Complex number and functions of a complex variable.

- $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
  - $z^n = r^n(\cos n\theta + i\sin n\theta)$.
- $z = x + iy$; $e^z = e^x(\cos y + i\sin y) = e^x e^{iy} \Rightarrow \begin{cases} r = e^x \\ \theta = y \end{cases}$
- $\ln z = \ln|z| + i\arg z \quad (z = x+iy)$
- Complex plane
  - $z = a+bi,\ w = c+di \Rightarrow |z-w| = \sqrt{(a-c)^2+(b-d)^2}$
- $\begin{cases} \text{Continuity at } z_0 \iff \lim\limits_{z\to z_0} f(z) = f(z_0) \\ \text{Differentiable at } z_0 \iff \lim\limits_{z\to z_0} \dfrac{f(z)-f(z_0)}{z-z_0} \text{ exists and is finite.} \end{cases}$
- Cauchy–Riemann Equations
  - $f(z) = f(x+iy) = \underbrace{u(x,y)}_{Re[f(z)]} + i\underbrace{v(x,y)}_{Im[f(z)]}$. $\iff \begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial v}{\partial x} = -\dfrac{\partial u}{\partial y}. \end{cases}$
  - $f(z) = \bar{z}$ is not differentiable.
- Complex Power Series
  - $\sum\limits_{n=0}^{\infty} a_n(z-z_0)^n$ where $z_0, a_n \in \mathbb{C}$ (z_0: Centre, a_n: coefficients.)
  - $R = \lim\limits_{n\to\infty}\left|\dfrac{a_n}{a_{n+1}}\right| \Rightarrow \begin{cases} \text{converges} & |z-z_0| < R \\ \text{diverges} & |z-z_0| > R \end{cases}$
    - $e^z = \sum\limits_{n=0}^{+\infty} \dfrac{1}{n!} z^n$ for all z.

$$\sin z = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1} \quad \text{all } z.$$

$$\cos z = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!} z^{2n} \quad \text{all } z.$$

$$\frac{1}{1-z} = \sum_{n=0}^{+\infty} z^n \quad |z|<1$$

$$\frac{1}{z+1} = \sum_{n=0}^{+\infty} (-1)^n z^n \quad |z|<1$$

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{2n!}$$

$$\sin z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n \quad |z|<1$$