Solar Flares & CMEs
Problem Set 2
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In[294]:=
```
Clear["Global`*"]
```

Problem 2

From 1 to 1.01 solar radii we use the VAL model used by Hurford and Gary 2004

Import tabulated data from the VAL model

In[295]:=
```
data = Transpose[Import[FileNameJoin[{NotebookDirectory[], "p2_modelB.csv"}]]];
```

Put units in terms of solar radii

In[296]:=
```
h0 = data[[1]] / 695700;
```

The number density is the second column

In[297]:=
```
N0 = data[[2]];
```

The plasma frequency is given as

In[298]:=
```
νp[n_] := 8980 √n
```

Construct a plot of this region

In[299]:=
```
ν0 = Transpose[{νp[N0], h0}];
pltrng = {{10^4, 5 × 10^10}, {5 × 10^-4, 10^3}};
p0 = ListLogLogPlot[ν0, PlotRange → pltrng, Joined → True, ImageSize → Large];
```

From 1.01 to 1.2 solar radii use model by Mann et al (1997)

The functional form given in the paper is

In[302]:=
$$N2 = Ns\, Exp\left[\frac{A}{Rs}\left(\frac{1}{R} - 1\right)\right];$$
$$A = \frac{\mu\, G\, Ms\, Mp}{k\, T};$$

with the corresponding values

In[304]:=
$$N2v = \{Ns \to Min[N0],\ \mu \to 0.6,\ G \to 6.674\times10^{-11},\ Ms \to 1.988\times10^{30},\ k \to 1.38\times10^{-23},\ T \to 2\times10^{6},\ Rs \to 6.957\times10^{8},\ Mp \to 1.672\times10^{-27}\};$$

Put equation in terms of frequency

In[305]:=
```
R2 = Part[Solve[νp[N2] == ν, R] // Quiet, 1, 1, 2] - 1 /. N2v
```

Out[305]=
$$-1 + \frac{1.33104\times10^{-7}}{1.33104\times10^{-7} + 1.92013\times10^{-8}\, Log\left[1.49587\times10^{-17}\, \nu^2\right]}$$

Find min and max frequency

In[306]:=
```
Rmn = Max[h0] + 1;
Rmx = 2.3;
νmx = νp[N2] /. N2v /. R → Rmn
νmn = νp[N2] /. N2v /. R → Rmx
```

Out[308]= 2.55252×10^{8}

Out[309]= 3.64545×10^{7}

Construct plot of the region

In[310]:=
```
p2 = LogLogPlot[R2 /. N2v, {ν, νmn, νmx}, PlotRange → pltrng, ImageSize → Large];
```

From 1.2-215 solar radii we use the model given by Leblanc et al (1998)

The functional form is given as

In[311]:=
$$N4 = \frac{\alpha}{R^2} + \frac{\beta}{R^4} + \frac{\gamma}{R^6};$$

The values given in the paper are

In[312]:=
$$N4v = \{\alpha \to 3.3\times10^{5},\ \beta \to 4.1\times10^{6},\ \gamma \to 8\times10^{7}\};$$

Put equation in terms of frequency

In[313]:=
```
R4 = Part[Solve[νp[N4] == ν, R] // Quiet, 2, 1, 2];
```

Find min and max frequency

In[314]:=
```
Rmn = 1.33;
Rmx = 300;
νmx = νp[N4] /. N4v /. R → Rmn
νmn = νp[N4] /. N4v /. R → Rmx
```

Out[316]= 3.58645×10^7

Out[317]= 17 196.6

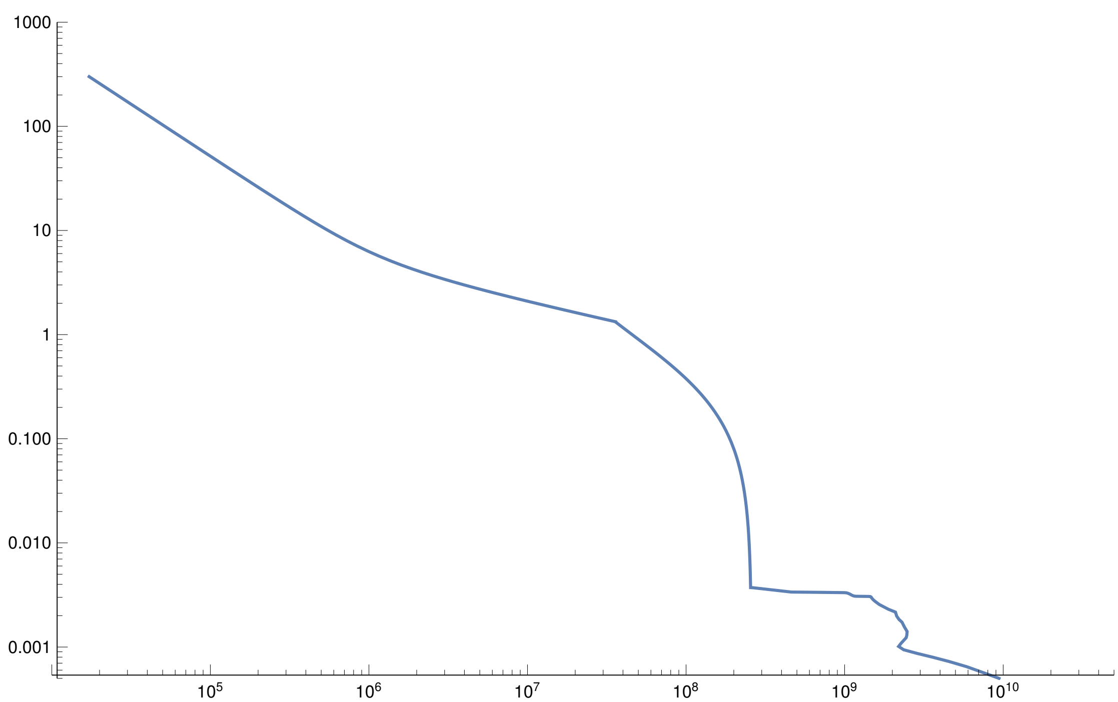
Construct plot of the region

In[318]:=
```
p4 = LogLogPlot[R4 /. N4v, {ν, νmn, νmx}, PlotRange → pltrng, ImageSize → Large];
```

Construct final plot

In[319]:=
```
Show[p0, p2, p4]
```

Out[319]=



In[320]:=
```
Clear["Global`*"]
```

Problem 3

Reproduce frequency drift feature

The position of the electron beam is given by the classical velocity, where we take the velocity to be 0.3c

In[321]:= $r = \frac{Rs + 0.3\, c\, tE}{Rs};$

The frequency of the emitted radiation as a function of radius is (in MHz)

In[322]:= $fE = 8980 \sqrt{ne} \Big/ 10^6;$

In the last problem, we found the number density as

In[323]:= $ne = \frac{\alpha}{r^2} + \frac{\beta}{r^4} + \frac{\gamma}{r^6} \;/.\; \{\alpha \to 3.3 \times 10^5, \beta \to 4.1 \times 10^6, \gamma \to 8 \times 10^7\};$

The time the signal arrives at earth is given by

In[324]:= $t == tE + \frac{(RE - Rs\, r)}{c};$

Solve the equation for tE

In[325]:= $tE = \mathrm{Part}\left[\mathrm{Solve}\left[t == tE + \frac{(RE - Rs\, r)}{c}, tE\right], 1, 1, 2\right]$ // FullSimplify

Out[325]= $\frac{-1.42857\, RE + 1.42857\, Rs + 1.42857\, c\, t}{c}$

The pertinent values for this equation are

In[326]:= $fEv = \{c \to 3 \times 10^{10}, RE \to 1.5 \times 10^{13}, Rs \to 6.96 \times 10^{10}\};$

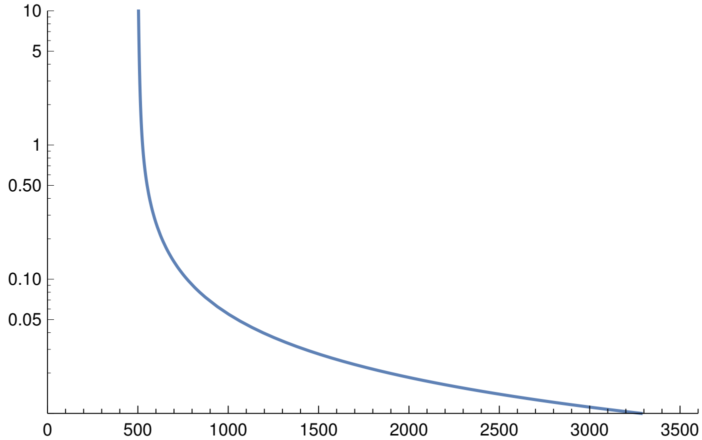
The start time is offset by the light travel time between the earth and the sn

In[327]:= $tStart = \frac{RE - Rs}{c} \;/.\; fEv$

Out[327]= 497.68

In[328]:=
```
p0 = LogPlot[{fE /. {c → 3 × 10^10, RE → 1.5 × 10^13, Rs → 6.96 × 10^10}},
  {t, tStart, 3600}, PlotRange → {{0, 3600}, {0.01, 10}}]
```

Out[328]=



Reproduce type-III radio burst dynamic spectrum

The time-evolution of the peak frequency of type-III radio bursts follows the empirical power law

In[329]:=
```
f = a t^b ;
fv = {a → 150, b → -0.6 };
```

The FWHM of these bursts is given by the empirical law

In[331]:=
```
Δf = C1 ν;
Δfv = {C1 → 0.57};
```

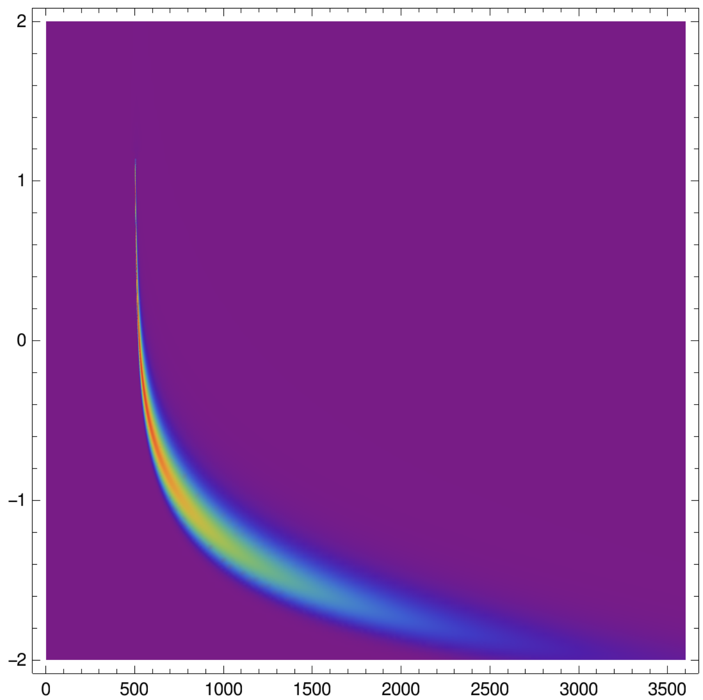
and the empirical frequency-dependent decay is

In[333]:=
```
τD = 10^γ (10^6 ν)^-δ /. ν → 0.1;
τDv = {γ → 7.71, δ → 0.95};
```

Plot the dynamic frequency using the curve developed in the first part of the question.

In[335]:=
```
S1 = Piecewise[{{Exp[- (4 Log[2] (ν - fE)^2)/Δf^2] Exp[-(t - tStart) / τD] /. ν → 10^ν, t > tStart},
    {0, t < tStart}}];
p1 = DensityPlot[S1 /. fv /. Δfv /. τDv /. fEv , {t, 0, 3600}, {ν, -2, 2},
  ColorFunction → "Rainbow", PlotRange → Full, PlotPoints → 100, MaxRecursion → 5]
```

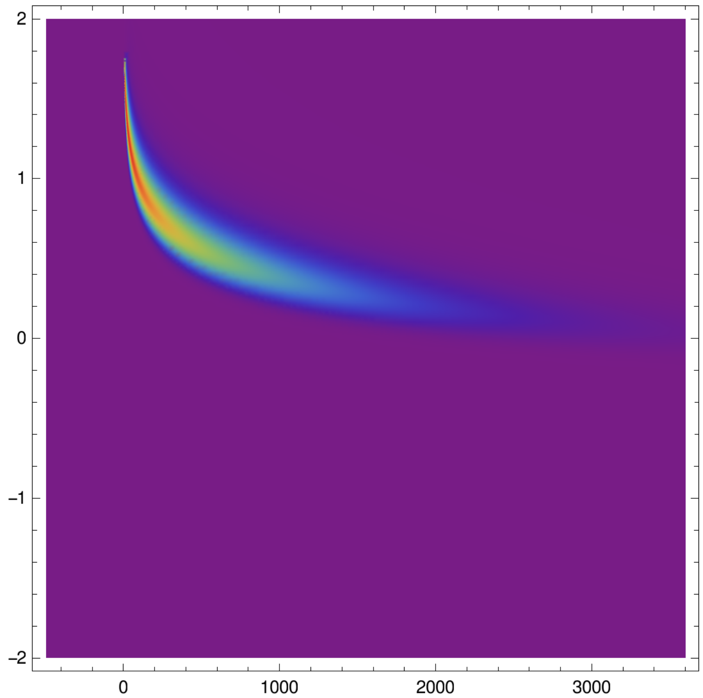
Out[336]=



Plot the dynamic spectrum using the empirical power law

In[337]:=
```
S2 = Piecewise[{{Exp[-(4 Log[2] (ν - f)^2)/Δf^2] Exp[-(t) / τD] /. ν → 10^ν, t > 0}, {0, t < 0}}];
p1 = DensityPlot[S2 /. fv /. Δfv /. τDv /. fEv , {t, -tStart, 3600}, {ν, -2, 2},
  ColorFunction → "Rainbow", PlotRange → Full, PlotPoints → 100, MaxRecursion → 5]
```

Out[338]=



Comparing to Figure 1 in the Problem Statement, the empirical law and the law derived from the plasma frequency in the heliosphere did not quite match. General shape seems to be correct.

In[339]:=
```
Clear["Global`*"]
```

Problem 4

Thermal Bremsstrahlung

Gary and Hurford 2004 gives the frequency at which thermal bremsstrahlung reaches optical depth unity as approximately

In[340]:=
```
νtb = 0.5 ne T^(-3/4) L^(1/2);
```

Use the scale height also provided in Gary and Hurford (2004)

In[341]:=
$$L = H0 \left(\frac{T}{T0}\right) \left(\frac{R}{Rs}\right)^2;$$
$$H0 = 0.1\,Rs;$$
$$T0 = 2\times 10^6;$$

Use the hydrostatic equilibrium model for number density given by Mann et al (1997)

In[344]:=
$$N2 = Ns\,Exp\left[\frac{A}{Rs}\left(\frac{Rs}{R} - 1\right)\right];$$
$$A = \frac{\mu\,G\,Ms\,Mp}{k\,T};$$
$$N2v = \{Ns \to 8.29\times 10^8,\ \mu \to 0.6,\ G \to 6.674\times 10^{-8},\ Ms \to 1.988\times 10^{33},\ k \to 1.38\times 10^{-16},\ T \to 2\times 10^6,\ Rs \to 6.957\times 10^{10},\ Mp \to 1.672\times 10^{-24}\};$$

Now we are free to calculate the frequency (in GHz)

In[347]:=
```
(vtb / 10^9 /. ne → N2 /. R → 1.005 Rs /. T → T0 /. N2v) // Framed
```

Out[347]= $\boxed{0.631177}$

Which is approximately where the brightness temperature starts to roll over in Figure 2 in the problem statement.

Thermal Gyrosychrotron

Dulk (1985) gives the optical unity frequency as

In[348]:=
$$N2v = \{Ns \to 1.21\times 10^{15},\ \mu \to 0.6,\ G \to 6.674\times 10^{-8},\ Ms \to 1.988\times 10^{33},\ k \to 1.38\times 10^{-16},\ T \to 1\times 10^8,\ Rs \to 6.957\times 10^{10},\ Mp \to 1.672\times 10^{-24}\};$$

In[349]:=
$$vtg = 475\left(\frac{N\,L}{B}\right)^{0.05} Sin[\theta]^{0.6}\,T^{0.5}\,B\ /.\ \theta \to 10\,\pi/180\ /.\ N \to N2$$

Out[349]=
$$71.6876\,B\,T^{0.5}\left(\frac{e^{\frac{G\,Mp\,Ms\left(-1+\frac{Rs}{R}\right)\mu}{k\,Rs\,T}}\,Ns\,R^2\,T}{B\,Rs}\right)^{0.05}$$

Hurford and Gary (2004) use the magnetic field

In[350]:=
$$B = 0.5\left(\frac{R}{Rs} - 1\right)^{1.5};$$

Evaluate frequency

In[351]:=
```
(vtg /. R → 1.003 Rs /. N2v) // Framed
```

Out[351]=
```
4685.38
```

This does not look right by many orders of magnitude.