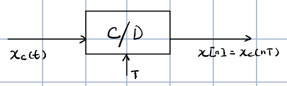
# Periodic Sampling

- Def: A method of obtaining a discrete-time respresentation of a continuous-time signal $x_c(t)$, where a sequence of samples $x[n]$ is obtained according to the relation

  $x[n] = x_c(nT)$ $\quad -\infty < n < \infty$

  where $T$ is the sampling period, $f_s = \frac{1}{T}$ is the sampling frequency, $w_s = \frac{2\pi}{T}$ in radians per second.
- System that implement the operation
  - continuous-to-discrete-time (C/D) converter or sampler:




  - Analog-to-digital (A/D) in practical setting.
- Sampling operation is not invertible if not restrict the input signals to the sampler.
- Sampling process:
  - the periodic impulse train: $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$
  - $x_s(t) = x_c(t)\,s(t)$

    $= x_c(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)$

$\delta(x) = \begin{cases} 1 & x=0 \\ 0 & x\neq 0 \end{cases}$

$x_s(t) = \sum_n (\delta(t-nT)\, x_c(t))$ $\quad t=nt$ $\rightarrow$ 对于给定n.
计 $t\neq nT$, $x_n(t)=0$
计 $t=nT$ $\quad \underline{x_{ns}(t)=x_c(nT)}$.

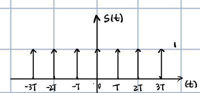
# Frequency-Domain Respresentation of Sampling

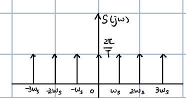
- Fourier transform for a continuous time signal $x(t)$.

  $X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$

  $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)\, e^{jwt}\, dt$
- $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$

  $S(jw) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(w-kw_s)$




- $x_s(t) = x_c(t)\,s(t)$ $\longrightarrow$ $X_s(jw) = \frac{1}{2\pi} X_c(jw) * S(jw) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(w-kw_s))$

  $= \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)$ $\longrightarrow$ $X_s(jw) = \sum_{n=-\infty}^{+\infty} x_c(nT)\, e^{-jnwT}$
- Nyquist Sampling Theorem.
  - Nyquist frequency: $W_N$ ; Nyquist rate: $2W_N$.

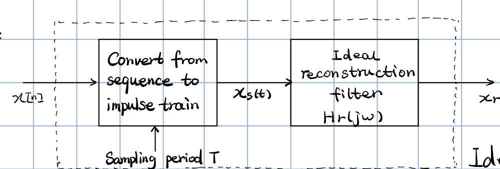
    $x_c(t)$ is a bandlimited signal with $X_c(jw) = 0$ for $|w| > W_N$. Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT)$, if $w_s = \frac{2\pi}{T} \geqslant 2W_N$
  - $X_s(jw) = X(e^{j\Omega})\big|_{\Omega=wT} = X(e^{jwT})$

    $X_s(jw) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-jnwT} = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-jn\Omega}$ $\quad (\Omega = wT)$
  - $X(e^{jwT}) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X_c(j(w-kw_s))$.
- $\delta(w) = \delta(-w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{iwt}\, dt$

# Reconstruction of A Bandlimited Signal From Its Samples

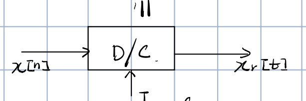
- If the modulated impulse train $x_s(t)$ is filtered by an appropriate lowpass filter, then the filter output will be exactly $x_c(t)$.

  $x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_r(t-nT) = x_c(t)$
- Process:




Ideal Reconstruction System

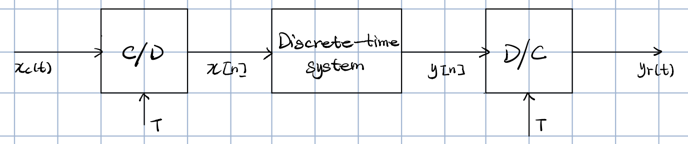
$\parallel$




Ideal D/C Converte.

- Ideal lowpass filter $H_r(jw) = \begin{cases} T & |w| \leq \frac{\pi}{T} \\ 0 & |w| > \frac{\pi}{T} \end{cases}$
  - corresponding impulse response: $h_r(t) = \frac{\sin(\pi t/T)}{\pi t/T}$
  - Output: $x_r(t) = \sum_{n=-\infty}^{\infty} x[n]\, \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T}$
  - $y[n] = x[n-\alpha T] \rightarrow y[n] = x_c(t-\alpha T)\big|_{t=nT}$

## Discrete-Time Processing of Continuous-Time Signals.




## Continuous-Time Processing of Discrete-Time Signals.