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Accretion by Neutron Stars: Accretion Disk and Rotating Magnetic Field

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Summary. We study the interaction of a thin Keplerian accretion disk with the magnetosphere of a rotating neutron star. In our model the neutron star's axis of rotation is perpendicular to and the magnetic dipole axis parallel to the disk plane. In such a configuration the disk can penetrate diamagnetically into the magnetosphere. The velocity difference between the disk material and the magnetosphere leads to a Kelvin-Helmholtz instability of the interface. This instability can grow to large amplitudes only within a narrow ring around the corotation radius. It can give rise to turbulent diffusion of the disk material into the magnetosphere. Part of this material will fall onto the neutron star, the rest will be flung out of the system. This can lead to an approximate balance between braking and acceleration of the neutron star. Application of the model to the pulsating X-ray source Her X-1 gives a natural explanation of the observed temporal variation of the pulse period.

Key words: accretion disk — magnetic neutron stars — pulsating X-ray source

1. Introduction

Regularly pulsating X-ray sources may well be rotating neutron stars with a strong magnetic field which circle around noncompact companions. The accretion of gas flowing over from the companion star provides the energy source for the X-rays, and the rotation of the magnetized neutron star is the clock-mechanism for the regular pulses.

This picture is strongly supported by the measurement of a spectral line of the regularly pulsating X-ray source Her X-1 by Kenziorra et al. (1977), which can be interpreted as a cyclotron line implying a magnetic field strength of $B = 5\ 10^{12}$ G near the surface. Such strong fields can exist on the surface of neutron stars, but not on white dwarfs.

The observed time variation of the pulse periods should also be considered: In Fig. 1 the periods of the regularly pulsating X-ray sources Her X-1 and Cen X-3 are plotted against time. A significant decrease of the period is evident. This has to be combined with estimates of the mass flow rate from the X-ray luminosity. Then the interesting conclusions can be drawn that the angular momentum transfer from the accreting material can produce the observed speed-up for a neutron star, but not for a white dwarf. The Keplerian angular momentum at the stellar radius would also be too small, and so it must be the neutron star's magnetic field which transmits angular momentum from the accreting matter to the star.

The region where the star's magnetic field begins to dominate the incoming gas flow is of great importance in determining the rotational state of the neutron star, as well as the accretion pattern. Therefore a description of the angular momentum transfer mechanism between the incoming gas and the stellar magnetic field is necessary.

It should be noted that the models, recently discussed (Arons and Lea, 1976; Lamb and Elsner, 1976), of spherically symmetric accretion by a nonrotating neutron star do not cover this important aspect of the accretion problem.

In this paper we want to explore some further aspects of a model proposed by Börner et al. (1973), where angular momentum effects are taken into account. We start out from the following picture: the accreting matter has angular momentum large enough to form a disk of gas around the neutron star.

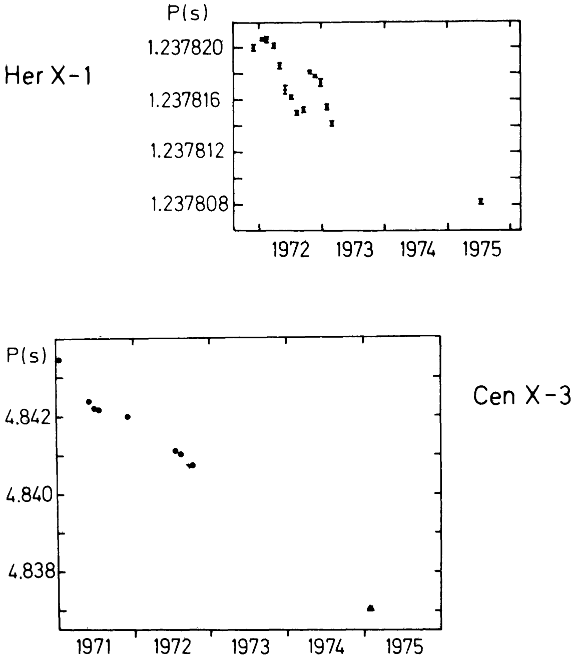

Fig. 1. Observed pulse period vs. time for Her X-1 (Joss et al., 1977) and Cen X-3 (Fabbiano and Schreier, 1976)

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The star's rotation axis is taken to be perpendicular to the plane of the disk, and the star's magnetic field axis is in the plane of the disk. We think that such a simple geometrical configuration is best suited to study the angular momentum transfer.

2. The Inner Edge of the Disk

The disk around the neutron star behaves like diamagnetic material in the star's magnetic field: currents on the disk's surface will be induced, which shield the disk material from the magnetic fields.

Because of the high electrical conductivity of the disk, we can assume that in a good approximation the neutron star's magnetic fields do not penetrate the disk; that the diamagnetic disk squeezes in between the field lines, which bend out of the way a bit. The magnetic dipole field of the neutron star thus does not change its shape under disk accretion (see Fig. 2). There is also, in this case, no flux swept up and pushed into a magnetospheric cavity, as in the case of spherically symmetric accretion.

This would seem like a very special configuration, whereas the general case with a dipole axis not in the disk plane is not covered. We want to suggest that in the general case the following picture applies, at least for small inclinations of the dipole out of the plane of the disk: the magnetosphere is still open, the disk is parallel to the far-away field lines. Further in the field lines bend and join to the original oblique dipole. Estimates of the energy of such a configuration show it to be energetically favourable compared to a configuration where the disk indents into the magnetosphere.

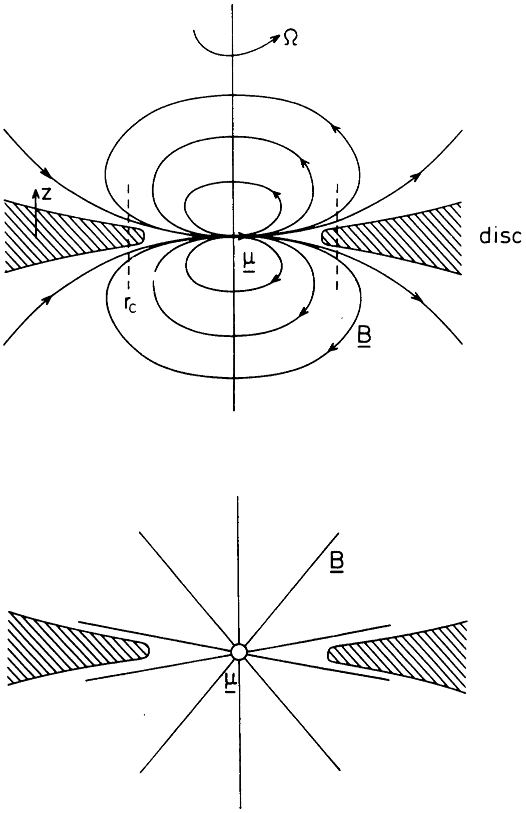

Fig. 2. Model of the magnetosphere-disk configuration. The upper part shows the side view of the magnetic dipole field, the lower part gives the projection of the field lines when the dipole is seen end-on.

The shape of the disk, and its existence as a stable configuration are determined by the properties of angular momentum transport inside [cf. e.g. Pringle and Rees (1972)]. Börner et al. (1973) have described a specific model for the viscosity in an almost Keplerian disk, which depended upon magnetic fields being carried in with the accreting plasma. For the moment we want to avoid the additional complications of the interaction of a frozen-in magnetic field of the disk with the neutron star's dipole field, and therefore assume that there is no magnetic field in the disk. (Effects of a disk field will have to be discussed later.)

For the vertical structure of the disk (z-direction) we assume hydrostatic equilibrium:

$$\frac{dp}{dz} = -\rho g \tag{1}$$

with

$$g = \frac{GM}{r^2}\frac{z}{r}, \tag{2}$$

where p is the gas pressure, ρ the mass density, M the mass of the neutron star, and G the Gravitational constant.

We integrate (1) for a disk which is isothermal in the z-direction, i.e. $p = \rho RT$, with $T(r)$, and obtain

$$p(r, z) = p(r, 0) \exp(-z^2/H^2), \tag{3}$$

where the scale height H is given by

$$H^2 = \frac{RTr}{GM} r^2 = r^2\left(\frac{c_s}{V_k}\right)^2. \tag{4}$$

Here, c_s is the velocity of sound and V_k the Keplerian velocity.

This hydrostatic equilibrium in the z-direction is maintained as long as there are no disturbances with timescales short compared to $\tau_s = H/c_s$.

Where the disk (here, as stated above, taken without magnetic fields) meets the neutron star's magnetic field, this outer boundary in the z-direction ($z = \pm d$, say) is determined by pressure equilibrium $p_g = p_{B'}$ i.e.

$$P_g(r, z = \pm d) = p(r, z = 0) \exp(-d^2/H^2) = \frac{B^2}{8\pi}. \tag{5}$$

Then the inner edge of the disk, $r_{M'}$ will be defined by

$$p_g(r_{M'}\, z = 0) = \frac{B^2(r_M)}{8\pi}. \tag{6}$$

The undisturbed disk by itself would continually move in towards the neutron star: the mass transport by viscosity in the disk would accumulate matter at the equilibrium radius $r_{M'}$ increasing the pressure there, such that a new equilibrium radius $\overline{r_M} < r_M$ was established. Finally, the disk would extend to the surface of the neutron star, and the matter would accrete in a ring. This would contradict the observation of regular X-ray pulses. Therefore, efficient mechanisms which mix magnetic field lines and disk matter, or bring particles from the disk onto the field lines, are necessary. A very plausible explanation for these pulses is that they are produced by hot spots at the magnetic poles of the star, which rotate through our field of vision. The question is, therefore, how the material in the disk can be guided by the magnetic field so that it accretes at the poles.

The pressure equilibrium as described in Eqs. (5) and (6) is actually more complicated, because for a dipole field with magnetic moment μ there is a dependence on the azimuthal angle φ between dipole axis and reference point

$$\mathbf{B} = 2\mu \frac{\cos\varphi}{r^3}\,\mathbf{i}_r + \mu \frac{\sin\varphi}{r_\odot}\,\mathbf{i}_\varphi, \tag{7}$$

and

$$p_B(r, \varphi) = \frac{\mu^2}{r^6}(1 + 3\cos^2\varphi). \tag{8}$$

Thus, between pole ($\varphi = 0$) and equator ($\varphi = \pi/2$), on a constant radius, the pressure varies by a factor of 4. In order to maintain pressure equilibrium for all angles at one specific radius, the disk has to adjust its vertical structure to the varying pressure. Any effect of the magnetic pressure variation will be most noticeable near r_M, because there even at the centre of the disk its structure is strongly influenced by the outside magnetic pressure. Whether such an effect can produce a strong mixing of disk material and magnetic field has not yet been investigated in detail. We want to remark only that the effect is probably less important for larger velocity differences between disk and magnetic field, since then any outside pressure variation tends to be smoothed out by the ram pressure of the disk material, which is larger than the gas pressure in the disk. For velocity differences less than the sound velocity in the disk, the pressure variation together with the equilibrium condition (Eq. 6) may be important for a possible instability of the boundary. As we shall see below there is a strong instability in this region for other reasons, to which the pressure variation may just add one more.

It will become clear in the following that the normal Ohmic diffusion of particles across the field lines cannot lead to a substantial accretion rate. Börner et al. (1973) have pointed out that the Kepler disk may corotate with the magnetic field near the inner edge of the disk, and that this layer will be subject to a Rayleigh-Taylor instability. The question of the formation of such a corotating layer has not been discussed, however, and also the angular momentum transport has not been described in detail.

We want to discuss here one specific mechanism which mixes disk matter with the magnetic field. Our starting point is the observation that the disk's boundary in the z-direction is subject to an instability, induced by the velocity difference between the corotating magnetic field ($V_\varphi = \Omega r$) and the Keplerian disk ($V_\varphi = (GM/r)^{1/2}$).

3. Instability of the Boundary

At the radius of corotation the matter inside the Keplerian disk rotates with the same velocity as the magnetosphere of the neutron star, therefore

$$\frac{GM}{r_c} = \Omega^2 r_c^2$$

or

$$r_c = \left(\frac{GM}{\Omega^2}\right)^{1/3}. \tag{9}$$

But away from r_c the plasma of the disk moves relative to the magnetosphere with a velocity

$$\Delta V = \left(\frac{GM}{r}\right)^{1/2} - \Omega r. \tag{10}$$

This difference velocity will give rise to a Kelvin-Helmholtz instability. The analysis of the instability which will occur at the interface between disk and the actual magnetosphere is an extremely complex problem. In this paper we shall therefore try to extend the results obtained for simpler geometries to our accretion disk.

As a first step we shall study the stability of a plane interface (defined by $z = 0$) between two identical gases both having density ρ and velocity of sound c_s. We shall further assume a planar flow (instead of cylindrical symmetry) and take the difference velocity constant and in the x-direction, $\Delta\mathbf{v} = (v, 0, 0)$.

We investigate the stability of this system against perturbations of the form

$$\xi = \xi_0 \exp\left[i(\omega t + k_x x + k_y y) - \kappa z\right]. \tag{11}$$

Now with $k = (k_x^2 + k_y^2)^{1/2}$ and $\mathbf{k}\Delta\mathbf{v} = kv\cos\phi$ one obtains the following dispersion relation (Blake, 1972):

$$\frac{(\omega - kv\cos\phi)^2}{(k^2c_s^2 - (\omega - kv\cos\phi)^2)^{1/2}} = \frac{\omega^2}{(k^2c_s^2 - \omega^2)^{1/2}}. \tag{12}$$

Defining a Mach number by $\mathscr{M} = (v/c_s)\cos\phi$ and writing $\omega/kc_s = y + \mathscr{M}/2$ then leads to

$$\frac{\left(y - \frac{\mathscr{M}}{2}\right)^4}{1 - \left(y - \frac{\mathscr{M}}{2}\right)^2} = \frac{\left(y + \frac{\mathscr{M}}{2}\right)^4}{1 - \left(y + \frac{\mathscr{M}}{2}\right)^2}. \tag{13}$$

The solutions of this equation are given by $y = 0$ and

$$y = \pm\left[1 + \frac{\mathscr{M}^2}{4} \pm (1 + \mathscr{M}^2)^{1/2}\right]^{1/2}. \tag{14}$$

For $\mathscr{M} > \mathscr{M}_c = 2\sqrt{2}$ all solutions are real, which means that in this case the system is stable. For $\mathscr{M} < \mathscr{M}_c$ one of the above solutions leads to exponentially growing (i.e. unstable) perturbations. For small values of $\mathscr{M}$ one finds approximately

$$\omega \approx \tfrac{1}{2}kv\cos\phi(1 - i). \tag{15}$$

The perturbations associated with this model satisfy the condition $|\xi| \to 0$ for $|z| \to \infty$ and thus are physically realistic. From $v\cos\phi < 2\sqrt{2}\,c_s$ the growth rate $\gamma(= -J m\omega)$ in Eq. (15) is also limited by

$$\gamma < \sqrt{2}\,kc_s. \tag{16}$$

From Eq. (15) one would conclude that the fastest growing modes are the ones with the shortest wavelength. This, however, is only true as long as the perturbations can be treated linearly. If nonlinear effects are taken into account one finds that the amplitudes are usually limited by a $\lesssim 2\pi/k$ (Ong and Roderick, 1972), therefore the longest wavelengths will eventually reach the largest amplitudes. Since the vertical density changes in the disk occurs over one scale height H we shall make the assumption

$$k_{\min} \approx 1/H$$

which will lead to

$$\gamma \leqq \sqrt{2}\,\frac{c_s}{H}.$$

Let us now return to the disk configuration. First we note that there exists a ring around the radius of corotation where $|\Delta v| < c_s$ holds. This ring is given by $|r - r_c| < \delta$ with

$$\delta/r_s = \tfrac{2}{3}c_s/\Omega r_c.$$

In this part of the disk all k-modes can grow exponentially. But as one moves away from this ring one has very soon $|\Delta v| \gg c_s$. In these regions only the waves with $\cos\phi \ll 1$ or k approximately perpendicular to $\Delta\mathbf{v}$ can grow. Since $\Delta\mathbf{v}$ is in the φ-direction, $\mathbf{k}$ has to be predominantly in the r-direction. These waves will move rapidly through regions of varying field orientation and, depending on their direction, the magnetic fields can have a highly stabilizing effect.

Therefore, we shall now discuss the influence of the magnetic field on the Kelvin-Helmholtz instability. We shall again make strong simplifications by investigating a configuration in which a nonmagnetic plasma (density ρ_1) streams past a homogeneously magnetized plasma with density ρ_2. According to Hasegawa (1975, p. 126) the instability will only occur for

$$(\mathbf{k}\Delta\mathbf{v})^2 > (\mathbf{k}\mathbf{v}_A)^2 \tag{17}$$

with $\mathbf{v}_A = \mathbf{B}/(4\pi\rho_2)^{1/2}$. From $\mathbf{k}\Delta\mathbf{v} \leqq kc_s$ and $\mathbf{k} \approx \mathbf{k}_r$ one thus obtains

$$\frac{B_r^2}{4\pi\rho_2} \leqq c_s^2. \tag{18}$$

Since the interface is given by pressure equilibrium (i.e. $B^2/8\pi = \rho_1 c_s^2$) and since around the magnetic poles $B^2 \approx B_r^2$ holds, we find the following approximate relation near the poles:

$$\frac{B_r^2}{4\pi\rho_2} \approx 2\,\frac{\rho_1}{\rho_2}\,c_s^2. \tag{19}$$

In general, one has $\rho_1 \gg \rho_2$ and therefore $B_r^2/4\pi\rho_2 \ll c_s^2$, which means that the region around the poles will be very efficiently stabilized. Depending on the ratio ρ_2/ρ_1, the regions near the magnetic equator can be unstable. But the rotating disk material will be in unstable regions only over times of the order of

$$\Delta t \approx \frac{\pi r}{\Delta v}. \tag{20}$$

Therefore the instabilities can only grow by a factor $\exp(\gamma\Delta t)$ with

$$\gamma\Delta t \approx \frac{c_s}{H}\frac{\pi r}{\Delta v}. \tag{21}$$

Since away from the region of corotation we have $\Delta v \geqq v_K$ we find $\gamma\Delta t \lesssim 1$. We thus conclude that the supersonic part of the disk will be stabilized.

On the other hand, in the subsonic regime around the radius of corotation, $\mathbf{k}\Delta\mathbf{v} < kc_s$ holds for all k-modes. Therefore all these modes are unstable. Because one has

$$\gamma\Delta t \geqq \frac{\pi r_c}{H} \gg 1,$$

these instabilities can grow until they are limited by nonlinear effects. The modes for which (17) is violated will be suppressed but all the others are unaffected by the magnetic field (e.g. the modes with $\mathbf{k} \perp \mathbf{v}_A$).

This very crude investigation indicates that a narrow ring around the radius of corotation will be unstable whereas the rest of the disk is stabilized by the interplay between the supersonic flow and the fields in the magnetosphere. In order to really prove the existence of such an instability one would have to do an analysis which takes the cylindrical geometry, the variation of $\Delta\mathbf{v}$ with r, and the change of the field orientation simultaneously into account. We shall postpone this complicated stability analysis to a later time.

Does the instability lead to turbulence? We have to leave that question open, although in the following we will make use of turbulence in a boundary layer. We are fairly confident that a boundary layer will become turbulent, since the Reynolds number of the flow must be quite large: Using for the kinematic viscosity ν a formula given by Linhardt (1961)

$$\nu = \frac{\sqrt{2}}{\pi}\frac{\sqrt{m_p}\,(kT)^{5/2}}{e^4\rho\ln\Lambda} \tag{22}$$

we find for the Reynolds number $R_e = lv/\nu$ (taking $l \approx H \approx 10^6$ cm, $v \leqq c_s \approx 10^7\,\mathrm{cm\,s^{-1}}$, $T = 10^6$ K, $n = 10^{20}\,\mathrm{cm^{-3}}$)

$$R_e \simeq 10^9.$$

In view of this high Reynolds number, combined with the instability of the boundary, it does not seem unreasonable to assume that at least a boundary layer is turbulent.

On the other hand we do not expect supersonic turbulence, shock waves, and other similarly exciting phenomena, because the instability is confined to subsonic shear flow and moves more or less along with the disk material.

4. Turbulent Diffusion into the Magnetosphere

The velocity difference Δv between disk matter and magnetosphere is less than the sound velocity c_s only within an interval $r_c - \delta \leqq r \leqq r_c + \delta$ around the radius of corotation r_c. The instability of the disk boundary is confined to this narrow region. Since at the interface between disk and magnetosphere the gas pressure equals the magnetic pressure, we can, fortunately, conclude that for $\Delta v < c_s$ the magnetic field can control the motion of the gas, i.e. also remove angular momentum efficiently.

The region where the velocity difference is smaller than the sound velocity is given by

$$r_c - \delta \leqq r \leqq r_c + \delta$$

with (23)

$$\delta = \frac{2}{3}\frac{c_s}{\Omega r_c}\,r_c,$$

which amounts to 10^6 cm for $T = 10^6$ K. Now, if the entire mass flow of $10^{17}\,\mathrm{g\,s^{-1}}$ is supplied by diffusion in the region between $r_c - \delta$ and r_c, one has to require diffusion velocities of the order of $3\,10^5\,\mathrm{cm\,s^{-1}}$.

Let us now study the structure of the diffusion layer. At the inner part, say at $z = z_0$ (see Fig. 3), the gas has a high density which makes it hard for the magnetic field to take up

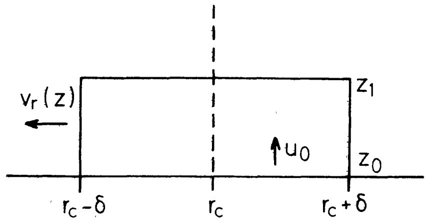

Fig. 3. Geometry of the diffusion region, $z = z_0$ is the magnetosphere-disk boundary

the angular momentum of the gas and therefore the radial velocity, v_r, will be small in this region. As the density decreases with larger depths, z, this radial velocity will increase. We denote by $v_r(z)$ the velocity of the material leaving the diffusion region, assume that the diffusion layer extends over an interval $z_0 \leqq z \leqq z_1$, and take as the simplest possible functional form for $v_r(z)$

$$v_r(z) = c_s \frac{z - z_0}{\Delta z} \quad \text{with} \quad \Delta z = z_1 - z_0. \tag{24}$$

If we take a density structure in the layer which depends only on z, we obtain the following equations for the density, ρ, and the diffusion velocity, u (D being the coefficient of diffusion):

$$u = -D \frac{d \ln \rho}{dz}, \tag{25}$$

$$\frac{d(\rho u)}{dz} \delta + \rho v_r(z) = 0; \tag{26}$$

and combining the two equations leads to

$$\frac{du}{dz} - \frac{u^2}{D} = -\frac{v_r(z)}{\delta}. \tag{27}$$

Then Eq. (27) can be written in dimensionless form

$$\frac{dw}{d\xi} = Aw^2 - B\xi \tag{28}$$

with

$$w = \frac{u}{c_s}, \qquad \xi = \frac{z - z_0}{\Delta z}, \tag{29}$$

$$A = \frac{c_s \Delta z}{D}, \qquad B = \frac{\Delta z}{\delta}. \tag{30}$$

Equation (24) gives a velocity field which is subsonic everywhere and has $v_r = c_s$ at the inner edge. Since the outflow is driven by gas pressure, $v_r \leqq c_s$ should always hold. But one could also consider flow fields with

$$v_r = \alpha c_s \frac{z - z_0}{\Delta z} \quad \text{and} \quad \alpha < 1.$$

This will lead again to Eq. (28), but Δz has to be replaced by $\Delta z/\alpha$, and D by D/α. If α is taken much smaller than unity, then the diffusion coefficient would have to be extremely large. For this reason we have restrained our discussion to the case $\alpha = 1$. The mass flow requirement together with $c_s = 10^7$ cm s^{-1} gives us the boundary condition $w(0) = 3\ 10^{-2}$. We have solved Eq. (28) numerically for various values of A and B. The density structure of the layer was then calculated from

$$\rho(\xi) = \rho(0) \exp \left(- \int_0^{\xi} A w d\xi' \right). \tag{31}$$

Table 1. Results from model calculations for the diffusion into the magnetosphere

A	B	D(cm^2 s^{-1})	Δz (cm)	$\overline{\rho v_r}/\rho_0 c_s$
50	0.2	4 10^{10}	2 10^5	0.18
100	0.7	7 10^{10}	7 10^5	0.043
150	1.7	1.1 10^{11}	1.7 10^6	0.021
200	2.8	1.4 10^{11}	2.8 10^6	0.0106

The range of possible combinations of the parameters A and B can be narrowed down by imposing conditions on velocity and density at the inner boundary of the layer, i.e. at $z = z_\perp$ or $\xi = 1$. We take the following conditions: $w(1) = 0$, and $\rho(1) \ll \rho(0)$. This then leads to $A \gtrsim 50$, and $B \gtrsim 0.2$. We have calculated models for different values of A. The results are shown in Table 1. The last column gives the average density of the outflow, a quantity which is of prime interest for our accretion models. Since there is approximate symmetry with respect to r_c, we expect roughly the same amount of matter flowing into the magnetosphere in the inner region ($r_c - \delta \leqq r \leqq r_c$) as in the outer region ($r_c \leqq r \leqq r_c + \delta$). These models require diffusion coefficients of 4 10^{10} cm^2 s^{-1} to 10^{11} cm^2 s^{-1}. The actual diffusion coefficient is, of course, determined by the physics of the transition region between disk and magnetosphere, but we are, at present, not able to calculate this quantity. We want, however, to point out that diffusion coefficients of the order of 10^{11} cm s^{-1} are only possible in a turbulent layer. Classical diffusion gives values which are many orders smaller – classical ohmic diffusion gives $D = 10^3$ cm^2 s^{-1}. Let us estimate the turbulent diffusion by setting

$$D = L v_{\text{turb}},$$

where the turbulent velocity is given by $v_{\text{turb}} = c_s \eta$, and L is a characteristic size of turbulent elements. $D = 10^{11}$ cm^2 s^{-1} can be achieved with $\eta = 0.1$ and $L = 10^5$ cm (0.1 scale height). Since, in the calculations, the whole diffusion layer has only a size of approximately 10^6 cm, we conclude that, even in the case of turbulent diffusion, D cannot become much larger than 10^{11} cm^2 s^{-1}.

5. Flow in the Magnetosphere

The turbulent diffusion discussed in the last paragraph brings disk matter into the magnetosphere. It will enter the magnetic region in the form of blobs. Since the coefficient for ohmic diffusion is extremely small ($D = 10^3$ cm^2 s^{-1}), these blobs will essentially behave like diamagnetic material.

The question now arises how such a blob moves through the magnetosphere. In Fig. 4 we have drawn schematically such a plasma blob. We have pressure equilibrium between the plasma

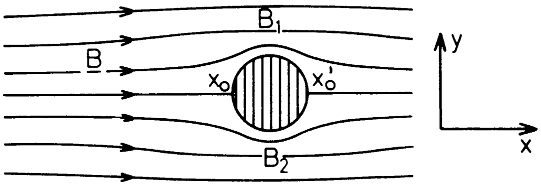

Fig. 4. Diamagnetic blob in an inhomogeneous magnetic field

and the magnetic field at the interface. The gas inside the blob will then move according to

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p, \tag{32}$$

where the pressure along the surface is given by $p = B^2/8\pi$.

Now if $B_1 > B_2$ there will exist a pressure gradient in the y-direction (cf. Fig. 4). But at x_0 and x_0' we have $B = 0$. Hence the steepest pressure gradients point everywhere towards x_0 and x_0'. The fluid element will flow towards and elongate along the x-axis, i.e. it will follow the field lines. The so-called "melon-seed effect" (Schlüter, 1965) can only operate if the palsma really is a melon seed (i.e. has strong cohesive forces).

Therefore, the diamagnetic fluid elements also follow the field lines. As they move towards the neutron star, they become more and more elongated and thinner, so that eventually ohmic diffusion will be effective in bringing the individual particles onto the field lines. Such a flow along the unperturbed magnetic field can only occur if the field is strong enough to take up all the excess angular momentum of the gas. If the density of the kinetic energy of the plasma becomes comparable to, or larger than the magnetic energy density, then the mass motion will wind up the magnetic field. This then would reduce the rate of accretion considerably.

We shall therefore now discuss the radial variation of the kinetic energy density. Inside the rigidly rotating magnetosphere the radial acceleration is given by

$$\frac{dv_r}{dt} = \Omega^2 r - \frac{GM}{r^2}, \tag{33}$$

which gives

$$v_r^2 = v_0^2 + \Omega^2 r_c^2 \left(x^2 + \frac{2}{x} - 3\right), \tag{34}$$

where $x = r/r_c$. Since $v_0 < c_s$ and $c_s \ll \Omega r_c$, we have almost everywhere

$$v_r \approx \Omega r_c \left(x^2 + \frac{2}{x} - 3\right)^{1/2}. \tag{35}$$

With the gas streaming along the dipole field we also have

$$\rho v_r r^3 = \rho_1 v_0 r_c^3. \tag{36}$$

The velocity difference between rigid rotation and Kepler motion is given by

$$\Delta v = \Omega r_c \left(x - \frac{1}{\sqrt{x}}\right). \tag{37}$$

From Eqs. (35) through (37) we obtain for the kinetic energy density

$$\tfrac{1}{2}\rho(\Delta v)^2 = \tfrac{1}{2}\rho_1 v_0 \Omega r_c \frac{\left(x - \frac{1}{\sqrt{x}}\right)^2}{\left(x^2 + \frac{2}{x} - 3\right)^{1/2} x^3}. \tag{38}$$

Table 2. Comparison between kinetic energy density (E_k) of material moving through the magnetosphere and energy density of the dipole field (E_m)

x	E_k/E_0	E_m/E_0
0.1	2260	10^6
0.2	195	$1.56\ 10^5$
0.3	44.4	$1.37\ 10^3$
0.4	14.8	224
0.5	5.98	64
0.6	2.64	21
0.7	1.21	8.5
0.8	0.528	3.81
0.9	0.182	1.88
0.95	0.076	1.36
1.05	0.056	0.74
1.1	0.096	0.56
1.2	0.146	0.33
1.3	0.170	0.21
1.4	0.178	0.13
1.5	0.182	0.09

For our model we took the following assumptions:

$$\rho_0 c_s^2 = \frac{B_0^2}{8\pi} = E_0$$

and

$$\Omega r_c = 100\ c_s.$$

Therefore we finally arrive at the equation

$$\tfrac{1}{2}\rho(\Delta v)^2 = 50 \frac{\rho_1 v_0}{\rho_0 c_s} E_0 \frac{\left(x - \frac{1}{\sqrt{x}}\right)^2}{\left(x^2 + \frac{2}{x} - 3\right)^{1/2} x^3} = 50 \frac{\rho_1 v_0}{\rho_0 c_s} E_k(x). \tag{39}$$

The magnetic energy density is given by

$$E_m(x) = E_0 x^{-6}.$$

The quantities $E_k(x)$ and $E_m(x)$ are shown in Table 2.

If one takes for $\rho_1 v_0$ the average values of ρv_r given in Table 1, one finds for $A = 50$ that the magnetic energy is dominating only inside the region $0.9 < x < 1.1$. We conclude from this that diffusion coefficients of $\approx 4\ 10^{10}\ \mathrm{cm^2\ s^{-1}}$ (corresponding to $A = 50$) cannot lead to the accretion required. For $A \geqq 100$, on the other hand, the magnetic energy dominates the kinetic energy everywhere between the neutron star and the radius of corotation, thus accretion can occur for all $D \geqq 7\ 10^{10}\ \mathrm{cm^2\ s^{-1}}$.

The part of the material which diffuses into the magnetosphere outside the radius of corotation will be accelerated radially outward. At some distance, x_{eq}, the kinetic energy equals the magnetic energy and beyond that point the material will leave the magnetosphere.

6. Consequences for Disk Accretion

Within the small distance δ inside or outside of the corotation radius r_c the same physical conditions prevail. Thus the model predicts, in addition to the mass flow $\dot{M}_{\rm in}$ inside of r_c, a mass flow $\dot{M}_{\rm out}$ into the magnetosphere outside of r_c of the same magnitude:

$$\dot{M}_{\rm in} \approx \dot{M}_{\rm out}.$$

The mass flow outside of r_c brakes the rotation of the neutron star, because the material in the magnetic field is brought into corotation and therefore gains angular momentum from the magnetic field. One finds

$$\delta J_{\rm brake} = \dot{M}_{\rm out}\Omega r_c^2(x_{\rm eq}^2 - 1). \tag{41}$$

This loss has to be compared with the gain from material falling onto the neutron star

$$\delta J_{\rm acc} = \dot{M}_{\rm in}\Omega r_c^2. \tag{42}$$

The observations of Her X-1 show a net average acceleration of $\Delta P/P = -3\ 10^{-6}\ {\rm yr}^{-1}$. The accreleration of a neutron star of $1\ M_\odot$, moment of inertia $I = 0.75\ 10^{45}\ {\rm g\ cm}^2$, from the transfer of the Keplerian angular momentum at r_c ($P = 1.24$ s, $\dot{M} = 10^{17}\ {\rm g\ s}^{-1}$) would correspond to $\Delta P/P = -1.2\ 10^{-4}\ {\rm yr}^{-1}$. This clearly shows that Her X-1 has almost achieved a strict balance between braking and acceleration:

$$(\delta J_{\rm acc} - \delta J_{\rm brake})/\delta J_{\rm brake} = 2.5\ 10^{-2}.$$

If we assume $\dot{M}_{\rm in} = \dot{M}_{\rm out}$ and take $x_{\rm eq} = 1.42$ (corresponding to $D = 1.2\ 10^{11}\ {\rm cm}^2\ {\rm s}^{-1}$) the net acceleration of the system will agree with the observed decrease of the period.

In 1972 there have also been observed, with Her X-1 short term period increases and decreases, superimposed on the small average acceleration (Giaconni, 1975): one has found, e.g. $\Delta P/P = -5\ 10^{-6}$ per 6 months. Within the terms of our model these fluctuations could be explained by small fluctuations in the mass flow through the disk. Since $\dot{M}_{\rm out}$ will stay the same, any additional mass δM will flow inside r_c, and transfer an angular momentum $\delta M\Omega r_c^2$ to the neutron star. From

$$\delta M\Omega r_c^2 = I\Delta\Omega = 0.75\ 10^{45}\ \frac{\Delta P}{P}\ \Omega, \tag{43}$$

$\delta M = 1.2\ 10^{23}$ g, which distributed over half a year is a small additional mass flow rate $\dot{M} = 10^{16}\ {\rm g\ s}^{-1}$. Similarly a small decrease in the mass flow rate of this order (10% of the average rate) will lead to a deceleration of similar magnitude.

The X-ray source Cen X-3 can probably also be explained by a model of this type. Its period, however, is 4.84 s, so the magnetic field at the corotation radius will be weaker by a factor of 16 than the field in the source Her X-1, if surface field strength and mass of the neutron star are the same. The parameters may, of course, be adapted to the case of Cen X-3, and one amusing coincidence should be noted: if we take the mass of Cen X-3 to be $1\ M_\odot$, but assume a weaker magnetic field at r_c, such that the outward flowing mass just reaches escape velocity at $r_e/r_c = 1.26$, and then separates from the field lines, then $(\delta J_{\rm acc} - \delta J_{\rm brake}) = (2 - (1.26)^2)\delta J_{\rm acc} = 0.4\delta J_{\rm acc}$. This leads to an average period decrease of $\Delta P/P = -3\ 10^{-4}\ {\rm yr}^{-1}$, which exactly corresponds to the observed value (Fabbiano and Schreier, 1976).

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