

$$q = \int \rho(\mathbf{x}') d^3x' \quad \text{Monopole (4.4)}$$

$$\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') d^3x' \quad \text{Dipole (4.8)}$$

$$Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\mathbf{x}') d^3x' \quad \text{Quadrupole (4.9)}$$

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \dots \right]$$

Multipole Expansion (4.10)

$$E_r = \frac{2p \cos \theta}{4\pi\epsilon_0}, \quad E_\theta = \frac{2p \sin \theta}{4\pi\epsilon_0} \quad \text{Dipole in } \hat{\mathbf{z}} \text{ (4.12)}$$

$$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0 |\mathbf{x} - \mathbf{x}_0|^3} \quad \text{E-field due to dipole } \mathbf{p} \text{ (4.13)}$$

$$W = \int \rho(\mathbf{x}) \Phi(\mathbf{x}) d^3x \quad \text{Charge in external field (4.21)}$$

$$W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_i \sum_j Q_{ij} \frac{\partial E_j}{\partial x_i}(0) + \dots$$

Energy multipole expansion (4.24)

$$\mathbf{P}(\mathbf{x}) = \sum_i N_i \langle \mathbf{p}_i \rangle \quad \text{Electric polarization (4.28)}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \text{Electric displacement (4.34)}$$

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad \text{Induced polarization (4.36)}$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad \text{Electric displacement (4.37)}$$

$$\epsilon = \epsilon_0(1 + \chi_e) \quad \text{Electric permittivity (4.38)}$$

$$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1 \times \mathbf{n}_{21}) = 0 \end{cases} \quad \text{Boundary conditions (4.40)}$$

$$\Phi_{\text{in}} = - \left( \frac{3}{\epsilon/\epsilon_0 + 2} \right) E_0 r \cos \theta$$

$$\Phi_{\text{out}} = -E_0 r \cos \theta + \left( \frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2} \right) E_0 \frac{a^3}{r^2} \cos \theta$$

Dielectric sphere in uniform  $\mathbf{E}$ -field (4.54)

$$W = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{D} d^3x \quad \text{Electrostatic Energy (4.89)}$$

$$W = -\frac{1}{2} \int_{V_1} \mathbf{P} \cdot \mathbf{E}_0 d^3x \quad \text{Dielectric placed in } \mathbf{E}_0 \text{ (4.93)}$$