Force-free magnetic fields — Problem set 5

1. a. Find a constant-α field satisfying the following boundary conditions

$$\begin{aligned} B_z(y, z=0) &= B_0 \cos\left(\frac{\pi}{L} y\right) && (1) \\ B_z(y, z=2L) &= 0 && (2) \\ B_y(y=-L, z) &= 0 && (3) \\ B_y(y=+L, z) &= 0 && (4) \end{aligned}$$

and x is an ignorable coordinate in whose direction the domain extends infinitely. Do this for all allowable values of α and use a flux function $f(y, z)$ to express your answer.

b. Compute the magnetic energy, per length in x, as a function of α.

c. For $\alpha > 0$ find the field line anchored to $\mathbf{x} = (\frac{1}{2}L + \varepsilon)\hat{\mathbf{y}}$, where $\varepsilon \ll L$. How does the location of the other footpoint (i.e. other point on the $z = 0$ plane) vary with α? How does the apex height of this field line vary with α?

d. Show that for certain values of α there are constant-α solutions with *homogeneous* versions of boundary conditions (1)–(4). Use a flux function to write these solutions.

e. For smallest[1] value of α from part d. show that the field contains, somewhere in the interior ($0 < z < 2L$, $|y| < L$), a straight magnetic field line parallel to the x axis. Find the field line which crosses the $x = 0$ plane a distance ε below this straight field line ($\varepsilon \ll L$).

f. For the same α from part e. construct a constant-α field subject to the inhomogeneous boundary conditions (1)–(4), but with an arbitrary amount of the solution from d. added on. Find the energy as a function of the homogeneous contribution.

[1] smallest in magnitude.