Evaluate **CAViaR** by Quantile Regression

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VALUE-AT-RISK (VAR) IS A MEASURE OF HOW MUCH A CERTAIN PORTFOLIO CAN LOSE WITHIN A GIVEN TIME PERIOD, FOR A GIVEN CONFIDENCE LEVEL

Current limitation and research direction

1. Particular quantile of future portfolio values conditional on current information
2. Distribution of portfolio returns typically changes over time
3. Challenge is to find a suitable model for time-varying conditional quantiles

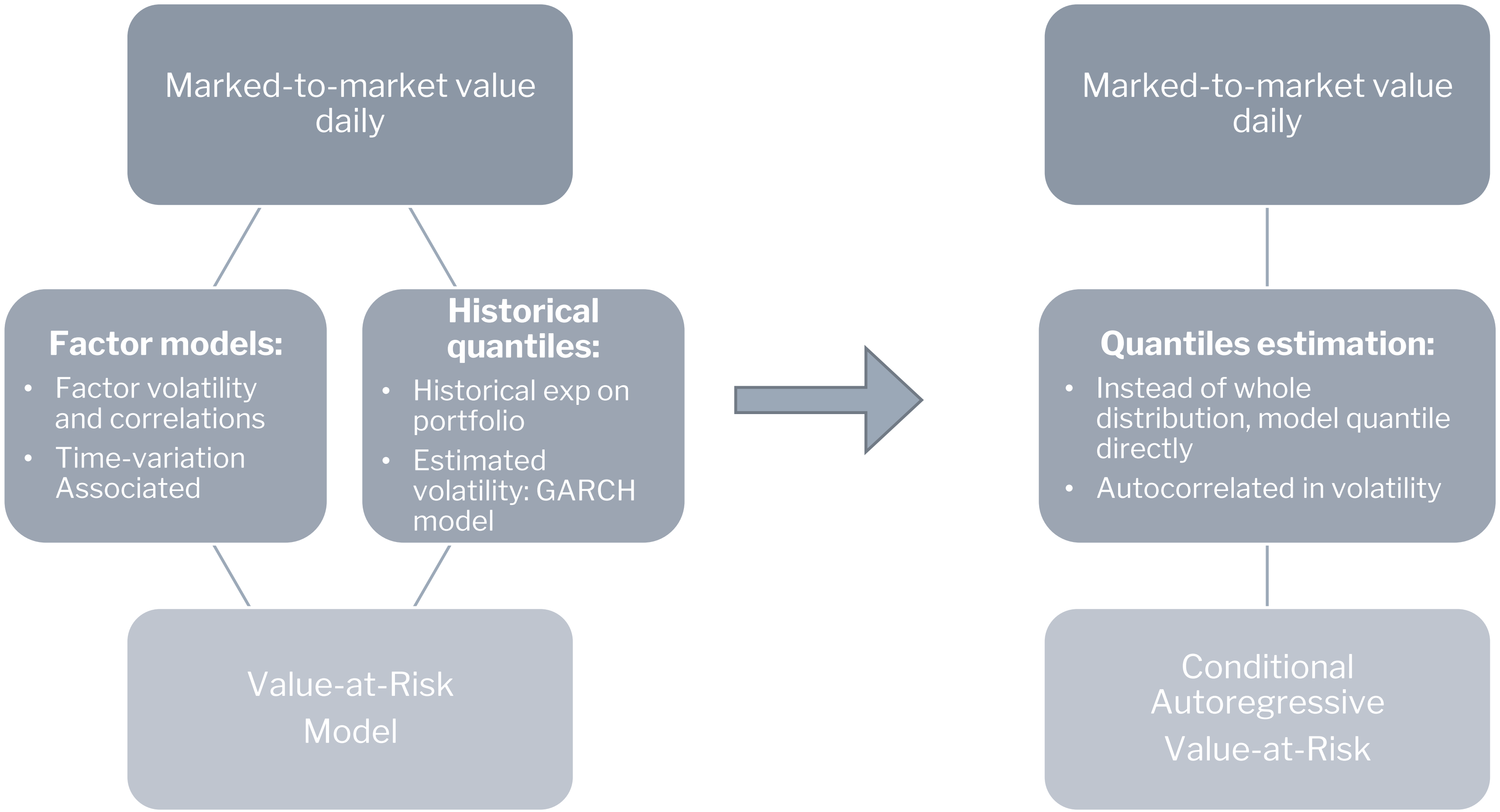
$$\Pr[\, y_t < -VaR_t | \Omega_t \,] = \theta$$

Issue to be addressed:

i. Calculating VaR_t as a function of variables known at time $t-1$ and parameters to be estimated
ii. Loss function and optimization algorithm to estimate the set of unknown parameters
iii. Quality test for the estimate.

Conditional Autoregressive Value at Risk

Var Model | **CAViar Model**



Our research direction

1. Explore the 4 different types of CAViaR models (Engle & Manganelli, 2004)
   i. Adaptive
   ii. Symmetric absolute value
   iii. Asymmetric slope
   iv. IGARCH(1, 1)
2. Fit the CAViaR models to SPY for performance comparison
3. Investigate different ways to fit the CAViaR model and compare statistically
4. Applications

   i. Stop loss in the trading strategy

   ii. Model deployment (displaying on the dashboard)

Literature review

- Symmetric absolute value (SAV)

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 |y_{t-1}|$$

- Asymmetric slope (AS)

$$f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3 \max(y_{t-1}, 0) + \beta_4 \min(y_{t-1}, 0)$$

- Adaptive (ADP)

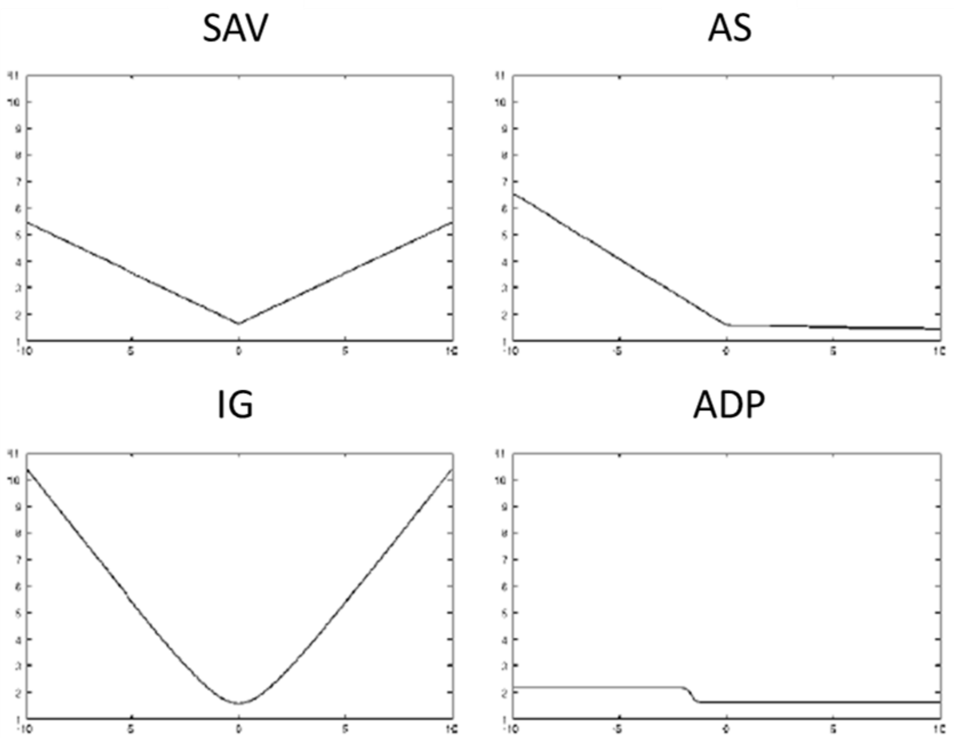
$$f_t(\beta_1) = f_{t-1}(\beta) + \beta_1 \{sigmoid\left(G\left(y_{t-1} - f_{t-1}(\beta)\right)\right) - \theta\}$$

$$where\ sigmoid(Gx) = \frac{1}{1 + \exp(Gx)} \approx I(x < 0)$$

- Indirect GARCH(1, 1) (IG)

$$f_t(\beta) = (\beta_1 + \beta_2 f_{t-1}^2(\beta) + \beta_3 y_{t-1}^2)^{1/2}$$

The News (y_{t-1}) Impact Curve



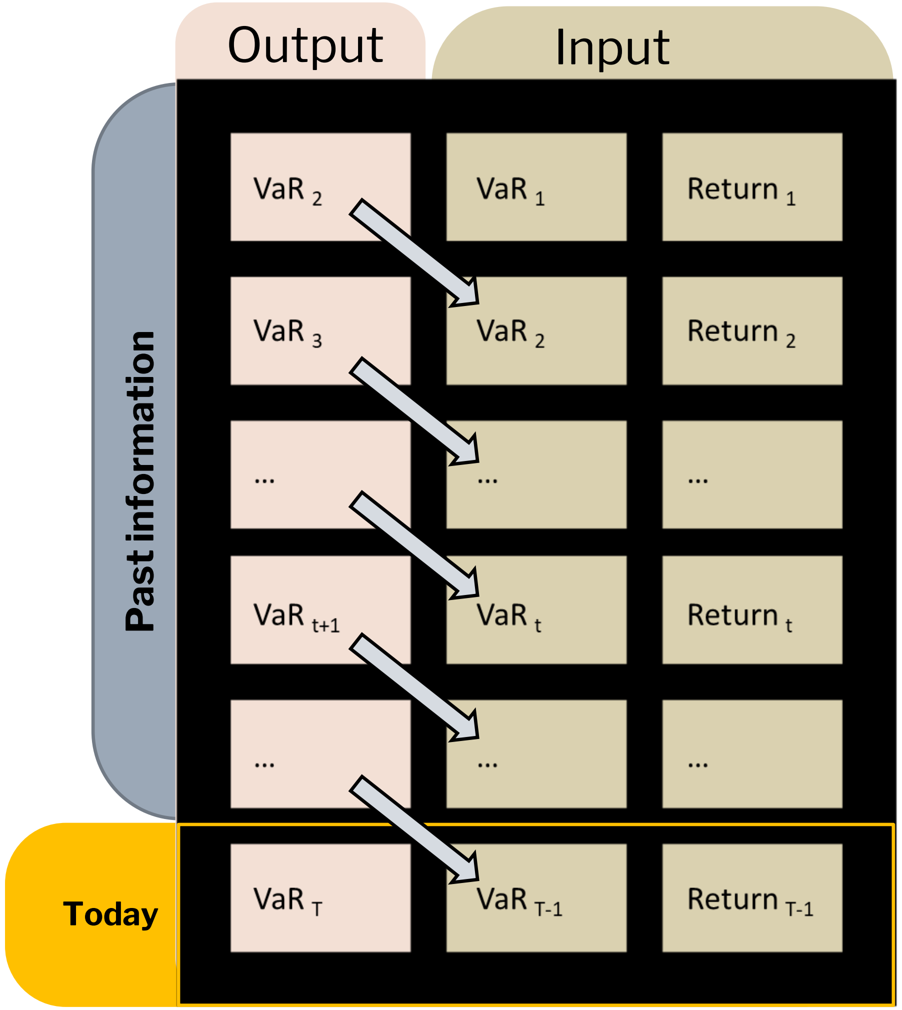
X-axis: y_{t-1}
Y-axis: $f_t(\beta)$
Fixed $f_{t-1}(\beta)$, i.e., the previous VaR estimate

Literature review - Quantile Regression

- Regression Quantiles by Koenker and Bassett (1978)

$$y_t = \mathbf{x}_t'\beta^0 + \varepsilon_{\theta t}, \qquad Quant_\theta(\varepsilon_{\theta t}|\mathbf{x}_t) = 0,$$

- Through quantile regression, to estimate the coefficients to predict the VaR at q% quantile
- So, we can use information yesterday (at time = T-1) to tell what will be the VaR today (at time = T)
- However, we don't know the VaR_t, for all t in {1, …, T}



Literature review - RQ Criterion Minimization (RQ)

$$\min_{\beta}.\sum_{t=1}^{T}\{I[y_t < f_t(\beta)] - \theta\} * (y_t - f_t(\beta)) \ ,$$

where:

-$f_t(\beta)$ = one of the CAViaR function

-$y_t = [\log(\text{Close}_t) - \log(\text{Close}_{t-1})] * 100$

-θ = quantile within [0, 1]

-$I[y_t - f_t(\beta)]$ = indicator function

Optimizer: The limited memory BFGS method (L-BFGS-B in Scipy.optimize)

Literature review - Negative Log-likelihood Minimization (MLE)

$$\min_{\tau,\beta}. T \log \tau + \tau^{-1} \sum_{t}^{T} \{\theta - I[y_t < f_t(\beta)]\} * (y_t - f_t(\beta)),$$

where:

- $y_t \sim SL(f_t(\beta), \tau, \theta)$, i. e., Asymmetric Skewed Laplace Distribution with $\tau > 0$

- $f_t(\beta)$ = one of the CAViaR function

- $y_t = [\log(Close_t) - \log(Close_{t-1})] * 100$

- θ = quantile within [0, 1]

- $I[y_t - f_t(\beta)]$ = indicator function

Optimizer: The limited memory BFGS method (L-BFGS-B in Scipy.optimize)

Why L-BFGS-B

- Non-linear and non-convex problems
- Efficiently estimating the hessian matrix
- Working well with non-differentiable objective function
- Able to handle the box-constraints

- Bounds:
  - $\tau \in (0, \inf)$
  - IGARCH: $\beta_1 \in (0, \inf)$, $\beta_2 \in (0, 1]$ $\beta_3 \in (0, 1]$
    - as negative values of beta may lead to convergence problem
  - $Adaptive$: $\beta_1 \in (-\inf, \inf)$
  - Asymmetric and symmetric: $\beta_1 \in (-\inf, \inf)$ and $\beta_j \in [-1, 1]$ for j =/= 1

Statistics of Beta

β is asymptotically normal:

$$\sqrt{T}(\hat{\beta}_\theta - \beta_\theta) \xrightarrow{d} \mathrm{N}\left(0, \theta(1-\theta)D_T^{-1}A_T D_T^{-1}\right),$$

Where:

- $\hat{A}_T = T^{-1}\theta(1-\theta)\sum_{t=1}^{T}\nabla^T f_t(\beta)\nabla f_t(\beta)$
- $\hat{D}_T = (2T\hat{c}_T)^{-1}\sum_{t=1}^{T} I(|y_t - f_t(\beta)| < \hat{c}_T)\nabla^T f_t(\beta)\nabla f_t(\beta)$
- $\hat{c}_T$ *is the bandwidth for the KNN kernel density estimation*

How to get?

- combining kernel-density estimation with heteroskedasticity-consistent covariance matrix estimation to estimate the asymptotic covariance matrix

How to set the bandwidth $\hat{c}_T$

The bandwidth set by Engle and Manganelli:

- Data independent approach (1999 version):
  - 0.1 for all assets and all confidence levels
- Data dependent approach (2004 version):
  - KNN(k=40) for quantile 0.01
  - KNN(k=60) for quantile 0.05

A rule of thumb used in Rubia & Sanchis-Marco (2013):

- KNN($k = \sqrt{T}$) for all quantile levels
- They implemented both measures and found out the result is not sensitive to the quantile dependent size of k.

Backtesting Methods (1)

Binomial Test

- Compare the observed number of exceptions with the expected one
- Rejected when there are too few/many observations

Traffic Light Test by the Basel Committee

- Compute the P(X $\leq k$ | quantile),

  where X ~ binom(n, quantile)
- Only too many exceptions lead to rejection
  - Return "red" if $\mathrm{P} > 0.99$
    - too many exceptions for a correct VaR model
  - Return "yellow" if $0.95 < \mathrm{P} \leq 0.99$
    - the violation counts are not exceedingly high
  - Return green if $\mathrm{P} \leq 0.95$

Zone	Number of violations	Cummulative probability	Increase in scaling factor
green	0	0.0000	0.0000
	1	0.0000	0.0000
	2	0.0003	0.0000
	3	0.0013	0.0000
	4	0.0046	0.0000
	5	0.0131	0.0000
	6	0.0314	0.0000
	7	0.0650	0.0000
	8	0.1186	0.0000
	9	0.1946	0.0000
	10	0.2909	0.0000
	11	0.4016	0.0000
	12	0.5175	0.0000
	13	0.6293	0.0000
	14	0.7288	0.0000
	15	0.8113	0.0000
	16	0.8750	0.0000
	17	0.9212	0.0000
yellow	18	0.9526	0.3774
	19	0.9729	0.4447
	20	0.9851	0.5120
	21	0.9922	0.5792
	22	0.9961	0.6467
	23	0.9981	0.7143
	24	0.9991	0.7822
	25	0.9996	0.8505
	26	0.9998	0.9192
red	27	0.9999	1.0000

Traffic Light Approach, the Basel Committee (1996)

Backtesting Methods (2)

Kupiec POF Test

- uses a likelihood ratio to test whether the probability of exceptions is synchronized with the probability p implied by the VaR confidence level.

Christoffersen Test

- test whether the sequence of violations $\{Z_t\}$ is iid.

DQ Test

- test for unbiasedness, independent hits, and independence of the quantile estimates.

Hit rate (%)

- simply count the number of violations and divided by the number of observations

Empirical Analysis - Implementation

- Data source: Yahoo Finance
- Size of in-samples: 15 years (3772 obs.)
  - 01/2001 – 12/2015
  - Including dot-com bubble and GFC 2008.
- Size of out-of-samples: 7+ years (1824 obs.)
  - 01/2016 – 03/2023
  - Including the China-US Trade war, COVID19 and the recent SVB bankrupt in the analysis.
- Tickers: SPY

Empirical Analysis - Implementation

SPY	Quantile = 0.01				Quantile = 0.05			
	ADP	AS	SAV	IG	ADP	AS	SAV	IG
Beta 1	-0.7704	-0.0735	-0.1183	0.2712	-0.6552	-0.0457	-0.0373	0.0430
S.E.	0.1197	0.0110	0.0244	0.0924	0.0413	0.0098	0.0462	0.0295
Pval.	0.0000	0.0000	0.0000	0.0017	0.0000	0.0000	0.2094	0.0725
Beta 2		0.9214	0.8773	0.8782		0.9265	0.8943	0.9115
S.E.		0.0088	0.0137	0.0299		0.0231	0.0559	0.0213
Pval.		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
Beta 3		-0.0403	-0.2742	0.5776		0.0034	-0.2003	0.2405
S.E.		0.0537	0.0255	0.2135		0.0519	0.0727	0.0448
Pval.		0.2262	0.0000	0.0034		0.4738	0.0029	0.0000
Beta 4		0.3139				0.2311		
S.E.		0.0343				0.0707		
Pval.		0.0000				0.0005		

Rejected if p ≤ 0.01

Rejected if p ≤ 0.05

Accepted

Empirical Analysis - Implementation

SPY		Quantile = 0.01				Quantile = 0.05			
		ADP	AS	SAV	IG	ADP	AS	SAV	IG
In	Hit (%)	0.0093	0.0101	0.0101	0.0103	0.0485	0.0504	0.0501	0.0504
Out	Hit (%)	0.0109	0.0153	0.0153	0.0126	0.0448	0.0366	0.0394	0.0405

Adaptive gives the closest out-of-sample VaR estimate according to the Hit(%).

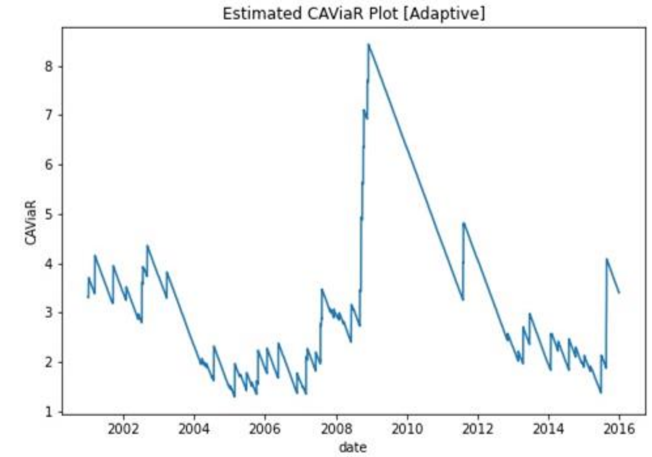
Empirical Analysis - Implementation

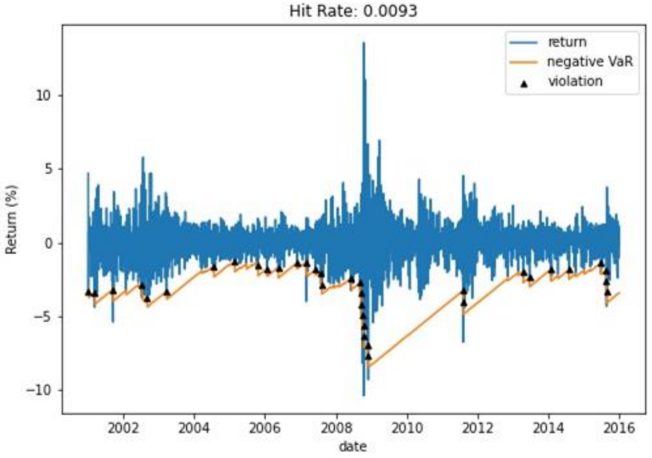
SPY		Quantile = 0.01				Quantile = 0.05			
		ADP	AS	SAV	IG	ADP	AS	SAV	IG
In	Binomial	0.7431	0.9347	0.9347	0.8058	0.7087	0.9107	0.9702	0.9107
	Traffic Light								
	Kupiec POF	0.6523	0.9635	0.9635	0.8350	0.6742	0.9168	0.9762	0.9168
	I.I.D. Test	0.0433	0.4007	0.0606	0.0073	0.4924	0.8440	0.6096	0.6344
	DQ Test	0.0000	0.6353	0.0000	0.0000	0.3845	0.7176	0.5805	0.2811
Out	Binomial	0.6378	0.0330	0.0330	0.2877	0.3342	0.0073	0.0363	0.0604
	Traffic Light								
	Kupiec POF	0.6922	0.0343	0.0343	0.2872	0.3025	0.0060	0.0305	0.0532
	I.I.D. Test	0.0009	0.4483	0.0736	0.2934	0.0384	0.3454	0.2234	0.0371
	DQ Test	0.0000	0.6353	0.0000	0.0000	0.3845	0.7176	0.5805	0.2811

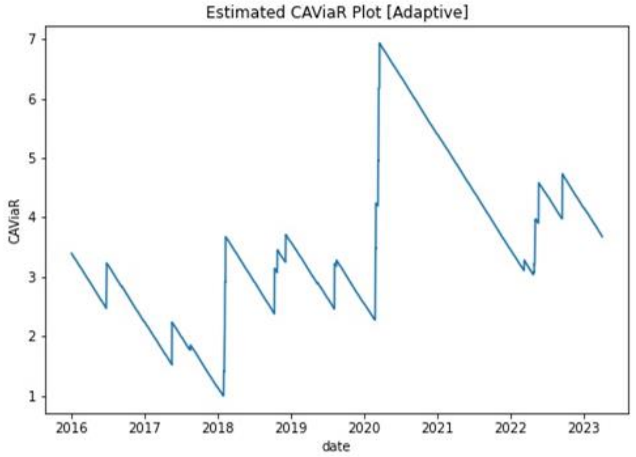
Rejected if $p \leq 0.05$

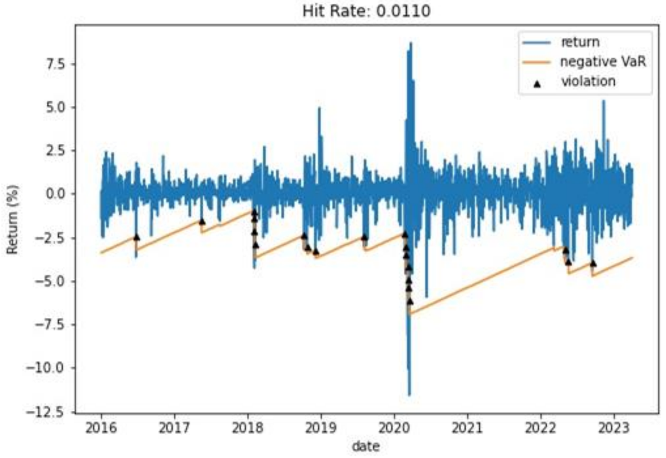
Rejected if $p \leq 0.01$

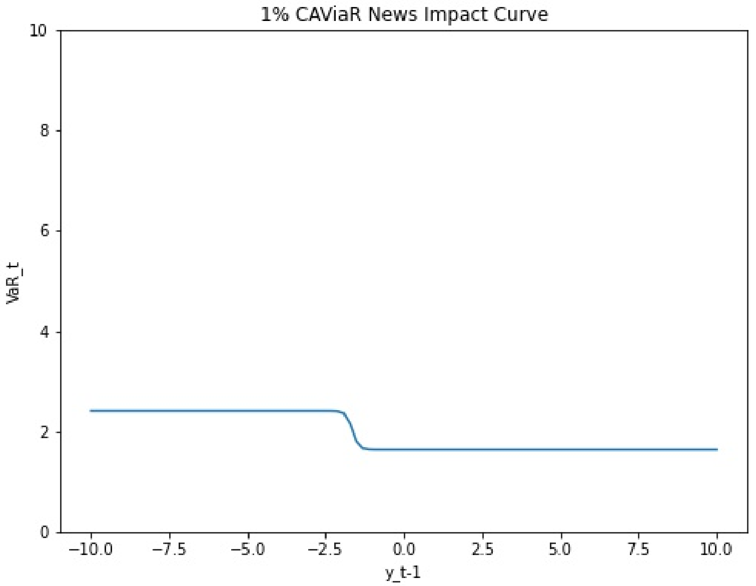
Empirical Analysis - Implementation





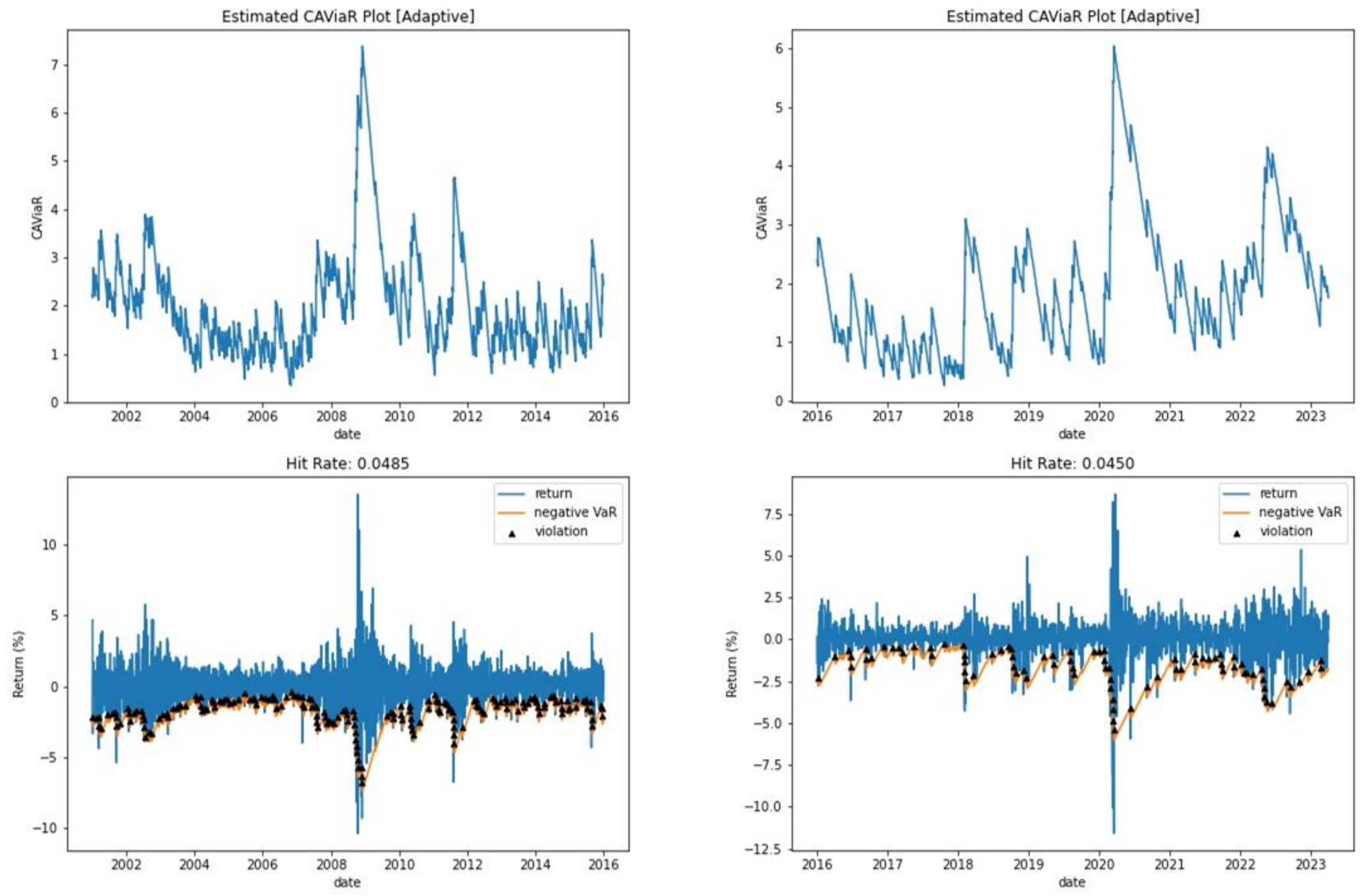


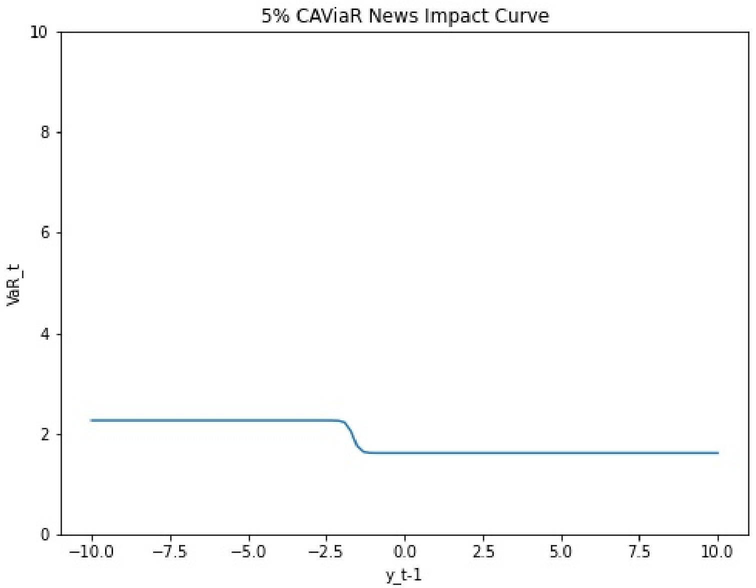




Ticker: SPY
Model: Adaptive
Quantile = 0.01
Minimization approach:
RQ Criterion Minimization

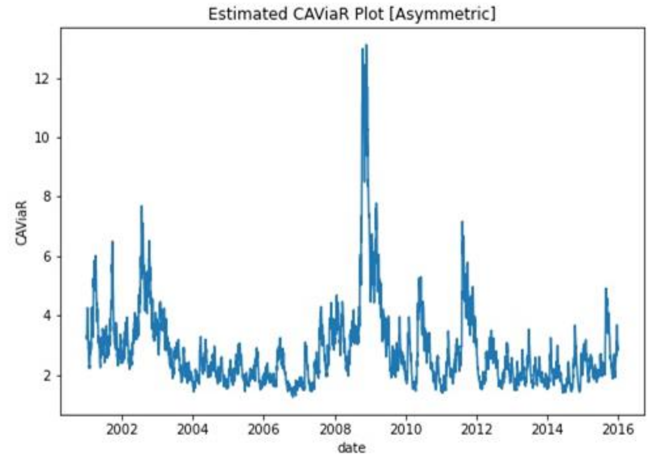
Empirical Analysis - Implementation

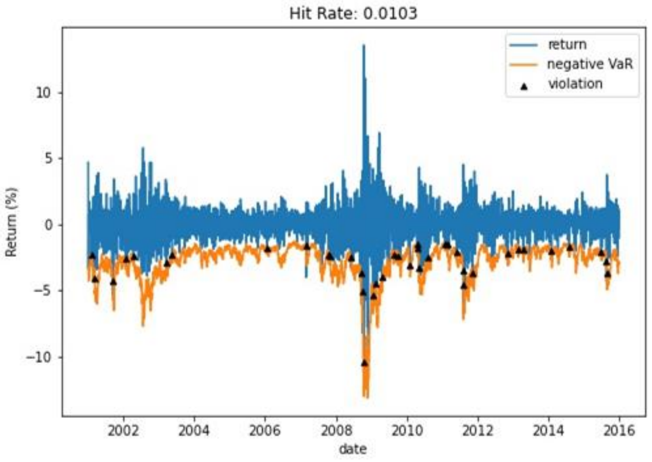


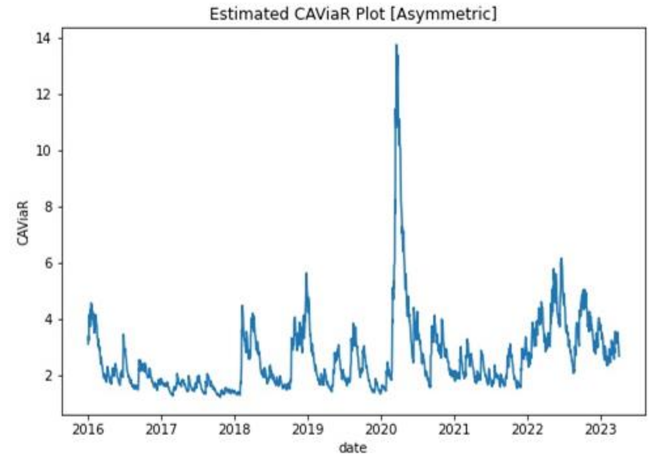


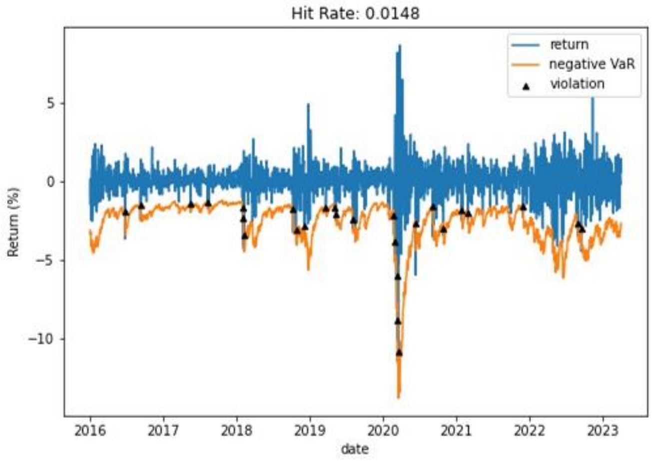
Ticker: SPY
Model: Adaptive
Quantile = 0.05
Minimization approach:
RQ Criterion Minimization

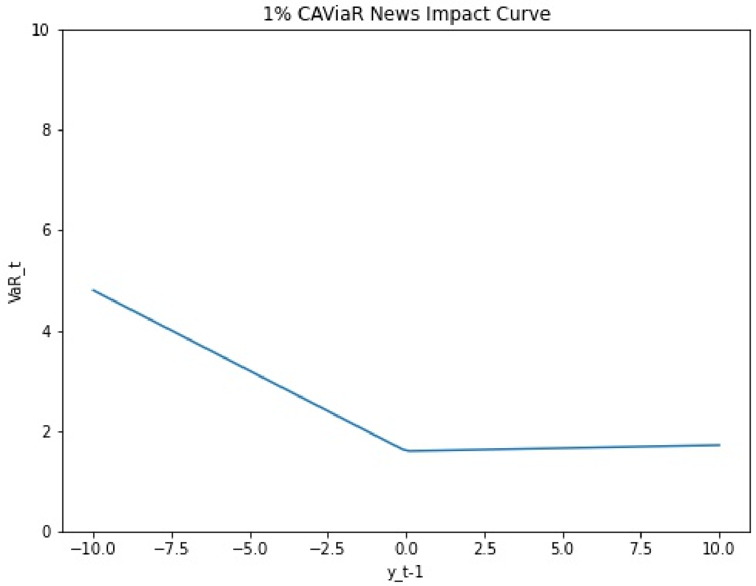
Empirical Analysis - Implementation





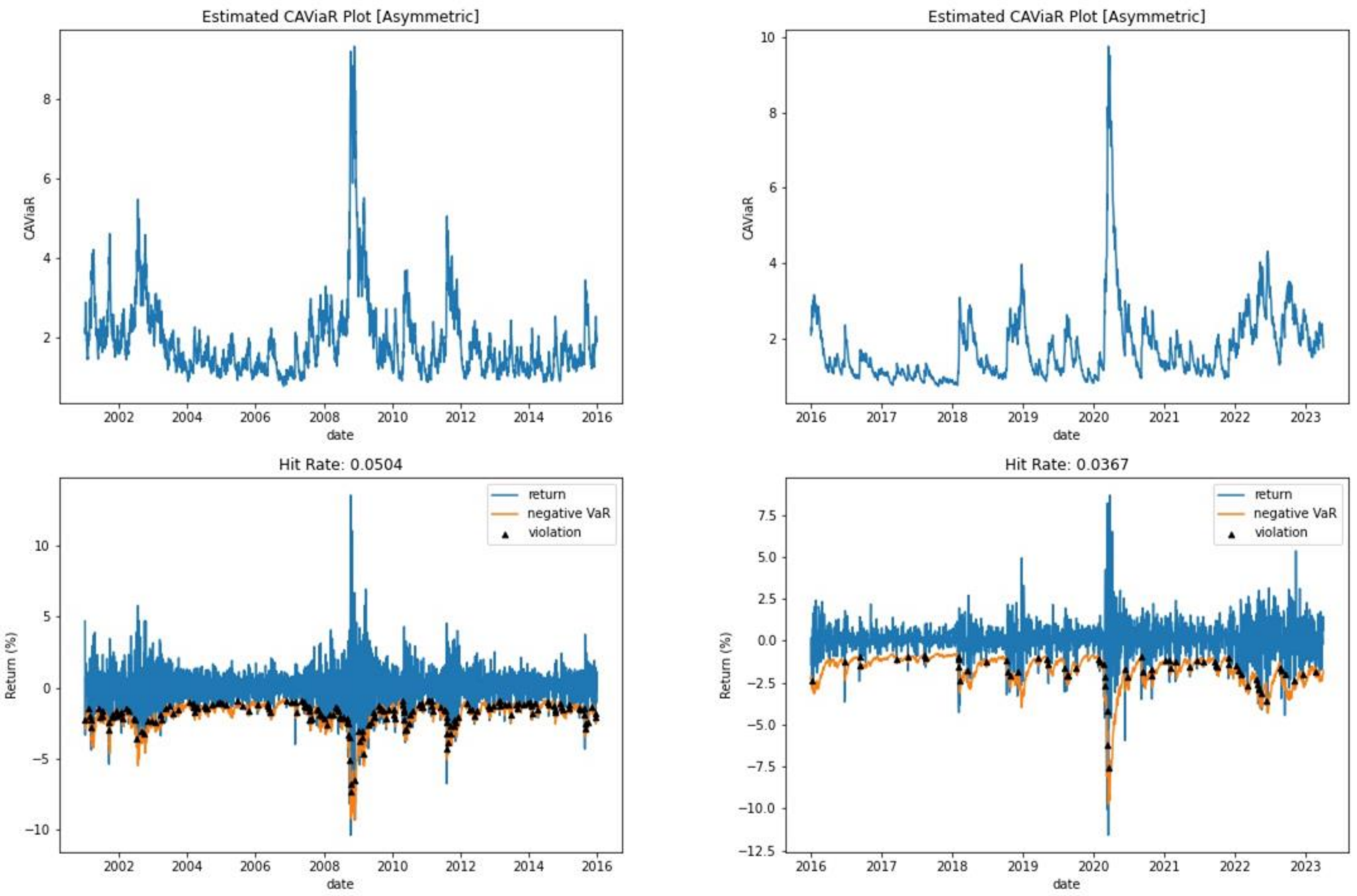


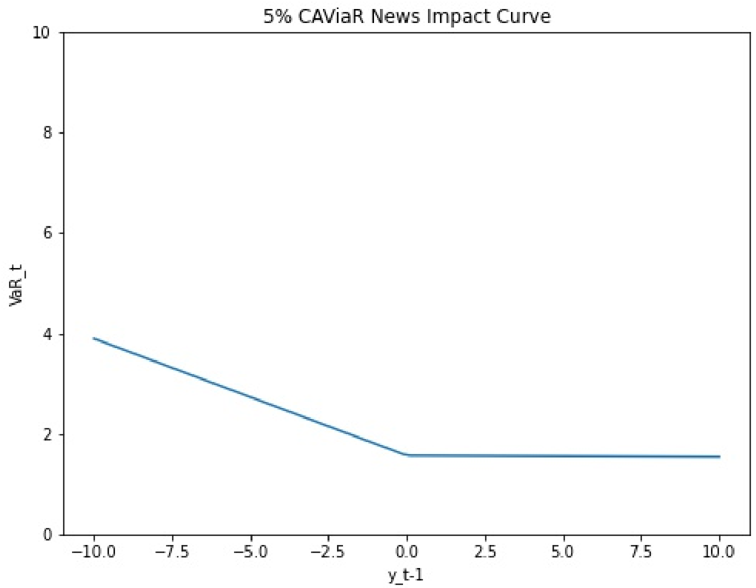




Ticker: SPY
Model: Asymmetric
Quantile = 0.01
Minimization approach:
RQ Criterion Minimization

Empirical Analysis - Implementation





Ticker: SPY
Model: Asymmetric
Quantile = 0.05
Minimization approach:
RQ Criterion Minimization

Experiment (1) – RQ v.s. MLE

30 times fitting with different random seeds

We compared the mean difference of each statistic by the Mann–Whitney U test

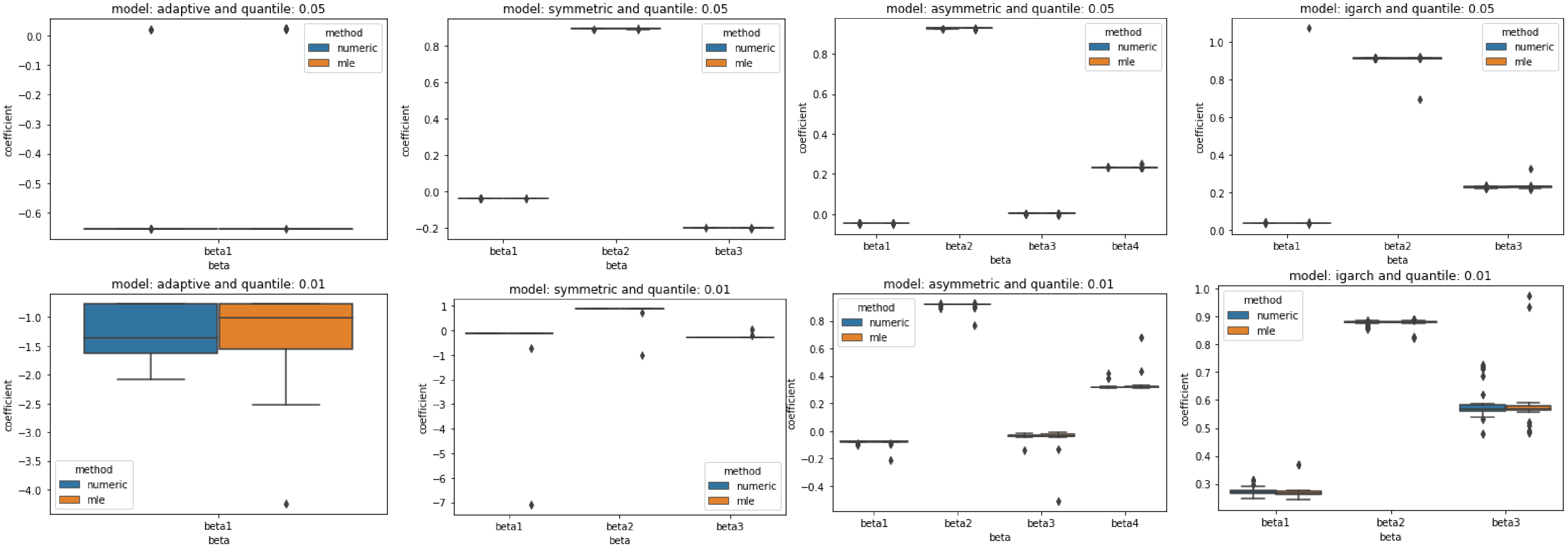
Doesn't show an apparent difference in performance except 0.05 adaptive

	Quantile = 0.05				Quantile = 0.01			
	adaptive	symmetric	asymmetric	igarch	adaptive	symmetric	asymmetric	igarch
loss	rejected	accepted	accepted	accepted	accepted	accepted	accepted	accepted
mle's	0.131943	0.126727	0.12414	0.126657	0.04029	0.035271	0.033553	0.034229
RQ's	0.130347	0.126727	0.12414	0.12625	0.039905	0.034448	0.033489	0.034221
hit_rate_in	rejected	accepted	rejected	rejected	accepted	rejected	accepted	accepted
mle's	0.049258	0.050124	0.050159	0.049576	0.009244	0.009897	0.01003	0.010277
RQ's	0.047526	0.050124	0.050247	0.050256	0.009288	0.010065	0.010074	0.010322
hit_rate_out	rejected	accepted	accepted	accepted	accepted	accepted	accepted	accepted
mle's	0.055428	0.039474	0.036842	0.04015	0.010216	0.015058	0.014675	0.012975
RQ's	0.042361	0.039474	0.036824	0.04057	0.009923	0.015351	0.014839	0.012975

Rejected when p ≤ 0.05

Using the random start approach to minimize the loss

Experiment (1) – RQ v.s. MLE



Beta doesn't have too much difference visually.

Experiment (2) - Best Start v.s. Random Start

Best start: select 5 out of 5000 then optimize the best 5 and pick the best one based on the RQ criterion

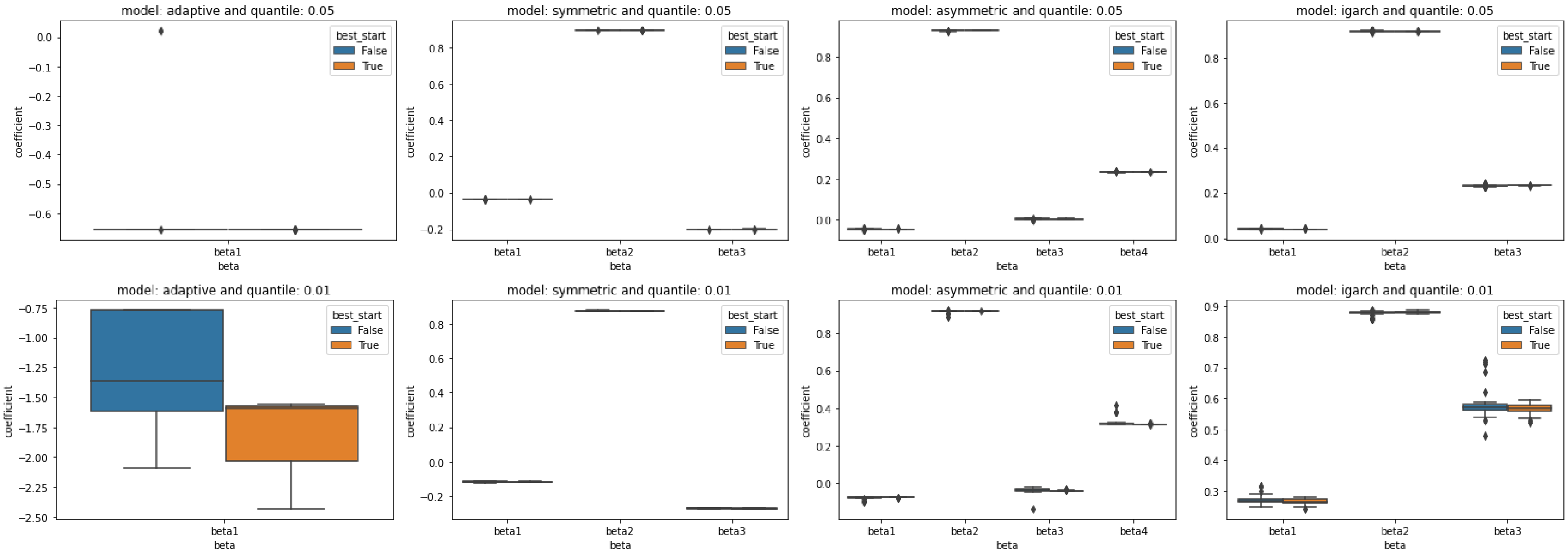
Random start: start with a random seed and optimize

	Quantile = 0.05				Quantile = 0.01			
	adaptive	symmetric	asymmetric	igarch	adaptive	symmetric	asymmetric	igarch
loss	accepted	rejected	rejected	rejected	rejected	rejected	rejected	accepted
Random's	0.130347	0.126727	0.12414	0.12625	0.039905	0.034448	0.033489	0.034221
Best's	0.128174	0.126727	0.124139	0.126249	0.038675	0.034448	0.033471	0.034215
hit_rate_in	accepted	accepted	accepted	accepted	rejected	accepted	accepted	accepted
Random's	0.047526	0.050124	0.050247	0.050256	0.009288	0.010065	0.010074	0.010322
Best's	0.048515	0.050124	0.050203	0.050203	0.008952	0.010039	0.010074	0.010277
hit_rate_out	accepted	accepted	rejected	accepted	rejected	accepted	rejected	accepted
Random's	0.042361	0.039474	0.036824	0.04057	0.009923	0.015351	0.014839	0.012975
Best's	0.044956	0.039474	0.036732	0.04057	0.008973	0.015351	0.015278	0.013012

Rejected when p ≤ 0.05

Using RQ's approach to minimize the loss

Experiment (2) - Best Start v.s. Random Start



Beta doesn’t have too much difference visually except adaptive (0.01)

Experiment (3) - Bounded v.s. Unbounded

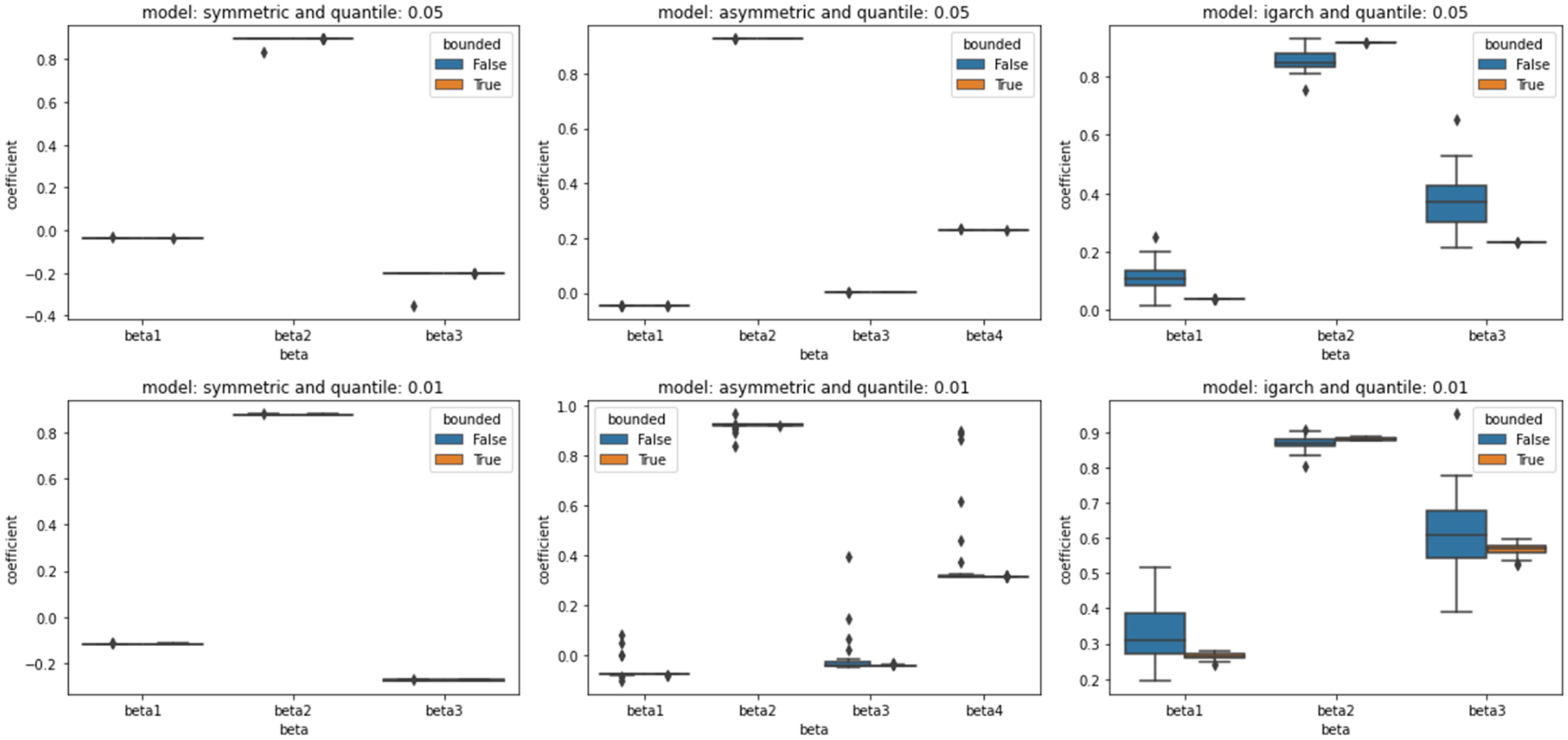
	Quantile = 0.05			Quantile = 0.01		
	symmetric	asymmetric	igarch	symmetric	asymmetric	igarch
loss	rejected	accepted	rejected	accepted	rejected	rejected
unbounded's loss	0.126788	0.124139	0.127106	0.034448	0.040231	0.034337
bounded's loss	0.126727	0.124139	0.126249	0.034448	0.033471	0.034215
hit_rate_in	accepted	rejected	accepted	accepted	accepted	accepted
unbounded's hit_rate_in	0.050424	0.050283	0.049867	0.010092	0.02158	0.010048
bounded's hit_rate_in	0.050124	0.050203	0.050203	0.010039	0.010074	0.010277
hit_rate_out	rejected	rejected	accepted	rejected	rejected	accepted
unbounded's hit_rate_out	0.039616	0.036749	0.040329	0.015342	0.030685	0.013096
bounded's hit_rate_out	0.039474	0.036732	0.04057	0.015351	0.015278	0.013012

Rejected when p ≤ 0.05

Using RQ's approach with best start to minimize the loss

As the adaptive specification is unbounded anyway.

Experiment (3) - Bounded v.s. Unbounded



Applications

Deploying in the Trading Strategy

Set everyday **-VaR** as the stop loss

Transaction cost: 0.2%

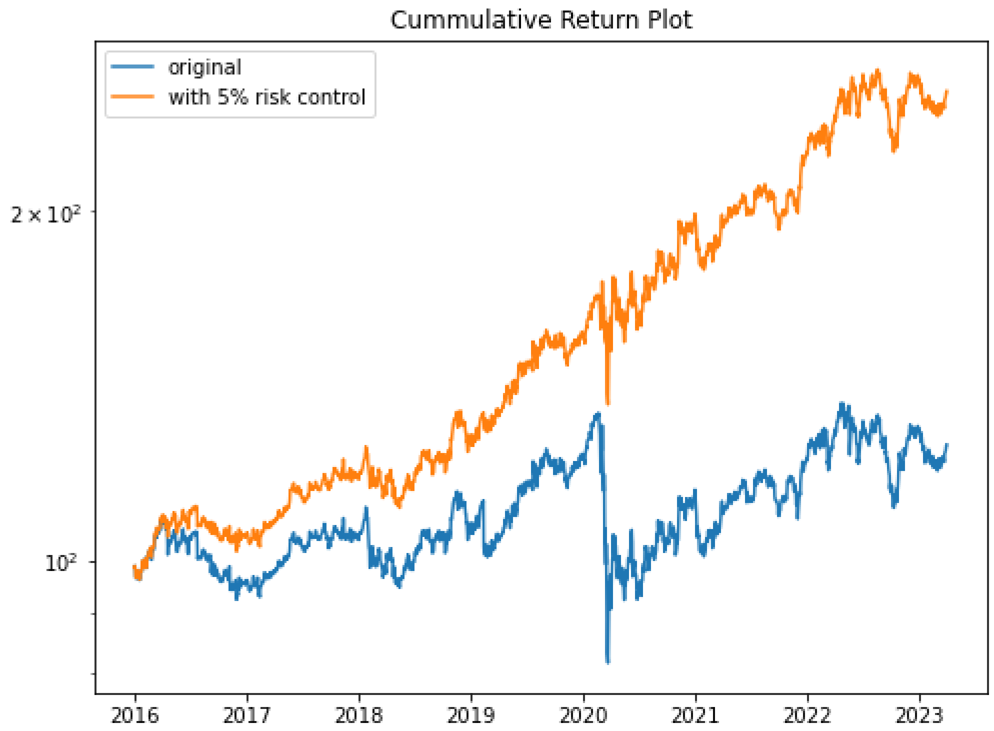
Notional = USD100

If log(low) – log(open) < -VaR:

- Sell
- Buy at close price

Else

- Passively strong hold



Ticker: KO
Model: Adaptive
Quantile = 0.05
Minimization approach: MLE

Deploying in the Trading Strategy

Tickers	Annualized Return (original)	Annualized Return (with CAViaR)
AAPL	23.08%	**40.91%**
JPM	5.24%	**18.93%**
KO	3.24%	**13.73%**
MSFT	20.63%	**32.91%**
SPY	8.10%	**15.98%**

Tickers	Cumulative Return (original)	Cumulative Return (with CAViaR)
AAPL	349.53%	**1096.77%**
JPM	44.72%	**250.69%**
KO	25.92%	**153.73%**
MSFT	288.69%	**684.17%**
SPY	75.77%	**192.40%**

Tickers	Maximum Drawdown (original)	Maximum Drawdown (with CAViaR)
AAPL	-40.37%	**-31.73%**
JPM	-47.64%	**-37.53%**
KO	-39.12%	**-21.60%**
MSFT	-41.00%	**-33.51%**
SPY	-36.14%	**-23.05%**

Outperform the passive strategy by a good risk management
(outperformance are shown in bold text)

Model: Adaptive
Quantile = 0.05
Minimization approach: MLE

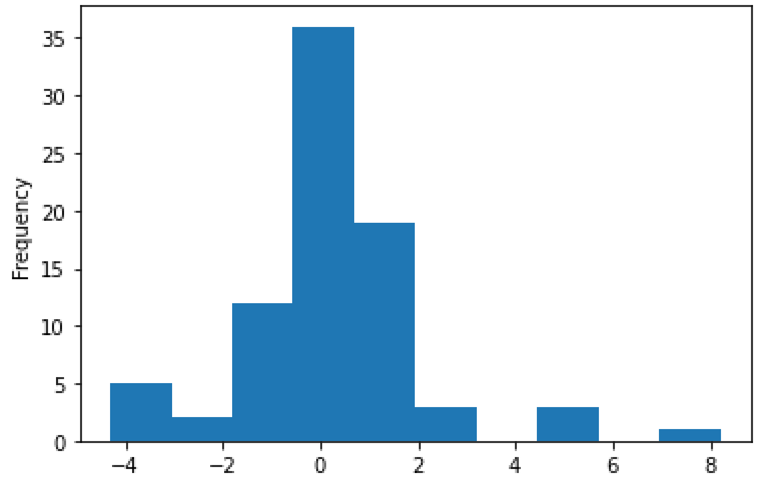
Deploying in the Trading Strategy

Mean Reversion

Statistic	Return at day after exceeding -VaR
count	81
mean	0.2794%
std	1.8743%
min	-4.3018%
25%	-0.4210%
50%	0.2153%
75%	1.0425%
max	8.2028%

For 95% quantile, the data also exhibits a similar effect.

Histogram of return at day after exceeding -VaR



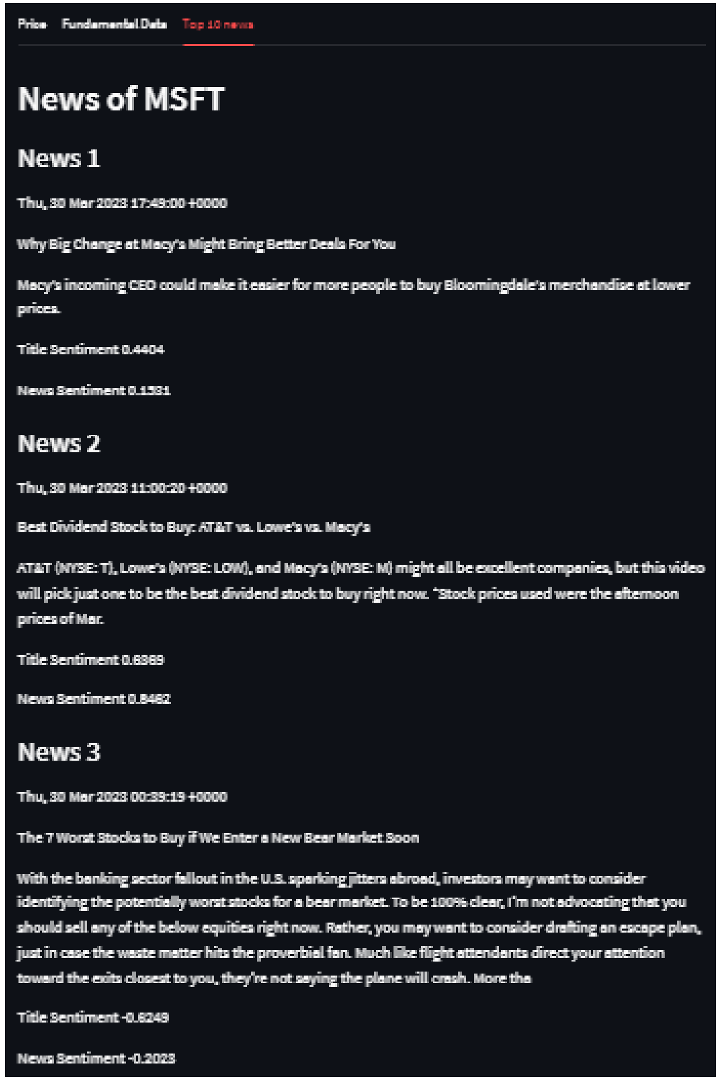
Ticker: SPY
Model: IGARCH
Quantile = 0.05
Minimization approach:
RQ Criterion Minimization

Dashboard

Libraries:
streamlit, localtunnel, alpha_vantage, plotly

Input:
- Stock Ticker
- model selection for CAViaR

Functionalities:
- stock price chart
- news sentiment analysis
- balance sheet
- VaR forecast by CAViaR, GARCH
- VaR statistical tests



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