| Midterm 2 Equations | PHSX519 Electro |
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| $\label{eq:continuous_def} \boldsymbol{\nabla} \cdot \mathbf{D} = \boldsymbol{\rho}_f, \boldsymbol{\nabla} \times \mathbf{E} = 0, \boldsymbol{\nabla} \cdot \mathbf{B} = 0,$ | $ abla 	imes \mathbf{H} = \mathbf{J}_f$ Maxwell's equations in matter |
| $\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{ \mathbf{x} - \mathbf{x}' ^3} d^3x'$ | Coulomb's Law (1.5) |
| $\delta(f(x)) = \sum_{i} \frac{1}{\left \frac{\mathrm{d}f}{\mathrm{d}x}(x_{i})\right } \delta(x - x_{i})$ | Delta function Rule 5 |
| $\oint_{S} \mathbf{E} \cdot \mathbf{n} \ da = \frac{1}{\epsilon_{0}} \int_{V} \rho(\mathbf{x}) d^{3}x$ | Gauss' Law (1.11) |
| $\mathbf{E} = -\nabla \Phi$ | Electric field in terms of scalar potential (1.16) |
| $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x'})}{ \mathbf{x} - \mathbf{x'} } d^3x'$ | Scalar potential in terms of charge density (1.17) |
| $\nabla^2 \Phi = -\rho/\epsilon_0$ | Poisson Equation (1.28) |
| $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int_V \rho(\mathbf{x'}) G_D(\mathbf{x}, \mathbf{x'}) d^3x' =$ | $\frac{1}{4\pi} \oint_{S} (\mathbf{x}') \frac{\partial G_{D}}{\partial n'} da'$ DBCs (1.44) |
| $\Phi(\mathbf{x}) = \langle \Phi \rangle_S + \frac{1}{4\pi\epsilon_0} \int_V \rho(\mathbf{x}') G_N(\mathbf{x}, \mathbf{x}')$ | $d^{3}x + \frac{1}{4\pi} \oint_{S} \frac{\partial \Phi}{\partial n'} G_{N} da' \qquad \text{NBCs (1.46)}$ |
| 2 | tude and position of image charge on sphere (2.4) |
| $\Phi = -E_0 \left(r - \frac{a^3}{r^2} \right) \qquad \qquad \text{Electric}$ | potential of conducting sphere in ${\bf E}=E_0{\bf \hat{z}}$ (2.14) |
| $\frac{1}{ \mathbf{x} - \mathbf{x}' } = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_{<}^{\ell}}{r_{<}^{\ell+1}} Y_{\ell}^{\ell}$ | $_{m}^{*}(\theta',\phi')Y_{\ell m}(\theta,\phi) \qquad \qquad \text{GFE: } r,\theta,\phi \text{ (3.70)}$ |
| $\frac{1}{ \mathbf{x} - \mathbf{x}' } = \frac{2}{\pi} \sum_{m = -\infty}^{\infty} \int_{0}^{\infty} dk e^{im(\phi - \phi')}$ | |
| Where $r_{<}(r_{>})$ $[\rho_{<}(\rho_{>})]$ is the smaller (| larger) of $ \mathbf{x} $ and $ \mathbf{x'} $ |
| $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} q_{lm} \frac{Y_{lm}}{r^{l+1}}$ | Potential outside spher of charge (4.1) |
| $q_{lm} = \int Y_{lm}^*(\theta', \phi') r'^l \rho(\mathbf{x'}) \ d^3x'$ | Multipole moments (4.3) |
| $q = \int \rho(\mathbf{x}') \ d^3x'$ | Monopole (4.4) |
| $\mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') \ d^3 x'$ | Dipole (4.8) |
| $Q_{ij} = \int (3x_i'x_j' - r'\delta_{ij})\rho(\mathbf{x}')d^3x'$ | Quadrupole (4.9) |
| $\mathbf{Q} = \int (3\hat{\mathbf{x}}\hat{\mathbf{x}} - 1)\rho(x')x'^2 \ d^3x'$ | Dana quadrupole 1D |
| $Q_{ij} = 3M_{ij} - \mathrm{Tr}(\mathbf{M})$ | Dana Quadrupole expression |
| $M_{ij} = \int x_i' x_j' \rho(x') d^3 x$ | Dana definition |
| $\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} \right]$ | $\left[\frac{j}{j} + \ldots\right]$ Multipole Expansion (4.10) |
| $E_T = \frac{2p\cos\theta}{4\pi\epsilon_0}, E_\theta = \frac{2p\sin\theta}{4\pi\epsilon_0}$ | Dipole in $\hat{\mathbf{z}}$ (4.12) |
| $\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0 \mathbf{x} - \mathbf{x}_0 ^3}$ | ${f E}$ -field due to dipole ${f p}$ (4.13) |
| $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ | Electric displacement (4.34) |
| $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$ $\mathbf{P} = (\epsilon - \epsilon_0) \mathbf{E}$ | Induced polarization (4.36) Better expression for polarization |
| $\mathbf{D} = \epsilon \mathbf{E}$ | Electric displacement (4.37) |
| $\epsilon = \epsilon_0 (1 + \chi_e)$ $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$ | Electric permittivity (4.38) Electric surface bound charge density (G. 4.11) |
| $ \rho_b = \mathbf{r} \cdot \mathbf{r} $ $ \rho_b = -\nabla \cdot \mathbf{P} $ | Electric volume bound charge density (G. 4.11) |
| $\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \end{cases}$ | Boundary conditions (4.40) |
| $\begin{cases} \Phi_{\mathrm{in}} = -\left(\frac{3}{\epsilon/\epsilon_0 + 2}\right) E_0 r \cos \theta \\ \Phi_{\mathrm{out}} = -E_0 r \cos \theta + \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0 \frac{a^3}{r^2} \end{cases}$ | Dielectric sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (4.54) |
| $W = \int \rho(\mathbf{x}) \Phi(\mathbf{x}) \ d^3x = \frac{\epsilon_0}{2} \int \mathbf{E} ^2 \ d^3x$ | Energy to bring charges from ∞ (4.83) |
| $W = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{D} \ d^3 x$ | Energy stored in electric field (4.89) |
| $\Delta W = -\frac{1}{2} \int_{V_1} \mathbf{P} \cdot \mathbf{E}_0 \ d^3 x$ | Dielectric placed in \mathbf{E}_0 (4.93) |
| $W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_{i} \sum_{j} Q_{ij} \frac{\partial E_{i}}{\partial x_{i}}$ | $\frac{j}{t}(0) + \dots$ Work multipole expsn. (4.24) |
| $\mathbf{N} = \mathbf{m} \times \mathbf{B}$ $\nabla \cdot \mathbf{J} = 0$ | Torque on magnetic dipole moment (5.1) Condition of magnetostatics (5.3) |
| $d\mathbf{B} = kI \frac{d\mathbf{l} \times \mathbf{x}}{ \mathbf{x} ^3}$ | Biot-Savart Law (5.4) |
| $\mathbf{F} = \int \mathbf{J}(\mathbf{x}) \times \mathbf{B}(\mathbf{x}) \ d^3x$ | Force on current dist. (5.12) |
| $\mathbf{N} = \int \mathbf{x} \times (\mathbf{J} \times \mathbf{B}) \ d^3 x$ | Torque on current dist. (5.13) |

| magnetic Theory I | Roy Smart | |
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| $\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$ | Ampère's law (5.25) | |
| $\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x})$ | Magnetic vector potential (5.27) | |
| $\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x'})}{ \mathbf{x} - \mathbf{x'} } d^3 x'$ | Magnetic vector potential of current distribution (5.32) | |
| $\mathbf{m} = \frac{1}{2} \int \mathbf{x'} \times \mathbf{J}(\mathbf{x'}) \ d^3 x$ | Magnetic moment definition (5.54) | |
| $\mathbf{m} = \frac{I}{2} \oint \mathbf{x} \times d\mathbf{l}$ | Magnetic moment of closed circuit (J. pg. 186) | |
| $ \mathbf{m} = I \times (Area)$ | Magnetic moment of plane loop (5.57) | |
| $\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{ \mathbf{x} ^3}$ | Dipole vector potential (5.55) | |
| $\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[\frac{3\mathbf{n}(\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{ \mathbf{x} ^3} \right]$ | Dipole induction (5.56) | |
| $\mathbf{F} = \mathbf{\nabla}(\mathbf{m} \cdot \mathbf{B})$ | Force on dipole (5.69) | |
| $\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$ | Magnetic field (5.81) | |
| $\mathbf{M} = (\mu/\mu_0 - 1)\mathbf{H}$ | Magnetization in linear media (G. 6.29) | |
| $\mathbf{B} = \mu \mathbf{H}$ | Linear condition (5.84) | |
| $\begin{cases} (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f \end{cases}$ | Interface BC (5.86) | |
| $\mathbf{H} = -\nabla \Phi_M$ | Magnetic scalar potential (5.93) | |
| $\mathbf{J}_M = \mathbf{\nabla} \times \mathbf{M}$ $\mathbf{K}_b = \mathbf{M} \times \mathbf{n}$ | Bound volume current density (G. 6.13) Bound surface current density (G. 6.14) | |
| $ \rho_M = -\nabla \cdot \mathbf{M} $ | Effective magnetic charge density (5.96) | |
| $\sigma_M = \mathbf{v} \cdot \mathbf{M}$ $\sigma_M = \mathbf{n} \cdot \mathbf{M}$ | Effective magnetic surface-charge density (5.99) | |
| $\Phi_M(\mathbf{x}) = \frac{\mathbf{m} \cdot \mathbf{x}}{4 - x^3}$ | | |
| $4\pi T^{\circ}$ | Magnetic scalar potential of dipole (J. pg. 196) | |
| $\mathbf{m} = \int \mathbf{M} \ d^3 x$ | Total magnetic moment (J. pg. 197) | |
| | on equation in terms of magnetic vector potential (5.101) | |
| $\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{\nabla}' \times \mathbf{M}(\mathbf{x}')}{ \mathbf{x} - \mathbf{x}' } \ d^3x$ | $x + \frac{\mu_0}{4\pi} \oint_S \frac{\mathbf{M}(\mathbf{x}') \times \mathbf{n}'}{ \mathbf{x} - \mathbf{x}' } da'$ Discontinuous M (5.103) | |
| $\Phi_M(r,\theta) = \frac{1}{3} M_0 a^2 \frac{r <}{r > 2} \cos \theta$ | Sphere with $\mathbf{M} = M_0 \hat{\mathbf{z}} \; [(r_<, r_>), \; (r, a)] \; (5.104)$ | |
| $\mathbf{M} = \frac{3}{\mu_0} \left(\frac{\mu - \mu_0}{\mu + 2\mu_0} \right) \mathbf{B}_0$ | Permeable sphere in uniform magnetic field ${f B}_0$ (5.115) | |
| $F = \int_S \mathbf{B} \cdot \mathbf{n} \ da$ | Magnetic flux (5.133) | |
| $\mathscr{E} = \oint_C \mathbf{E}' \cdot d1$ | Electromotive force (5.134) | |
| $\mathscr{E} = -k \frac{\mathrm{d}F}{\mathrm{d}t}$ | Faraday's Law (5.135) | |
| $W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} \ d^3x = \frac{1}{2\mu_0} \int \mathbf{B} ^2 \ d^3x$ Energy to ramp current from zero (4.83)::(5.148) | | |
| $W = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \ d^3 x$ | Magnetic energy in fields (4.89) :: (5.148) | |
| $\frac{\mathrm{d}W}{\mathrm{d}t} = \int \mathbf{H} \cdot \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \ d^3x$ | Power in magetic field (5.147) | |
| $\Delta W = \frac{1}{2} \int_{V_1} \mathbf{M} \cdot \mathbf{B}_0 \ d^3 x$ | Energy to place permeable object in ${\bf B}_0$ (4.93)::(5.150) | |
| $W = \frac{\mu_0}{2} \int \mathbf{H} ^2 d^3x = \frac{\mu_0}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \int \Phi_{M_j}(\mathbf{x}) \rho_{M_i}(\mathbf{x}) d^3x \qquad N \text{ ferromagnets (HW 8.2b)}$ | | |
| $W = \frac{1}{2} \sum_{i=1}^{N} L_i I_i^2 + \sum_{i=1}^{N} \sum_{j>i}^{N} M_{ij} I_i^2$ | $I_i I_j$ Inductive energy (5.152) | |
| $M_{ij}=rac{1}{I_{j}}F_{ij}$ | Mutual inductance (5.156) | |
| $B_z(\rho, z) \approx B_z(0, z) - \left(\frac{\rho^2}{4}\right) \left[\frac{\partial^2}{\partial z^2}\right]$ | $B_z(\rho,z) \approx B_z(0,z) - \left(\frac{\rho^2}{4}\right) \left[\frac{\partial^2 B_z(0,z)}{\partial z^2}\right] + \dots$ | |
| $B_{\rho}(\rho,z) \approx -\left(\frac{\rho}{2}\right) \frac{\partial B_{z}(0,z)}{\partial z} + .$ | $B_{\rho}(\rho, z) \approx -\left(\frac{\rho}{2}\right) \frac{\partial B_{z}(0, z)}{\partial z} + \dots$ B_{z} known $\mathbf{J} = 0, \ \phi$ -sym, near origin (J Pr. 5.4) | |
| $A_{\phi}(\rho, z) = \frac{\mu_0 I a}{\pi} \int_0^{\infty} \cos(kz) I_1(k\rho_{<}) K_1(k\rho_{>}) dk = \frac{\mu_0 I a}{2} \int_0^{\infty} J_1(ka) J_1(k\rho) dk$ A of current ring (J. PR. 5.10) | | |
| $\tau = I\alpha = -\frac{\partial W}{\partial \theta} = \mathbf{r} \times \mathbf{F}$ | Mechanical torque | |
| $I = \int r^2 \rho(x, y, z) \ d^3 x$ | Moment of inertia | |
| | | |
| $\frac{\partial^2 W}{\partial q^2} > 0 \Rightarrow \text{Stable}, \frac{\partial^2 W}{\partial q^2} = 0 \Rightarrow \text{Saddle}, \frac{\partial^2 W}{\partial q^2} < 0 \Rightarrow \text{Unstable}$ | | |
| Wiggly | Decaying | |

| | Wiggly | Decaying |
|-------------------|--|--|
| x,y,z | $e^{\pm ik_n x}$, $A\cos(k_n x) + B\sin(k_n x)$ | $e^{\pm k_n x}$, $A \cosh(k_n x) + B \sinh(k_n x)$ |
| ρ, ϕ, z | $e^{im\phi}, AJ_m(k_n\rho) + BY_m(k_n\rho)$ | $AI_{m}(k_{n}\rho) + BK_{m}(k_{n}\rho)$ |
| ρ , ϕ | $e^{im\phi}$ | $A_0 + B_0 \ln \rho + \sum A_m \rho^m + B_m \rho^{-m}$ |
| r , θ | $P_{\ell}(\cos \theta)$ | $A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$ |
| r, θ, ϕ | $Y_{\ell m}(\theta,\phi)$ | $A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$ |