In[271]:=
$$tt = \frac{1}{a}\,\text{ArcTanh}[T/X];$$
$$xt = \sqrt{X^2 - T^2};$$

In[273]:=
$$\text{first} = t == tt;$$
$$\text{second} = x == \sqrt{X^2 - T^2};$$

In[275]:=
$$\text{trans} = \text{FullSimplify}\left[\text{Solve}\left[\text{first} \;\&\&\; \text{second},\ \{X, T\}\right],\ \text{Assumptions} \to a \in \text{Reals} \;\&\&\; t \in \text{Reals} \;\&\&\; x \in \text{Reals}\right]$$

Out[275]=
$$\left\{\left\{X \to -x\,\text{Cosh}[a\,t],\ T \to -x\,\text{Sinh}[a\,t]\right\},\ \left\{X \to x\,\text{Cosh}[a\,t],\ T \to x\,\text{Sinh}[a\,t]\right\}\right\}$$

In[276]:=
$$Xt = \text{FullSimplify}\left[\text{trans}[[2,1,2]],\ \text{Assumptions} \to a \in \text{Reals} \;\&\&\; t \in \text{Reals} \;\&\&\; x \in \text{Reals}\right]$$
$$Tt = \text{FullSimplify}\left[\text{trans}[[2,2,2]],\ \text{Assumptions} \to a \in \text{Reals} \;\&\&\; t \in \text{Reals} \;\&\&\; x \in \text{Reals}\right]$$

Out[276]=
$$x\,\text{Cosh}[a\,t]$$

Out[277]=
$$x\,\text{Sinh}[a\,t]$$

In[278]:=
$$dT = \text{FullSimplify}[\{D[Tt, t],\ D[Tt, x]\} /. x \to xt /. t \to tt]$$
$$dX = \text{FullSimplify}[\{D[Xt, t],\ D[Xt, x]\} /. x \to xt /. t \to tt]$$

Out[278]=
$$\left\{\frac{a\sqrt{-T^2 + X^2}}{\sqrt{1 - \frac{T^2}{X^2}}},\ \frac{T}{\sqrt{1 - \frac{T^2}{X^2}}\ X}\right\}$$

Out[279]=
$$\left\{\frac{a\,T\sqrt{1 - \frac{T^2}{X^2}}\ X}{\sqrt{-T^2 + X^2}},\ \frac{1}{\sqrt{1 - \frac{T^2}{X^2}}}\right\}$$

In[280]:=
$$\text{StreamPlot}\left[\{dT[[2]],\ dT[[1]]\} /. a \to 1,\ \{x, -3, 3\},\ \{t, -3, 3\}\right];$$
$$\text{StreamPlot}\left[\{dX[[2]],\ dX[[1]]\} /. a \to 1,\ \{x, -3, 3\},\ \{t, -3, 3\}\right];$$

In[282]:=
$$eT = \text{FullSimplify}[\{D[tt, T],\ D[xt, T]\}]$$
$$eX = \text{FullSimplify}[\{D[tt, X],\ D[xt, X]\}]$$

Out[282]=
$$\left\{\frac{X}{a\left(-T^2 + X^2\right)},\ -\frac{T}{\sqrt{-T^2 + X^2}}\right\}$$

Out[283]=
$$\left\{\frac{T}{a\,T^2 - a\,X^2},\ \frac{X}{\sqrt{-T^2 + X^2}}\right\}$$

In[284]:=
$$\text{StreamPlot}[\{eT[[2]],\ eT[[1]]\} /. a \to 1,\ \{x, 0.01, 3\},\ \{t, 0.01, 3\}];$$
$$\text{StreamPlot}[\{eX[[2]],\ eX[[1]]\} /. a \to 1,\ \{x, 0.01, 3\},\ \{t, 0.01, 3\}];$$

Part b.

In[286]:=
$$\eta = \{\{-1, 0\}, \{0, 1\}\}$$

Out[286]=
$$\{\{-1, 0\}, \{0, 1\}\}$$

In[287]:=
```
(g = FullSimplify[{{eT.η.eT, eT.η.eX}, {eX.η.eT, eX.η.eX}}] // FullSimplify) //
 MatrixForm
```

Out[287]//MatrixForm=

$$\begin{pmatrix} -\frac{T^4+\frac{X^2}{a^2}-T^2\,X^2}{\left(T^2-X^2\right)^2} & \frac{T\,X\left(1+a^2\,(T-X)\,(T+X)\right)}{a^2\,\left(T^2-X^2\right)^2} \\ \frac{T\,X\left(1+a^2\,(T-X)\,(T+X)\right)}{a^2\,\left(T^2-X^2\right)^2} & \frac{X^2}{-T^2+X^2}-\frac{T^2}{\left(a\,T^2-a\,X^2\right)^2} \end{pmatrix}$$

In[288]:=
```
(gInv = {{dT.η.dT, dT.η.dX}, {dX.η.dT, dX.η.dX}} // FullSimplify) // MatrixForm
```

Out[288]//MatrixForm=

$$\begin{pmatrix} -a^2\,X^2-\frac{T^2}{T^2-X^2} & T\,X\left(-a^2+\frac{1}{-T^2+X^2}\right) \\ T\,X\left(-a^2+\frac{1}{-T^2+X^2}\right) & -\frac{X^2+a^2\,\left(T^4-T^2\,X^2\right)}{T^2-X^2} \end{pmatrix}$$

In[289]:=
```
g.gInv // FullSimplify
```

Out[289]=
```
{{1, 0}, {0, 1}}
```

In[290]:=
```
dg = {D[g, T], D[g, X]} // FullSimplify
```

Out[290]=

$$\left\{\left\{\left\{\frac{2\,T\,X^2\left(2+a^2\,(T-X)\,(T+X)\right)}{a^2\,\left(T^2-X^2\right)^3},\ -\frac{X\left(3\,T^2+a^2\,T^4+X^2-a^2\,X^4\right)}{a^2\,\left(T^2-X^2\right)^3}\right\},\right.\right.$$
$$\left.\left.\left\{-\frac{X\left(3\,T^2+a^2\,T^4+X^2-a^2\,X^4\right)}{a^2\,\left(T^2-X^2\right)^3},\ \frac{2\,T\left(T^2+X^2+a^2\,T^2\,X^2-a^2\,X^4\right)}{a^2\,\left(T^2-X^2\right)^3}\right\}\right\},$$
$$\left\{\left\{-\frac{2\,X\left(X^2+T^2\left(1+a^2\,(T-X)\,(T+X)\right)\right)}{a^2\,\left(T^2-X^2\right)^3},\ \frac{T\left(T^2+a^2\,T^4+3\,X^2-a^2\,X^4\right)}{a^2\,\left(T^2-X^2\right)^3}\right\},\right.$$
$$\left.\left.\left\{\frac{T\left(T^2+a^2\,T^4+3\,X^2-a^2\,X^4\right)}{a^2\,\left(T^2-X^2\right)^3},\ -\frac{2\,T^2\,X\left(2+a^2\,(T-X)\,(T+X)\right)}{a^2\,\left(T^2-X^2\right)^3}\right\}\right\}\right\}$$

In[291]:=
```
Γ[γ_, β_, μ_] =
  1/2 ((gInv[[1, γ]] dg[[μ, 1, β]] + gInv[[1, γ]] dg[[β, 1, μ]] - gInv[[1, γ]] dg[[1, β, μ]]) +
     (gInv[[2, γ]] dg[[μ, 2, β]] + gInv[[2, γ]] dg[[β, 2, μ]] -
       gInv[[2, γ]] dg[[2, β, μ]])) // FullSimplify;
```

In[292]:=
```
Γ[1, 1, 1] // FullSimplify
```

Out[292]=

$$\frac{T\,X^2}{\left(T^2-X^2\right)^2}$$

In[293]:=
```
Γ[1, 1, 2] // Expand // FullSimplify
```

Out[293]=

$$-\frac{X^3}{\left(T^2-X^2\right)^2}$$

In[294]:=
```
Γ[1, 2, 1] // Expand // FullSimplify
```

Out[294]=

$$-\frac{X^3}{\left(T^2-X^2\right)^2}$$

In[295]:=
```
Γ[1, 2, 2] // Expand // FullSimplify
```

Out[295]=

$$\frac{-T^3+2\,T\,X^2}{\left(T^2-X^2\right)^2}$$

In[296]:=
```
Γ[2, 1, 1] // Expand // FullSimplify
```

Out[296]=
$$\frac{2\,T^2\,X - X^3}{\left(T^2 - X^2\right)^2}$$

In[297]:=
```
Γ[2, 1, 2] // FullSimplify
```

Out[297]=
$$-\frac{T^3}{\left(T^2 - X^2\right)^2}$$

In[303]:=
```
Γ[2, 2, 1] // Expand // FullSimplify
```

Out[303]=
$$-\frac{T^3}{\left(T^2 - X^2\right)^2}$$

In[302]:=
```
Γ[2, 2, 2] // Expand // FullSimplify
```

Out[302]=
$$\frac{T^2\,X}{\left(T^2 - X^2\right)^2}$$