3T2: Fourier Transform properties (2 of 2)

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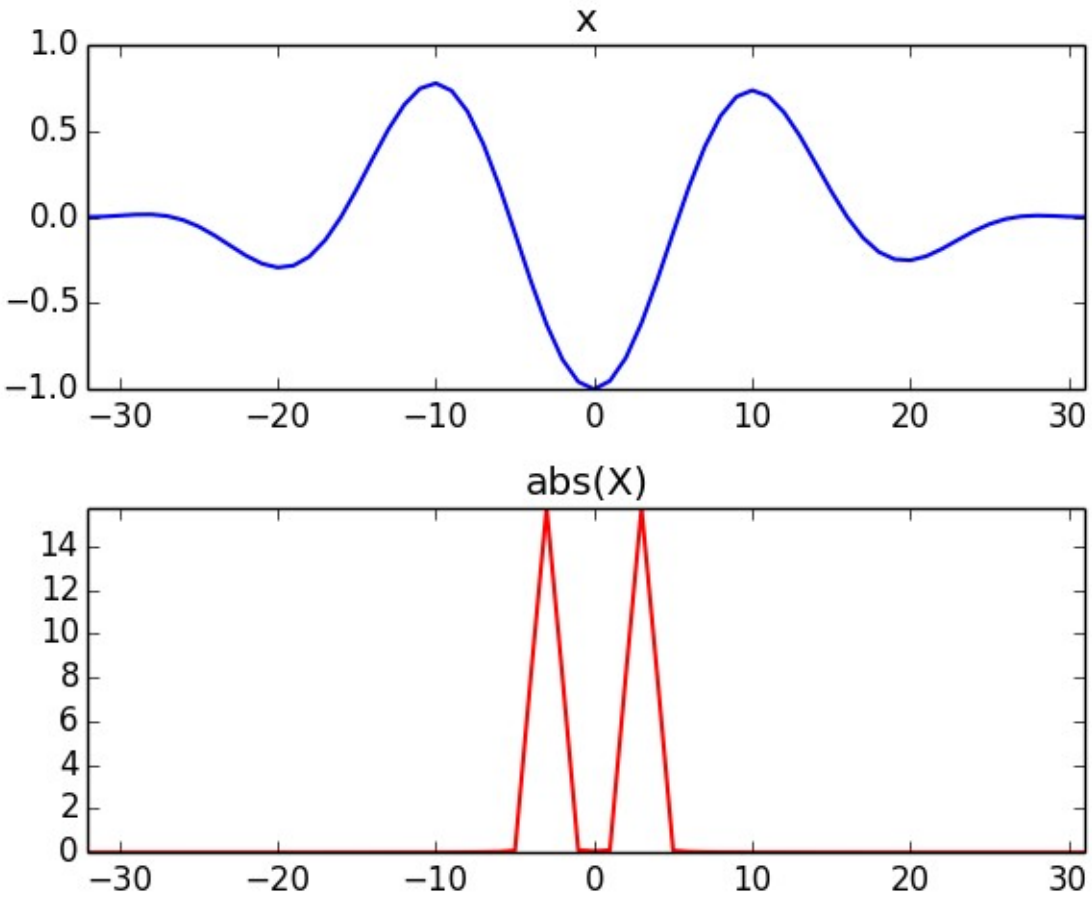
Energy conservation

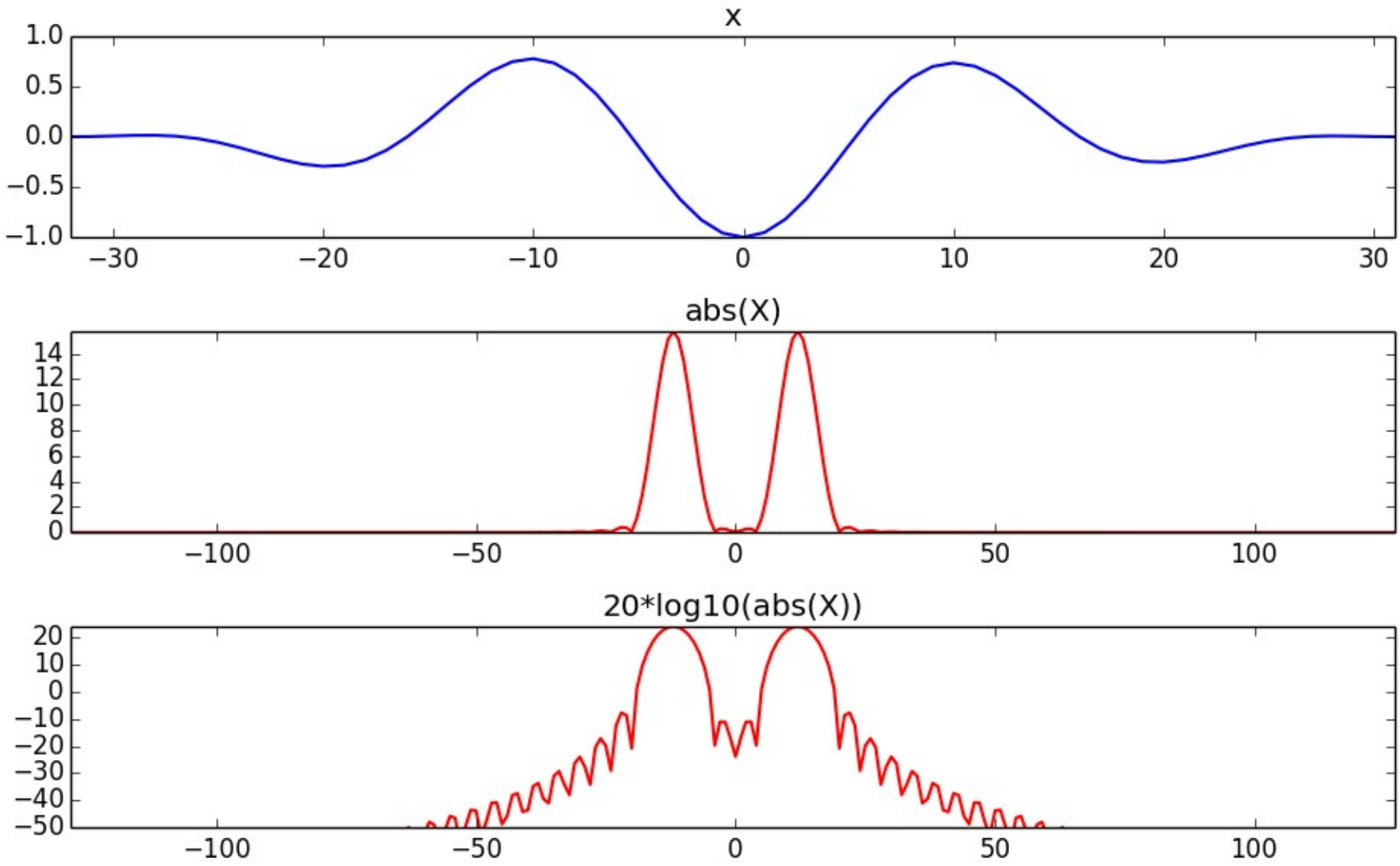
$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2$$



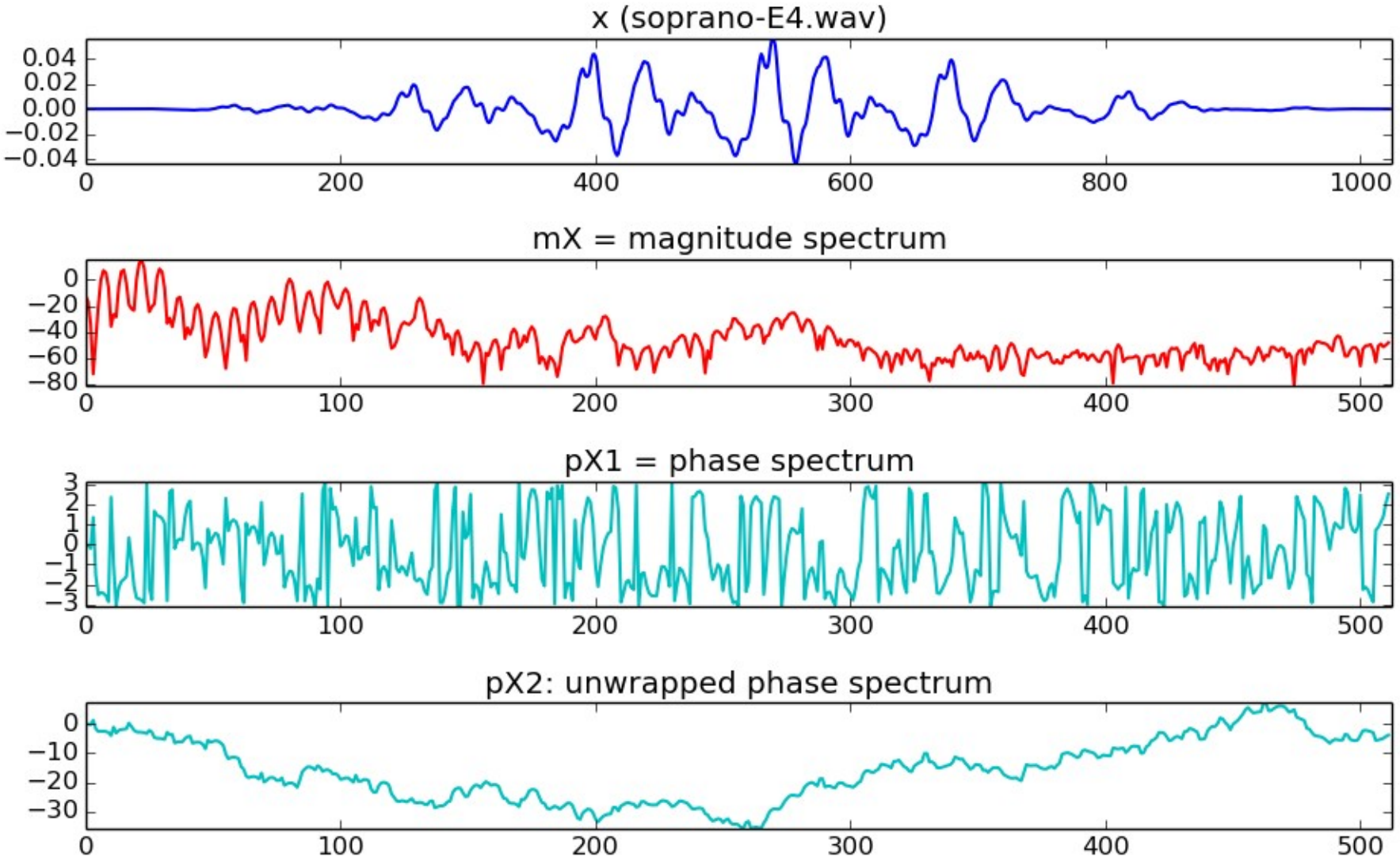
$$\sum_{n=N/2}^{N/2-1} |x[n]|^2 = 11.81182$$

$$\frac{1}{N} \sum_{k=N/2}^{N/2-1} |X[k]|^2 = 11.81182$$

Amplitude in decibels (dB)

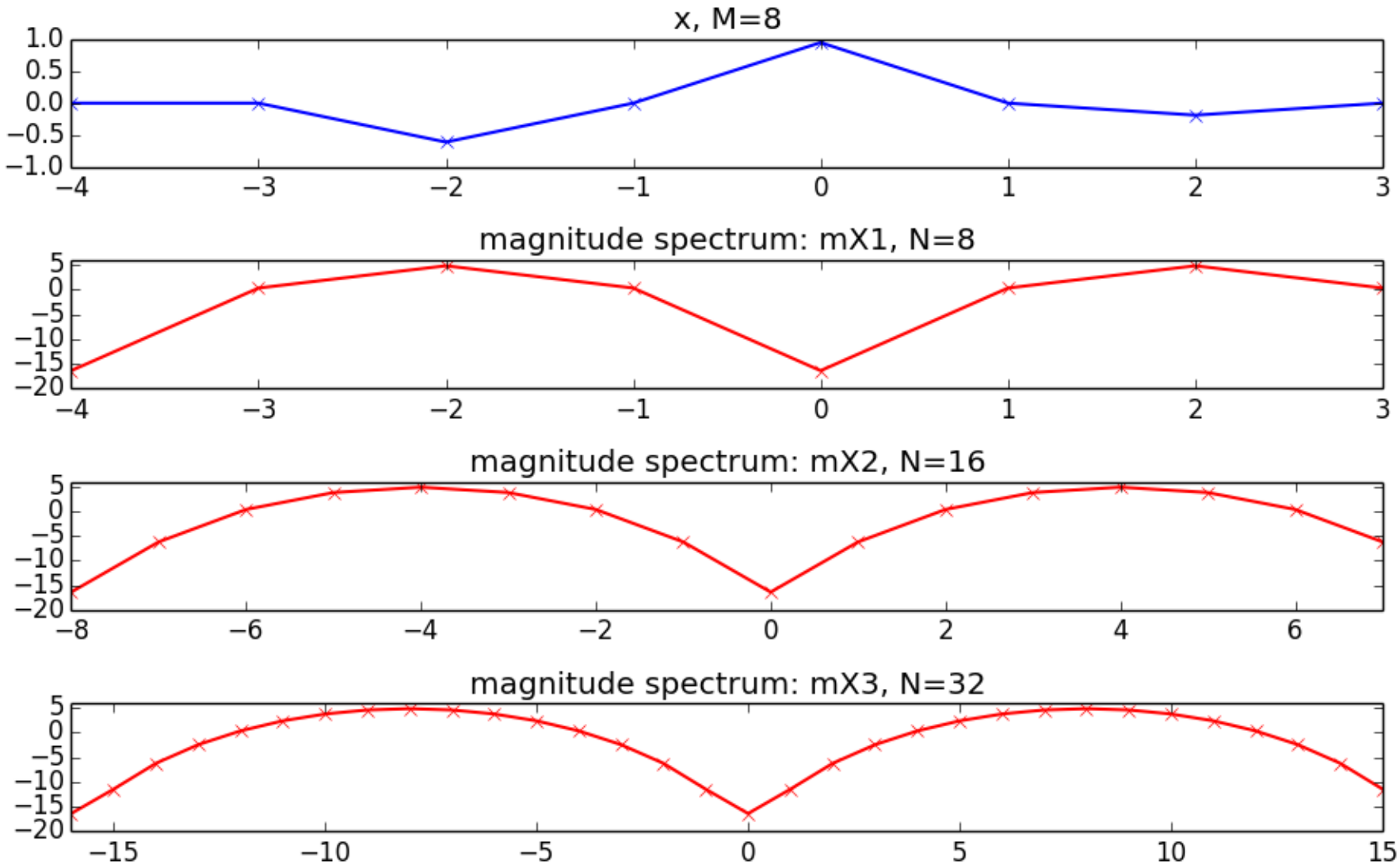


Phase unwrapping



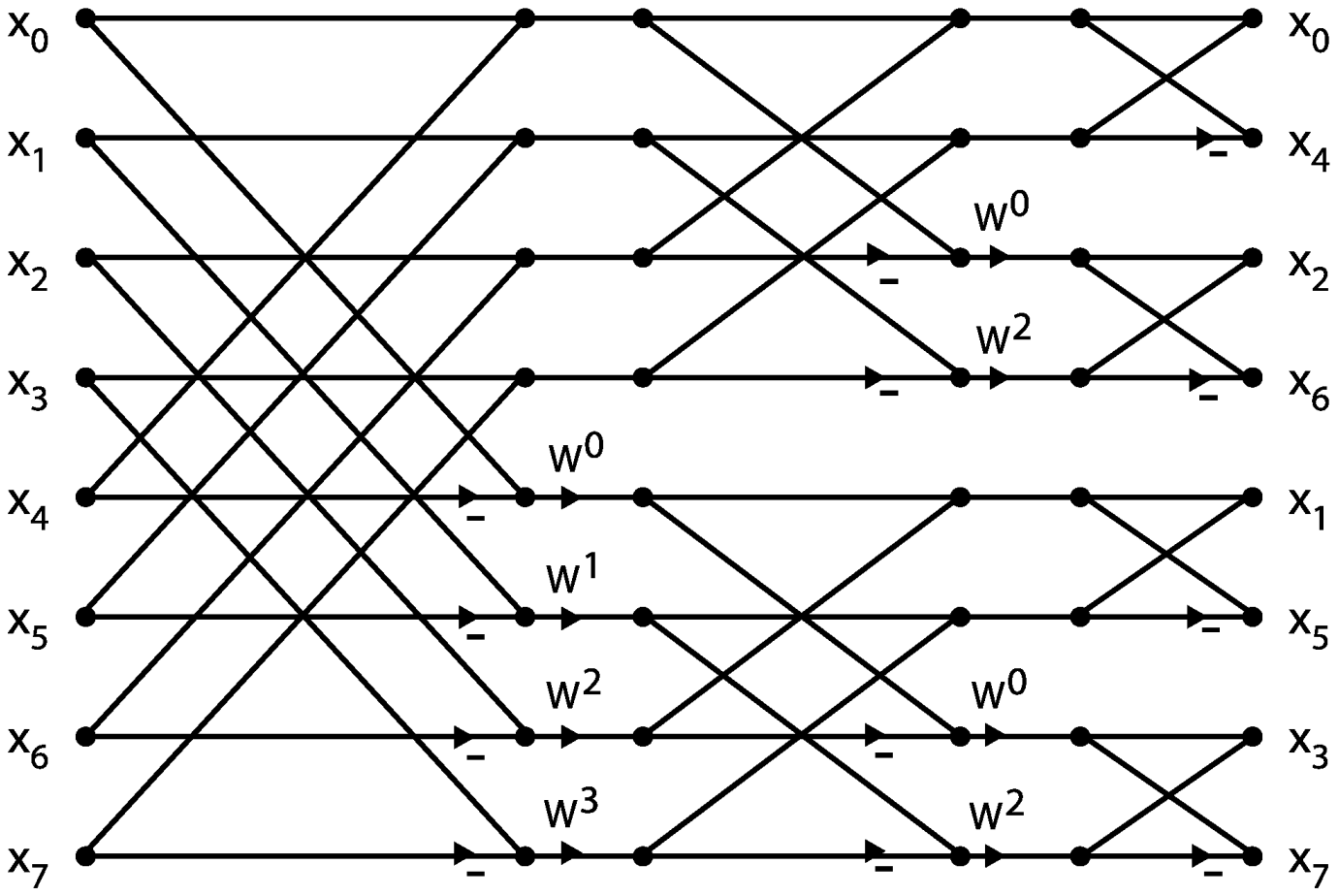
Zero-padding

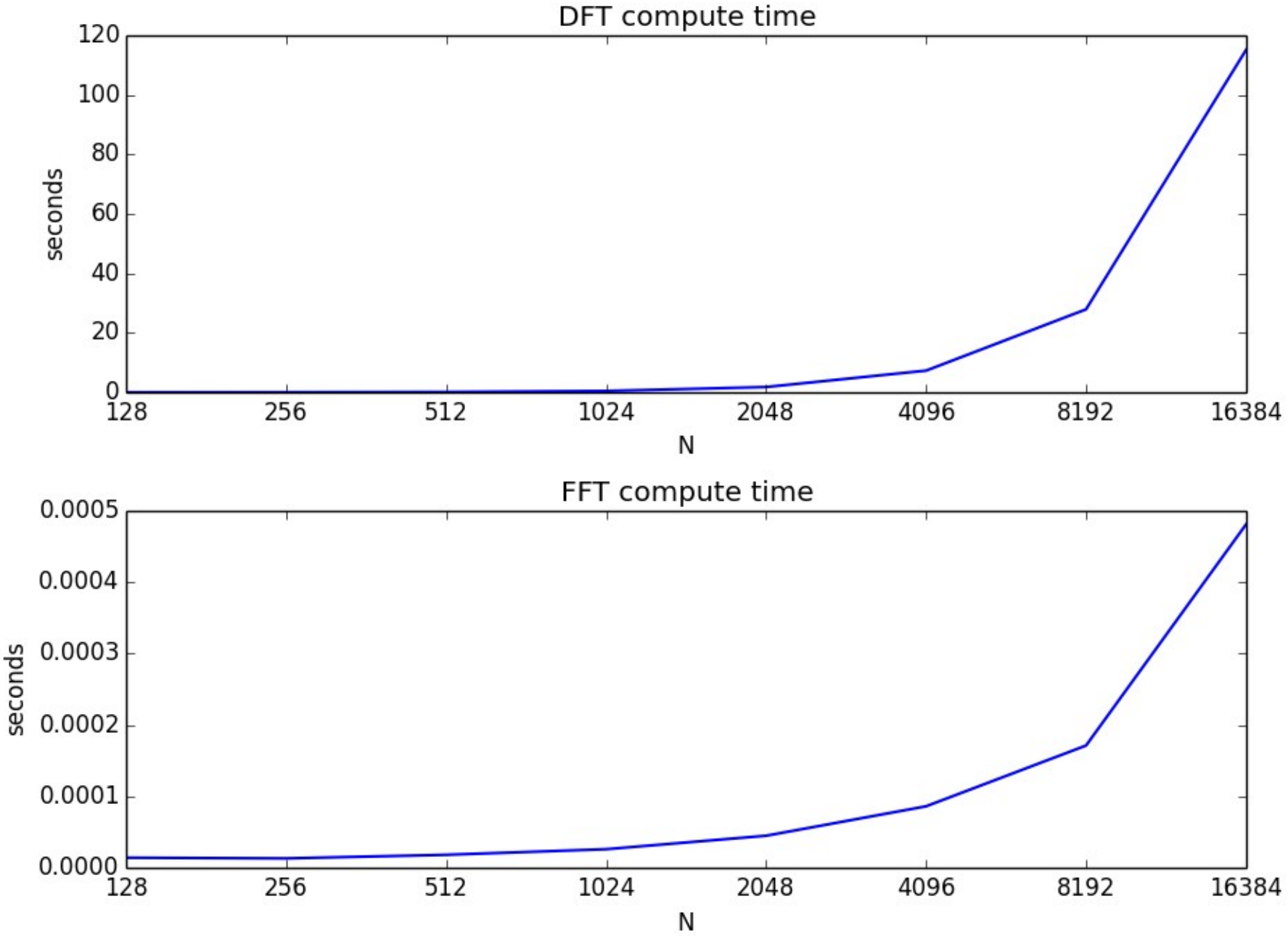
zero padding $\leftrightarrow$ interpolation



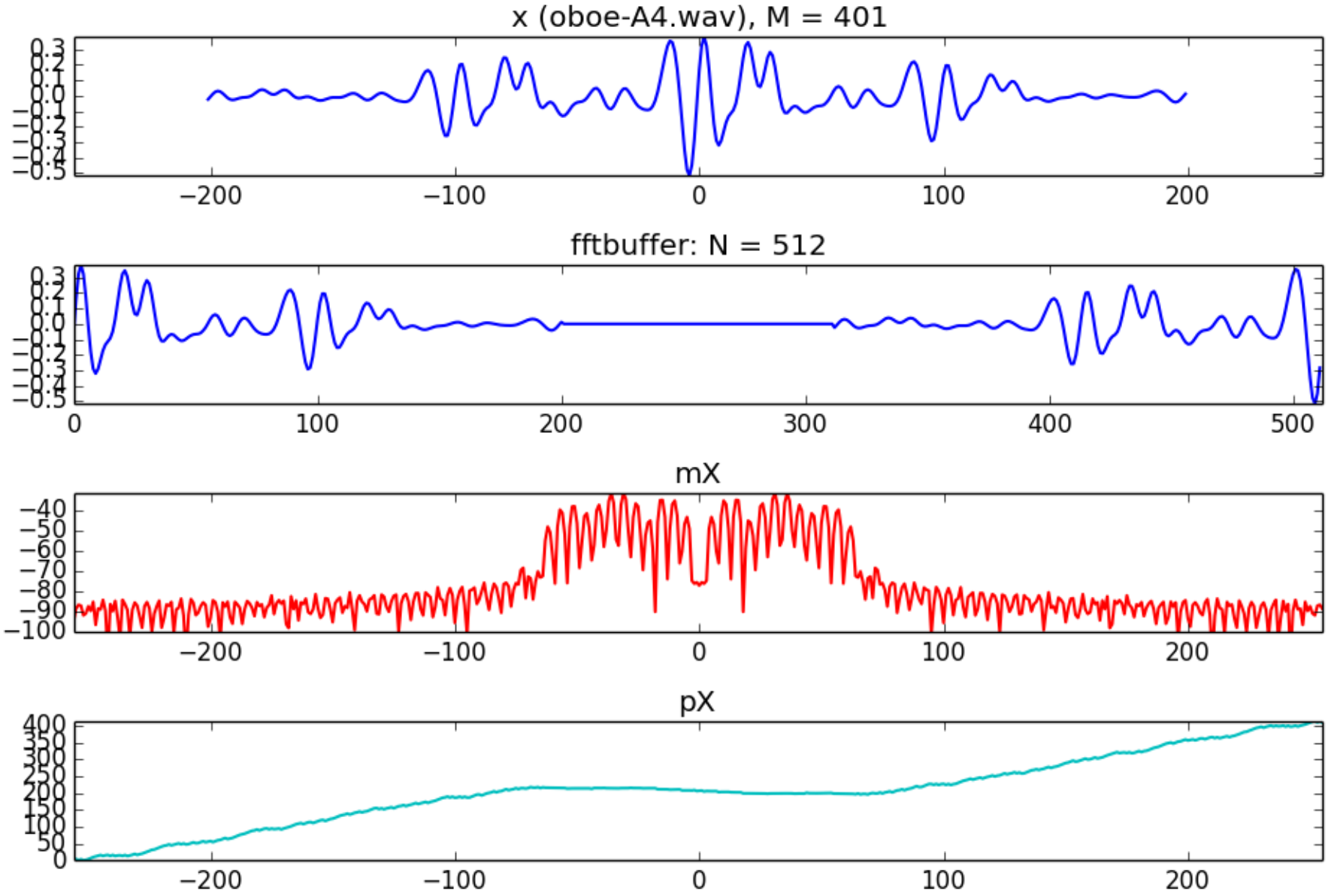
Fast Fourier Transform

Cooley-Tukey algorithm: breaks down recursively the DFT of a power of 2 size into two pieces of size N/2.

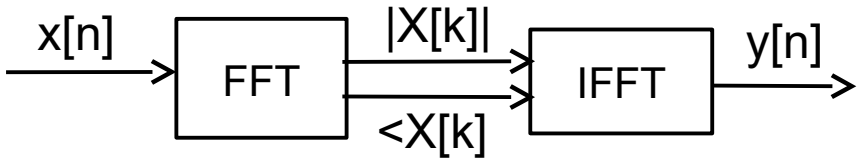


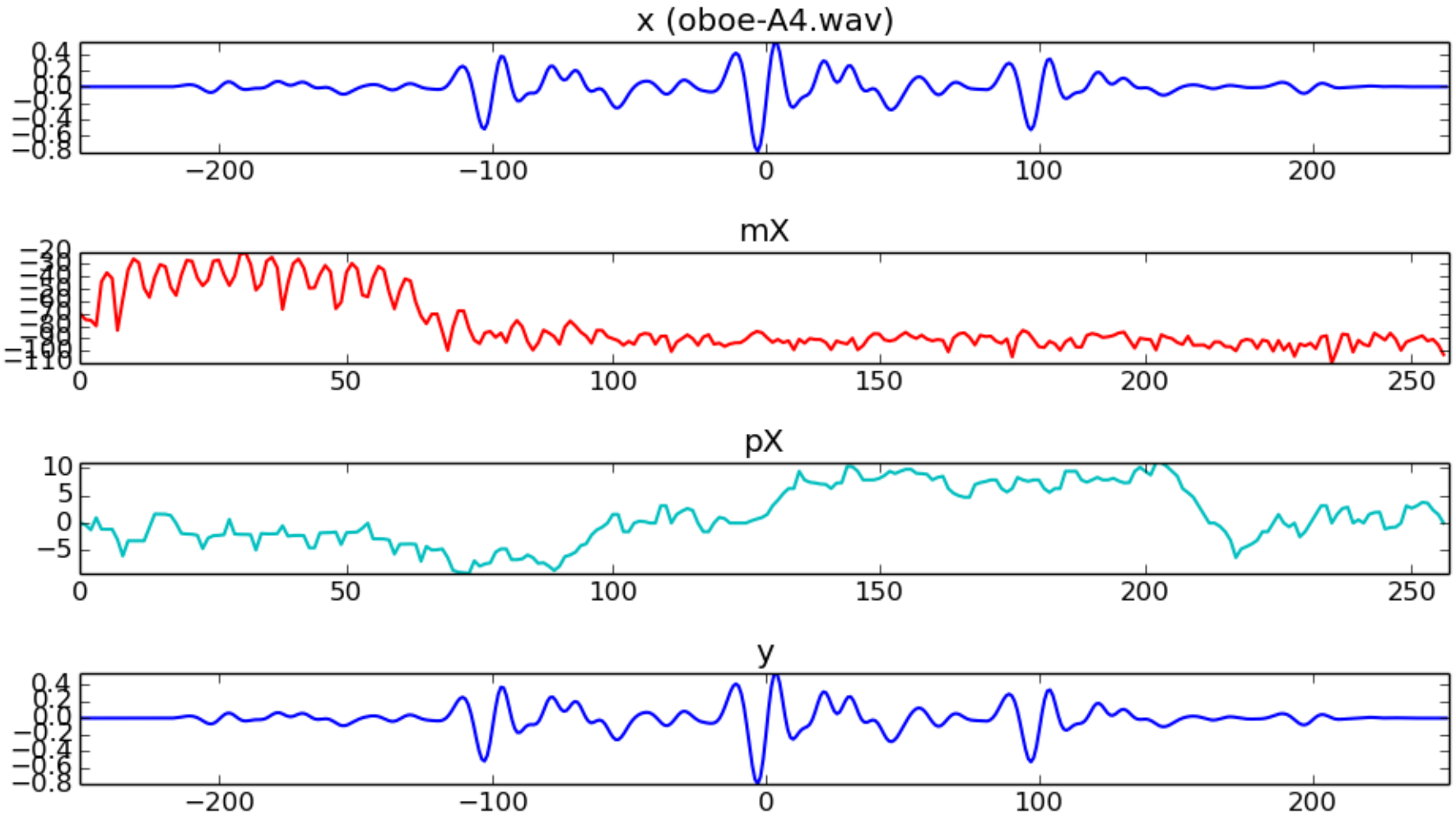

DFT compute time
seconds
120
100
80
60
40
20
0
128
256
512
1024
2048
4096
8192
16384
N
FFT compute time
seconds
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
128
256
512
1024
2048
4096
8192
16384
N

FFT and zero-phase windowing



Analysis/synthesis





References and credits

- More information on:
  https://en.wikipedia.org/wiki/Discrete_Fourier_transform
  https://en.wikipedia.org/wiki/Fast_Fourier_transform
- Sounds from:
  http://www.freesound.org/people/xserra/packs/13038/
- Reference for the DFT by Julius O. Smith:
  https://ccrma.stanford.edu/~jos/mdft/
- Slides released under CC Attribution-Non Commercial-Share Alike license and code under Affero GPL license; available from https://github.com/MTG/sms-tools

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