$\boldsymbol{\nabla}\cdot\mathbf{D}=\rho_f,\quad \boldsymbol{\nabla}\times\mathbf{E}=0,\quad \boldsymbol{\nabla}\cdot\mathbf{B}=0,\quad \boldsymbol{\nabla}\times\mathbf{H}=\mathbf{J}_f$ Maxwell's equations in matter

$\mathbf{E}(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\int\rho(\mathbf{x}')\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}d^3x'$ Coulomb's Law (1.5)

$\delta(f(x))=\sum_i\frac{1}{\left|\frac{\mathrm{d}f}{\mathrm{d}x}(x_i)\right|}\delta(x-x_i)$ Delta function Rule 5

$\oint_S\mathbf{E}\cdot\mathbf{n}\,da=\frac{1}{\epsilon_0}\int_V\rho(\mathbf{x})d^3x$ Gauss' Law (1.11)

$\mathbf{E}=-\nabla\Phi$ Electric field in terms of scalar potential (1.16)

$\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x'$ Scalar potential in terms of charge density (1.17)

$\nabla^2\Phi=-\rho/\epsilon_0$ Poisson Equation (1.28)

$\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\int_V\rho(\mathbf{x}')G_D(\mathbf{x},\mathbf{x}')d^3x'=\frac{1}{4\pi}\oint_S(\mathbf{x}')\frac{\partial G_D}{\partial n'}da'$ DBCs (1.44)

$\Phi(\mathbf{x})=\langle\Phi\rangle_S+\frac{1}{4\pi\epsilon_0}\int_V\rho(\mathbf{x}')G_N(\mathbf{x},\mathbf{x}')d^3x+\frac{1}{4\pi}\oint_S\frac{\partial\Phi}{\partial n'}G_N\,da'$ NBCs (1.46)

$\frac{1}{|\mathbf{x}-\mathbf{x}'|}=4\pi\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}\frac{1}{2\ell+1}\frac{r_<^\ell}{r_>^{\ell+1}}Y^*_{\ell m}(\theta',\phi')Y_{\ell m}(\theta,\phi)$ GFE: r,θ,ϕ (3.70)

$\frac{1}{|\mathbf{x}-\mathbf{x}'|}=\frac{2}{\pi}\sum_{m=-\infty}^{\infty}\int_0^\infty dk e^{im(\phi-\phi')}\cos[k(z-z')]I_m(k\rho_<)K_m(k\rho_>)$ GFE (3.148)

Where $r_<(r_>)$ $[\rho_<(\rho_>)]$ is the smaller (larger) of $|\mathbf{x}|$ and $|\mathbf{x}'|$

$\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{4\pi}{2l+1}q_{lm}\frac{Y_{lm}}{r^{l+1}}$ Potential outside spher of charge (4.1)

$q_{lm}=\int Y^*_{lm}(\theta',\phi')r'^l\rho(\mathbf{x}')\,d^3x'$ Multipole moments (4.3)

$q=\int\rho(\mathbf{x}')\,d^3x'$ Monopole (4.4)

$\mathbf{p}=\int\mathbf{x}'\rho(\mathbf{x}')\,d^3x'$ Dipole (4.8)

$Q_{ij}=\int(3x_i'x_j'-r'\delta_{ij})\rho(\mathbf{x}')d^3x'=3M_{ij}-\mathrm{Tr}(\mathbf{M}\delta_{ij})$ Quadrupole (4.9)

$M_{ij}=\int x_i'x_j'\rho(x')\,d^3x$ Dana definition

$\Phi(\mathbf{x})=\frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}+\frac{\mathbf{p}\cdot\mathbf{x}}{r^3}+\frac{1}{2}\sum_{i,j}Q_{ij}\frac{x_ix_j}{r^5}+\ldots\right]$ Multipole Expansion (4.10)

$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ Electric displacement(4.34)

$\mathbf{P}=\epsilon_0\chi_e\mathbf{E}$ Induced polarization (4.36)

$\mathbf{P}=(\epsilon-\epsilon_0)\mathbf{E}$ Better expression for polarization

$\mathbf{D}=\epsilon\mathbf{E}$ Electric displacement (4.37)

$\epsilon=\epsilon_0(1+\chi_e)$ Electric permittivity (4.38)

$\sigma_b=\mathbf{P}\cdot\hat{\mathbf{n}},\quad \rho_b=-\boldsymbol{\nabla}\cdot\mathbf{P}$ Electric bound charge density (G. 4.11)

$\begin{cases}(\mathbf{D}_2-\mathbf{D}_1)\cdot\mathbf{n}_{21}=\sigma\\(\mathbf{E}_2-\mathbf{E}_1)\times\mathbf{n}_{21}=0\end{cases}$ Boundary conditions (4.40)

$\begin{cases}\Phi_{\text{in}}=-\left(\frac{3}{\epsilon/\epsilon_0+2}\right)E_0r\cos\theta\\\Phi_{\text{out}}=-E_0r\cos\theta+\left(\frac{\epsilon/\epsilon_0-1}{\epsilon/\epsilon_0+2}\right)E_0\frac{a^3}{r^2}\cos\theta\end{cases}$ Dielectric sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (4.54)

$W=\int\rho(\mathbf{x})\Phi(\mathbf{x})\,d^3x=\frac{1}{2}\int\mathbf{E}\cdot\mathbf{D}\,d^3x$ Energy to bring charges from ∞ (4.83,89)

$\Delta W=-\frac{1}{2}\int_{V_1}\mathbf{P}\cdot\mathbf{E}_0\,d^3x$ Dielectric placed in $\mathbf{E}_0$ (4.93)

$W=q\Phi(0)-\mathbf{p}\cdot\mathbf{E}(0)-\frac{1}{6}\sum_i\sum_jQ_{ij}\frac{\partial E_j}{\partial x_i}(0)+\ldots$ Work multipole expsn. (4.24)

$\mathbf{N}=\mathbf{p}\times\mathbf{E}$ Torque on electric dipole (G. 4.4)

$\mathbf{F}=(\mathbf{P}\cdot\boldsymbol{\nabla})\mathbf{E}$ Force on electric dipole (G. 4.5)

$\Phi=-E_0\left(r-\frac{a^3}{r^2}\right)$ Electric potential of conducting sphere in $\mathbf{E}=E_0\hat{\mathbf{z}}$ (2.14)

$E_r=\frac{2p\cos\theta}{4\pi\epsilon_0},\quad E_\theta=\frac{2p\sin\theta}{4\pi\epsilon_0}$ Electric dipole at origin in $\hat{\mathbf{z}}$ (4.12)

$\mathbf{E}(\mathbf{x})=\frac{3\mathbf{n}(\mathbf{p}\cdot\mathbf{n})-\mathbf{p}}{4\pi\epsilon_0|\mathbf{x}-\mathbf{x}_0|^3}$ **E**-field due to dipole **p** (4.13)

$\mathbf{N}=\mathbf{m}\times\mathbf{B}$ Torque on magnetic dipole moment (5.1)

$\nabla\cdot\mathbf{J}=0$ Condition of magnetostatics (5.3)

$d\mathbf{B}=kI\frac{d\mathbf{l}\times\mathbf{x}}{|\mathbf{x}|^3}$ Biot-Savart Law (5.4)

$\oint_C\mathbf{B}\cdot d\mathbf{l}=\mu_0I$ Ampère's law (5.25)

$\mathbf{B}(\mathbf{x})=\nabla\times\mathbf{A}(\mathbf{x})$ Magnetic vector potential (5.27)

$\mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x'$ Magnetic vector potential of current distribution (5.32)

$\mathbf{m}=\frac{1}{2}\int\mathbf{x}'\times\mathbf{J}(\mathbf{x}')\,d^3x$ Magnetic moment definition (5.54)

$\mathbf{m}=\frac{I}{2}\oint\mathbf{x}\times d\mathbf{l}$ Magnetic moment of closed circuit (J. pg. 186)

$|\mathbf{m}|=I\times(\text{Area})$ Magnetic moment of plane loop (5.57)

$\mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$ Dipole vector potential (5.55)

$\mathbf{B}(\mathbf{x})=\frac{\mu_0}{4\pi}\left[\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{m})-\mathbf{m}}{|\mathbf{x}|^3}\right]$ Dipole induction (5.56)

$\mathbf{F}=\boldsymbol{\nabla}(\mathbf{m}\cdot\mathbf{B})$ Force on dipole (5.69)

$\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$ Magnetic field (5.81)

$\mathbf{M}=(\mu/\mu_0-1)\mathbf{H}$ Magnetization in linear media (G. 6.29)

$\mathbf{B}=\mu\mathbf{H}$ Linear condition (5.84)

$\begin{cases}(\mathbf{B}_2-\mathbf{B}_1)\cdot\mathbf{n}=0\\\mathbf{n}\times(\mathbf{H}_2-\mathbf{H}_1)=\mathbf{K}_f\end{cases}$ Interface BC (5.86)

$\mathbf{H}=-\boldsymbol{\nabla}\Phi_M$ Magnetic scalar potential (5.93)

$\mathbf{J}_M=\boldsymbol{\nabla}\times\mathbf{M},\quad \mathbf{K}_b=\mathbf{M}\times\mathbf{n}$ Bound current density (G. 6.13,14)

$\rho_M=-\boldsymbol{\nabla}\cdot\mathbf{M},\quad \sigma_M=\mathbf{n}\cdot\mathbf{M}$ Effective magnetic charge density (5.96,99)

$\Phi_M(\mathbf{x})=\frac{\mathbf{m}\cdot\mathbf{x}}{4\pi r^3}$ Magnetic scalar potential of dipole (J. pg. 196)

$\mathbf{m}=\int\mathbf{M}\,d^3x$ Total magnetic moment (J. pg. 197)

$\nabla^2\mathbf{A}=-\mu_0\mathbf{J}_M$ Poisson equation in terms of magnetic vector potential (5.101)

$\mathbf{A}(\mathbf{x})=\frac{\mu_0}{4\pi}\int_V\frac{\boldsymbol{\nabla}'\times\mathbf{M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x+\frac{\mu_0}{4\pi}\oint_S\frac{\mathbf{M}(\mathbf{x}')\times\mathbf{n}'}{|\mathbf{x}-\mathbf{x}'|}da'$ Discontinuous **M** (5.103)

$\Phi_M(r,\theta)=\frac{1}{3}M_0a^2\frac{r_<}{r_>^2}\cos\theta$ Sphere with $\mathbf{M}=M_0\hat{\mathbf{z}}$ $[(r_<,r_>),(r,a)]$ (5.104)

$\mathbf{M}=\frac{3}{\mu_0}\left(\frac{\mu-\mu_0}{\mu+2\mu_0}\right)\mathbf{B}_0$ Permeable sphere in uniform magnetic field $\mathbf{B}_0$ (5.115)

$F=\int_S\mathbf{B}\cdot\mathbf{n}\,da$ Magnetic flux (5.133)

$\mathscr{E}=\oint_C\mathbf{E}'\cdot d\mathbf{l}$ Electromotive force (5.134)

$\mathscr{E}=-k\frac{\mathrm{d}F}{\mathrm{d}t}$ Faraday's Law (5.135)

$W=\frac{1}{2}\int\mathbf{J}\cdot\mathbf{A}\,d^3x$ Energy to ramp current from zero (4.83)::(5.149)

$W=\frac{1}{2}\int\mathbf{H}\cdot\mathbf{B}\,d^3x$ Magnetic energy in fields (4.89)::(5.148)

$\frac{\mathrm{d}W}{\mathrm{d}t}=\int\mathbf{H}\cdot\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t}\,d^3x$ Power in magetic field (5.147)

$\Delta W=\frac{1}{2}\int_{V_1}\mathbf{M}\cdot\mathbf{B}_0\,d^3x$ Energy to place permeable object in $\mathbf{B}_0$ (4.93)::(5.150)

$\frac{\mathrm{d}W}{\mathrm{d}t}=-\int\mathbf{J}\cdot\mathbf{E}\,d^3x$ Change in energy due to EMF

$W=\frac{\mu_0}{2}\int|\mathbf{H}|^2\,d^3x=\frac{\mu_0}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\int\Phi_{M_j}(\mathbf{x})\rho_{M_i}(\mathbf{x})\,d^3x$ N ferromagnets (HW 8.2b)

$W=\frac{1}{2}\sum_{i=1}^{N}L_iI_i^2+\sum_{i=1}^{N}\sum_{j>i}^{N}M_{ij}I_iI_j$ Inductive energy (5.152)

$M_{ij}=\frac{1}{I_j}F_{ij}$ Mutual inductance (5.156)

$B_z(\rho,z)\approx B_z(0,z)-\left(\frac{\rho^2}{4}\right)\left[\frac{\partial^2B_z(0,z)}{\partial z^2}\right]+\ldots$

$B_\rho(\rho,z)\approx-\left(\frac{\rho}{2}\right)\frac{\partial B_z(0,z)}{\partial z}+\ldots$ B_z known $\mathbf{J}=0$, $\phi-$sym, near origin (J Pr. 5.4)

$A_\phi(\rho,z)=\frac{\mu_0Ia}{\pi}\int_0^\infty\cos(kz)I_1(k\rho_<)K_1(k\rho_>)\,dk=\frac{\mu_0Ia}{2}\int_0^\infty J_1(ka)J_1(k\rho)\,dk$
A of current ring (J. PR. 5.10)

$\mathbf{A}_{\text{dip}}=\frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\Rightarrow\mathbf{B}_{\text{dip}}=\frac{\mu_0m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}}+\sin\theta\hat{\boldsymbol{\theta}})$ Field of dipole (G 5.87,88)

$\tau=I\alpha=-\frac{\partial W}{\partial\theta}=\mathbf{r}\times\mathbf{F}$ Mechanical torque

$I=\int r^2\rho(x,y,z)\,d^3x$ Moment of inertia

$\frac{\partial^2W}{\partial q^2}>0\Rightarrow\text{Stable},\quad\frac{\partial^2W}{\partial q^2}=0\Rightarrow\text{Saddle},\quad\frac{\partial^2W}{\partial q^2}<0\Rightarrow\text{Unstable}$

$(a+x)^n=a^n+na^{n-1}x+\frac{n(n-1)}{2!}a^{n-2}x^2+\ldots=$ Binomial Expansion

$f(x)\approx f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots$ Taylor Series

$$\begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix}=\begin{pmatrix}\cos\phi&\sin\phi&0\\-\sin\phi&\cos\phi&0\\0&0&1\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\\hat{\mathbf{y}}\\\hat{\mathbf{z}}\end{pmatrix},\quad\begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi&\sin\theta\sin\phi&\cos\theta\\\cos\theta\cos\phi&\cos\theta\sin\phi&-\sin\theta\\-\sin\phi&\cos\phi&0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{x}}\\\hat{\mathbf{y}}\\\hat{\mathbf{z}}\end{pmatrix}$$

$$\begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}=\begin{pmatrix}\sin\theta&0&\cos\theta\\\cos\theta&0&-\sin\theta\\0&1&0\end{pmatrix}\begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix},\quad\begin{pmatrix}\hat{\boldsymbol{\rho}}\\\hat{\boldsymbol{\phi}}\\\hat{\mathbf{z}}\end{pmatrix}=\begin{pmatrix}\rho/\sqrt{\rho^2+z^2}&z/\sqrt{\rho^2+z^2}&0\\0&0&1\\z/\sqrt{\rho^2+z^2}&-\rho/\sqrt{\rho^2+z^2}&0\end{pmatrix}\begin{pmatrix}\hat{\mathbf{r}}\\\hat{\boldsymbol{\theta}}\\\hat{\boldsymbol{\phi}}\end{pmatrix}$$

$\Phi_i(r,\theta.\phi)=\left[A_ir^l+B_i^{-(l+1)}\right]P_l(\cos\theta),\quad\mathbf{K}=-\frac{K_0}{\sin\theta}\left[\cos\theta P_l(\cos\theta)-P_{l+1}(\cos\theta)\right]\hat{\boldsymbol{\phi}}$

$B_I=\left(\frac{1}{l\mu_{II}+(l+1)\mu_I}\right)\left[A_Ia^{2l+1}(\mu_I-\mu_{II})+K_0la^{l+2}\mu_{II}+B_{II}(2l+1)\mu_{II}\right]$

$A_{II}=\left(\frac{a^{-(2l+1)}}{l\mu_{II}+(l+1)\mu_I}\right)\left[B_{II}(l+1)(\mu_{II}-\mu_I)+A_I(2l+1)\mu_Ia^{2l+1}-K_0(l+1)\mu_Ia^{l+2}\right]$

	Wiggly	Decaying
x,y,z	$e^{\pm ik_nx},\ A\cos(k_nx)+B\sin(k_nx)$	$e^{\pm k_nx},\ A\cosh(k_nx)+B\sinh(k_nx)$
ρ,ϕ,z	$e^{im\phi},\ AJ_m(k_n\rho)+BY_m(k_n\rho)$	$AI_m(k_n\rho)+BK_m(k_n\rho)$
ρ,ϕ	$e^{im\phi}$	$A_0+B_0\ln\rho+\sum A_m\rho^m+B_m\rho^{-m}$
r,θ	$P_\ell(\cos\theta)$	$A\left(\frac{r}{a}\right)^\ell+B\left(\frac{r}{a}\right)^{-(\ell+1)}$
r,θ,ϕ	$Y_{\ell m}(\theta,\phi)$	$A\left(\frac{r}{a}\right)^\ell+B\left(\frac{r}{a}\right)^{-(\ell+1)}$