Optimization of User Equipment Random Access Delay in LEO Satellite Communication Systems via Synchronization Signal Block Periodicity

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ABSTRACT

Low Earth Orbit (LEO) satellite network has been a promising technology due to the wide spread coverage area and high data throughput. However, with the high speed of satellites, frequent handover is unavoidable for user equipments (UEs) on the ground, causing significant interruption time and signaling overhead. Thus, we introduce a quasi-earth-fixed satellite beam scheme to solve the continually handover from the UEs at the cell edge. In this scheme, we allocate the satellite beams to the ground cells in order to maximize overall throughput. After that, we also propose a UE cell selection algorithm, which base on both position information and UE measurement.

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CHAPTER 1

INTRODUCTION

Non terrestrial network (NTN) has become a promising technique in next generation network. It provides network connectivity to area that traditional network platform cannot reach. For instance, forests, oceans, and deserts. Various platforms are used to provide network services in NTN, such as GEO, MEO, LEO satellites, UAVs, and drones. Among these platforms, LEO satellites are the most actively discussed. These satellites are located at altitude 500 to 2000 kilometers. The strongest advantage is that they provide global coverage, low latency, and high throughput compared to MEO or GEO satellites.

However, to acheive LEO satellite network service, there are some key challanges that need to be resolved. One of the challanges is the unavoidable frequent handover [1]. The high speed of LEO satellite forces user equipments (UEs) on the ground to switch the serving satellites frequently, as shown in Figure 1. In such scenario, the signalling overhead led by handover signals and the handover interruption time are big issues. 3GPP has discussed some solutions to deal with these issues. By using quasi-earth-fixed cell and satellite switch with resynchronization [2], the frequent handovers are avoided and the signalling overhead is reduced. For UEs that have already accessed to the network, they can receive the upcoming serving satellite information from the previous one. Also, with the help of the ephemeris data of satellies and the position information of UEs, the matching between satellites and UEs has been largely improved [3]. Nontheless, for those UEs who have not access to the network, the random access procedure is the only way for them to get into the network. To establish the connection between satellites and UEs, satellites need to transmit synchronization signal block (SSB) to ground for UEs to capture. Once UEs have successfully receive SSBs, the random access procedure starts and the UEs are able to access to the satellite network. Typically in terrestrial network, ground stations send SSB to the serving area every 20 milliseconds. However, the traditional SSB specifications in LEO satellite communication system do not work because the power budget in LEO satellite is so tight that the power is not enough to send all the serving cells in such a short periodicity. Thus, in this thesis we will find out how to deal with this issue by adjust the SSB periodicity and transmitted power.

Cell Diameter Size (km)	UE Speed (km/hr)	Satellite Speed (km/s)	Time to HO (s)
50 (lower bound)	+500	7.56 (NOTE 1)	6.49
	-500		6.74
	+1200		6.33
	- 1200		6.92
	Neglected		6.61
1000 (upper bound)	+500		129.89
	-500		134.75
	+1200		126.69
	- 1200		138.38
	Neglected		132.28

Figure 1: Time to handover for min/max cell diameter and varying UE speed

CHAPTER 2

BACKGROUND AND RELATED WORK

2.1 Random Access Procedure

The random access procedure in 5G is a critical mechanism that allows a User Equipment (UE) to establish initial uplink synchronization and connect to the network (gNB). The procedure is essential for first-time access, handovers, and re-establishing connections after inactivity or loss of synchronization. Here are some typical steps in contention-based random access:

- Preamble Transmission (Msg1): The UE randomly selects a PRACH (Physical Random Access Channel) preamble and transmits it to the gNB. This "signature" indicates the UE's request for network access.

- Random Access Response (Msg2): The gNB detects the preamble and responds with a Random Access Response (RAR), which includes timing adjustment, a temporary identity, and an uplink resource grant. This enables the UE to align its timing with the network and prepare for further communication.

- RRC Connection Request (Msg3): Using the granted resources, the UE sends a connection request, which includes its identity and the reason for establishing the connection, such as initial access or handover.

- Contention Resolution (Msg4): The gNB sends a contention resolution message confirming which UE has successfully completed the random access. If multiple UEs used the same preamble, only the correct one will be acknowledged, resolving the contention.

2.2 Synchronization Signal Block

2.2.1 Components of SSB

The Synchronization Signal Block (SSB) in 5G New Radio (NR) is a critical structure for initial access between the user equipment (UE) and the base station (gNB). Each SSB is composed of three main elements:

- Primary Synchronization Signal (PSS): The PSS enables the UE to obtain symbol timing and perform coarse frequency synchronization. It allows the

UE to find the starting point of a radio frame and resolves the physical layer cell identity group.

- Secondary Synchronization Signal (SSS): The SSS complements PSS by providing additional information to finalize the cell identification and determines the frame timing, which refines synchronization accuracy for the UE.
- Physical Broadcast Channel (PBCH): The PBCH conveys essential cell-specific information, including system configuration parameters (such as the System Frame Number), which the UE needs for further connection setup after synchronization.

These components jointly allow the UE to perform downlink synchronization, cell identification, and to decode key system information for network access.

2.2.2 SSB Configuration

A series of SSBs called *SSB burst* are sent in a half frame (5ms), the number of SSBs in a SSB burst is determined depends on the carrier frequency and the subcarrier spacing of the transmitted signals. Each cell has a SSB periodicity, defined as the time interval the SSB burst be transmitted to the cell, as shown in Figure 2.

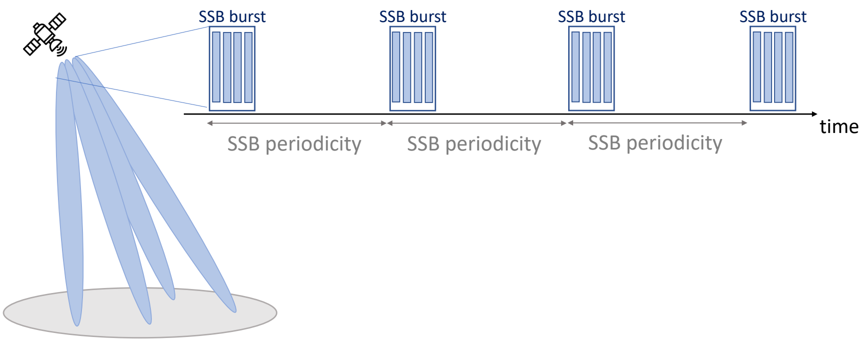

Figure 2: Illustration of SSB configuration.

2.2.3 SSB in LEO Satellite Network

In terrestiral network (TN), the base station (gNB) transmits SSBs periodically in time and across different spatial directions through beam sweeping, enabling the UE to detect the best SSB and select the optimal beam for communication. In NTN, the SSBs have the same function but the coverage area of each satellite is much bigger than TN, which means each satellite has to provide service to more cells. Moreover, the long distance from satellite to ground and the power budget of each satellite forces us to properly allocate the power of the SSB. Thus, it is essential for us to manage the SSB transmitted power and the periodicity of each cell.

2.3 Related Work

CHAPTER 3

SYSTEM MODEL

3.1 System Overview

This chapter introduces the LEO satellite communication system, as illustrated in Figure 3. We consider one of the satellites in the LEO satellite communication system. It consists of M beams, represented by $\mathcal{M} = \{m \mid m = 1, 2, \ldots, M\}$. In the coerage area of the satellite, it contains K cells, indexed by $\mathcal{K} = \{k \mid k = 1, 2, \ldots, K\}$. To achieve optimal coverage and minimize overlap, all ground cells are arranged in a regular hexagonal grid, ensuring uniform cell size. There are U user equipments (UEs) in the coverage area, indexed by $\mathcal{U} = \{u \mid u = 1, 2, \ldots, U\}$.

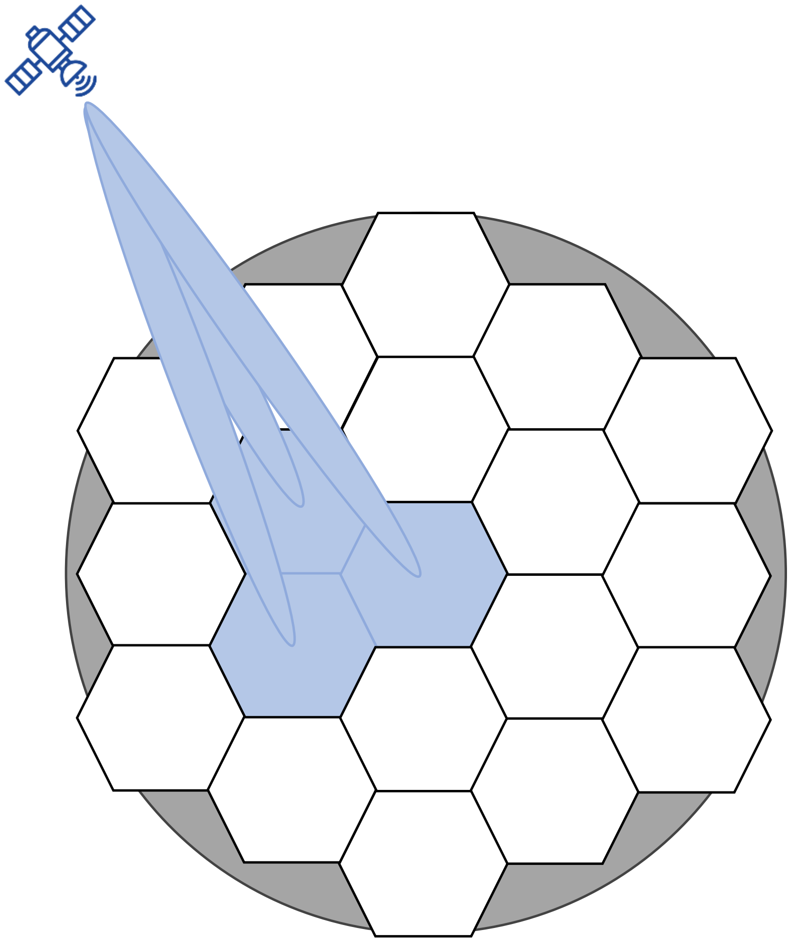

Figure 3: Illustration of satellite beams and cells.

We define a short time T^{slot} as the time duration of a time slot, and T^{total} as the total time that the satellite serves the area. Thus, there are T^{total}/T^{slot} time slots in total service time, denoted as $\mathcal{T} = \{t \mid t = 1, 2, \ldots, T^{total}/T^{slot}\}$.

In this thesis, we adopt the quasi-earth-fixed scheme. Unlike the earth-moving cell scheme—where the coverage areas of satellite beams move as the LEO satellites orbit—this approach directs satellite beams so that each beam consistently covers the same geographical cell for a given period. Thus, the coverage area of each satellite beam remains fixed relative to the ground during that interval. Throughout, we assume each satellite beam is oriented toward the center of its designated cell.

The locations of UEs wishing to access the network are assigned randomly within their corresponding cells, and the population of each cell are generated according to area population density statistics.

3.2 Channel Model

3.2.1 Free Space Path Loss

In the LEO satellite system, the free space path loss from the satellite to cell k is expressed as follows [4]:

$$L_k = \left(\frac{\lambda}{4\pi d_k}\right)^2 \tag{3.1}$$

where λ is the wavelength, and d_k is the distance between the satellite and the center of the k-th cell.

3.2.2 Shadowed-Rician Fading Channel

The shadowed-Rician fading model is suitable for satellite communication systems because it accurately reflects the physical propagation environment, capturing both the presence of a strong line-of-sight (LoS) signal and the effects of shadowing from obstacles [5]. Let h_k denote the channel gain between the satellite and the k-th cell. The cumulative distribution function (CDF) of the channel gain is:

$$F_{h_k}(x) = K \sum_{n=0}^{\infty} \frac{(m)_n \, \delta^n \, (2b)^{1+n}}{(n!)^2} \, \zeta\left(1+n, \frac{x}{2b}\right) \tag{3.2}$$

where $K = \left(\frac{2bm}{2bm+\Omega}\right)^m / 2b$, $\delta = \Omega/(2bm+\Omega)/2b$, Ω is the average power of the LoS component, $2b$ is the average power of the multipath component except the LoS component, and m is the Nakagami parameter. $\Gamma(\cdot)$ is the Gamma function, and the Pochhammer symbol is defined as $(x)_n = \Gamma(x+n)/\Gamma(x)$. The lower incomplete Gamma function is defined as $\zeta(a, x) = \int_0^x t^{a-1} \exp(-t) dt$.

3.2.3 Antenna Radiation Pattern

We introduce the antenna radiation pattern in [6]:

$$G(\phi_{k,u}) = G_{max}\left[\frac{J_1\left(\mu(\phi_{k,u})\right)}{2\mu(\phi_{k,u})} + 36\frac{J_3\left(\mu(\phi_{k,u})\right)}{\mu(\phi_{k,u})^3}\right]^2 \tag{3.3}$$

where $\phi_{k,u}$ is the boresight angle between the user position and the cell center with respect to the satellite, G_{max} is the maximum antenna gain, $\mu(\phi)$ is defined as $2.07123 \cdot \sin(\phi)/\sin(\phi_{3dB})$, ϕ_{3dB} is the 3 dB half-power beamwidth angle of the antenna, and $J_1(\cdot)$, $J_3(\cdot)$ are the Bessel functions of the first kind of orders 1 and 3, respectively.

3.2.4 UE Received Power

In this thesis, the transmitted power of each beam is fixed during one epoch. The transmitted power to the k-th cell at the $\mathcal{S}_i$-th epoch is denoted as $P_k[\mathcal{S}_i]$. In the $\mathcal{S}_i$-th epoch, the received power of the u-th user in the k-th cell at the t-th time slot $\hat{P}_{k,u}[t]$ can be calculated as follows:

$$\hat{P}_{k,u}[t] = P_k[\mathcal{S}_i] \cdot L_k \cdot h_k \cdot G(\phi_{k,u}) \tag{3.4}$$

3.3 Joint Power-Periodicity Optimization on SSB

In this section, we will discuss how the satellite beam transmitted power and the SSB periodicity affect UE cell search delay and formulate an optimization problem. During the total service time, it is essential to adjust the satellite beam transmitted power and the SSB periodicity because the system environment changes in real-time. The most significant change is the elevation angle of the satellite. It affects not only the transmission distance, but also the channel fading effect. However, if we adjust the power and periodicity too frequently, the computation complexity rises and the signaling overhead increases. Thus, we adjust the satellite beam transmitted power and the SSB periodicity in a certain number of time slots, defined as an epoch. We define one epoch equals to N time slots, so there are $T^{total}/T^{slot}/N$ epochs in total. The set of all epochs within the service duration is represented as $\widetilde{\mathcal{S}} = \{\mathcal{S}_i \mid i = 0, 1, 2, \ldots, (T^{total}/T^{slot}/N - 1)\}$, where each epoch S_i encompasses the time slots from iN to $(i+1)N-1$. By adjusting system parameters only at epoch boundaries, this approach balances real-time responsiveness against the complexity and signaling overhead associated with too frequent updates, ensuring system stability and efficient resource allocation throughout the mission duration.

3.3.1 Synchronization Signal Block Periodicity

During the total service time, the satellite transmits SSBs to the serving cells at specific time intervals, depending on the SSB periodicity configured for each cell. With longer SSB periodicity, the satellite beam has more time slots to serve other cells, and can allocate more power to each SSB signal. Thus, we adjust the SSB periodicity every epoch. During each epoch, the SSB periodicity of each cell is fixed. We then define the SSB periodicity of the cell k at the $\mathcal{S}_i$-th epoch as the number of slots between two consecutive SSBs that transmit to the k-th cell, denoted as $\theta_k[\mathcal{S}_i]$. Since there are at most M beams serving the cells in one satellite and one beam can only transmit one SSB to a cell in a time slot, we must insure that there are enough time slots to transmit all SSBs in one cycle. That is,

$$\sum_k \frac{1}{\theta_k[\mathcal{S}_i]} \leq M, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.5}$$

3.3.2 Satellite Power Budget

In LEO satellite communication systems, satellites primarily rely on solar power for data transmission. Each satellite has a specific power budget, determined by the amount of solar energy it can harvest. Since the distance between space and ground is typically several hundred kilometers, transmitted signals must be strong enough to achieve the minimum signal-to-noise ratio (SNR) required on the user equipment (UE) side. Therefore, the efficient allocation of power resources to each beam under a limited power budget is a critical topic in LEO satellite communication system design. We define a power budget for the satellite. Let $P_k[\mathcal{S}_i]$ denote the transmitted power to the k-th cell at epoch $\mathcal{S}_i$. The aggregate transmit power of all beams on the satellite must satisfy:

$$\sum_m \frac{P_k[\mathcal{S}_i]}{\theta_k[\mathcal{S}_i]} \leq P^{total}, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.6}$$

where P^{total} is the maximum transmit power for a satellite in one time slot.

3.4 UE Cell Searching Delay

The cell searching delay $\alpha_u[\mathcal{S}_i]$ for the u-th UE is a random variable, defined as the time duration between the start of SSB measurement and the successful reception of SSB at the $\mathcal{S}_i$-th epoch, as shown in Figure 4. $\alpha_u[\mathcal{S}_i]$ can be decomposed into two parts: $\beta_u[\mathcal{S}_i]$ (initial waiting time at the $\mathcal{S}_i$-th epoch) and $\gamma_u[\mathcal{S}_i]$ (additional delay due to failed attempts at the $\mathcal{S}_i$-th epoch). $\beta_u[\mathcal{S}_i]$ is a random variable that represents the time from the start of SSB measurement to the arrival of the first SSB, and $\gamma_u[\mathcal{S}_i]$ is another random variable that represents the

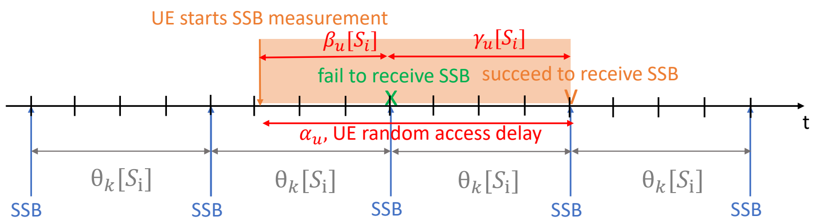

Figure 4: Illustration of random access delay at $\mathcal{S}_i$-th epoch

time from the first SSB arrival to the successful SSB reception. Since the UE can start SSB measurement at any time, $\beta_u[\mathcal{S}_i]$ follows uniform distribution from 0 to $\theta_k[\mathcal{S}_i]$, where k represents the serving cell of u. $\gamma_u[\mathcal{S}_i]$ is a multiple of $\theta_k[\mathcal{S}_i]$, depending on the number of SSB reception failures $Q_u[\mathcal{S}_i]$, which is a discrete random variable. If the received SSB power at the t-th time slot $\hat{P}_{m,u}[t]$ is less than the threshold P^{th}, the UE fails to measure SSB. The probability that the received SSB power is less than P^{th} at the t-th time slot is denoted as $R_u[t]$. The mathematical formulation is as follows:

$$\alpha_u[\mathcal{S}_i] = \beta_u[\mathcal{S}_i] + \gamma_u[\mathcal{S}_i] \tag{3.7}$$

$$F_{\beta_u[\mathcal{S}_i]}(x) = \begin{cases} \frac{x}{\theta_k[\mathcal{S}_i]}, & 0 \leq x < \theta_k[\mathcal{S}_i] \\ 1, & x \geq \theta_k[\mathcal{S}_i] \\ 0, & \text{otherwise} \end{cases} \tag{3.8}$$

$$\gamma_u[\mathcal{S}_i] = Q_u[\mathcal{S}_i] \cdot \theta_k[\mathcal{S}_i] \tag{3.9}$$

where $F_{\beta_u[\mathcal{S}_i]}(x)$ is the CDF of $\beta_u[\mathcal{S}_i]$. To calculate the Probability Mass Function (PMF) of $Q_u[\mathcal{S}_i]$, we denote the u-th UE fails to measure n SSBs at $t_1, t_2, \ldots, t_n$ time slots, and successfully receives SSB at time slot t_{n+1}. The PMF of $Q_u[\mathcal{S}_i]$ can be express as follows:

$$\Pr\{Q_u[\mathcal{S}_i] = n\} = (1 - R_u[t_{n+1}]) \prod_{i=1}^{n} R_u[t_i] \tag{3.10}$$

$R_u[t]$ can be expressed by F_{h_k} as follows:

$$\begin{aligned} R_u[t] &= \Pr\{\hat{P}_{k,u}[t] < P^{th}\} \\ &= \Pr\{h_k < \frac{P^{th}}{P_k[\mathcal{S}_i] \cdot L_k \cdot G(\phi_{m,u})}\} \\ &= F_{h_k}(\frac{P^{th}}{P_k[\mathcal{S}_i] \cdot L_k \cdot G(\phi_{m,u})}) \end{aligned} \tag{3.11}$$

3.5 Problem Formulation

This section formulates the optimization problem based on recent 3GPP standardization discussions. The main challenge is to provide SSBs to as many cells as possible with limited satellite power. Extending the SSB periodicity for some cells increases UE random access delay, but it saves more power so that there would be other cells to be served. On the other hand, shortening the SSB periodicity reduces the UE cell searching delay, but the serving cells might be less because of the power budget. The transmitted SSB power also affects the success probability of the cell searching delay. The trade-off among power allocation, SSB periodicity, and UE cell searching delay is modeled as follows:

$$\min_{P,\theta} \sum_{u} \sum_{\mathcal{S}_i \in \widetilde{\mathcal{S}}} \mathbb{E}[\alpha_u[\mathcal{S}_i]] \tag{3.12a}$$

subject to

$$\sum_{k} \frac{P_k[\mathcal{S}_i]}{\theta_k[\mathcal{S}_i]} \leq P^{total}, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.12b}$$

$$P_k[\mathcal{S}_i] \geq 0, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.12c}$$

$$\theta_k[\mathcal{S}_i] \leq N, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.12d}$$

$$\theta_k[\mathcal{S}_i] \in \mathbb{N}^+, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.12e}$$

$$\sum_{k} \frac{1}{\theta_k[\mathcal{S}_i]} \leq M, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{3.12f}$$

where $\mathbb{N}^+$ is the set with all positive integers. The mathematical fomula between transmitted power and received power is described in equation (3.4). And the mathematical derivation between the received power $\hat{P}_{m,u}[t]$, $\theta_k[s]$ and $\alpha_u[s]$ is described from equation (3.7) to equation (3.11).

CHAPTER 4

PROPOSED ALGORITHM

4.1 Recap: Alternating Optimization with Genetic Algorithm and Simulated Annealing (GASA)

Alternating optimization is a widely used strategy for non-convex problems that involve multiple variables or mixed types of parameters. The core idea is to decompose the original challenging joint optimization into two or more easier subproblems, which are solved iteratively. In our problem, we first fix the power allocation P and optimize the SSB periodicity θ. Then, we fix the SSB periodicity θ and optimize the power allocation P. This process repeats until convergence. Thus, the problem (3.12) can be devided to two subproblems.

4.1.1 Fix Power Optimize SSB Periodicity

$$\min_{\theta} \sum_{u} \sum_{\mathcal{S}_i \in \widetilde{\mathcal{S}}} \mathbb{E}[\alpha_u[\mathcal{S}_i]] \tag{4.1a}$$

subject to

$$\theta_k[\mathcal{S}_i] \leq N, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{4.1b}$$

$$\theta_k[\mathcal{S}_i] \in \mathbb{N}^+, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{4.1c}$$

$$\sum_{k} \frac{1}{\theta_k[\mathcal{S}_i]} \leq M, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{4.1d}$$

We use Genetic Algorithm to solve this problem:

Algorithm 1 Fix Power, Optimize SSB Periodicity

Require: Given the transmission power $P_{k,\mathcal{S}_i}$ for each cell k at each epoch $\mathcal{S}_i$, as well as the user distribution.

Ensure: Find and assign the optimal SSB periodicity $\theta_{k,\mathcal{S}_i}$ for each cell, such that the user access delay is minimized while the periodicity configuration for all cells meets the available resource constraints.

```
Generate initial population of candidate solutions with size POP_SIZE.
Each individual is a vector of integers θ_k[S_i] within constraints.
for generation = 1 to max_generations do
    for each individual in population do
        Compute E[α_u[S_i]] by (3.7)-(3.11)
        Assign fitness = 1/E[α_u[S_i]]
    end for
    Select parents based on fitness
    Apply crossover to produce offspring
    Apply mutation to offspring within constraints
    Calculate fitness for offspring
    Select next generation population from parents and offspring
end for
return individual with best fitness as optimal periodicity vector
```

4.1.2 Fix SSB Periodicity Optimize Power

$$\min_{P} \sum_{u} \sum_{\mathcal{S}_i \in \widetilde{\mathcal{S}}} \mathbb{E}[\alpha_u[\mathcal{S}_i]] \tag{4.2a}$$

$$\sum_{m} \frac{P_k[\mathcal{S}_i]}{\theta_k[\mathcal{S}_i]} \leq P^{total}, \quad \forall \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{4.2b}$$

$$P_k[\mathcal{S}_i] \geq 0, \quad \forall k \in \mathcal{K}, \mathcal{S}_i \in \widetilde{\mathcal{S}} \tag{4.2c}$$

We use Simulated Annealing to solve this problem:

Algorithm 2 Fix SSB Periodicity, Optimize Power

Require: Given the transmission power $\theta_{k,\mathcal{S}_i}$ for each cell k at each epoch $\mathcal{S}_i$, as well as the user distribution.

Ensure: Find and assign the optimal SSB periodicity $P_{k,\mathcal{S}_i}$ for each cell, such that the user access delay is minimized while the periodicity configuration for all cells meets the available resource constraints.

```
Initialize current solution P_current with feasible power allocation
Calculate current_delay E[α_u[S_i]] for P_current
Initialize temperature T ← T_max, cooling rate 0 < α < 1
while T > T_min do
    Generate new solution P_new by perturbing P_current within constraints
    Calculate new_delay for P_new
    if new_delay < current_delay then
        Accept P_new: P_current ← P_new, current_delay ← new_delay
    else
        Accept P_new with probability exp(−(new_delay − current_delay)/T)
    end if
    Update temperature T ← α × T
end while
return P_current as optimized power allocation
```

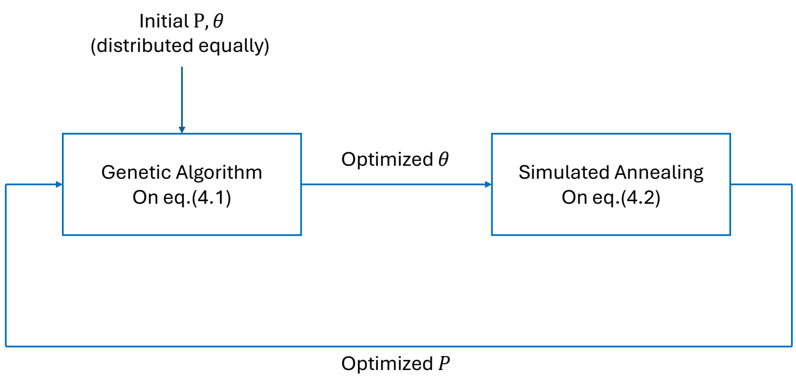

Figure 5: GASA diagram flow

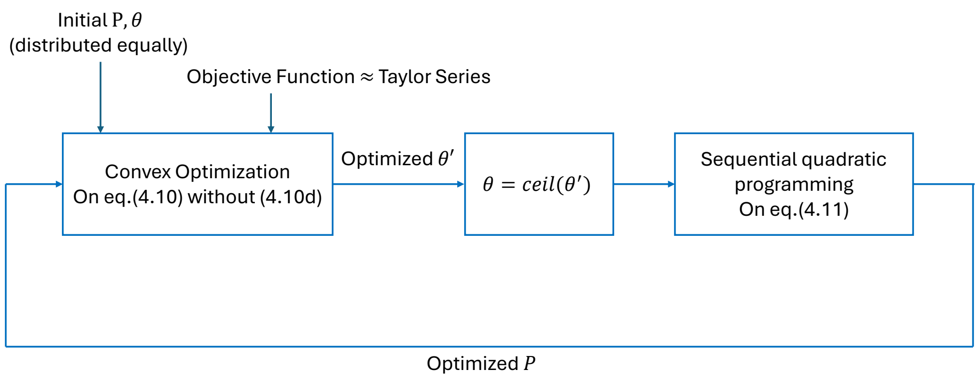

Figure 6: Taylor series expansion diagram flow

4.2 Taylor Expansion on Objective Function

In this method, we use Taylor expansion on F_{h_k} to approximate the non-linear term into linear term. Below are the calculations:

$$F(x) = K\sum_{n=0}^{\infty}\frac{(m)_n d^n (2b)^{tn}}{(n!)^2}\zeta(1+n, \frac{x}{2b}) \tag{4.3}$$

$$F'(x) = K\sum_{n=0}^{\infty}\frac{(m)_n d^n (2b)^{tn}}{(n!)^2}\left(\frac{x}{2b}\right)^n e^{\frac{x}{2b}}\cdot\frac{1}{2b} \tag{4.4}$$

Given a real number a, the first order Taylor series of a real function $f(x)$ can be expressed as:

$$f(x) \approx f(a) + f'(a)\cdot(x-a) \tag{4.5}$$

Then the expectation of the number of SSB reception failures $Q_u[S_i]$ can be expressed as follows:

$$\begin{aligned}\mathbb{E}[Q_u[S_i]] &= \frac{F\left(\frac{P_k^{\text{th}}}{P_k\cdot L_k}\right)}{1-F\left(\frac{P_k^{\text{th}}}{P_k\cdot L_k}\right)}\\ &\approx \frac{F(a)+F'(a)\cdot\left(\frac{P_k^{\text{th}}}{P_k\cdot L_k}-a\right)}{1-\left[F(a)+F'(a)\cdot\left(\frac{P_k^{\text{th}}}{P_k\cdot L_k}-a\right)\right]}\end{aligned} \tag{4.6}$$

where k is the cell that the u-th user connected to. The expectation of user cell searching delay can be expressed as follows:

$$E[\alpha_u] = E[\beta_u] + E[\gamma_u] = \left(\frac{1}{2}+\frac{P_kC+E_k}{P_k(1-C)-E_k}\right)\theta_k \tag{4.7}$$

where $C = F(a) - F'(a) \cdot a$, $E_k = \frac{F'(a)P^{th}}{L_k}$. We introduce the population density of the k-th cell as D_k. Then the cell searching delay for all UE can expressed as follows:

$$\sum_u E[\alpha_u] = \sum_k D_k \left(\frac{1}{2} + \frac{P_k C + E_k}{P_k(1-C) - E_k}\right)\theta_k \tag{4.8}$$

We can rewrite the problem from 3.12 as follows:

$$\min_{P,\theta} \frac{1}{\sum_k D_k} \sum_k D_k \left(\frac{1}{2} + \frac{P_k C + E_k}{P_k(1-C) - E_k}\right)\theta_k \tag{4.9a}$$

$$\text{subject to} \tag{4.9b}$$

$$P_k > 0, \quad \forall k \in \mathcal{K} \tag{4.9c}$$

$$\theta_k \le N, \quad \forall k \in \mathcal{K} \tag{4.9d}$$

$$\theta_k \in \mathbb{N}^+, \quad \forall k \in \mathcal{K} \tag{4.9e}$$

$$\sum_k P_k \le P_{\text{total}} \tag{4.9f}$$

$$\sum_k \frac{1}{\theta_k} \le M \tag{4.9g}$$

We use alternating optimization to seperate equation (4.9) into two subproblems. With fixed power, we optimize SSB periodicity:

$$\min_{\theta} \frac{1}{\sum_k D_k} \sum_k D_k \left(\frac{1}{2} + \frac{P_k C + E_k}{P_k(1-C) - E_k}\right)\theta_k \tag{4.10a}$$

$$\text{subject to} \tag{4.10b}$$

$$\theta_k \le N, \quad \forall k \in \mathcal{K} \tag{4.10c}$$

$$\theta_k \in \mathbb{N}^+, \quad \forall k \in \mathcal{K} \tag{4.10d}$$

$$\sum_k \frac{P_k}{\theta_k} \le P_{\text{total}} \tag{4.10e}$$

$$\sum_k \frac{1}{\theta_k} \le M \tag{4.10f}$$

And with fixed SSB periodicity, we optimize power:

$$\min_{P} \frac{1}{\sum_k D_k} \sum_k D_k \left(\frac{1}{2} + \frac{P_k C + E_k}{P_k(1-C) - E_k}\right)\theta_k \tag{4.11a}$$

$$\text{subject to} \tag{4.11b}$$

$$P_k > 0, \quad \forall k \in \mathcal{K} \tag{4.11c}$$

$$\sum_k \frac{P_k}{\theta_k} \le P_{\text{total}} \tag{4.11d}$$

For problem (4.10), it satisfies convex optimization problem if we ignore constraint (4.10d). Thus, we compute the problem (4.10) without constraint (4.10d) by cvx in matlab, and then ceil the θ to make sure all θ satisfy constraint (4.10d).

For problem (4.11), it is not a convex optimization problem. Thus I use sequential quadratic programming (SQP) to calculate the result.

4.3 Result

The result is shown in Table 1. We distribute power and SSB periodicity for all cells as our baseline. That is,

$$P_k = P_{total}/M, \quad \forall k \in \mathcal{K} \tag{4.12}$$

$$\theta_k = K/M, \quad \forall k \in \mathcal{K} \tag{4.13}$$

The result of GASA is not quite well. The gain is calculated as $\frac{\text{(delay of baseline}-\text{delay of proposed method)}}{\text{delay of baseline}}$. The average delay is not better than baseline and it does not converge through iteration, as shown in Figure 5. The result of Taylor expansion is better than baseline.

Table 1: Comparison of different solution method

	Equally Distributed	GASA	Taylor
delay (number of time slots)	18.6805	21.9090	13.6312
gain	0%	-17%	27%

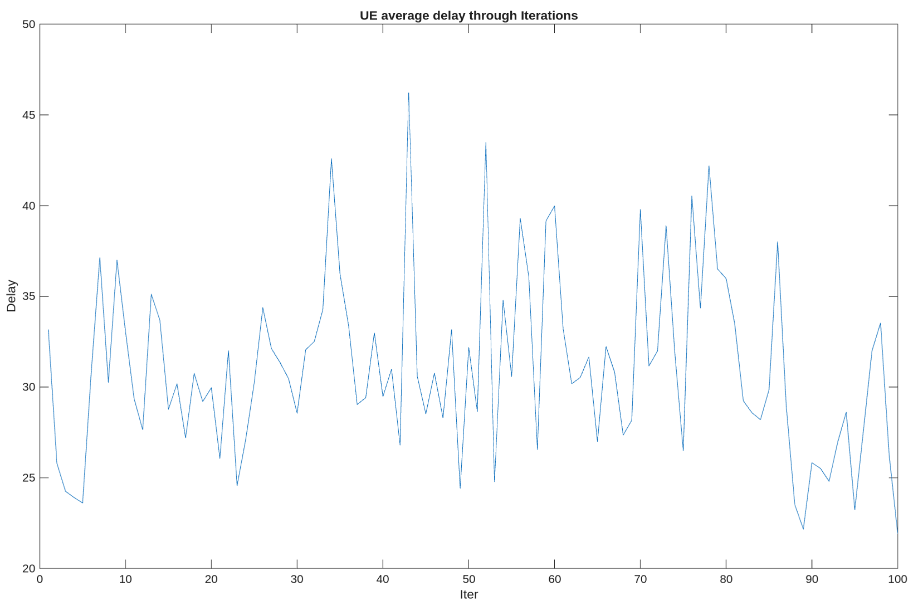

Figure 7: Average delay of GASA through iterations.

4.4 Future Work

1. I have found the reason why GASA has bad result. I will modify it and compare the result with other methods.
2. To solve equation (4.11), maybe Dinkelbach algorithm can work.
3. I am still figuring out how to determine a in Taylor series.
4. I only consider one epoch in the algorithm above. The relation between different epochs still need to be considered.

CHAPTER 5

PERFORMANCE EVALUATION

All figures should be of the same width whenever possible for consistency.

5.1 Simulation Setup

We use BONMON [**?**] to solve the optimization problem. The simulation setup follows that in [**?**].

5.2 Simulation Results

CHAPTER 6

CONCLUSION AND FUTURE WORK

REFERENCES

[1] "Solutions for nr to support non-terrestrial networks (ntn)," 3rd Generation Partnership Project (3GPP), Technical Report TR 38.821, 2023, release 16. Online Available at: https://www.3gpp.org/dynareport/38821.htm

[2] "Nr; nr and ng-ran overall description; stage-2," 3rd Generation Partnership Project (3GPP), Technical Specification TS 38.300, 2024, release 18. Online Available at: https://www.3gpp.org/dynareport/38300.htm

[3] 3GPP, "Radio Resource Control (RRC) protocol specification," 3rd Generation Partnership Project (3GPP), Technical Report TR 38.331, March 2024, version 18.1.0. Online Available at: https://www.3gpp.org/ftp/Specs/archive/38_series/38.331/

[4] J. Wang, C. Jiang, L. Kuang, and R. Han, "Satellite multi-beam collaborative scheduling in satellite aviation communications," *IEEE Transactions on Wireless Communications*, vol. 23, no. 3, pp. 2097–2111, 2024.

[5] D.-H. Jung, J.-G. Ryu, W.-J. Byun, and J. Choi, "Performance analysis of satellite communication system under the shadowed-rician fading: A stochastic geometry approach," *IEEE Transactions on Communications*, vol. 70, no. 4, pp. 2707–2721, 2022.

[6] S.-H. Chen, L.-H. Shen, K.-T. Feng, L.-L. Yang, and J.-M. Wu, "Energy-efficient joint handover and beam switching scheme for multi-leo networks," in *2024 IEEE 99th Vehicular Technology Conference (VTC2024-Spring)*, 2024, pp. 1–7.