4T1: The Short-Time Fourier Transform (1 of 2)

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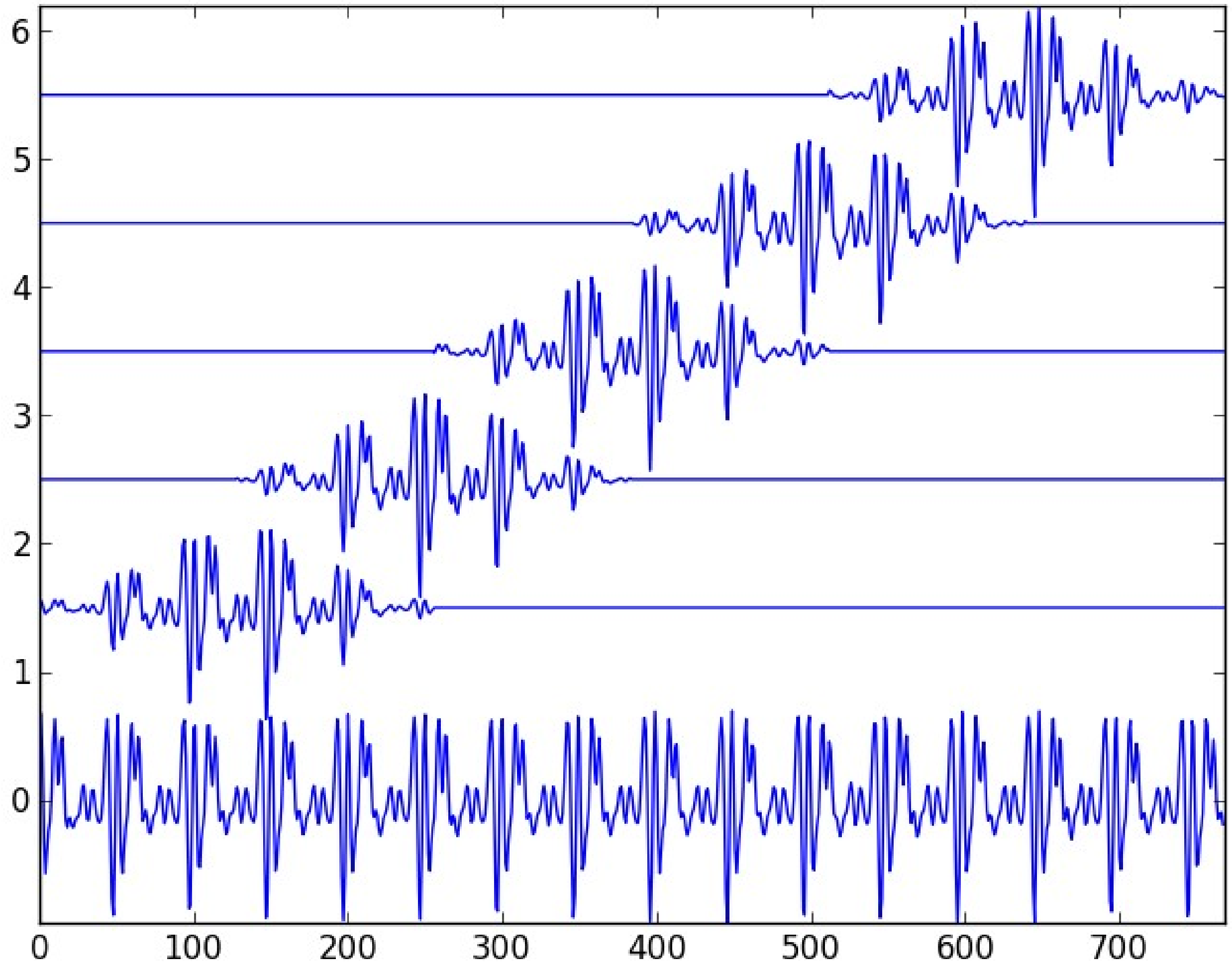
Short-time Fourier Transform

$$X_l[k] = \sum_{n=-N/2}^{N/2-1} w[n]\,x[n+lH]\,e^{-j2\pi kn/N} \qquad l = 0, 1, \ldots,$$

w: analysis window
l: frame number
H: hop-size

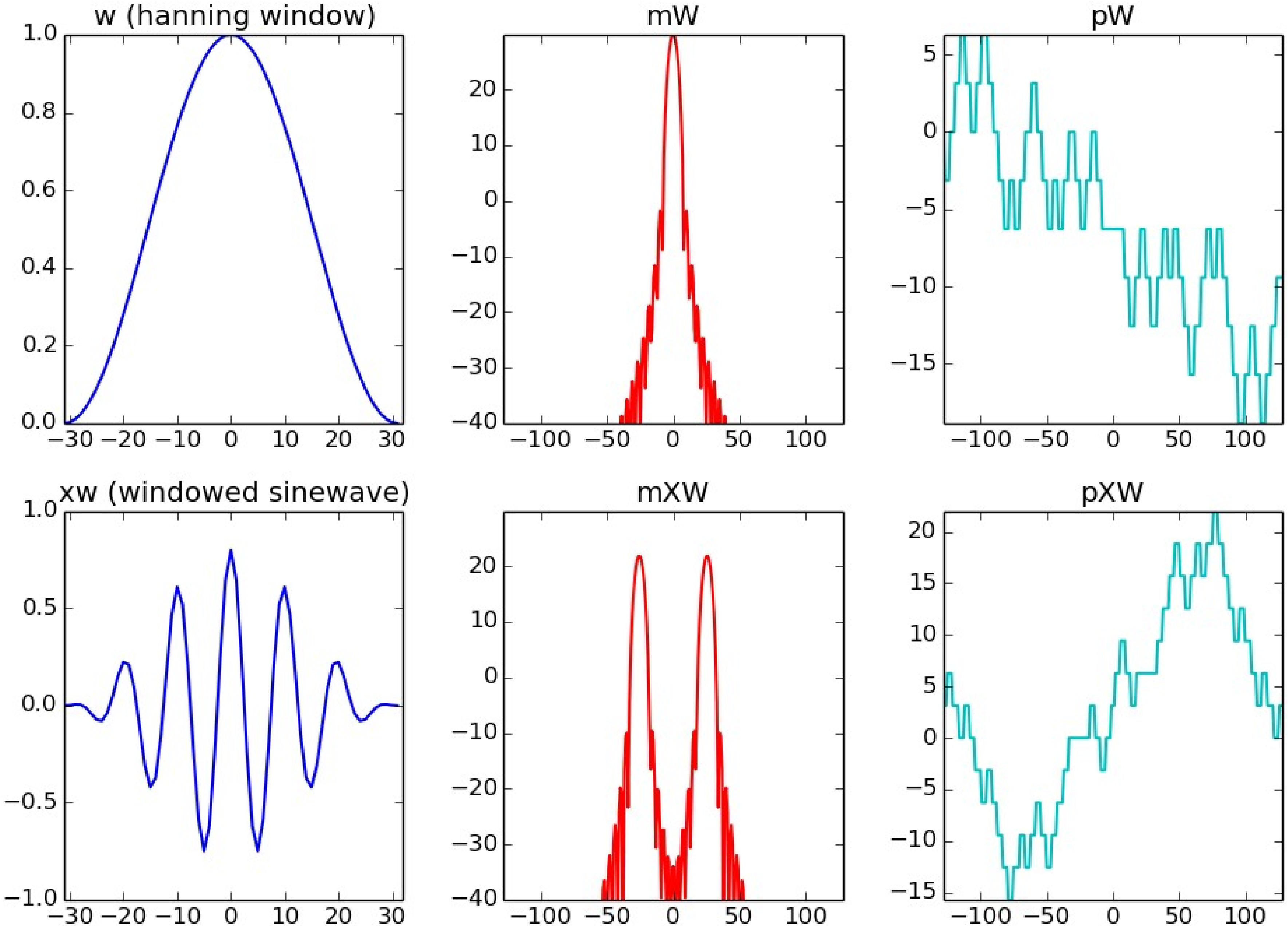
$$xw_l[n]=w[n]x[n+lH] \qquad l=0,1,\ldots,$$



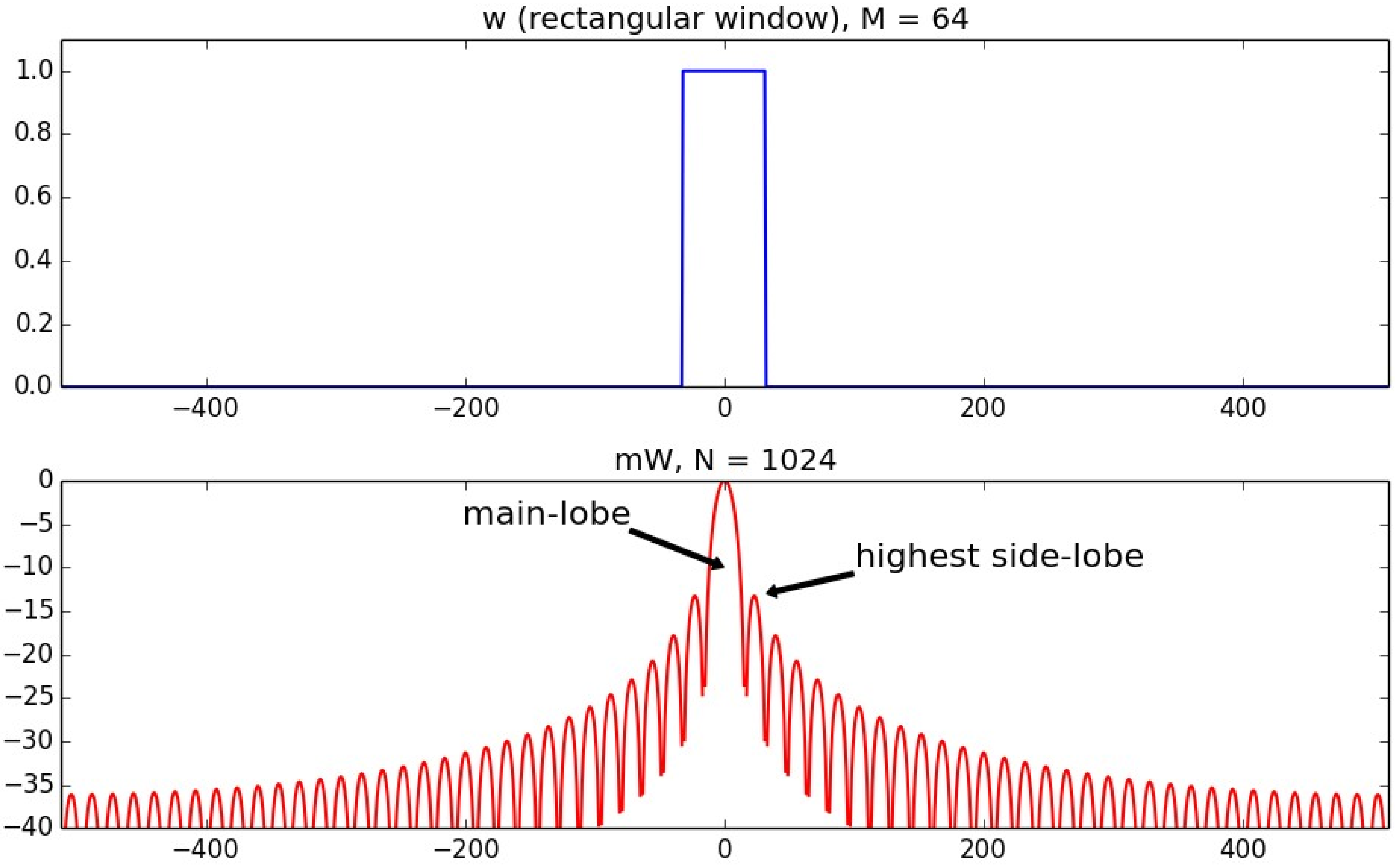
Transform of a windowed sinewave

$$x[n]=A_0\cos(2\pi k_0 n/N)=\frac{A_0}{2}e^{j2\pi k_0 n/N}+\frac{A_0}{2}e^{-j2\pi k_0 n/N}$$

$$\begin{aligned}
X[k]&=\sum_{n=-N/2}^{N/2-1} w[n]x[n]e^{-j2\pi kn/N}\\
&=\sum_{n=-N/2}^{N/2-1} w[n]\left(\frac{A_0}{2}e^{j2\pi k_0 n/N}+\frac{A_0}{2}e^{-j2\pi k_0 n/N}\right)e^{-j2\pi kn/N}\\
&=\sum_{n=-N/2}^{N/2-1} w[n]\frac{A_0}{2}e^{j2\pi k_0 n/N}e^{-j2\pi kn/N}+\sum_{n=-N/2}^{N/2-1} w[n]\frac{A_0}{2}e^{-j2\pi k_0 n/N}e^{-j2\pi kn/N}\\
&=\frac{A_0}{2}\sum_{n=-N/2}^{N/2-1} w[n]e^{-j2\pi(k-k_0)n/N}+\frac{A_0}{2}\sum_{n=-N/2}^{N/2-1} w[n]e^{-j2\pi(k+k_0)n/N}\\
&=\frac{A_0}{2}W[k-k_0]+\frac{A_0}{2}W[k+k_0]
\end{aligned}$$


w (hanning window)
mW
pW
xw (windowed sinewave)
mXW
pXW

Analysis window



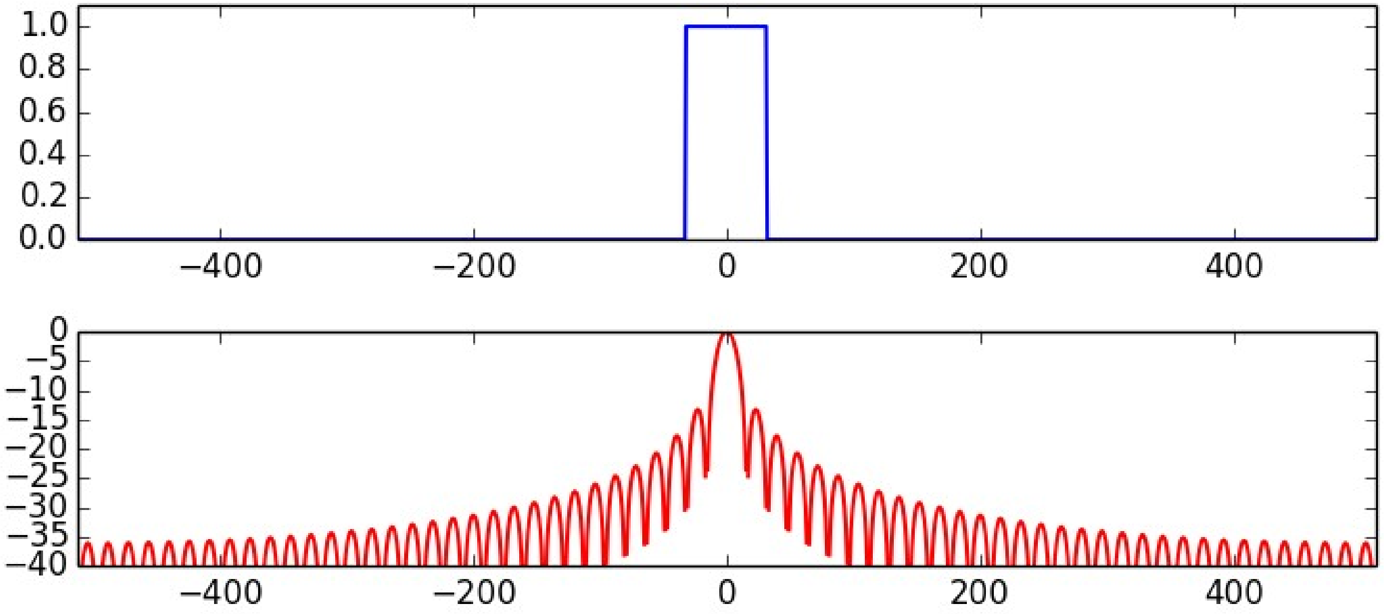
Window functions in Scipy

barthann (M[, sym])	Return a modified Bartlett-Hann window.
bartlett (M[, sym])	Return a Bartlett window.
blackman (M[, sym])	Return a Blackman window.
blackmanharris (M[, sym])	Return a minimum 4-term Blackman-Harris window.
bohman (M[, sym])	Return a Bohman window.
boxcar (M[, sym])	Return a boxcar or rectangular window.
chebwin (M, at[, sym])	Return a Dolph-Chebyshev window.
flattop (M[, sym])	Return a flat top window.
gaussian (M, std[, sym])	Return a Gaussian window.
general-gaussian (M, p, sig[, sym])	Return a window with a generalized Gaussian shape.
hamming (M[, sym])	Return a Hamming window.
hann (M[, sym])	Return a Hann window.
kaiser (M, beta[, sym])	Return a Kaiser window.
nuttall (M[, sym])	Return a minimum 4-term Blackman-Harris window according to Nuttall.
parzen (M[, sym])	Return a Parzen window.
slepian (M, width[, sym])	Return a digital Slepian window.
triang (M[, sym])	Return a triangular window.

Rectangular window

$$w[n] = 1, \quad n = -M/2, \ldots, 0, \ldots M/2$$
$$= 0, \quad n = \text{elsewhere}$$

$$W[k] = \frac{\sin(\pi k)}{\sin(\pi k/M)}$$

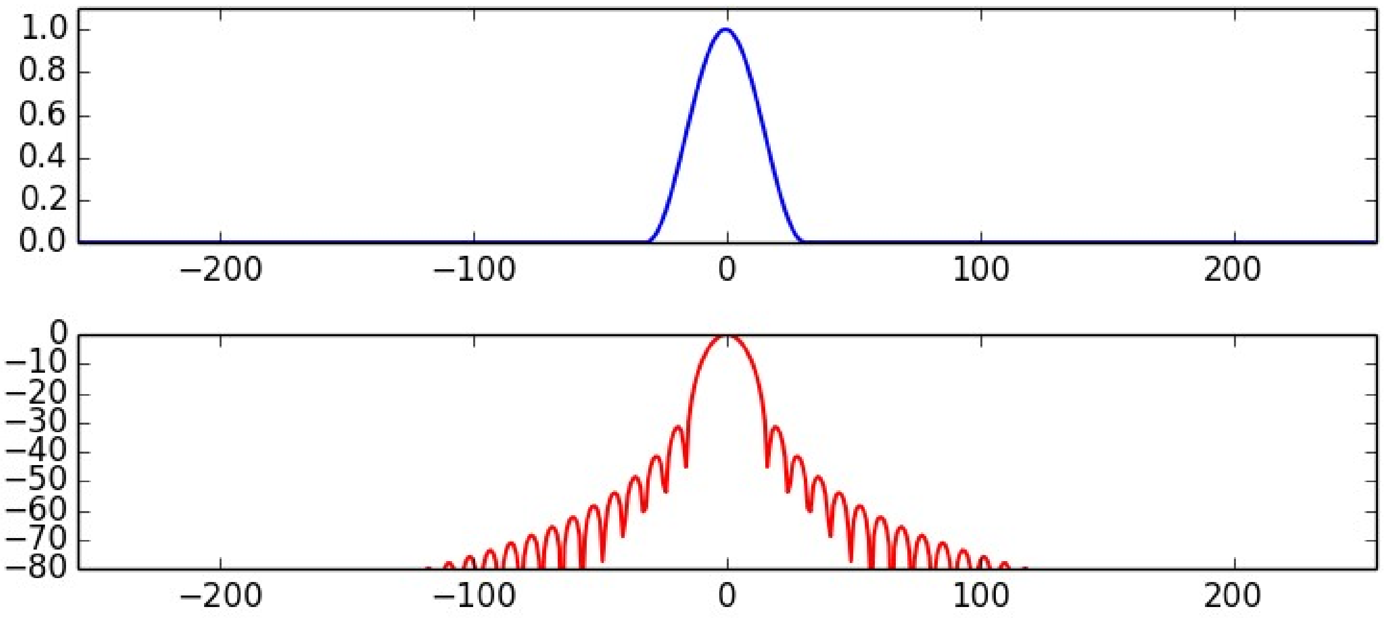


main-lobe width: 2 bins
side-lobe level: -13.3 dB

Hanning window

$$w[n] = .5 + .5\cos(2\pi n/M), \quad n = -M/2, \ldots, 0, \ldots M/2$$

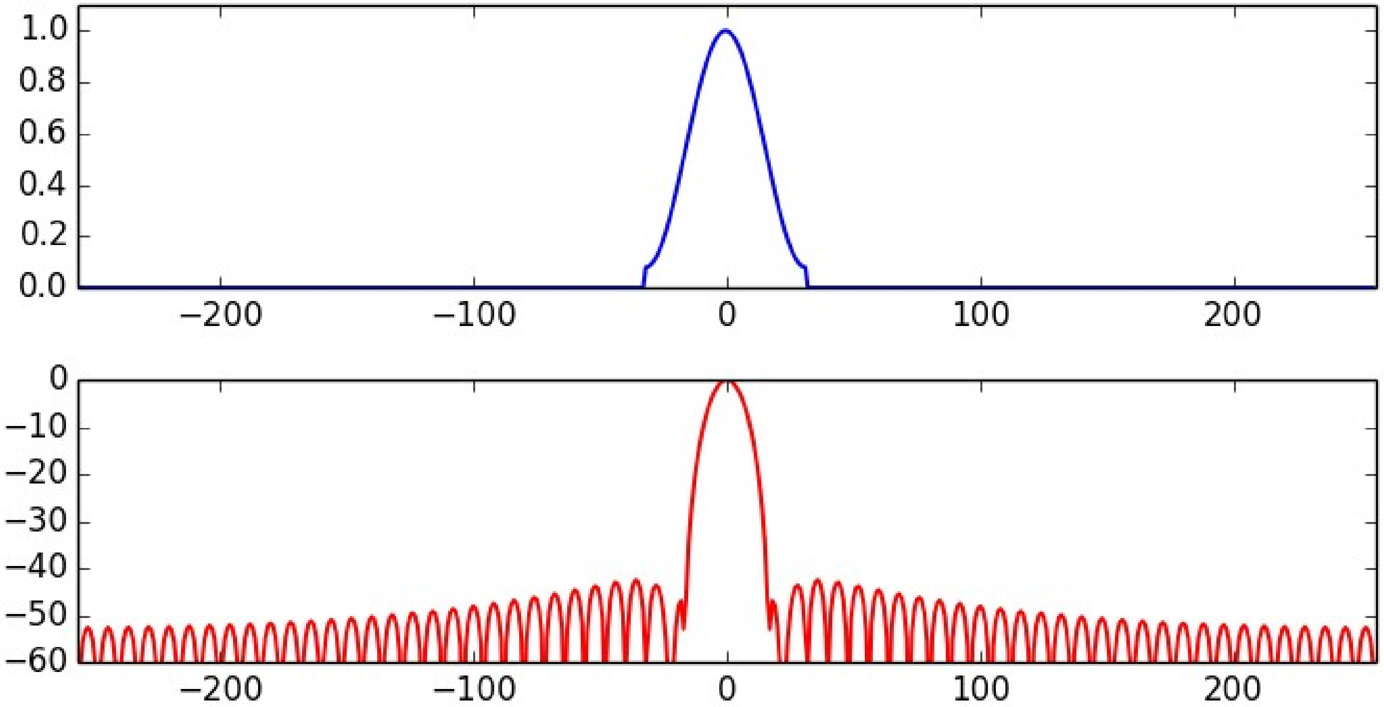
$$W[k] = .5\,D[k] + .25\,(D[k-1] + D[k+1]) \quad \text{where } D[k] = \frac{\sin(\pi k)}{\sin(\pi k/M)}$$



main-lobe width: 4 bins
side-lobe level: -31.5 dB

Hamming window

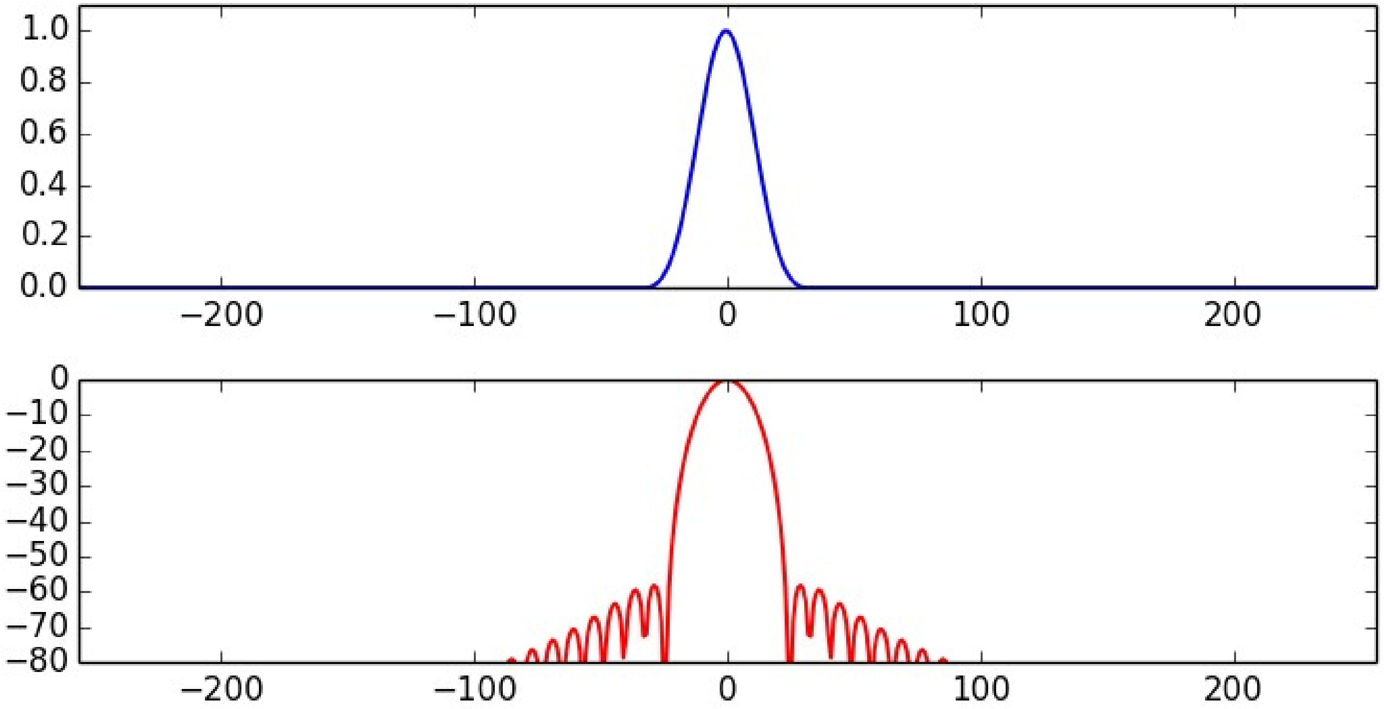
$$w[n] = .54 + .46\cos(2\pi n/M), \quad n = -M/2, \ldots, 0, \ldots M/2$$



main-lobe width: 4 bins
side-lobe level: -42.7 dB

Blackman window

$$w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$$

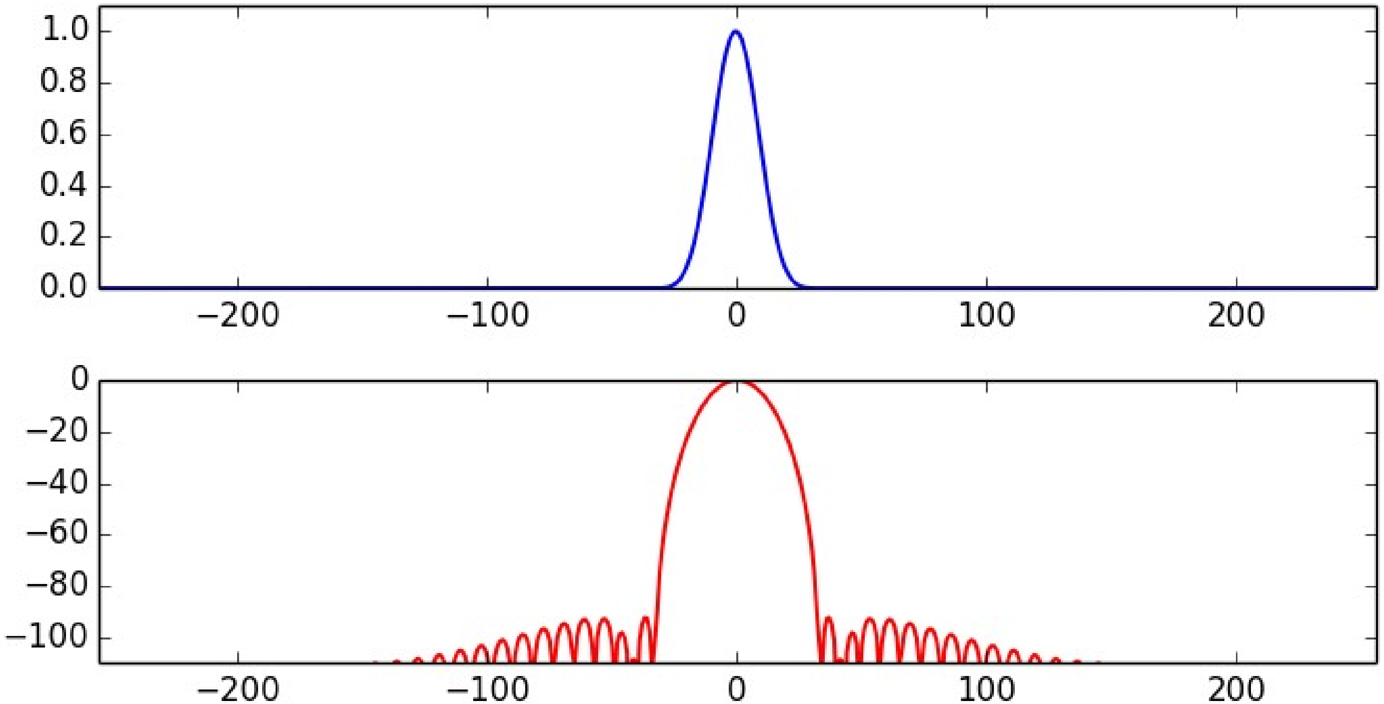


main-lobe width: 6 bins
side-lobe level: -58 dB

Blackman-Harris window

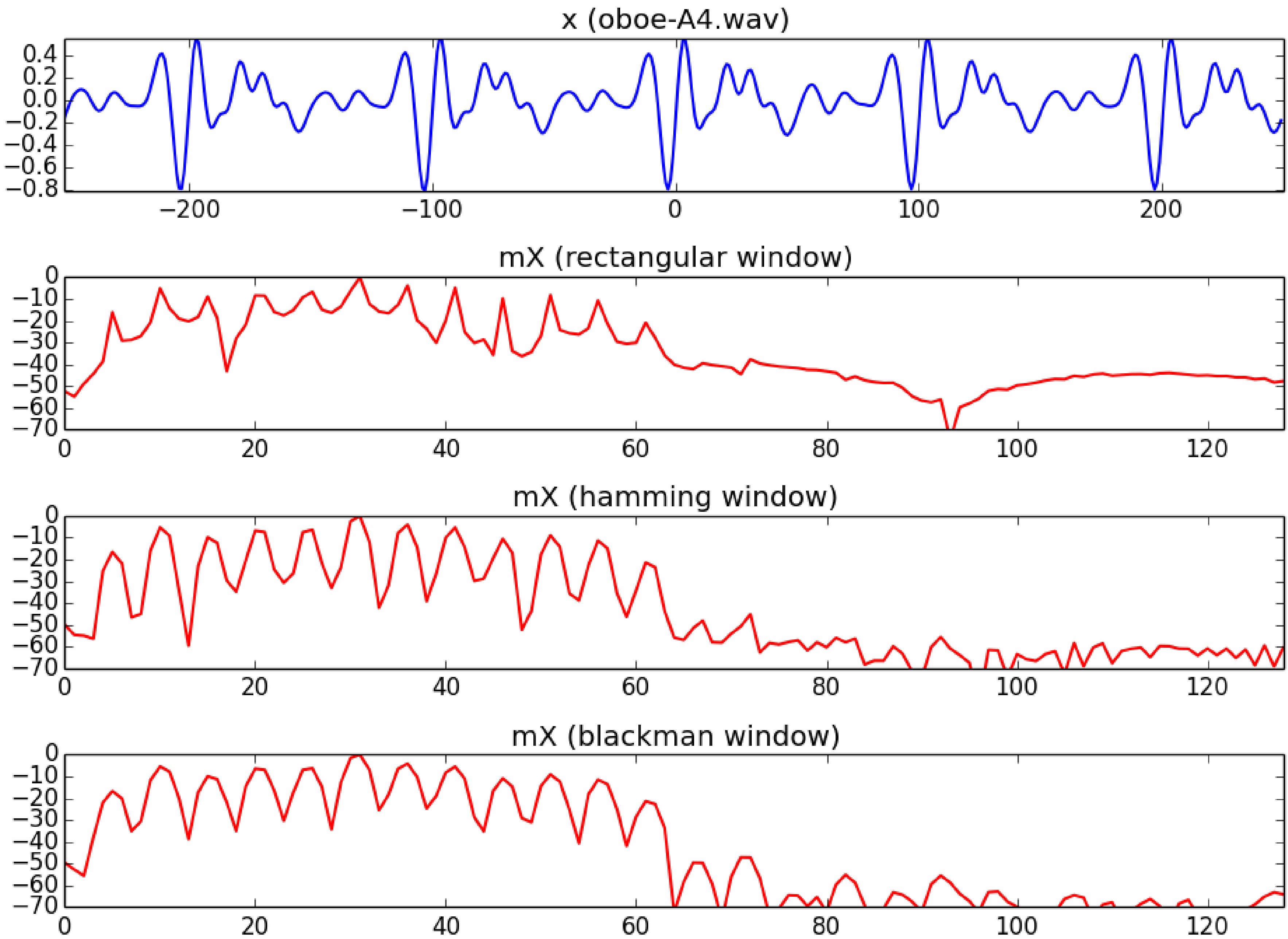
$$w(n)=\frac{1}{M}\sum_{l=0}^{3}\alpha_l\cos(2nl\pi/M),\quad n=-M/2,\ldots 0,\ldots M/2$$

where $\alpha_0=0.35875, \alpha_1=0.48829, \alpha_2=0.14128, \alpha_3=0.01168$



$\text{main lobe width}: 8\,\text{bins}$

$\text{side-lobe level}: -92\text{dB}$



x (oboe-A4.wav)
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−200
−100
0
100
200
mX (rectangular window)
0
−10
−20
−30
−40
−50
−60
−70
0
20
40
60
80
100
120
mX (hamming window)
mX (blackman window)

References and credits

- More information in:
  https://en.wikipedia.org/wiki/STFT
  https://en.wikipedia.org/wiki/Window_function
- Reference on the STFT by Julius O. Smith:
  https://ccrma.stanford.edu/~jos/sasp/
- Sounds from:
  http://www.freesound.org/people/xserra/packs/13038/
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