Midterm 2 Equations	PHSX519 Electron
$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \rho(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{ \mathbf{x} - \mathbf{x}' ^3} d^3x'$	Coulomb's Law (1.5)
$\delta(f(x)) = \sum_{i} \frac{1}{\left \frac{\mathrm{d}f}{\mathrm{d}x}(x_{i})\right } \delta(x - x_{i})$	Delta function Rule 5
$\oint_S \mathbf{E} \cdot \mathbf{n} \ da = \frac{1}{\epsilon_0} \int_V \rho(\mathbf{x}) d^3x$	Gauss' Law (1.11)
$ abla  imes \mathbf{E} = 0$ $\mathbf{E} = -\nabla \Phi$ Elect	Curl of electric field (1.14) ric field in terms of scalar potential (1.16)
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x'})}{ \mathbf{x} - \mathbf{x'} } d^3x'  \text{Scalar}$	potential in terms of charge density (1.17)
	lectric field of a surface distribution (1.22)
$\nabla^2 \Phi = -\rho/\epsilon_0$ $\nabla^2 \Phi = 0$	Poisson Equation (1.28)
	Laplace Equation (1.29)
	een's function for Poisson's equation (1.40)
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int_V \rho(\mathbf{x}') G_D(\mathbf{x}, \mathbf{x}') d^3 x' =$	111 - 011
$\Phi(\mathbf{x}) = \langle \Phi \rangle_S + \frac{1}{4\pi\epsilon_0} \int_V \rho(\mathbf{x}') G_N(\mathbf{x}, \mathbf{x}')$	$d^3x + \frac{1}{4\pi} \oint_S \frac{\partial \Phi}{\partial n'} G_N da'$ NBCs (1.46)
2 " 2 "	Energy stored in electric field (1.54)
$q' = -\frac{a}{y}q$ , $y' = \frac{a^2}{y}$ Magnitude an	d position of image charge on sphere (2.4)
$\Phi = -E_0 \left( r - \frac{a^3}{r^2} \right) \qquad \text{Electric potential}$	ial of conducting sphere in $\mathbf{E} = E_0 \hat{\mathbf{z}}$ (2.14)
$\frac{1}{ \mathbf{x} - \mathbf{x}' } = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_{<}^{\ell}}{r_{>}^{\ell+1}} Y_{>}^{\ell}$	$_{\ell m}^{*}(\theta',\phi')Y_{\ell m}(\theta,\phi) \qquad \qquad \text{GFE (3.70)}$
Where $r < (r >)$ is the smaller (larger) of	$ \mathbf{x} $ and $ \mathbf{x'} $
$q = \int \rho(\mathbf{x'}) \ d^3x'$	Monopole (4.4)
$\mathbf{p} = \int \mathbf{x'} \rho(\mathbf{x'}) \ d^3x'$	Dipole (4.8)
$Q_{ij} = \int (3x_i'x_j' - r'\delta_{ij})\rho(\mathbf{x}')d^3x'$	Quadrupole (4.9)
$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i}{r} \right]$	$\left[\frac{x_j}{5} + \ldots\right]$ Multipole Expansion (4.10)
$E_{T}=rac{2p\cos heta}{4\pi\epsilon_{0}}, E_{ heta}=rac{2p\sin heta}{4\pi\epsilon_{0}}$	Dipole in $\hat{\mathbf{z}}$ (4.12)
$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{n}(\mathbf{p} \cdot \mathbf{n}) - \mathbf{p}}{4\pi\epsilon_0  \mathbf{x} - \mathbf{x}_0 ^3}$	<b>E</b> -field due to dipole $\mathbf{p}$ (4.13)
$W = \int \rho(\mathbf{x}) \Phi(\mathbf{x}) \ d^3x$	Charge in external field (4.21)
$W = q\Phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{6} \sum_{i} \sum_{j} Q_{ij} \frac{\partial E}{\partial x}$	$\frac{(j)}{i}(0) + \dots$ Work multipole expsn. (4.24)
$\mathbf{P}(\mathbf{x}) = \sum_{i} N_{i} \left\langle \mathbf{p}_{i}  ight angle$	Electric polarization (4.28)
$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$	Electric displacement $(4.34)$
$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$	Induced polarization (4.36)
$\mathbf{D} = \epsilon \mathbf{E}$ $\epsilon = \epsilon_0 (1 + \chi_e)$	Electric displacement (4.37) Electric permittivity (4.38)
$\begin{cases} (\mathbf{D}_2 - \mathbf{D}_1) \cdot \mathbf{n}_{21} = \sigma \\ (\mathbf{E}_2 - \mathbf{E}_1) \times \mathbf{n}_{21} = 0 \end{cases}$	Boundary conditions (4.40)
$\Phi_{\rm in} = -\left(\frac{3}{\epsilon/\epsilon_0 + 2}\right) E_0 r \cos \theta$	
$\Phi_{\text{out}} = -E_0 r \cos \theta + \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0 \frac{a^3}{r^2}$	Dielectric sphere in $\mathbf{E}=E_0\mathbf{\hat{z}}$ (4.54) - $\cos\theta$
$W = \frac{1}{2} \int \mathbf{E} \cdot \mathbf{D} \ d^3 x$	Electrostatic Energy (4.89)
$\Delta W = -\frac{1}{2} \int_{V_1} \mathbf{P} \cdot \mathbf{E}_0 \ d^3 x$	Dielectric placed in $\mathbf{E}_0$ (4.93)
$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$ $\nabla \cdot \mathbf{J} = 0$	Torque on magnetic dipole moment (5.1)  Condition of magnetostatics (5.3)
$d\mathbf{B} = kI \frac{d\mathbf{l} \times \mathbf{x}}{ \mathbf{x} ^3}$	Biot-Savart Law (5.4)
$\mathbf{F} = \int \mathbf{J}(\mathbf{x}) \times \mathbf{B}(\mathbf{x}) \ d^3x$	Force on current dist. (5.12)
$\mathbf{N} = \int \mathbf{x} \times (\mathbf{J} \times \mathbf{B}) \ d^3 x$	Torque on current dist. (5.13)

$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$	Ampère's law (5.25)
$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x})$	Magnetic vector potential (5.27)
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}')}{ \mathbf{x} - \mathbf{x}' } d^3 x'$	MVP, current dist. (5.32)
$\mathbf{m} = \frac{1}{2} \int \mathbf{x'} \times \mathbf{J}(\mathbf{x'}) d^3x$	Magnetic moment def. $(5.54)$
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{x}}{ \mathbf{x} ^3}$	Dipole vector potential (5.55)
$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \left[ \frac{3\mathbf{n}(\mathbf{n} \cdot \mathbf{m}) - \mathbf{m}}{ \mathbf{x} ^3} \right]$	Dipole induction (5.56)
$\mathbf{m} = \frac{I}{2} \oint \mathbf{x} \times d\mathbf{l}$	Mag. moment of closed circuit
$ \mathbf{m}  = I \times (Area)$	Moment of plane loop (5.57)
$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$	Force on dipole (5.69)
$\mathbf{M(x)} = \sum_{i} N_i \left\langle \mathbf{m}_i \right\rangle$	Magnetization (5.76)
$\mathbf{J}_M = \mathbf{\nabla} \times \mathbf{M}$	Bound current density (5.79)
$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$	Magnetic field (5.81)
$\nabla \times \mathbf{H} = \mathbf{J}$ $\nabla \cdot \mathbf{B} = 0$	Macroscopic equations in matter (5.82)
$\mathbf{B} = \mu \mathbf{H}$	Linear condition (5.84)
$\begin{cases} (\mathbf{B}_2 - \mathbf{B}_1) \cdot \mathbf{n} = 0 \\ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K} \end{cases}$	Interface BC (5.86)
$\mathbf{H} = -\nabla \Phi_M$	Magnetic scalar potential (5.93)
$\nabla^2 \Phi_M = 0$	Magnetostatic Laplace Equation
$\nabla^2 \Phi_M = -\rho_M$	Magnetostatic Poisson Equation (5.95)
$\rho_M = -\nabla \cdot \mathbf{M}$ $\mathbf{m} \cdot \mathbf{x}$	Effective magnetic charge density (5.96)
$\Phi_{M}(\mathbf{x}) = \frac{\mathbf{m} \cdot \mathbf{x}}{4\pi r^{3}}$	Mag. potential of dipole
$\sigma_M = \mathbf{n} \cdot \mathbf{M}$ E $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_M$	Effective magnetic surface-charge density (5.99)
	Poisson equation for ${\bf A}$ (5.101) $\mu_0$ , ${\bf M}({\bf x}') \times {\bf n}'$ ,
$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{\nabla}' \times \mathbf{M}(\mathbf{x}')}{ \mathbf{x} - \mathbf{x}' } d\mathbf{x}'$	$3x + \frac{1}{4\pi} \oint_S \frac{1}{ \mathbf{x} - \mathbf{x}' } da'$
$\mathbf{K}_M = \mathbf{M} \times \mathbf{n}$	Effective surface current
$\Phi_M(r,\theta) = \frac{1}{3} M_0 a^2 \frac{r <}{r > 2} \cos \theta$	Uniformly magnetized sphere (5.104)
$\mathbf{M} = \frac{3}{\mu_0} \left( \frac{\mu - \mu_0}{\mu + 2\mu_0} \right) \mathbf{B}_0$	Sphere in uniform field (5.115)
$F = \int_S \mathbf{B} \cdot \mathbf{n} \ da$	Magnetic flux (5.133)
$\mathscr{E} = \oint_C \mathbf{E'} \cdot d1$	Electromotive force (5.134)
$\mathscr{E} = -k \frac{\mathrm{d}F}{\mathrm{d}t}$	Faraday's Law (5.135)
$W = \frac{1}{2} \int \mathbf{H} \cdot \mathbf{B} \ d^3 x$	Total magnetic energy (5.148)
$W = \frac{1}{2} \int \mathbf{J} \cdot \mathbf{A} \ d^3 x$	Energy in terms of potential (5.149)
$\Delta W = \frac{1}{2} \int_{V_1} \mathbf{M} \cdot \mathbf{B}_0 \ d^3 x$	Change in energy for placing object (5.150)
$F_{\xi} = \left(\frac{\partial W}{\partial \xi}\right)_J$	Generalized force (5.151)
$W = \frac{1}{2} \sum_{i=1}^{N} L_i I_i^2 + \sum_{i=1}^{N} \sum_{j>i}^{N} M_i$	$ij I_i I_j$ Inductive energy (5.152)
$M_{ij} = \frac{1}{I_j} F_{ij}$	Mutual inductance (5.156)
$\tau = I\alpha = -\frac{\partial W}{\partial \theta}$	Mechanical torque

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		Wiggly	Decaying
	x,y,z	$e^{\pm ik_n x}$ , $A\cos(k_n x) + B\sin(k_n x)$	$e^{\pm k_n x}$ , $A \cosh(k_n x) + B \sinh(k_n x)$
	$ ho,\phi,z$	$e^{im\phi},\ AJ_m(k_n\rho)+BY_m(k_n\rho)$	$AI_{m}(k_{n}\rho)+BK_{m}(k_{n}\rho)$
	$ ho,\phi$	$e^{im\phi}$	$A_0 + B_0 \ln \rho + \sum A_m \rho^m + B_m \rho^{-m}$
	$r, \theta$	$P_{\ell}(\cos  heta)$	$A\left(\frac{r}{a}\right)^{\ell} + B\left(\frac{r}{a}\right)^{-(\ell+1)}$
ſ	$r, \theta, \phi$	$Y_{\theta} = (\theta, \phi)$	$A\left(\frac{r}{\ell}\right)^{\ell} + B\left(\frac{r}{\ell}\right)^{-(\ell+1)}$