5T2: Sinusoidal model (2 of 3)

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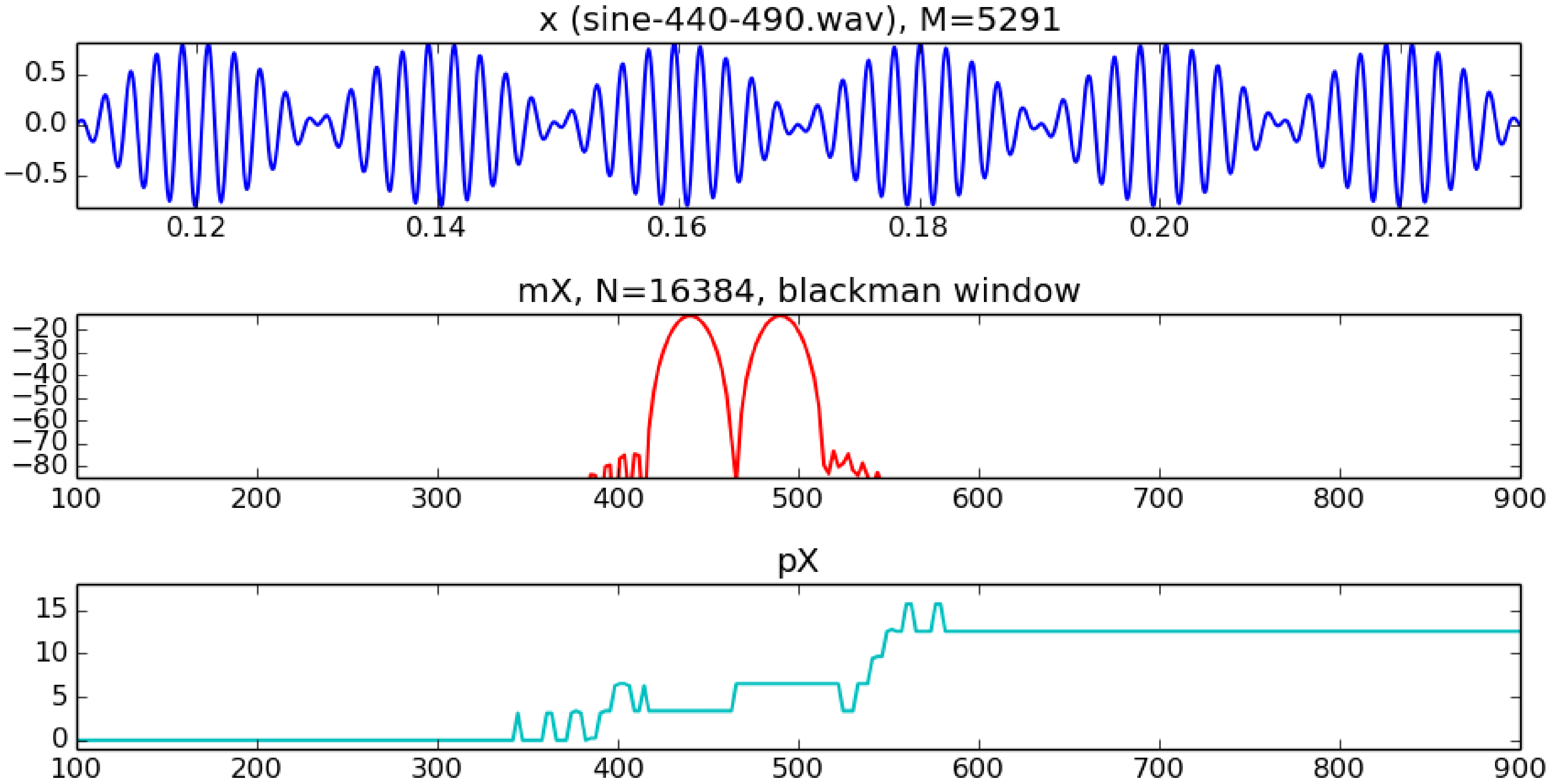
Sinusoidal model

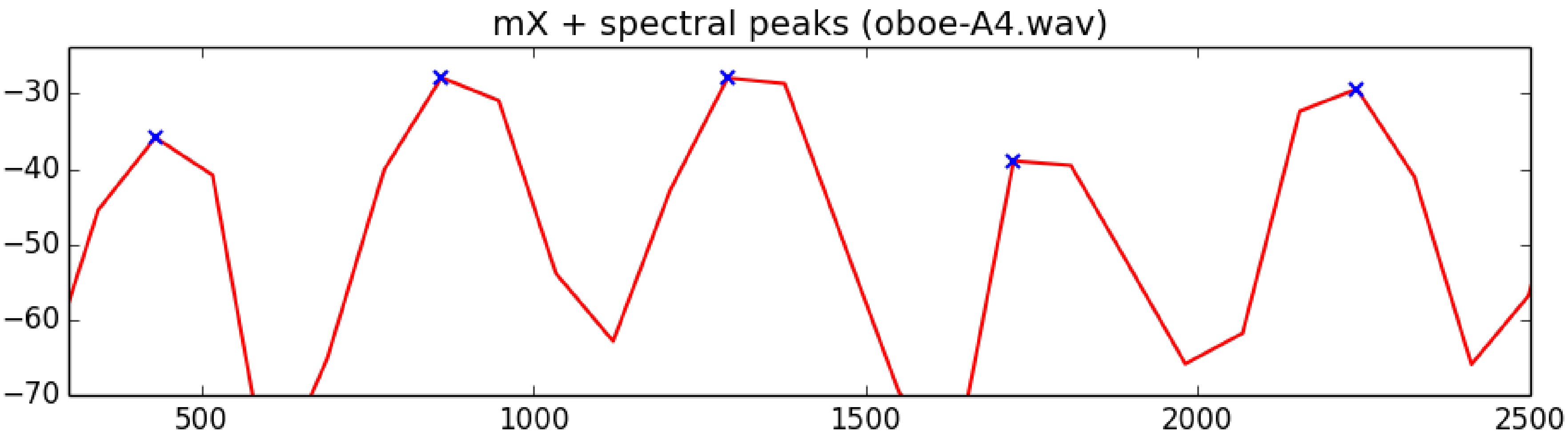
$$y[n]=\sum_{r=1}^{R} A_r[n]\cos(2\pi f_r[n]n)$$

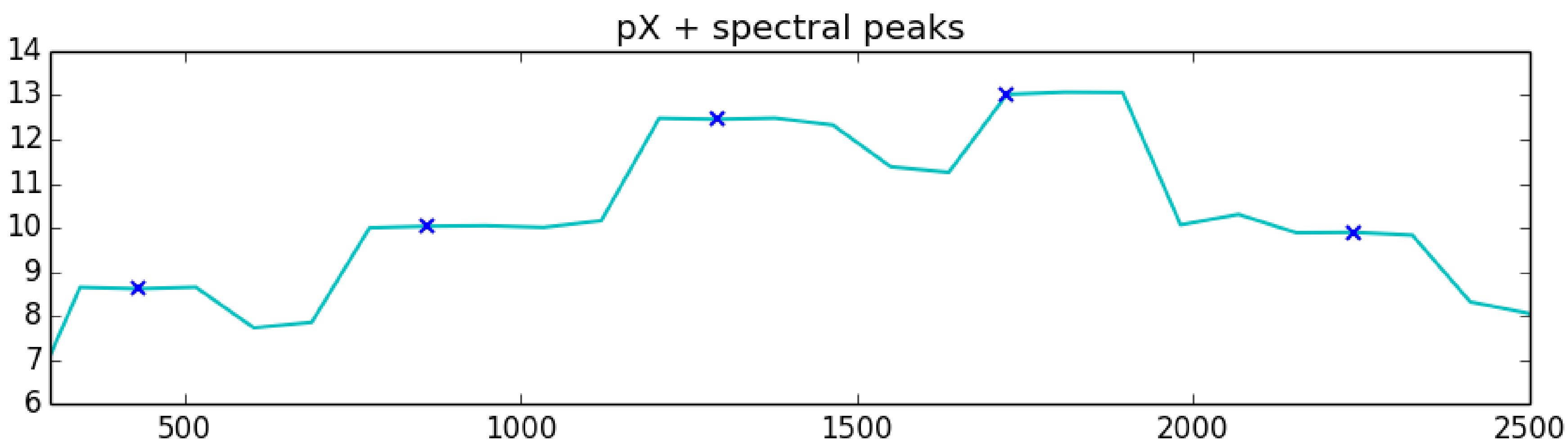


Peak detection

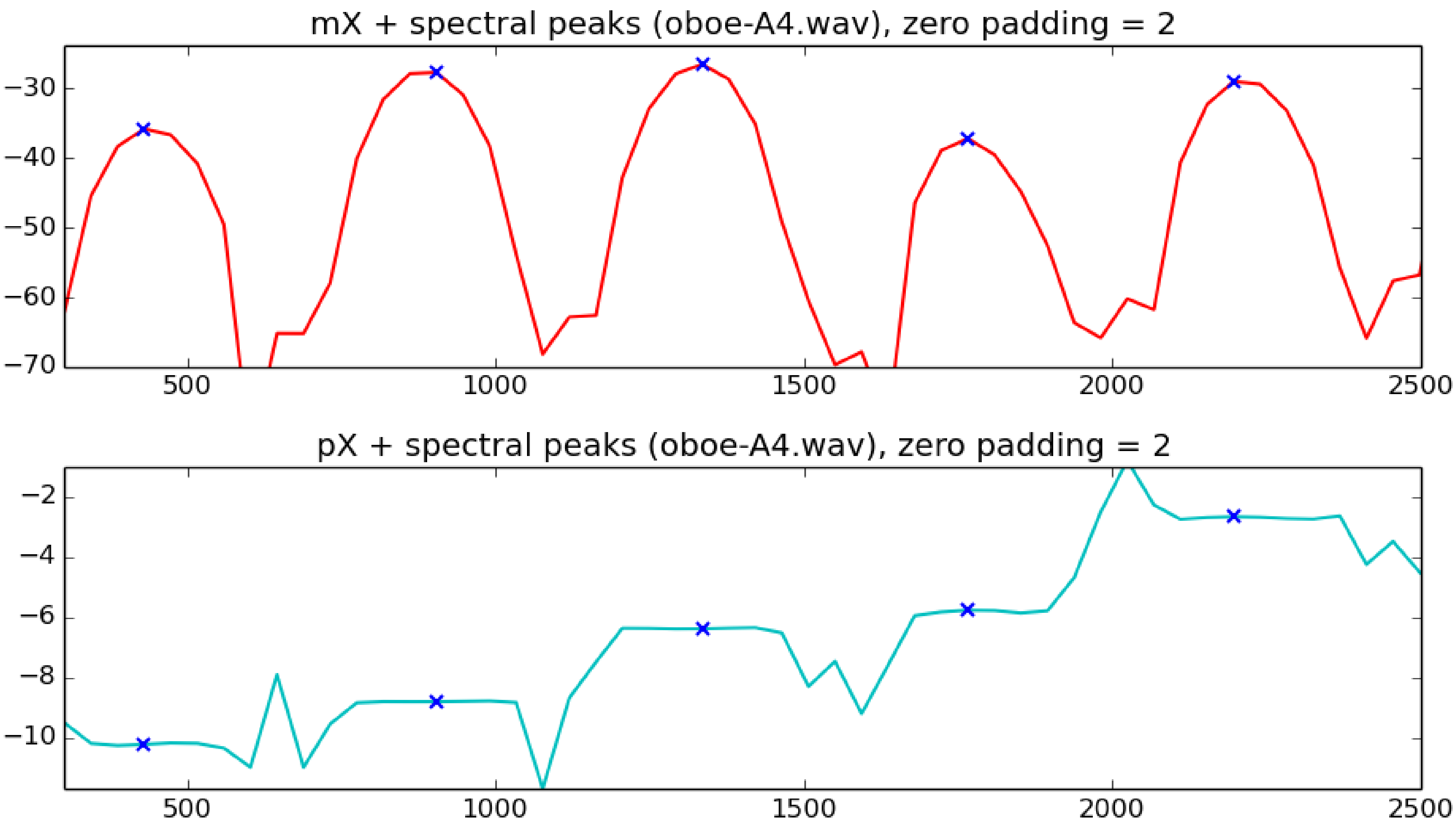
$$p_r = mX[k_0] \quad \text{when} \quad mX[k_0-1] < mX[k_0] > mX[k_0+1]$$

mX = magnitude spectrum ; r = peak number ; k_0 = peak location





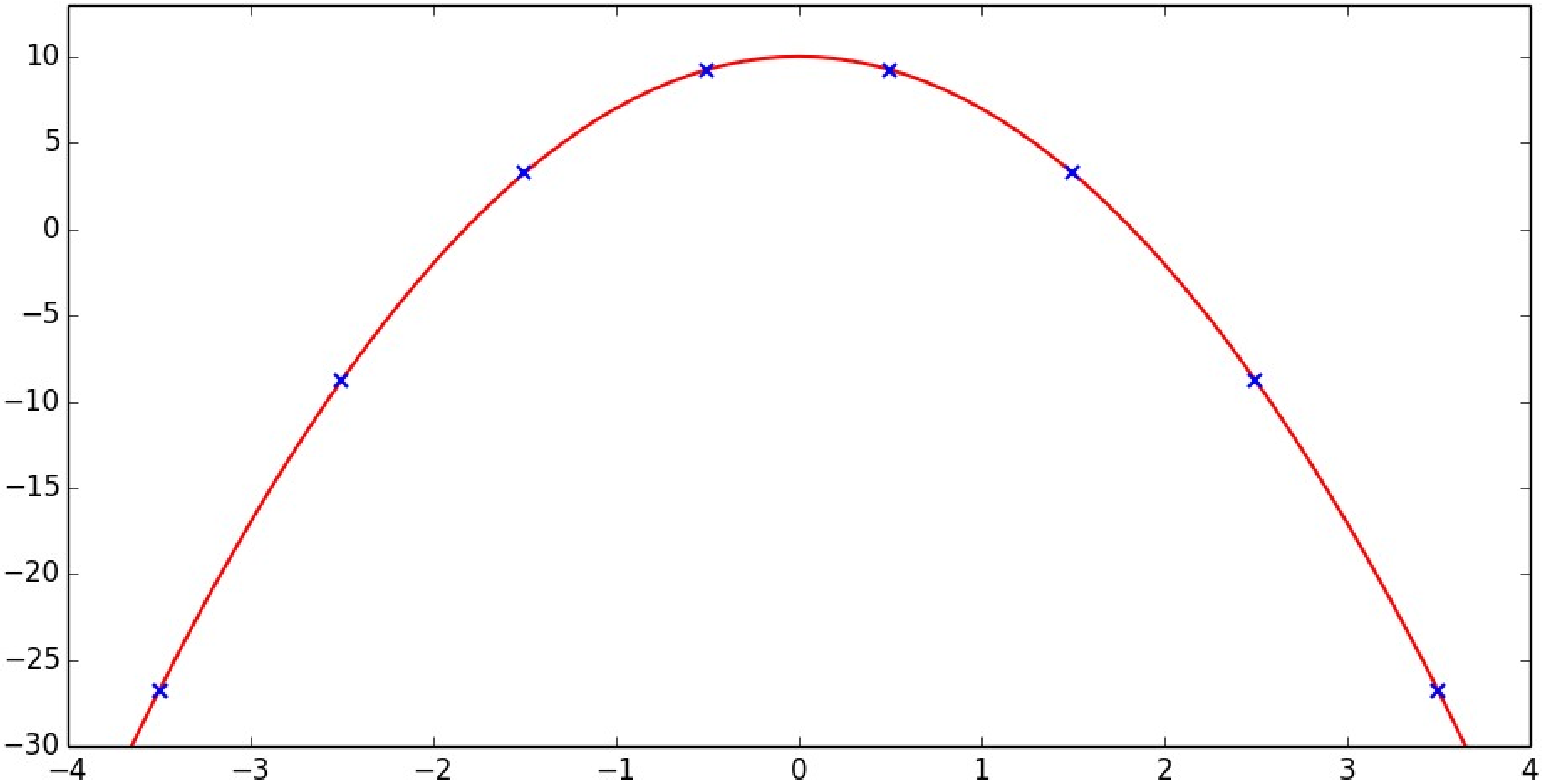
Peak detection with zero-padding



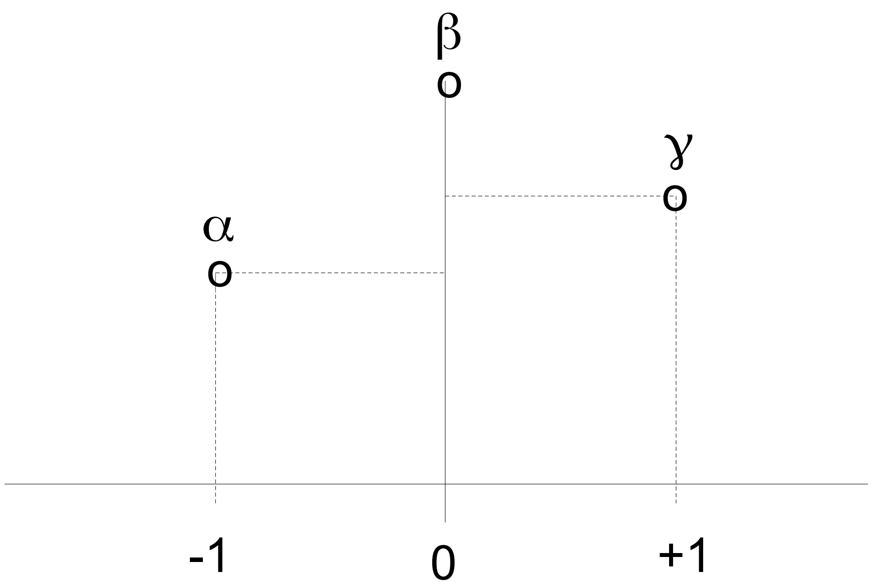
Parabola

$$x[n] = a(n-p)^2 + b$$

p: center of parabola
a: concavity measure
b: offset



Peak interpolation

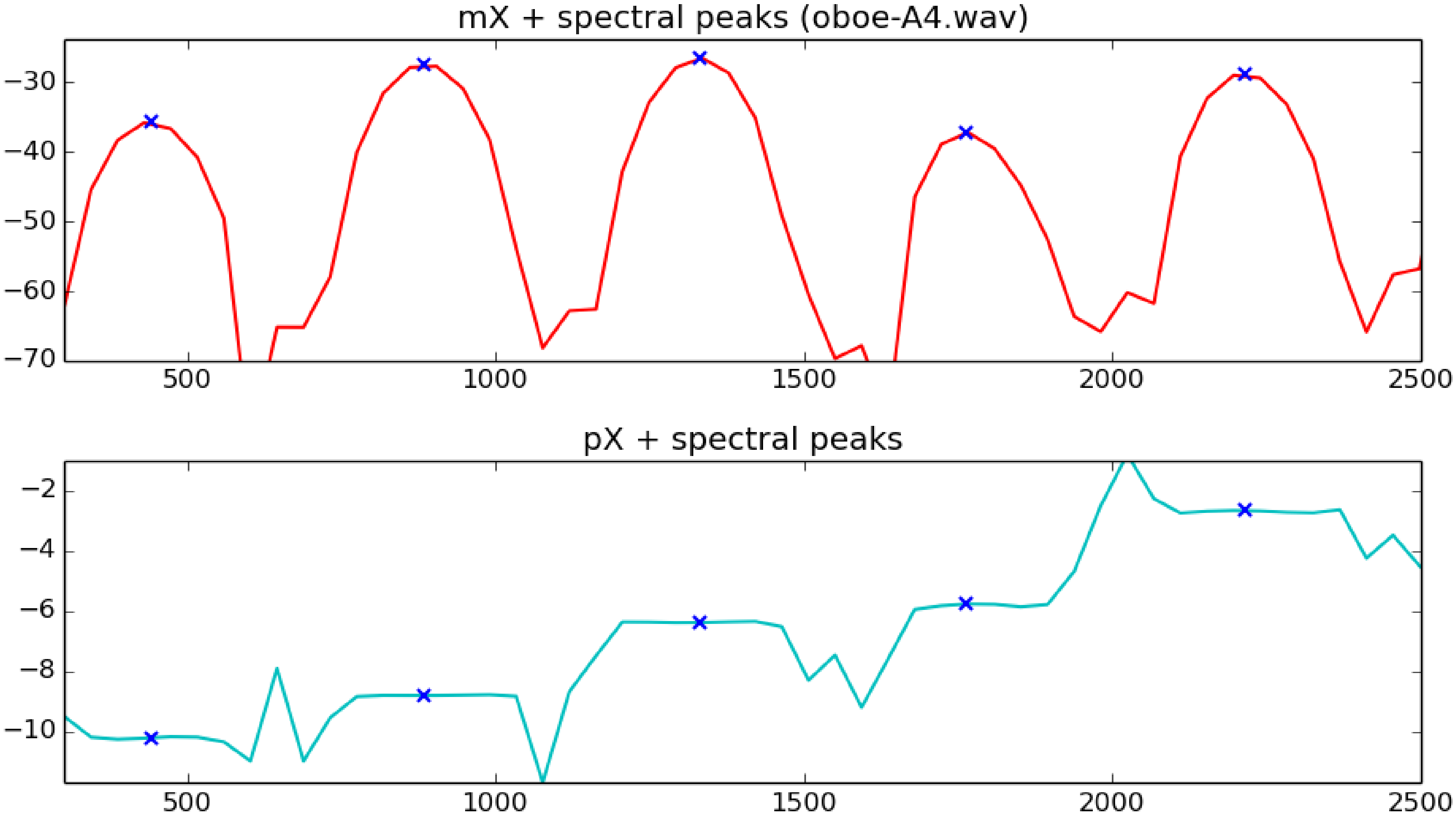


$$
\begin{aligned}
x[-1] &= \alpha = mX[k_\beta - 1] \\
x[0] &= \beta = mX[k_\beta] \\
x[1] &= \gamma = mX[k_\beta + 1]
\end{aligned}
$$

center of the parabola: $\hat{k}_p = k_p + \frac{1}{2}\frac{\alpha-\gamma}{(\alpha-2\beta+\gamma)}$

amplitude: $\hat{a} = \beta - \frac{1}{4}(\hat{k}_p - k_p)(\alpha-\gamma)$

Peak detection with interpolation



Sinusoidal parameters from peaks

$$\hat{k}_p = k_p + \frac{0.5 * (|X[k_p - 1]| - |X[k_p + 1]|)}{|X[k_p - 1]| - 2 * |X[k_p]| + |X[k_p + 1]|}$$

$$f_p = \frac{f_s * \hat{k}_p}{N}$$

$$A_p = |X[k_p]| - 0.25 * (|X[k_p - 1]| - |X[k_p + 1]|) * (\hat{k}_p - k_p)$$

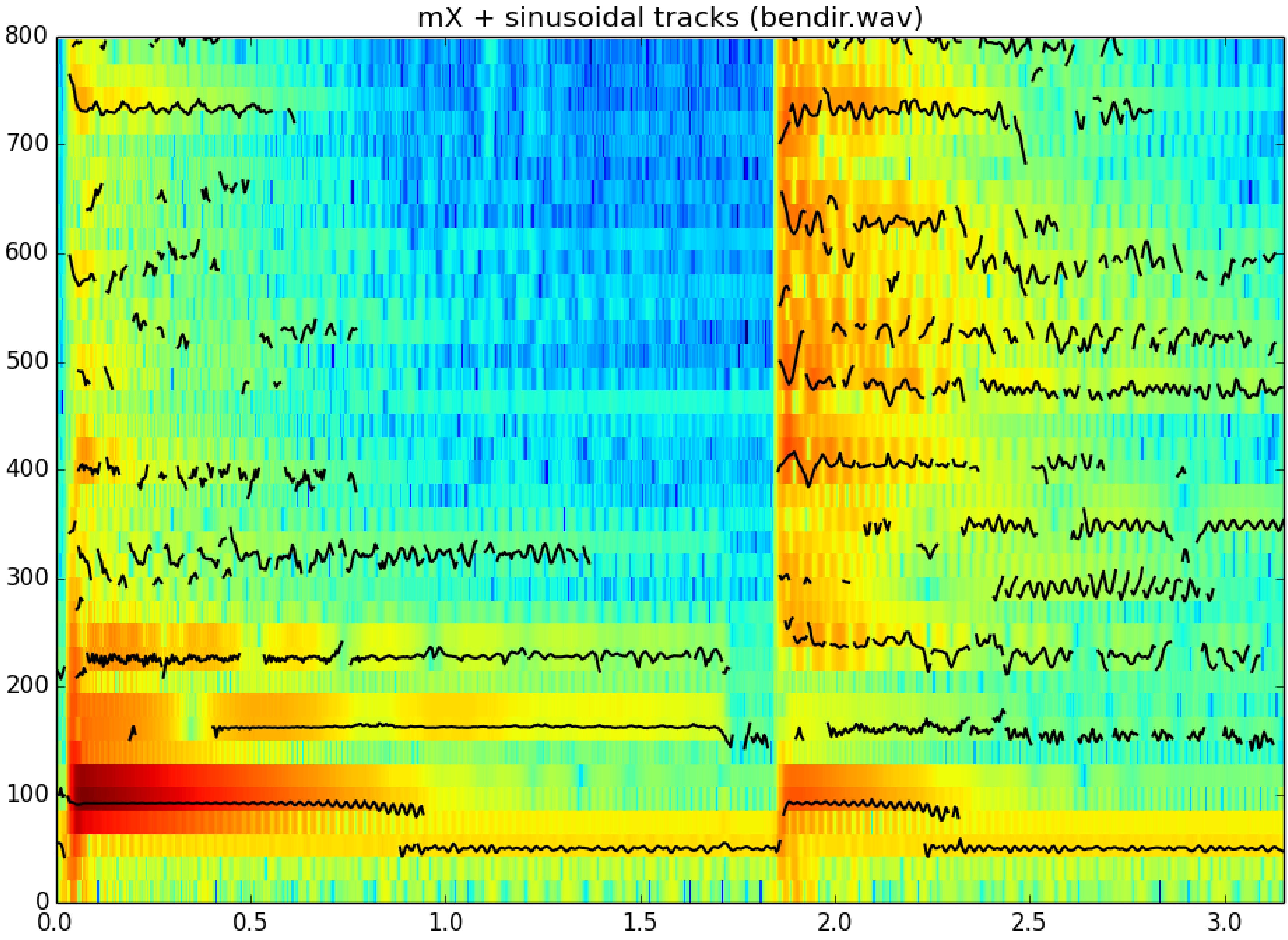
$$ph_p = \sphericalangle X[\hat{k}_p]$$

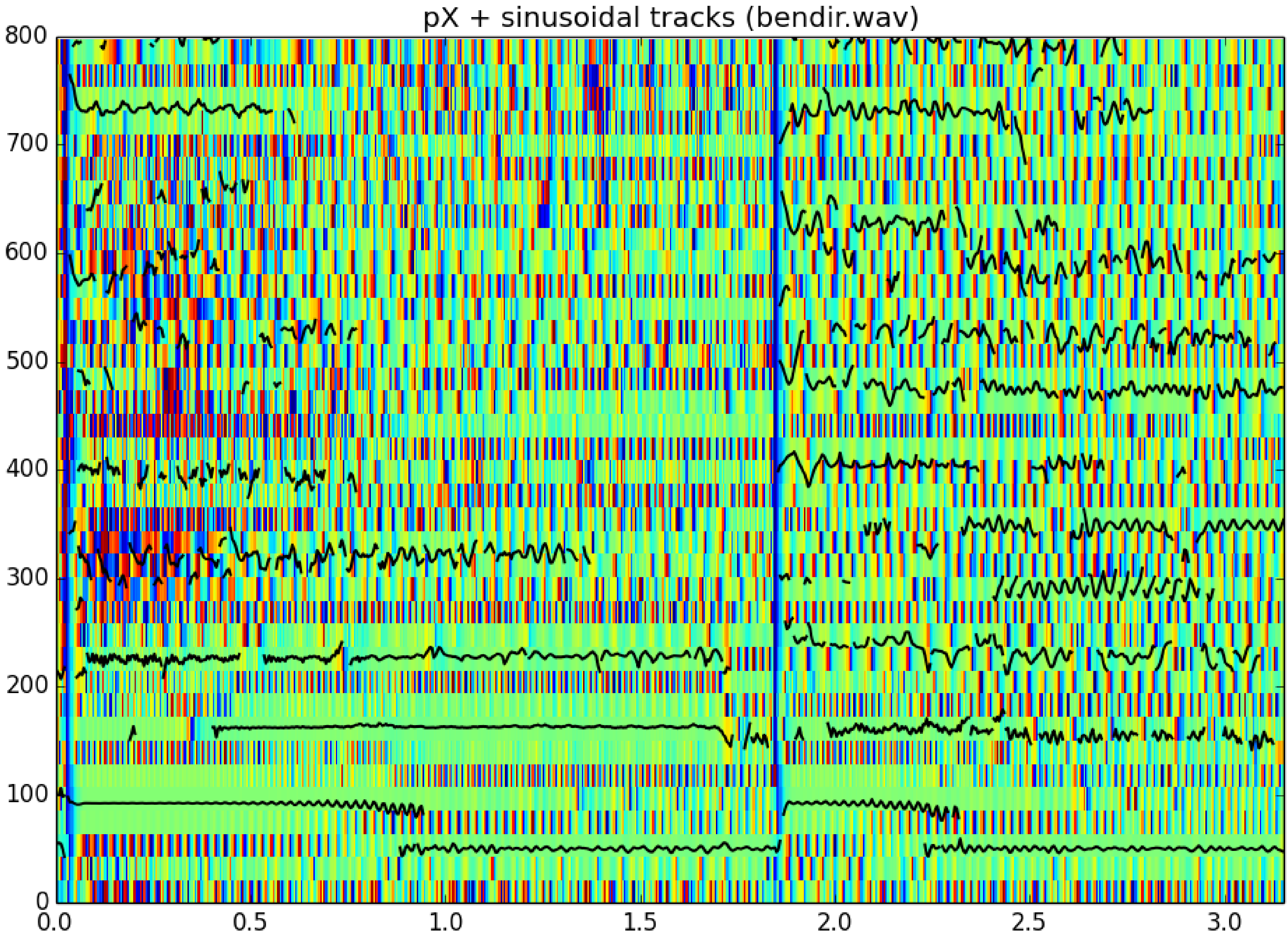
Sinewaves in spectrogram

- Sinusoid $\longrightarrow$ **stable** peak track in spectrogram
- Stability identified by
  - frequency and amplitude in succesive frames
  - phase derivative in time/freq

Condition for a peak f_p of frame l to be part of a track t:

$$f_p[l] = f_t[l] \ \text{ if } \ \left(\left|f_p[l] - f_t[l-1]\right| < \ \textit{threshold}\right)$$
$$\text{and if exists} \ \ f_t[l-2], f_t[l-3], \ldots, f_t[l-L]$$


mX + sinusoidal tracks (bendir.wav)
800
700
600
500
400
300
200
100
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0


pX + sinusoidal tracks (bendir.wav)
800
700
600
500
400
300
200
100
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

References and credits

- More information in: http://en.wikipedia.org/wiki/Sinusoidal_model
- Reference on sinusoidal modeling by Julius O. Smith: https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html
- Sounds from: http://www.freesound.org/people/xserra/packs/13038/
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