```
a = α1 + β1 == α2 + β2
b = n1 (α1 - β1) == n2 (α2 - β2)
c = α2 E^(I k2 d) + β2 E^(-I k2 d) == α3 E^(I k3 d)
f = n2 (α2 E^(I k2 d) - β2 E^(-I k2 d)) == n3 α3 E^(I k3 d)
```

$$\alpha1 + \beta1 == \alpha2 + \beta2$$

$$\mathrm{n1}\ (\alpha1 - \beta1) == \mathrm{n2}\ (\alpha2 - \beta2)$$

$$e^{i\, d\, k2}\, \alpha2 + e^{-i\, d\, k2}\, \beta2 == e^{i\, d\, k3}\, \alpha3$$

$$\mathrm{n2}\ \left(e^{i\, d\, k2}\, \alpha2 - e^{-i\, d\, k2}\, \beta2\right) == e^{i\, d\, k3}\, \mathrm{n3}\, \alpha3$$

```
sols = Solve[a && b && c && f, {β1, α2, β2, α3}] // ExpToTrig // FullSimplify
```

$$\left\{\left\{\beta1 \to \frac{\alpha1\ \left(\mathrm{n2}\ (\mathrm{n1}-\mathrm{n3})\ \mathrm{Cos}[d\, k2] + i\ \left(\mathrm{n2}^2 - \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]\right)}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]},\right.\right.$$

$$\alpha2 \to \frac{2\ \mathrm{n1}\ (\mathrm{n2}+\mathrm{n3})\ \alpha1}{e^{2\, i\, d\, k2}\ (\mathrm{n1}-\mathrm{n2})\ (\mathrm{n2}-\mathrm{n3}) + (\mathrm{n1}+\mathrm{n2})\ (\mathrm{n2}+\mathrm{n3})},$$

$$\beta2 \to \frac{e^{i\, d\, k2}\ \mathrm{n1}\ (\mathrm{n2}-\mathrm{n3})\ \alpha1}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]},$$

$$\left.\left.\alpha3 \to \frac{2\ e^{-i\, d\, k3}\ \mathrm{n1}\, \mathrm{n2}\, \alpha1}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]}\right\}\right\}$$

```
coeffs = {α2, α3, β1, β2} /. sols
```

$$\left\{\left\{\frac{2\ \mathrm{n1}\ (\mathrm{n2}+\mathrm{n3})\ \alpha1}{e^{2\, i\, d\, k2}\ (\mathrm{n1}-\mathrm{n2})\ (\mathrm{n2}-\mathrm{n3}) + (\mathrm{n1}+\mathrm{n2})\ (\mathrm{n2}+\mathrm{n3})},\right.\right.$$

$$\frac{2\ e^{-i\, d\, k3}\ \mathrm{n1}\, \mathrm{n2}\, \alpha1}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]},$$

$$\frac{\alpha1\ \left(\mathrm{n2}\ (\mathrm{n1}-\mathrm{n3})\ \mathrm{Cos}[d\, k2] + i\ \left(\mathrm{n2}^2 - \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]\right)}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]},$$

$$\left.\left.\frac{e^{i\, d\, k2}\ \mathrm{n1}\ (\mathrm{n2}-\mathrm{n3})\ \alpha1}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]}\right\}\right\}$$

```
Ra = coeffs[[1, 3]] / α1
Ta = coeffs[[1, 2]] / α1
```

$$\frac{\mathrm{n2}\ (\mathrm{n1}-\mathrm{n3})\ \mathrm{Cos}[d\, k2] + i\ \left(\mathrm{n2}^2 - \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]}$$

$$\frac{2\ e^{-i\, d\, k3}\ \mathrm{n1}\, \mathrm{n2}}{\mathrm{n2}\ (\mathrm{n1}+\mathrm{n3})\ \mathrm{Cos}[d\, k2] - i\ \left(\mathrm{n2}^2 + \mathrm{n1}\, \mathrm{n3}\right)\ \mathrm{Sin}[d\, k2]}$$

```
R = Refine[Ra* Ra, {Element[n1, Reals], Element[n2, Reals],
    Element[n3, Reals], Element[k2, Reals], Element[d, Reals]}] // FullSimplify
```

$$T = \frac{n3}{n1} \text{Refine}[\text{Ta}^* \text{Ta}, \{\text{Element}[n1, \text{Reals}], \text{Element}[n2, \text{Reals}], \text{Element}[n3, \text{Reals}], \text{Element}[k2, \text{Reals}], \text{Element}[k3, \text{Reals}], \text{Element}[d, \text{Reals}]\}] \text{ // FullSimplify}$$

$$\frac{n2^2\,(n1-n3)^2\,\text{Cos}[d\,k2]^2 + \left(n2^2 - n1\,n3\right)^2\,\text{Sin}[d\,k2]^2}{n2^2\,(n1+n3)^2\,\text{Cos}[d\,k2]^2 + \left(n2^2 + n1\,n3\right)^2\,\text{Sin}[d\,k2]^2}$$

$$\frac{4\,n1\,n2^2\,n3}{n2^2\,(n1+n3)^2\,\text{Cos}[d\,k2]^2 + \left(n2^2 + n1\,n3\right)^2\,\text{Sin}[d\,k2]^2}$$

```
R + T // FullSimplify
```

1

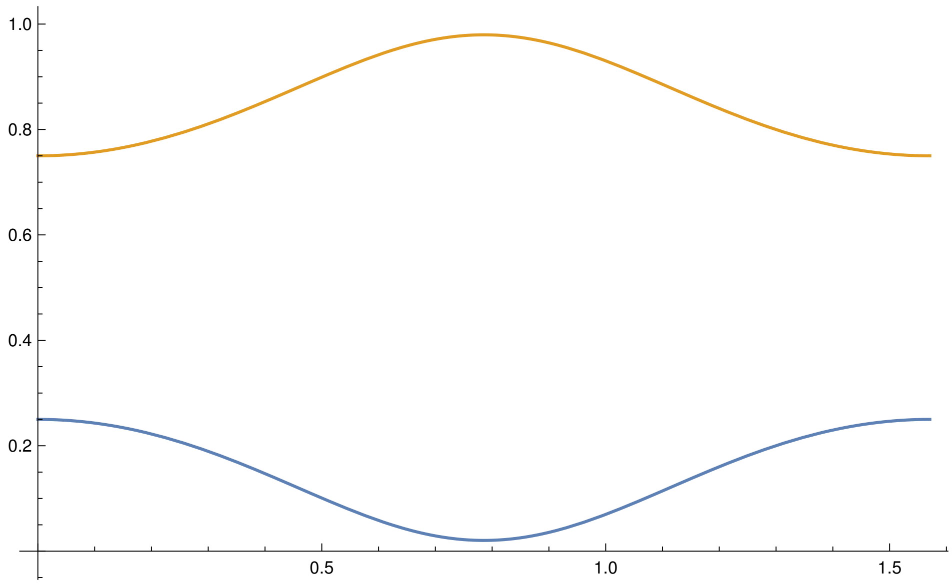
$$R = R\ /.\ k2 \to \frac{n2\,\omega}{C}\ /.\ d \to 1\ /.\ C \to 1$$

$$T = T\ /.\ k2 \to \frac{n2\,\omega}{C}\ /.\ k3 \to \frac{n3\,\omega}{C}\ /.\ d \to 1\ /.\ C \to 1$$

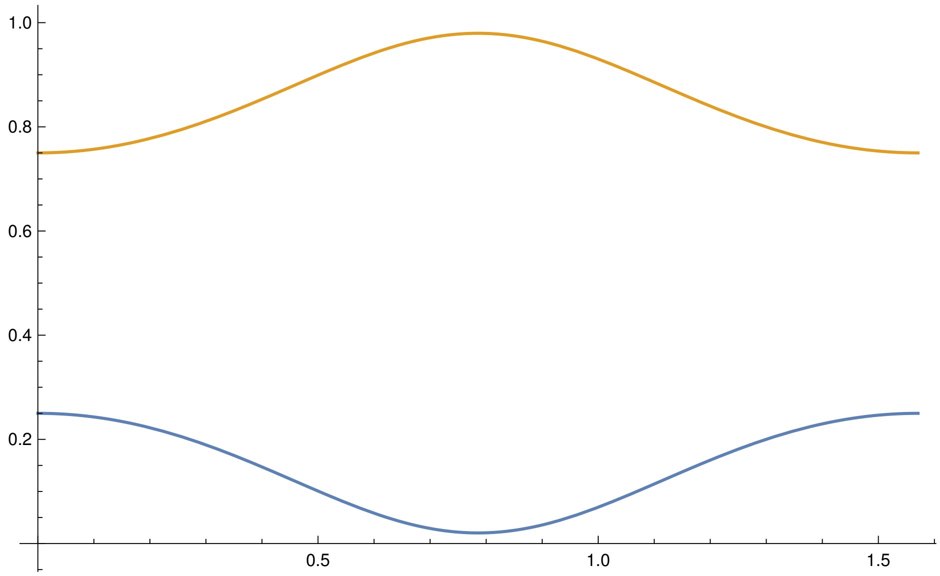
$$\frac{n2^2\,(n1-n3)^2\,\text{Cos}[n2\,\omega]^2 + \left(n2^2 - n1\,n3\right)^2\,\text{Sin}[n2\,\omega]^2}{n2^2\,(n1+n3)^2\,\text{Cos}[n2\,\omega]^2 + \left(n2^2 + n1\,n3\right)^2\,\text{Sin}[n2\,\omega]^2}$$

$$\frac{4\,n1\,n2^2\,n3}{n2^2\,(n1+n3)^2\,\text{Cos}[n2\,\omega]^2 + \left(n2^2 + n1\,n3\right)^2\,\text{Sin}[n2\,\omega]^2}$$

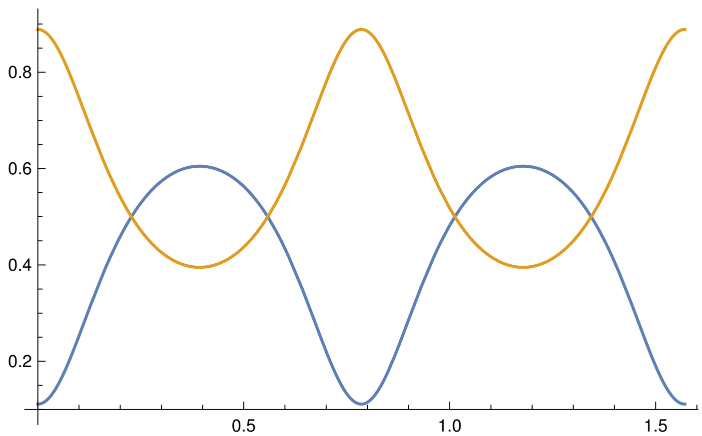
```
Plot[{R /. n1 → 1 /. n2 → 2 /. n3 → 3, T /. n1 → 1 /. n2 → 2 /. n3 → 3}, {ω, 0, Pi/2}]
```



```
Plot[{R /. n1 → 3 /. n2 → 2 /. n3 → 1, T /. n1 → 3 /. n2 → 2 /. n3 → 1}, {ω, 0, Pi/2}]
```




```
Plot[{R /. n1 → 2 /. n2 → 4 /. n3 → 1, T /. n1 → 2 /. n2 → 4 /. n3 → 1}, {ω, 0, Pi/2}]
```




```
Solve[R == 0, d]
{}
```