Problem 7.6

Part a

```
gauss = a1 + b1 == a2
ampere = a1 - b1 == n a2
```

$a1 + b1 == a2$

$a1 - b1 == a2\, n$

```
Solve[gauss && ampere, {b1, a2}] // FullSimplify
```

$$\left\{\left\{b1 \to \frac{a1 - a1\, n}{1 + n},\ a2 \to \frac{2\, a1}{1 + n}\right\}\right\}$$

Part b

```
first = β^2 - α^2/4 == ω^2/c^2 Re[n^2]
second = β α == ω^2/c^2 Im[n^2]
```

$$-\frac{\alpha^2}{4} + \beta^2 == \frac{\omega^2\, \mathrm{Re}[n^2]}{c^2}$$

$$\alpha\, \beta == \frac{\omega^2\, \mathrm{Im}[n^2]}{c^2}$$

```
sol = Solve[first && second, {α, β}]
FullSimplify[sol, Assumptions → Element[ω, Reals] &&
   Element[c, Reals] && Element[ω, Positive] && Element[c, Positive]]
```

$$\left\{\left\{\alpha \to -2\, i\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Re}[n]}{c}\right],\ \beta \to \frac{2\, i\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Re}[n]}{c}\right]\, \mathrm{Im}[n]^2}{\mathrm{Im}[n^2]}\right\},\right.$$

$$\left\{\alpha \to 2\, i\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Re}[n]}{c}\right],\ \beta \to \frac{i\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Re}[n]}{c}\right]\, \left(-n\, \mathrm{Conjugate}[n] + \mathrm{Re}[n^2]\right)}{\mathrm{Im}[n^2]}\right\},$$

$$\left\{\alpha \to -2\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Im}[n]}{c}\right],\ \beta \to -\frac{2\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Im}[n]}{c}\right]\, \mathrm{Re}[n]^2}{\mathrm{Im}[n^2]}\right\},$$

$$\left.\left\{\alpha \to 2\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Im}[n]}{c}\right],\ \beta \to \frac{2\, \mathrm{Abs}\left[\frac{\omega\, \mathrm{Im}[n]}{c}\right]\, \mathrm{Re}[n]^2}{\mathrm{Im}[n^2]}\right\}\right\}$$

Part c

```
n = Sqrt[1 + I x]
```

$$\sqrt{1 + i\, x}$$

```
u = Re[n] // ComplexExpand // FullSimplify
v = Im[n] // ComplexExpand // FullSimplify
```

$$\left(1+x^2\right)^{1/4} \mathrm{Cos}\left[\frac{1}{2} \mathrm{Arg}[1+i\, x]\right]$$

$$\left(1+x^2\right)^{1/4} \mathrm{Sin}\left[\frac{1}{2} \mathrm{Arg}[1+i\, x]\right]$$

```
u = v /. Arg[1 + I x] → ArcTan[x] // FullSimplify
v = v /. Arg[1 + I x] → ArcTan[x] // FullSimplify
```

$$\left(1+x^2\right)^{1/4} \mathrm{Sin}\left[\frac{\mathrm{ArcTan}[x]}{2}\right]$$

$$\left(1+x^2\right)^{1/4} \mathrm{Sin}\left[\frac{\mathrm{ArcTan}[x]}{2}\right]$$

```
u = Normal[Series[u, {x, Infinity, 0}]]
v = Normal[Series[v, {x, Infinity, 0}]]
```

$$\frac{1}{\sqrt{2}\sqrt{\frac{1}{x}}}$$

$$\frac{1}{\sqrt{2}\sqrt{\frac{1}{x}}}$$

```
n = u + I v
```

$$\frac{1+i}{\sqrt{2}\sqrt{\frac{1}{x}}}$$

```
R = FullSimplify[(Abs[(1 - n)/(1 + n)])^2 // ComplexExpand,
  Assumptions → Element[x, Reals] && Element[x, Positive]]
T = FullSimplify[(4 Re[n])/(Abs[1 + n]^2) // ComplexExpand,
  Assumptions → Element[x, Reals] && Element[x, Positive]]
```

$$\frac{1}{-1+\frac{2\sqrt{x}\,(1+\mathrm{Abs}[x])}{\sqrt{x}+(-1)^{3/4}\,x+\sqrt{x^3}-(-1)^{1/4}\,\mathrm{Abs}[x]}}$$

$$\frac{2\,(x+i\,\mathrm{Abs}[x])}{x+i\,\mathrm{Abs}[x]+(-1)^{1/4}\sqrt{x}\,(1+\mathrm{Abs}[x])}$$

```
Series[R, {x, Infinity, 1}] // FullSimplify
Series[T, {x, Infinity, 1}] // FullSimplify
```

$$1-2\sqrt{2}\sqrt{\frac{1}{x}}+\frac{4}{x}-2\sqrt{2}\left(\frac{1}{x}\right)^{3/2}+O\left[\frac{1}{x}\right]^2$$

$$2\sqrt{2}\sqrt{\frac{1}{x}}-\frac{4}{x}+2\sqrt{2}\left(\frac{1}{x}\right)^{3/2}+O\left[\frac{1}{x}\right]^{5/2}$$