4.8)
$Q_{ij} = \int (3x'_i x'_j - r' \delta_{ij}) \rho(\mathbf{x}') d^3 x' = 3$	$3M_{ij} - \text{Tr}(\mathbf{M}\delta_{ij})$ Quadrupole (4.9)
$M_{ij} = \int x_i' x_j' \rho(x') d^3x$	Dana definition
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \right]$	$\left[\frac{x_i x_j}{r^5} + \ldots\right]$ Multipole Expansion (4.10)
$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$	Electric displacement (4.34) Induced polarization (4.36)
$\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$	Better expression for polarization
$\mathbf{D} = \epsilon \mathbf{E}$ $\epsilon = \epsilon_0 (1 + \chi_e)$	Electric displacement (4.37) Electric permittivity (4.38)
$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}, \rho_b = -\nabla \cdot \mathbf{P}$	Electric bound charge density (G. 4.11)
$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \end{cases}$	Boundary conditions (4.40)
$\begin{cases} \Phi_{\rm in} = -\left(\frac{3}{\epsilon/\epsilon_0 + 2}\right) E_0 r \cos \theta \\ \Phi_{\rm out} = -E_0 r \cos \theta + \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0 \end{cases}$	$\frac{a^3}{r^2}\cos\theta$ Dielectric sphere in $\mathbf{E}=E_0\mathbf{\hat{z}}$ (4.54)
$W = \int \rho(\mathbf{x})\Phi(\mathbf{x}) \ d^3x = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{D} \ d^3x$	Energy to bring charges from ∞ (4.83,89)
$\Delta W = -\frac{1}{2} \int_{V_1} \mathbf{P} \cdot \mathbf{E}_0 \ d^3x$	Dielectric placed in \mathbf{E}_0 (4.93)
$W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_{i} \sum_{j} Q_{ij} \frac{\partial}{\partial t}$	$\frac{\partial E_j}{\partial x_i}(0) + \dots$ Work multipole expsn. (4.24)
$\mathbf{N} = \mathbf{p} \times \mathbf{E}$ $\mathbf{F} = (\mathbf{P} \cdot \nabla)\mathbf{E}$	Torque on electric dipole (G. 4.4) Force on electric dipole (G. 4.5)
$\Phi = -E_0 \left(r - \frac{a^3}{r^2} \right)$ Elec	tric potential of conducting sphere in ${\bf E}=E_0{\bf \hat{z}}$ (2.14)
$E_T = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}, E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$	Electric dipole at origin in $\hat{\mathbf{z}}$ (4.12)
$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0 \mathbf{x} - \mathbf{x}_0 ^3}$	E -field due to dipole \mathbf{p} (4.13)
$\mathbf{N} = \mathbf{m} \times \mathbf{B}$ $\nabla \cdot \mathbf{J} = 0$	Torque on magnetic dipole moment (5.1) Condition of magnetostatics (5.3)
$d\mathbf{B} = kI \frac{d\mathbf{l} \times \mathbf{x}}{ \mathbf{x} ^3}$	Biot-Savart Law (5.4)
$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$	Ampère's law (5.25)
$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x})$	Magnetic vector potential (5.27)
$4\pi^{-J} \mathbf{x} - \mathbf{x}' $	agnetic vector potential of current distribution (5.32)
$\mathbf{m} = \frac{1}{2} \int \mathbf{x'} \times \mathbf{J}(\mathbf{x'}) \ d^3x$	Magnetic moment definition (5.54)
$\mathbf{m} = \frac{I}{2} \oint \mathbf{x} \times d\mathbf{l}$	Magnetic moment of closed circuit (J. pg. 186)
$ \mathbf{m} = I \times (\text{Area})$	Magnetic moment of plane loop (5.57)
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{ \mathbf{x} ^3}$	Dipole vector potential (5.55)
$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[\frac{3\mathbf{n}(\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{ \mathbf{x} ^3} \right]$	Dipole induction (5.56)
$\mathbf{F} = \mathbf{\nabla}(\mathbf{m} \cdot \mathbf{B})$	Force on dipole (5.69)
$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$	Magnetic field (5.81)
$\mathbf{M} = (\mu/\mu_0 - 1)\mathbf{H}$ $\mathbf{B} = \mu\mathbf{H}$	Magnetization in linear media (G. 6.29) Linear condition (5.84)
$\begin{cases} (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f \end{cases}$	Interface BC (5.86)
$ \left(\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f \right) $ $ \mathbf{H} = -\mathbf{\nabla} \Phi_M $	Magnetic scalar potential (5.93)
$\mathbf{J}_{M} = \mathbf{\nabla} \times \mathbf{M}, \mathbf{K}_{b} = \mathbf{M} \times \mathbf{n}$ $\rho_{M} = -\mathbf{\nabla} \cdot \mathbf{M}, \sigma_{M} = \mathbf{n} \cdot \mathbf{M}$	Bound current density (G. 6.13,14) Effective magnetic charge density (5.96,99)
$\Phi_M(\mathbf{x}) = \frac{\mathbf{m} \cdot \mathbf{x}}{4\pi r^3}$	Magnetic scalar potential of dipole (J. pg. 196)
$4\pi r^3$ $\mathbf{m} = \int \mathbf{M} \ d^3 x$	Total magnetic moment (J. pg. 197)
2	equation in terms of magnetic vector potential (5.101)
$\Phi_M(r,\theta) = \frac{1}{3} M_0 a^2 \frac{r_{<}}{r_{<}^2} \cos \theta$	Sphere with $\mathbf{M} = M_0 \hat{\mathbf{z}} \ [(r_{<}, r_{>}), \ (r, a)] \ (5.104)$
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omagnetic Theory I	Roy Smart
$\mathbf{M} = \frac{3}{\mu_0} \left(\frac{\mu - \mu_0}{\mu + 2\mu_0} \right) \mathbf{B}_0$	dermeable sphere in uniform magnetic field ${f B}_0$ (5.115)
$F = \int_{\mathcal{C}} \mathbf{B} \cdot \mathbf{n} da$	Magnetic flux (5.133)
$\mathscr{E} = \oint_C \mathbf{E}' \cdot d\mathbf{l}$	Electromotive force (5.134)
$\mathscr{E} = -k \frac{\mathrm{d}F}{\mathrm{d}t}$	Faraday's Law (5.135)
$W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} \ d^3 x$	Energy to ramp current from zero (4.83)::(5.149)
$W = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \ d^3x$	Magnetic energy in fields (4.89)::(5.148)
$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \mathbf{H} \cdot \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} d^3x$	Power in magetic field (5.147)
1 / 0	energy to place permeable object in ${f B}_0$ (4.93)::(5.150)
$\frac{\mathrm{d}W}{\mathrm{d}t} = -\int \mathbf{J} \cdot \mathbf{E} \ d^3x$	Change in energy due to EMF
$dt = \frac{\mu_0}{2} \int \mathbf{H} ^2 d^3x = \frac{\mu_0}{2} \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{i=1}^{N} \int_{i=1}^{$	$\int \Phi_{M_{\hat{I}}}(\mathbf{x}) ho_{M_{\hat{I}}}(\mathbf{x}) \ d^3x \qquad N ext{ ferromagnets (HW 8.2b)}$
$W = \frac{1}{2} \sum_{i=1}^{N} L_i I_i^2 + \sum_{i=1}^{N} \sum_{j>i}^{N} M_{ij} I_i I_j$	Inductive energy (5.152)
1=1 1=13/1	Mutual inductance (5.156)
$M_{ij} = \frac{1}{I_j} F_{ij}$ $\mathbf{A}_{\text{dip}} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \Rightarrow \mathbf{B}_{\text{dip}} = \frac{\mu_0 m}{4\pi r^3}$,
4/1 / 4/1/	$(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$ Field of dipole (G 5.87,88)
$\mathbf{B}(z) = \frac{\mu_0 I}{2} \left[\frac{a^2}{(a^2 + z^2)^{3/2}} \right] \hat{\mathbf{z}}$	On-axis magnetic field of current loop (G. 5.41)
$\mathbf{B} = rac{\mu_0 NI}{L} \mathbf{\hat{z}}$	Magnetic field inside solenoid (5.59)
$\mathbf{B}=rac{\mu_0 NI}{2\pi ho} oldsymbol{\hat{\phi}}$	Magnetic field inside toroidal coil (G. 5.60)
$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$	Continuity Equation (6.3)
$\mathbf{E} = -\mathbf{\nabla}\Phi - rac{\partial\mathbf{A}}{\partial t}$	Potentials in dynamic systems (6.9)
$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -\rho/\epsilon_0$	Inhomogenous wave equation in Φ (6.15)
$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu_0 \mathbf{J}$	Inhomogenous wave equation in ${\bf A}$ (6.16)
$\nabla \cdot \mathbf{A}' + \frac{1}{c^2} \frac{\partial \Phi}{\partial t} = 0$	Lorenz gauge condition (6.17)
$u = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$	Total energy density (6.106)
$rac{\partial u}{\partial t} + oldsymbol{ abla} \cdot \mathbf{S} = - \mathbf{J} \cdot \mathbf{E}$	Differential continuity equation (6.108)
$S = E \times H$	Poynting vector definition (6.109)
$\mathbf{g} = \frac{1}{c^2} (\mathbf{E} \times \mathbf{H})$	Electromagnetic momentum density (6.118)
$\mathbf{J} = \sigma \mathbf{E}, V = IR$	Ohm's Law (G. 7.3,4)
$P = IV = I^2R$	Joule heating law (G. 7.7)
$\frac{\partial^2 V}{\partial t^2} = \frac{1}{\mathcal{L}\mathcal{C}} \frac{\partial^2 V}{\partial z^2} = c^2 \frac{\partial^2 V}{\partial z^2}$	Wave formulation of telegrapher's equations (L. 23)
V(z,t) = f(t-z/c) + g(t+z/c)	Voltage solutions to telegrapher's equations (L. 25)
-	Current solutions to telegrapher's equations (L. 30)
$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)^n}{2!}$	
$f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}$	$-(x-a)^2 + \dots$ Taylor Series
$\begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{z}} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix},$	$ \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} $
$\begin{pmatrix} \hat{\mathbf{f}} \\ \hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} \end{pmatrix} = \begin{pmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{\rho}} \\ \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{z}} \end{pmatrix},$	$\begin{pmatrix} \hat{\rho} \\ \hat{\phi} \\ \hat{z} \end{pmatrix} = \begin{pmatrix} \rho/\sqrt{\rho^2 + z^2} & z/\sqrt{\rho^2 + z^2} & 0 \\ 0 & 0 & 1 \\ z/\sqrt{\rho^2 + z^2} & -\rho/\sqrt{\rho^2 + z^2} & 0 \end{pmatrix} \begin{pmatrix} \hat{\mathbf{f}} \\ \hat{\theta} \\ \hat{\phi} \end{pmatrix}$
Wiggly	Decaying

	Wiggly	Decaying
x, y, z	$e^{\pm ik_n x}, A\cos(k_n x) + B\sin(k_n x)$	$e^{\pm k_n x}, A \cosh(k_n x) + B \sinh(k_n x)$
ρ, ϕ, z	$e^{im\phi}, AJ_m(k_n\rho) + BY_m(k_n\rho)$	$AI_{m}(k_{n}\rho) + BK_{m}(k_{n}\rho)$
ρ, ϕ	$e^{im\phi}$	$A_0 + B_0 \ln \rho + \sum A_m \rho^m + B_m \rho^{-m}$
r, θ	$P_{\ell}(\cos \theta)$	$A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$
r, θ, ϕ	$Y_{\ell m}(\theta,\phi)$	$A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$