2021.5.20. 习题课：Chap 11 梳理：机械波 & 电磁波

§1. 波动

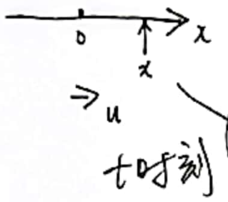
机械波两个条件 $\begin{cases} \text{波源} \\ \text{介质} \end{cases}$ 波源振动 → 介质相互作用 → 振动传播 → 波

$\begin{cases} \text{横波：振动方向与传播方向正交 （固. \cancel{气液}） eg. 吉他弦} \\ \text{纵波：振动方向与传播方向平行 （固.气.液） eg. 声音传播.} \end{cases}$

平面简谐波

假设波向右传播. 在 $x=0$ 处振动（t 时刻）

$y(x,t)$.　　$y(0,t) = A\cos(\omega t+\varphi_0)$.



t 时刻 x 点振动：由 $x=0$ 的点在 $\left(t-\frac{x}{u}\right)$ 时刻的振动传过来的.

$$\Rightarrow y(x,t) = A\cos\left[\omega\left(t-\frac{x}{u}\right)+\varphi_0\right] = A\cos(\omega t-kx+\varphi_0)$$

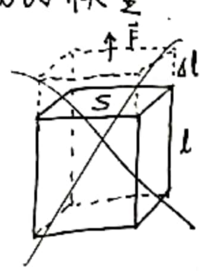
1. 波速：相速度 $u=\frac{\omega}{k}=\frac{\lambda}{T}=f\lambda$.
2. 群速度 $u_g=\frac{d\omega}{dk}$. 波包中心的前进速度.

§2/3 一维波动方程~~及其通解~~

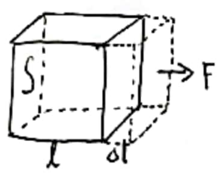
$$\frac{\partial^2}{\partial x^2}y - \frac{1}{u^2}\frac{\partial^2}{\partial t^2}y = 0.$$

§3. 导出

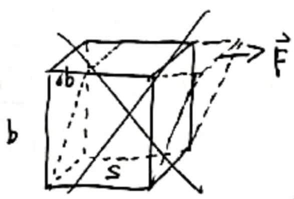
1. 杨氏模量：拉伸应变



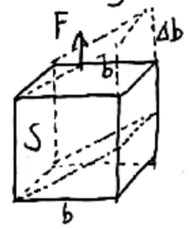
$$\frac{F}{S} = E\frac{\Delta l}{l} = E\frac{\partial y}{\partial x}.$$



2. 切变模量：剪切应变



$$\frac{F}{S} = G\frac{\Delta b}{b} = G\frac{\partial}{\partial}$$

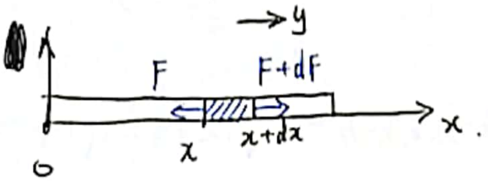


一般 $G < E$.

eg. 地震：横波比纵波慢

均匀弹性棒

(1) 纵波 $\leftrightarrow$



振动幅度 y

$dm = \rho S dx$

$$F(x) = ES\frac{\partial y}{\partial x}\Big|_x.$$

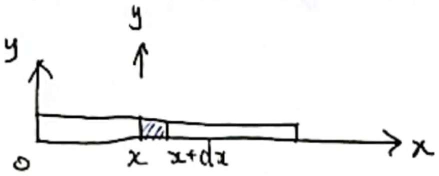
合力 $F(x+dx) - F(x) = ES\frac{\partial y}{\partial x}\Big|_{x+dx} - ES\frac{\partial y}{\partial x}\Big|_x = ES\frac{\partial^2 y}{\partial x^2}dx$

$$= dm\cdot\frac{\partial^2 y}{\partial t^2} = \rho S\frac{\partial^2 y}{\partial t^2}dx$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 y}{\partial t^2} = 0.$$

纵波波速 $u = \sqrt{\frac{E}{\rho}}$

(2) 横波 $\updownarrow$



$$F(x) = GS\frac{\partial y}{\partial x}\Big|_x.$$

合力 $F(x+dx) - F(x) = GS\frac{\partial y}{\partial x}\Big|_{x+dx} - GS\frac{\partial y}{\partial x}\Big|_x = GS\frac{\partial^2 y}{\partial x^2}dx$

$$= dm\frac{\partial^2 y}{\partial t^2} = \rho S\frac{\partial^2 y}{\partial t^2}dx$$

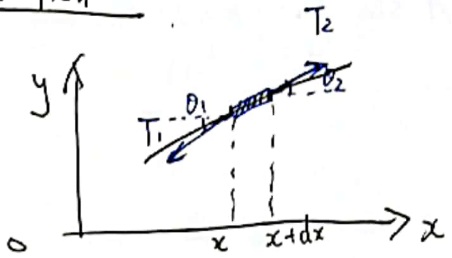
$$\Rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 y}{\partial t^2} = 0.$$

横波波速 $u = \sqrt{\frac{G}{\rho}}$.

☆ 一般 $G < E$. $\Rightarrow u_{横} < u_{纵}$

Scanned with CamScanner

弦的横波



$$T_1\cos\theta_1 = T_2\cos\theta_2 = T$$

$$dm = \eta dx$$

合力 $T_2\sin\theta_2 - T_1\sin\theta_2 = T\tan\theta_2 - T\tan\theta_1$

$$= T\left.\frac{\partial y}{\partial x}\right|_{x+dx} - T\left.\frac{\partial y}{\partial x}\right|_{x} = T\frac{\partial^2 y}{\partial x^2}dx$$

$$= dm\frac{\partial^2 y}{\partial t^2} = \eta\frac{\partial^2 y}{\partial t^2}dx$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} - \frac{\eta}{T}\frac{\partial^2 y}{\partial t^2} = 0.$$

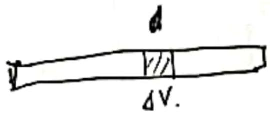
横波波速 $u = \sqrt{\frac{T}{\eta}}$.

§4 波场中的能量

$$y(x,t) = A\cos(\omega t - kx)$$

动能 $\Delta E_k = \frac{1}{2}\Delta m\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\rho\Delta V A^2\omega^2\sin^2(\omega t - kx)$

势能 $\Delta E_p =$



类比 弹簧 —— 介质棒 (纵波)

$F = kx$ —— $F = ES\frac{\Delta l}{l}$.

k —— $\frac{ES}{l}$

x —— Δl

$E_p = \frac{1}{2}kx^2$ —— $E_p = \frac{1}{2}\frac{ES}{l}(\Delta l)^2 = \frac{1}{2}E\left(\frac{\Delta l}{l}\right)^2 V.$

$$\Delta E_p = \frac{1}{2}E\left(\frac{\partial y}{\partial x}\right)^2\Delta V.$$

$$\Rightarrow \Delta E_p = \frac{1}{2}E\Delta V\cdot A^2k^2\sin^2(\omega t - kx)$$

$u = \sqrt{\frac{E}{\rho}} = \frac{\omega}{k}$ $\Rightarrow \Delta E_p = \frac{1}{2}\Delta V\rho\omega^2A^2\sin^2(\omega t - kx) = \Delta E_k$. 相互拉扯作用.

(同步) 每个质元能量不守恒，不断吸收、放出能量 Page 3

总机械能 $\Delta E = \Delta E_k + \Delta E_p = \rho \Delta V \omega^2 A^2 \sin^2(\omega t - kx)$

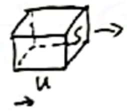
(1) 能量密度：波场中单位体积的能量

$$w = \frac{\Delta E}{\Delta V} = \rho \omega^2 A^2 \sin^2(\omega t + kx)$$

(2) 平均能量密度：一个周期内能量密度对时间平均值

$$\bar{w} = \frac{1}{T}\int_0^T w(x,t)\,dt = \frac{1}{2}\rho\omega^2 A^2$$

(3) 平均能流：单位时间内通过面积S的能量. ~~设波速为u~~



$$\bar{P} = \bar{w}\,\underbrace{u}_{\text{波速}}\,S$$

(4) 平均能流密度（波的强度）：单位时间内通过单位面积的能量

$$I = \bar{w} u = \frac{1}{2}\rho\omega^2 A^2 u = \frac{1}{2} Z \omega^2 A^2$$

特性阻抗 $Z = \rho u$

5. 电磁波

Maxwell Eqns. $\rightarrow \begin{cases} \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0 \\ \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0 \end{cases}$ 真空中光速 $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

沿 x 轴传播

$\nabla \cdot \vec{E} = 0$
$\nabla \cdot \vec{B} = 0$
$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$
$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\vec{E}$

$$\begin{cases} \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0 \\ \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{B} = 0 \end{cases} \Rightarrow \begin{cases} \vec{E} = \vec{E}_0 \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \\ \vec{B} = \vec{B}_0 \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \end{cases}$$

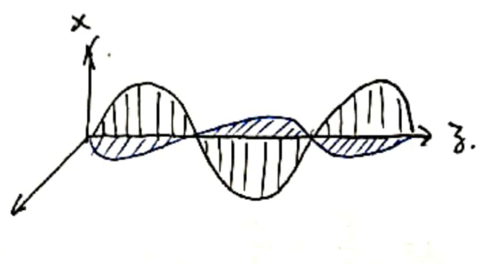
① 横波：振动方向与传播方向垂直.

$\because$ Gauss 定理 $\nabla \cdot \vec{E} = \vec{k} \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = \vec{k} \cdot \vec{B} = 0$. $\vec{k}$ 为传播方向

② $\vec{E}$ 与 $\vec{B}$ 正交.

since $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \Rightarrow \vec{k} \times \vec{E}_0 = \omega \vec{B}_0$



③ $E = cB$.

$$\begin{cases} E = E_0 \cos(\omega t - kx) \\ B = B_0 \cos(\omega t - kx). \end{cases}$$

(* 振荡偶极子： $x \to r$. $E \to E_\theta$.

$$E_0 = \frac{\omega^2 p_0 \sin\theta}{4\pi\varepsilon_0 c^2 r}$$).

电场能量密度 $w_e = \frac{1}{2}\varepsilon E^2$

磁场能量密度 $w_m = \frac{1}{2\mu} B^2 = \frac{1}{2}\mu H^2$

(1) 电磁场能量密度

$$w = w_e + w_m = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right)$$

(2) 能流密度

$$S = wu = \frac{1}{2\sqrt{\varepsilon\mu}}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right) = \frac{1}{\mu} EB$$

$$= \frac{1}{\mu} E_0 B_0 \cos^2(\omega t - kx).$$

表明 能流密度变化的频率是电磁场的 2倍

(3) 能流密度矢量 $\vec{S} = \frac{1}{\mu}\vec{E} \times \vec{B}$

(Poynting 矢量)

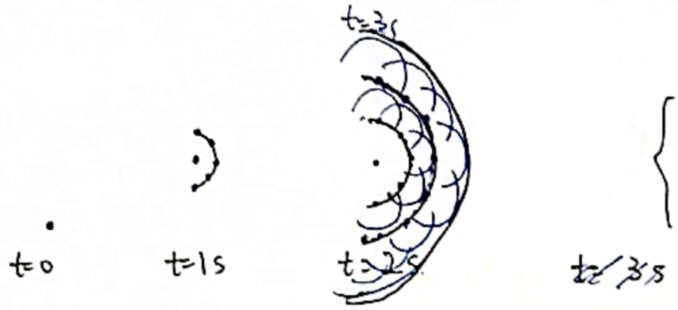
(辐射强度)

(4) 平均能流密度 $\bar{S} = \frac{1}{2\mu} E_0 B_0$

§6. 波的衍射、反射与折射

1. Hygens原理

在波传播过程中，波阵面上每一点可以看作发射子波的波源
其后任一时刻，这些子波的包络线成为新的波阵面

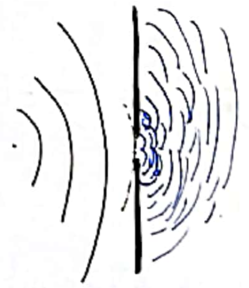


优点：解释波的传播、衍射、反射、折射
缺点：无法解释不同方向传播的强度分布

方向性

2. 衍射

3. 反射、折射

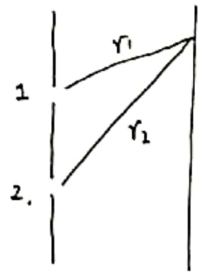


§7. 波的叠加

线性波动方程，强度较弱 ⇒ 可叠加原理，独立性

- 相干波：频率相同、振动方向相同、相位差固定。
- 同相位的相干波源产生两列波，波程差 $\delta = r_1 - r_2$.

干涉
- 相干加强 $\delta = k\lambda$ $k=0, \pm1, \pm2, \cdots$
- 相干减弱 $\delta = (k+\frac{1}{2})\lambda$ $k=0, \pm1, \pm2, \cdots$



驻波：两列频率相同，振动方向相同，传播方向相反的波

$y_1(x,t) = A\cos(\omega t - kx)$

$y_2(x,t) = A\cos(\omega t + kx)$

$y = y_1 + y_2 = \cancel{2A\cos kt}\ 2A\cos kx\cos\omega t = y(x,t)$.

每个质点 x 都作频率为 ω 的简谐运动，但振幅 $|2A\cos kx|$ 不同。

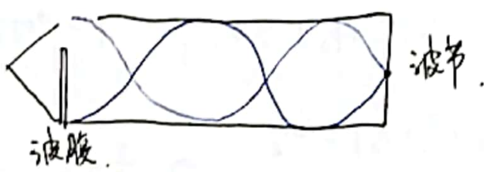
- 波腹：$|\cos kx| = 1$. $x = N\frac{\lambda}{2}$. $(k = \frac{2\pi}{\lambda})$.
- 波节：$|\cos kx| = 0$ $x = (N+\frac{1}{2})\frac{\lambda}{2}$. $N = 0, \pm1, \pm2, \cdots$

无长距离的能量传播：波节只有势能，波腹只有动能

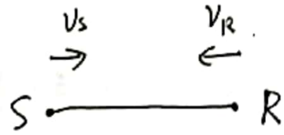
半波损失

固定端反射：反射波相位比入射波多 π. → 波节 (疏→密)

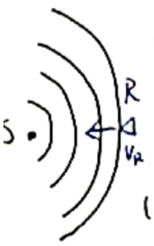
自由端反射：无事发生 → 波腹. (密→疏)

一端开口的管子 

§8. Doppler效应



波源 S 与 R 相对于介质运动 ⇒ $f_R \neq f_s$.

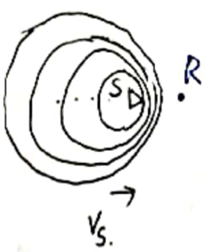


(1) 波源静止，R运动.

R 单位时间接到的波数 $f_R = \frac{u + v_R}{\lambda} = \frac{u+v_R}{u} f_s$.

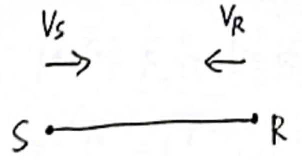
(2) S动，R静止.

S 发出的波实际波长 $\lambda_R = \frac{u}{f_s} - \frac{v_s}{f_s}$



$\Rightarrow f_R = \frac{u}{\lambda_R} = \frac{u}{u - v_s} f_s$.

(3) S运动 R运动.

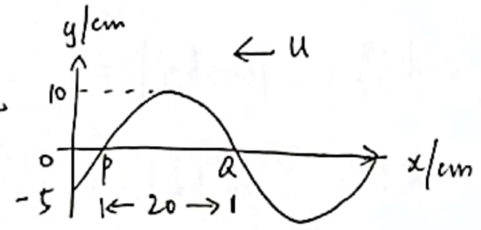


S运动 ⇒ 介质 $f_W = \frac{u}{u-V_s} f_s$

R运动 ⇒ 观察得 $f_R = \frac{u+V_R}{u} f_W = \frac{u+V_R}{u-V_s} f_s$.

习题

11-9. 沿 x轴负方向传播, $T=2s$. $t=\frac{1}{3}s$时



(1) O点振动表达式.

(2) 波动表达式

(3) Q点振动表达式.

(4) Q坐标.

$\boxed{\text{补充 } A=10\text{ cm.}}$

$A = 10$ cm

解: (1). O点 $\cancel{y_0^{(\frac{1}{3})} = A\cos(\omega\frac{1}{3}+\phi_0)}$. $\omega = \frac{2\pi}{T} = \pi$ rad/s

$$\begin{cases} y_0^{(t)} = A\cos(\omega t+\phi_0) \\ v_0(t) = -A\omega\sin(\omega t+\phi_0) \end{cases} \quad \begin{cases} y_0^{(\frac{1}{3})} = 10\cos(\frac{\pi}{3}+\phi_0) = -5. \\ v_0(\frac{1}{3}) = -10\pi\sin(\frac{\pi}{3}+\phi_0) > 0. \end{cases}$$ 下一时刻, 朝y正方向移动.

$$\Rightarrow \begin{cases} \cos(\frac{\pi}{3}+\phi_0) = -\frac{1}{2} \\ \sin(\frac{\pi}{3}+\phi_0) < 0 \end{cases} \Rightarrow \frac{\pi}{3}+\phi_0 = -\frac{2}{3}\pi.$$

$$\phi_0 = -\pi$$

∴ O点振动表达式 $y_0(t) = 10\cos(\pi t - \pi)$. $\begin{pmatrix} y: \text{cm} \\ t: \text{s} \end{pmatrix}$.

(2) 波动 $y(x,t) = A\cos(\omega t + kx + \phi_0)$.

$$\lambda = 40\text{cm} \begin{cases} A = 10\text{cm}. \\ \omega = \pi \text{ rad/s}. \\ k = \frac{2\pi}{\lambda} = \frac{\pi}{20} \text{ cm}^{-1} \\ y(0,t)\text{时 } \phi_0 = -\pi \end{cases} \Rightarrow y(x,t) = 10\cos(\pi t + \frac{\pi}{20}x - \pi).$$

$$\begin{pmatrix} x,y: \text{cm} \\ t: \text{s} \end{pmatrix}.$$

(3) $$\begin{cases} y_Q(t) = A\cos(\omega t+\phi_1) \\ v_Q(t) = -A\omega\sin(\omega t+\phi_1) \end{cases} \Rightarrow \begin{cases} y_Q(\frac{1}{3}) = 10\cos(\frac{\pi}{3}+\phi_1) = 0 \\ v_Q(\frac{1}{3}) = -10\pi\sin(\frac{\pi}{3}+\phi_1) < 0 \end{cases} \Rightarrow \begin{cases} \cos(\frac{\pi}{3}+\phi_1) = 0 \\ \sin(\frac{\pi}{3}+\phi_1) > 0 \end{cases} \Rightarrow \frac{\pi}{3}+\phi_1 = \frac{1}{2}\pi.$$

$$\phi_1 = \frac{\pi}{6}$$

$y_Q(t) = 10\cos(\pi t + \frac{\pi}{6})$.

(4) $y(x_Q, t) = 10\cos(\pi t + \frac{\pi}{20}x_Q - \pi)$

$= 10\cos(\pi t + \frac{\pi}{6})$.

$\Rightarrow \frac{\pi}{6} = +\frac{\pi}{20}x_Q - \pi + 2N\pi$.

$\frac{7}{6}\pi = 2N\pi + \frac{\pi}{20}x_Q$.

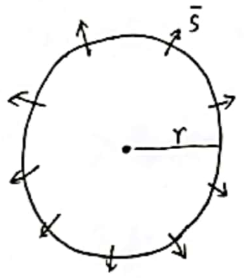
$N=0$. $\frac{\pi}{20}x_Q = \frac{7}{6}\pi$. $\Rightarrow x_Q = \frac{70}{3}$ cm.

11-12. 波源功率 $\bar{P} = 35000$ W 发射球面电磁波

某点测得波平均能量密度 $\bar{w} = 7.8\times10^{-15}\ J/m^3$. 求该处距波源距离 r.

电磁波波速 $c = 3.0\times10^{8}$ m/s.

解:



球面表面积 $A = 4\pi r^2$.

平均能流密度 $\bar{S} = \bar{w}c$.

$\bar{P} = \bar{S}\cdot 4\pi r^2 = \bar{w}c\cdot 4\pi r^2$.

$\Rightarrow r = \sqrt{\frac{\bar{P}}{4\pi \bar{w} c}} = 3.45\times10^{4}$ m.

11-20 激光为圆柱形光束，截面直径 $d = 2$mm. 功率 $\bar{P} = 10$mW.

求最大电场强度 E_0 和最大磁感应强度 B_0.

解:

平均能流密度 $\bar{S} = \frac{\bar{P}}{\pi(\frac{d}{2})^2}$.

能流密度 $S = \frac{1}{\mu_0}EB$

$E = E_0\cos(\omega t - kx)$.

$B = B_0\cos(\omega t - kx)$.

平均能流密度 $\bar{S} = \frac{1}{2\mu_0}E_0B_0$

$E_0 = cB_0$.

$\bar{S} = \frac{c}{2\mu_0}B_0^2 = \frac{4\bar{P}}{\pi d^2}$

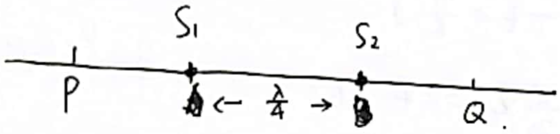
$= \frac{1}{2\mu_0 c}E_0^2$

$\Rightarrow B_0 = \sqrt{\frac{8\mu_0\bar{P}}{\pi c d^2}}$

$E_0 = \sqrt{\frac{8c\mu_0\bar{P}}{\pi d^2}}$.

11-23 相干波源 S_1、S_2 相距 $\frac{\lambda}{4}$. S_1 相位比 S_2 相位超前 $\frac{\pi}{2}$

单个波强度 I_0. 求 P、Q 两点强度.



解:

S_1: $y_1 = A\cos(\omega t + \frac{\pi}{2})$.

S_2: $y_2 = A\cos(\omega t)$.

P点:

$y_{1p} = A\cos(\omega t - k r_{1p} + \frac{\pi}{2})$ $\quad k = \frac{2\pi}{\lambda}$

$y_{2p} = A\cos(\omega t - k(r_{1p} + \frac{\lambda}{4}))$.

$\varphi_{1p} - \varphi_{2p}$ 相位差 $\Delta\varphi_p = \frac{\pi}{2} + k\frac{\lambda}{4} = \pi$.

相干抵消. 合振幅为 0. 强度 $I_p = 0$.

Q点: $y_{1Q} = A\cos(\omega t - k r_{1Q} + \frac{\pi}{2})$

$y_{2Q} = A\cos(\omega t - k(r_{1Q} - \frac{\lambda}{4}))$

$\varphi_{1Q} - \varphi_{2Q}$ 相位差 $\Delta\varphi_Q = \frac{\pi}{2} - k\frac{\lambda}{4} = 0$.

相干增强. 合振幅为 $2A$. 强度 $I_Q = 4A^2 = 4I_0$.

11-30 一根弦上有沿 x 轴正向传播的~~简谐波~~ 余弦波. $f=50\ Hz$.

$t=0$ 时 $x_1=0.5m$ 处 位移 $+\frac{A}{2}$. $A=0.04\ m$.

沿 y 轴负向运动. $u=100\ m/s$.

到 $x_2=10m$ 处 固定端被反射.

(1) 求 入射波、反射波的表达式.

(2) 合成波在 $0\le x\le 10m$ 区间内的波腹和波节.

$\omega=2\pi f=100\pi\ rad/s.$

解: (1) 入射波 $y_1=A\cos(\omega t-kx+\varphi_0)$ $u=\frac{\omega}{k}$

反射波 $y_2=A\cos(\omega t+kx+\varphi_0+\pi)$. $k=\frac{\omega}{u}=\pi\ m^{-1}$.

$y_1(x_1=0.5,t=0)=A\cos(-0.5\pi+\varphi_0)=\frac{A}{2}.$

$v_1(x_1=0.5,t=0)=-A\omega\sin(-0.5\pi+\varphi_0)<0.$

$\begin{cases}\cos(\varphi_0-\frac{\pi}{2})=\frac{1}{2}\\ \sin(\varphi_0-\frac{\pi}{2})>0\end{cases}\Rightarrow \varphi_0-\frac{\pi}{2}=\frac{\pi}{3}$

$\varphi_0=\frac{5\pi}{6}.$

入射波 $y_1(x,t)=0.04\cos(100\pi t-\pi x+\frac{5\pi}{6})$. $\left(\begin{matrix}y,x: m\\ t: s\end{matrix}\right)$

反射波 $y_2(x,t)=0.04\cos(100\pi t\underset{\text{反向}}{\underset{\uparrow}{+}}\pi x+\underset{\text{半波损失}}{\underset{\uparrow}{\frac{11\pi}{6}}})$

(2) $y=y_1+y_2=0.04\left[\cos(100\pi t-\pi x+\frac{5\pi}{6})-\cos(100\pi t+\pi x+\frac{5\pi}{6})\right]$

$\cos(\alpha-\beta)-\cos(\alpha+\beta)=2\sin\alpha\sin\beta$

$=0.08\sin(100\pi t+\frac{5}{6}\pi)\sin\pi x.$

波节: $\sin\pi x=0.\Rightarrow \pi x=N\pi \Rightarrow x=N.$ ~~N=0,1,2, ... 10.~~

$x=0,1,2,\cdots,10$ (m).

波腹 $\sin\pi x=\pm1 \Rightarrow \pi x=\frac{2N+1}{2}\pi \Rightarrow x=\frac{2N+1}{2}.$

$x=\frac{1}{2},\frac{3}{2},\frac{5}{2},\cdots,\frac{19}{2}$ (m).

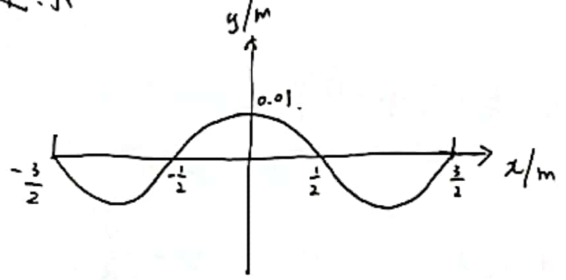
11-31 两端固定的 3.0 m 弦上激起驻波，3个波腹，振幅 1.0 cm

波速 100 m/s.

(1) 驻波频率；(2) 分解成两个行波表达式.

$\lambda = 2m$

解：建系



(1) $f = \cancel{\frac{\omega}{2\pi}} = \frac{u}{\lambda} = 50\ Hz$.

(2) 驻波 $y = 0.01\cos\left(\frac{2\pi}{\lambda}x\right)\cos\left(\frac{2\pi}{T}t\right)$.

$= 0.01\cos(\pi x)\cos(100\pi t) = y_1 + y_2$.

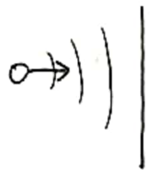
$2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$.

$$\Rightarrow \begin{cases} y_1 = 0.005\cos(\pi x - 100\pi t) \\ y_2 = 0.005\cos(\pi x + 100\pi t) \end{cases} \quad \begin{pmatrix} t: s \\ y, x: m \end{pmatrix}.$$

11-35 蝙蝠超声导航.

波源频率 $f_s = 39\ kHz$. 朝墙壁飞去. 速度 $v = \frac{340}{40}\ m/s$.

求蝙蝠接收到的反射波频率.



解：过程1：墙壁接收 $f_R = \frac{u}{u-V_s}f_s = \frac{u}{u-v}f_s$

过程2：墙壁反射 $f_{RR} = \frac{u+V_R}{u}f_R = \frac{u+v}{u}f_R$

$= \frac{u+v}{u-v}f_s$.