2T1: The Discrete Fourier Transform (1 of 2)

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Discrete Fourier Transform

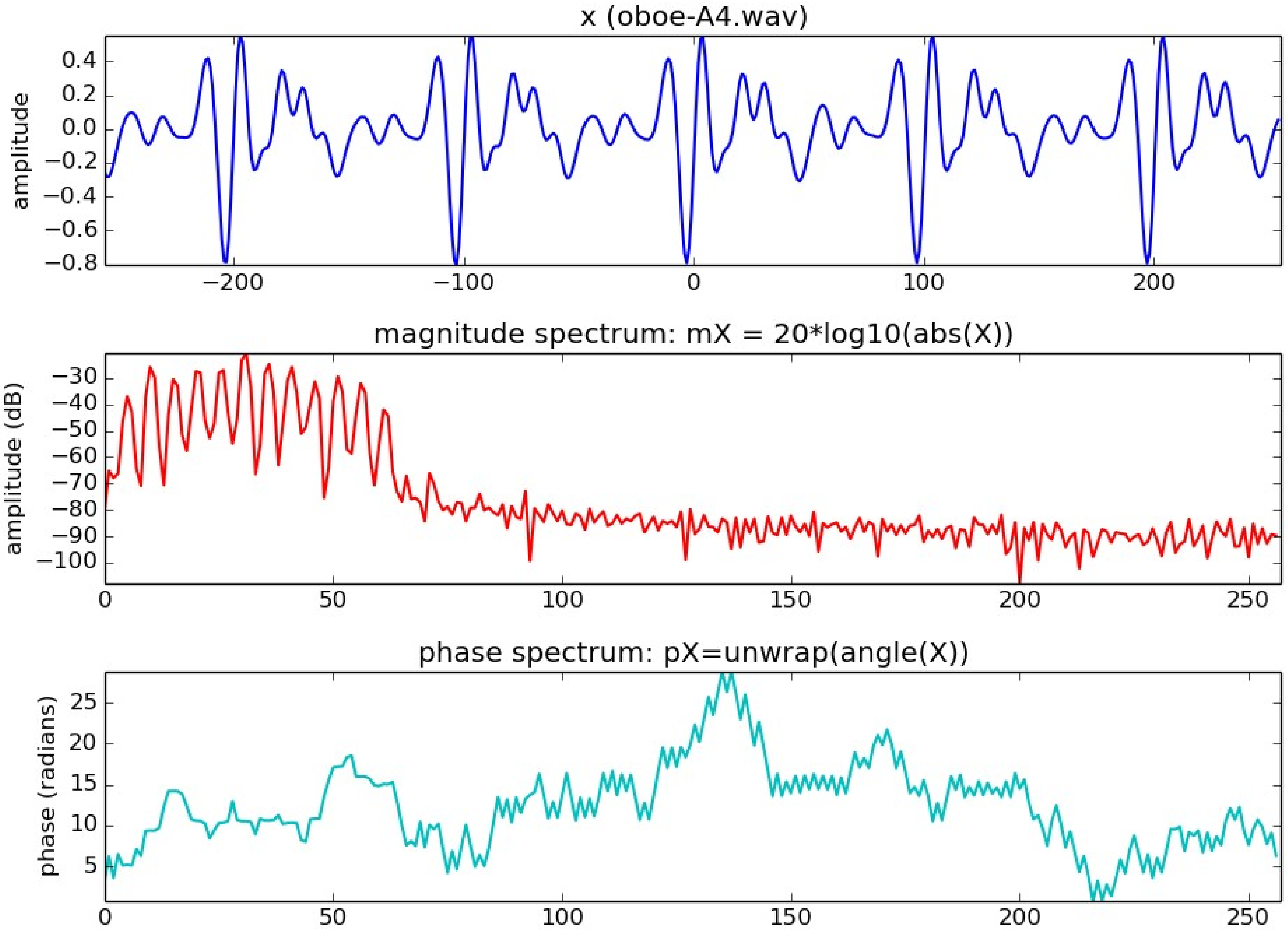
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn/N} \quad k = 0, \ldots, N-1$$

n : discrete time index (normalized time, $T = 1$)

k : discrete frequency index

$\omega_k = 2\pi k/N$: frequency in radians per second

$f_k = f_s k/N$: frequency in Hz (f_s : sampling rate)



x (oboe-A4.wav)
amplitude
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−200
−100
0
100
200
magnitude spectrum: mX = 20*log10(abs(X))
amplitude (dB)
−30
−40
−50
−60
−70
−80
−90
−100
0
50
100
150
200
250
phase spectrum: pX=unwrap(angle(X))
phase (radians)
25
20
15
10
5
0
50
100
150
200
250

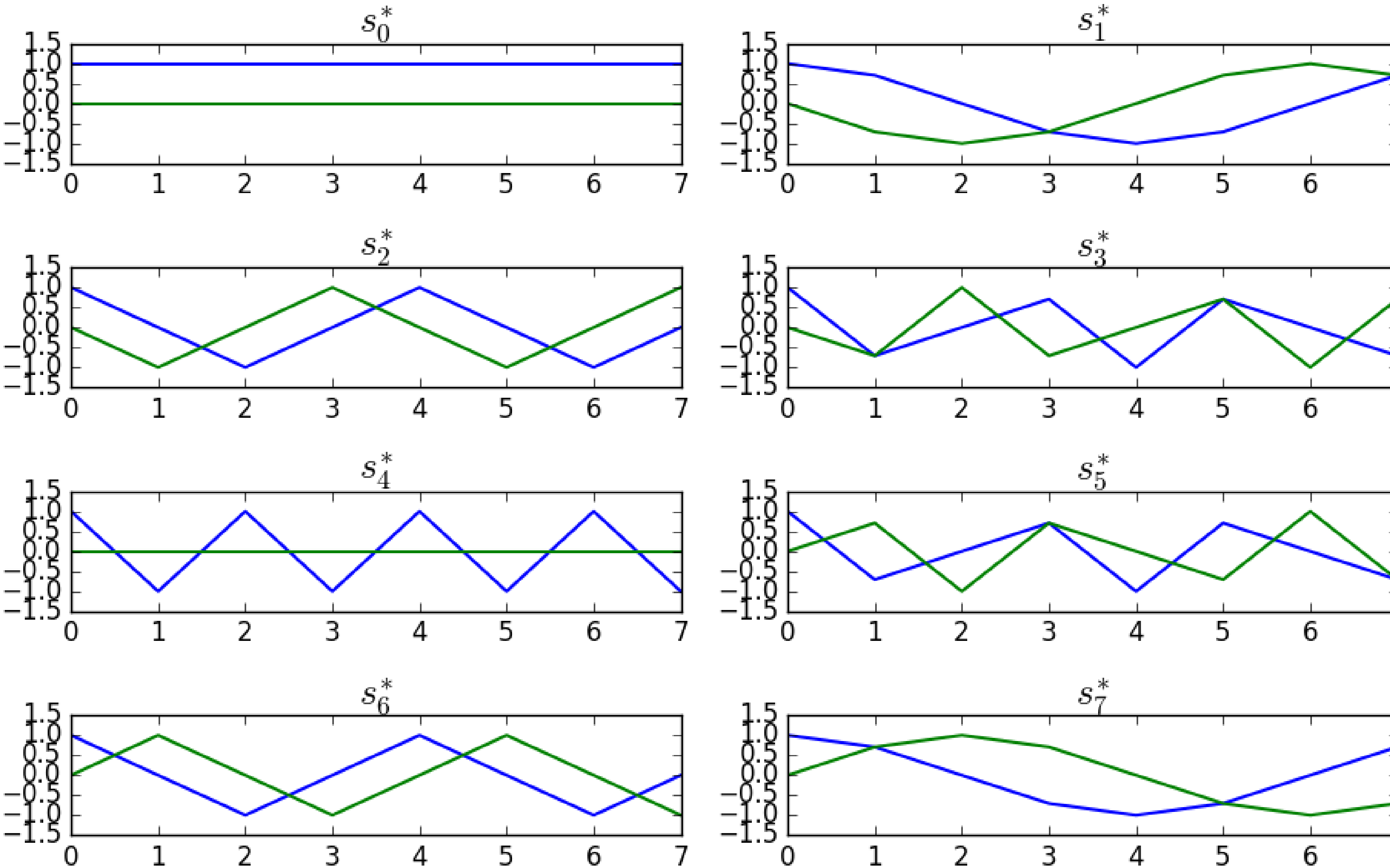
DFT: complex exponentials

$$s_k^* = e^{-j2\pi kn/N} = \cos(2\pi kn/N) - j\sin(2\pi kn/N)$$

for $N = 4$, thus for $n = 0,1,2,3\,;\ \ k = 0,1,2,3$

$$s_0^* = \cos(2\pi \times 0 \times n/4) - j\sin(2\pi \times 0 \times n/4) = [1,1,1,1]$$
$$s_1^* = \cos(2\pi \times 1 \times n/4) - j\sin(2\pi \times 1 \times n/4) = [1,-j,-1,j]$$
$$s_2^* = \cos(2\pi \times 2 \times n/4) - j\sin(2\pi \times 2 \times n/4) = [1,-1,1,-1]$$
$$s_3^* = \cos(2\pi \times 3 \times n/4) - j\sin(2\pi \times 3 \times n/4) = [1,j,-1,-j]$$

DFT: complex exponentials



DFT: scalar product

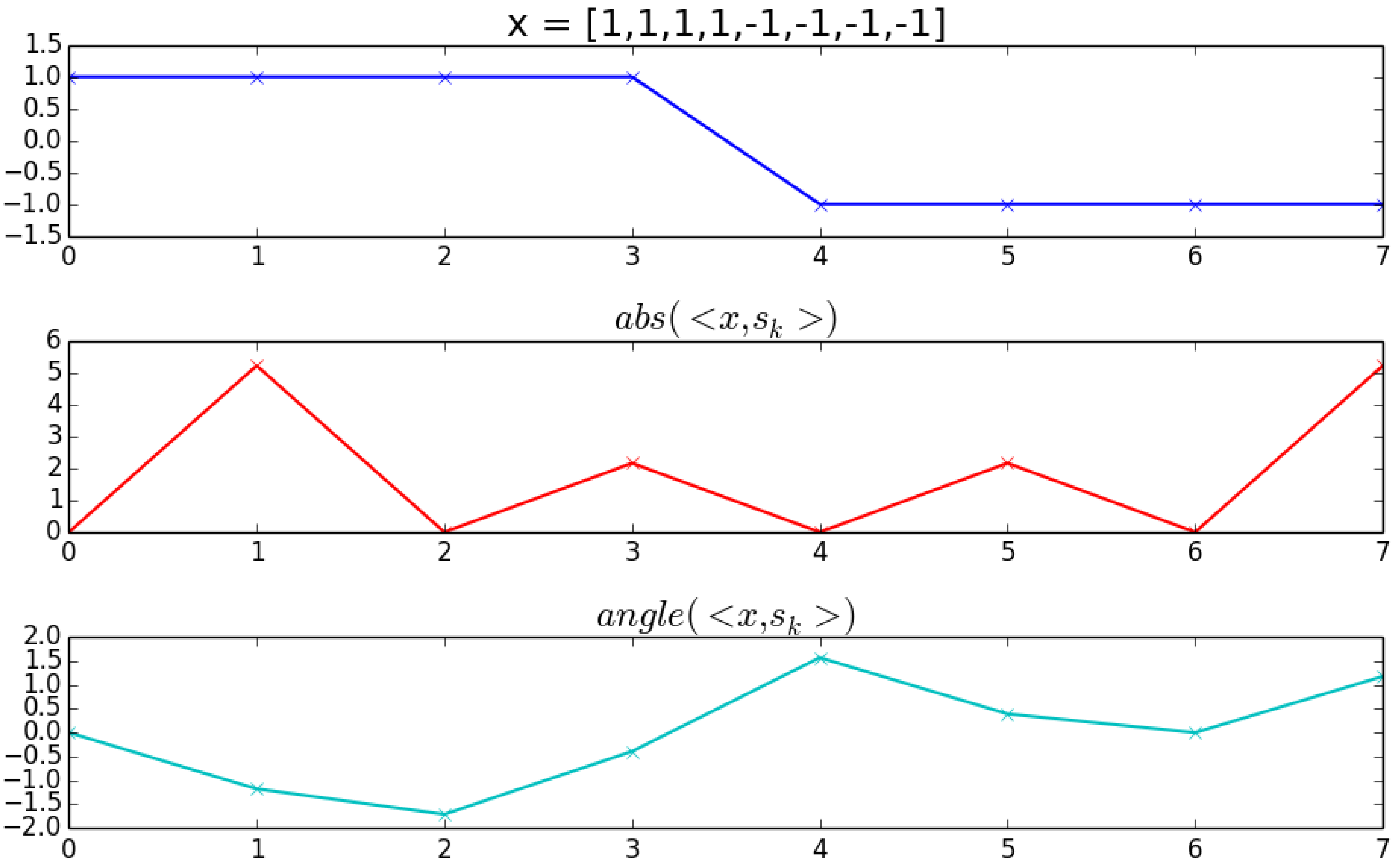
$$\langle x, s_k \rangle = \sum_{n=0}^{N-1} x[n] s_k^*[n] = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn/N}$$

Example:

$$x[n] = [1, -1, 1, -1]; N = 4$$

$$\langle x, s_0 \rangle = 1 \times 1 + (-1) \times 1 + 1 \times 1 + (-1) \times 1 = 0$$
$$\langle x, s_1 \rangle = 1 \times 1 + (-1) \times (-j) + 1 \times (-1) + (-1) \times j = 0$$
$$\langle x, s_2 \rangle = 1 \times 1 + (-1) \times (-1) + 1 \times 1 + (-1) \times (-1) = 4$$
$$\langle x, s_3 \rangle = 1 \times 1 + (-1) \times j + 1 \times (-1) + (-1) \times (-j) = 0$$

DFT: scalar product



References and credits

- More information in: https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- Reference on the mathematics of the DFT from Julius O. Smith: https://ccrma.stanford.edu/~jos/mdft/
- Sounds from: http://www.freesound.org/people/xserra/packs/13038
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